EXAMPLE 10 Finding direction angles

Find the direction cosines and angles for the vector $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, and show that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$.

SOLUTION

Since $\|\mathbf{v}\| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$, we have

$$\cos \alpha = \frac{v_1}{\|\mathbf{v}\|} = \frac{2}{\sqrt{29}} \Longrightarrow \alpha \approx 68.2°$$

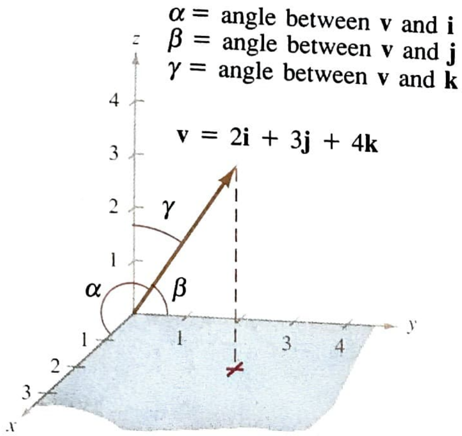
$$\cos \beta = \frac{v_2}{\|\mathbf{v}\|} = \frac{3}{\sqrt{29}} \Longrightarrow \beta \approx 56.1°$$

$$\cos \gamma = \frac{v_3}{\|\mathbf{v}\|} = \frac{4}{\sqrt{29}} \Longrightarrow \gamma \approx 42.0°.$$

Furthermore, the sum of the squares of the direction cosines is

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \frac{4}{29} + \frac{9}{29} + \frac{16}{29} = \frac{29}{29} = 1.$$

(See Figure 14.12.)



Direction Angles of $\mathbf{v}$

FIGURE 14.12

EXERCISES for Section 14.1

In Exercises 1–4, plot the points on the same three-dimensional coordinate system.

1. (a) $(2, 1, 3)$ (b) $(-1, 2, 1)$
2. (a) $(3, -2, 5)$ (b) $\left(\frac{3}{2}, 4, -2\right)$
3. (a) $(5, -2, 2)$ (b) $(5, -2, -2)$
4. (a) $(0, 4, -5)$ (b) $(4, 0, 5)$

In Exercises 5–8, find the lengths of the sides of the triangle with the indicated vertices, and determine whether the triangle is a right triangle, an isosceles triangle, or neither.

5. $(0, 0, 0)$, $(2, 2, 1)$, $(2, -4, 4)$
6. $(5, 3, 4)$, $(7, 1, 3)$, $(3, 5, 3)$
7. $(1, -3, -2)$, $(5, -1, 2)$, $(-1, 1, 2)$
8. $(5, 0, 0)$, $(0, 2, 0)$, $(0, 0, -3)$

In Exercises 9 and 10, find the coordinates of the midpoint of the line segment joining the given points.

9. $(5, -9, 7)$ and $(-2, 3, 3)$
10. $(4, 0, -6)$ and $(8, 8, 20)$

In Exercises 11–14, find the general form of the equation of the sphere.

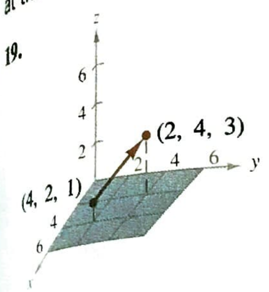
11. Center $(0, 2, 5)$, radius 2
12. Center $(4, -1, 1)$, radius 5
13. Endpoints of a diameter are $(2, 0, 0)$ and $(0, 6, 0)$
14. Center $(-2, 1, 1)$, tangent to the xy-coordinate plane

In Exercises 15–18, find the center and radius of the sphere.

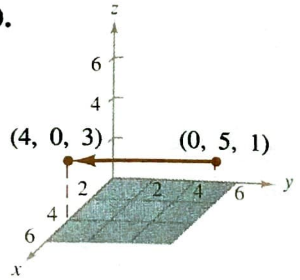
15. $x^2 + y^2 + z^2 - 2x + 6y + 8z + 1 = 0$
16. $x^2 + y^2 + z^2 + 9x - 2y + 10z + 19 = 0$
17. $9x^2 + 9y^2 + 9z^2 - 6x + 18y + 1 = 0$
18. $4x^2 + 4y^2 + 4z^2 - 4x - 32y + 8z + 33 = 0$

In Exercises 19–22, (a) find the component form of the vector **v** and (b) sketch the vector with its initial point at the origin.

