Math 21 – Review for Exam #1

Part One: Parametric equations and plane curves; polar coordinates	
1	For parametric equations: $x = \frac{1}{t}$ and $y = 2t^3 + 4$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ $at\ t = 1.$
2	For parametric equations $x = \sec\theta\ and\ y = \cos^2\theta$, find the tangent line to the curve at $\theta = 0.$
3	Find the equation of the tangent line to $x = \frac{1}{2t+1}$, $y = 3t - 2$ at $t = 1.$
4	Convert the rectangular equation to polar form: $x^2 + y^2 - 3x + 2y = 0$
5	Convert the polar equation to rectangular form: $r = \frac{3}{5\sin\theta - 2\cos\theta}$
6	A) Find the arc length of $x = t - \frac{t^3}{9}$; $y = \frac{t^2}{\sqrt{3}} + 1$, for $0 \le t \le 1$ B) Find the arc length of $x = 1 - \sin t$; $y = 1 - \cos t$, $for\ 0 \le t \le 2\pi$ C) Find the arc length of $x = t^2, y = 4t^3, 0 \le t \le 1$ (solution at end of solution document)
Part Two: Vectors ; Lines and Planes	
1	For $\boldsymbol{u} = \langle 3, -2, 0\rangle\ and\ \boldsymbol{v} = \langle -1, 2, -3\rangle$ find A) $\boldsymbol{u} \cdot \boldsymbol{v}$ B) $(2\boldsymbol{u} - 3\boldsymbol{v}) \cdot \boldsymbol{v}$ C) $proj_{\boldsymbol{u}}\boldsymbol{v}$ D) $\boldsymbol{u} \times \boldsymbol{v}$ E) the angle between $\boldsymbol{u}\ and\ \boldsymbol{v}$ F)$\|\boldsymbol{u}\|$
2	Find the area of the triangle with vertices: $P(2, -1, 3), Q(3, 0, 2), and\ R(0, 2, 1).$
3	Find the parametric equations of the line through $P(1, -2, 3)$ and $Q(0, 2, 5).$
4	Find the equation of the plane through points $P(0, 2, -1), Q(1, 0, 3) and\ R(2, -1, 1).$
5	Find the distance from point $P(2, 3, -1)$ to the plane $3x - y + 2z - 2 = 0$
6	Find the distance between point $Q(1, 2, -1)$ and line $x = 3 + 2t; y = -1 + 3t; z = -2t$.
7	Find the angle between planes $2x - 3y + 2z = 0\ and\ 3x - y + z - 1 = 0.$
8	Is vector $\langle 3, -5, 2\rangle$ parallel to $\langle 12, -20, -8\rangle$? Explain why or why not.
Part Three: Surfaces in Space	
1	Find the trace of surface $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = -1$ in the following planes and categorize the curve: A) in the $xy - plane$ B) in the $xz - plane$ C) in the $yz - plane$ D) in the plane $z = 4$ E) in the plane $z = 2$ F) in the plane $z = 5$
2	The 3D-cylinder $x^2 + z^2 = 4$ A) has trace in the plane $y = 4$, given by _____. B) the rulings of the 3D-cylinder $x^2 + z^2 = 4$ are parallel to which axis?