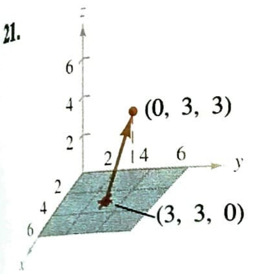
19.



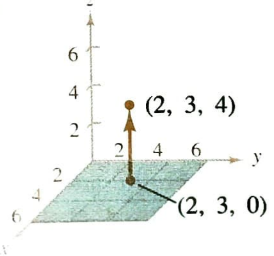
20.



21.



22.



In Exercises 23 and 24, the initial and terminal points of a vector **v** are given. (a) Sketch the directed line segment, (b) find the component form of the vector, and (c) sketch the vector with its initial point at the origin.

23. Initial point $(-1, 2, 3)$, terminal point $(3, 3, 4)$

24. Initial point $(2, -1, -2)$, terminal point $(-4, 3, 7)$

In Exercises 25 and 26, sketch each scalar multiple of **v**.

25. $\mathbf{v} = \langle 1, 2, 2 \rangle$

(a) $2\mathbf{v}$ (b) $-\mathbf{v}$

(c) $\frac{3}{2}\mathbf{v}$ (d) $0\mathbf{v}$

26. $\mathbf{v} = \langle 2, -2, 1 \rangle$

(a) $-\mathbf{v}$ (b) $2\mathbf{v}$

(c) $\frac{1}{2}\mathbf{v}$ (d) $\frac{5}{2}\mathbf{v}$

In Exercises 27–32, find the indicated vector, given $\mathbf{u} = \langle 1, 2, 3 \rangle$, $\mathbf{v} = \langle 2, 2, -1 \rangle$, and $\mathbf{w} = \langle 4, 0, -4 \rangle$.

27. $\mathbf{u} - \mathbf{v}$

28. $\mathbf{u} - \mathbf{v} + 2\mathbf{w}$

29. $2\mathbf{u} + 4\mathbf{v} - \mathbf{w}$

30. $5\mathbf{u} - 3\mathbf{v} - \frac{1}{2}\mathbf{w}$

31. $\mathbf{z}$, where $2\mathbf{z} - 3\mathbf{u} = \mathbf{w}$

32. $\mathbf{z}$, where $2\mathbf{u} + \mathbf{v} - \mathbf{w} + 3\mathbf{z} = \mathbf{0}$

In Exercises 33–36, determine which of the vectors are parallel to **z**.

33. $\mathbf{z} = \langle 3, 2, -5 \rangle$

(a) $\langle -6, -4, 10 \rangle$ (b) $\left\langle 2, \frac{4}{3}, -\frac{10}{3} \right\rangle$

(c) $\langle 6, 4, 10 \rangle$ (d) $\langle 1, -4, 2 \rangle$

34. $\mathbf{z} = \frac{1}{2}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{3}{4}\mathbf{k}$

(a) $6\mathbf{i} - 4\mathbf{j} + 9\mathbf{k}$ (b) $-\mathbf{i} + \frac{4}{3}\mathbf{j} - \frac{3}{2}\mathbf{k}$

(c) $12\mathbf{i} + 9\mathbf{k}$ (d) $\frac{3}{4}\mathbf{i} - \mathbf{j} + \frac{9}{8}\mathbf{k}$

35. **z** has initial point $(1, -1, 3)$ and terminal point $(-2, 3, 5)$

(a) $-6\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$ (b) $4\mathbf{j} + 2\mathbf{k}$

36. **z** has initial point $(3, 2, -1)$ and terminal point $(-1, -3, 5)$

(a) $\langle 0, 5, -6 \rangle$ (b) $\langle 8, 10, -12 \rangle$

In Exercises 37–40, use vectors to determine whether the given points lie on a straight line.

37. $(0, -2, -5)$, $(3, 4, 4)$, $(2, 2, 1)$

38. $(1, -1, 5)$, $(0, -1, 6)$, $(3, -1, 3)$

39. $(1, 2, 4)$, $(2, 5, 0)$, $(0, 1, 5)$

40. $(0, 0, 0)$, $(1, 3, -2)$, $(2, -6, 4)$

In Exercises 41 and 42, use vectors to show that the given points form the vertices of a parallelogram.

41. $(2, 9, 1)$, $(3, 11, 4)$, $(0, 10, 2)$, $(1, 12, 5)$

42. $(1, 1, -3)$, $(9, -1, 2)$, $(11, 2, 1)$, $(3, 4, -4)$

In Exercises 43 and 44, the vector **v** and its initial point are given. Find the terminal point.

43. $\mathbf{v} = \langle 3, -5, 6 \rangle$, initial point $(0, 6, 2)$

44. $\mathbf{v} = \left\langle 0, \frac{1}{2}, -\frac{1}{3} \right\rangle$, initial point $\left(3, 0, -\frac{2}{3}\right)$

In Exercises 45–50, find the magnitude of **v**.

45. $\mathbf{v} = \langle 0, 0, 0 \rangle$

46. $\mathbf{v} = \langle 1, 0, 3 \rangle$

47. **v** has $(1, -3, 4)$ and $(1, 0, -1)$ as its initial and terminal points, respectively.

48. **v** has $(0, -1, 0)$ and $(1, 2, -2)$ as its initial and terminal points, respectively.

49. $\mathbf{v} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$

50. $\mathbf{v} = -4\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$

In Exercises 51–54, find a unit vector (a) in the direction of **u** and (b) in the direction opposite that of **u**.

51. $\mathbf{u} = \langle 2, -1, 2 \rangle$

52. $\mathbf{u} = \langle 6, 0, 8 \rangle$

53. $\mathbf{u} = \langle 3, 2, -5 \rangle$

54. $\mathbf{u} = \langle 8, 0, 0 \rangle$

In Exercises 55–58, determine the values of c that satisfy the given equation. Let $\mathbf{u} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

55. $\|c\mathbf{u}\| = 1$

56. $\|c\mathbf{v}\| = 1$

57. $\|c\mathbf{v}\| = 5$

58. $\|c\mathbf{u}\| = 3$

In Exercises 59–62, use vectors to find the point that lies two-thirds of the way from P to Q.

59. $P = (4, 3, 0)$, $Q = (1, -3, 3)$

60. $P = (-2, 1, 6)$, $Q = (6, 1, 4)$

61. $P = (1, 2, 5)$, $Q = (6, 8, 2)$

62. $P = (-9, -8, 5)$, $Q = (12, 3, -1)$

In Exercises 63 and 64, write the component form of $\mathbf{v}$ and sketch it.

63. $\mathbf{v}$ lies in the yz-plane, has magnitude 2, and makes an angle of 30° with the positive y-axis.

64. $\mathbf{v}$ lies in the xz-plane, has magnitude 5, and makes an angle of 45° with the positive z-axis.

In Exercises 65 and 66, find (a) $\mathbf{u} \cdot \mathbf{v}$, (b) $\mathbf{u} \cdot \mathbf{u}$, (c) $\|\mathbf{u}\|^2$, (d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v}$, and (e) $\mathbf{u} \cdot (2\mathbf{v})$.

65. $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{v} = \mathbf{i} - \mathbf{k}$

66. $\mathbf{u} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\mathbf{v} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

In Exercises 67–70, find the angle θ between the given vectors.

67. $\mathbf{u} = \langle 1, 1, 1 \rangle$, $\mathbf{v} = \langle 2, 1, -1 \rangle$

68. $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, $\mathbf{v} = -3\mathbf{i} + 2\mathbf{j}$

69. $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}$, $\mathbf{v} = -2\mathbf{j} + 3\mathbf{k}$

70. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$

In Exercises 71–74, determine whether $\mathbf{u}$ and $\mathbf{v}$ are orthogonal, parallel, or neither.

71. $\mathbf{u} = \mathbf{j} + 6\mathbf{k}$, $\mathbf{v} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$

72. $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$

73. $\mathbf{u} = \langle 2, -3, 1 \rangle$, $\mathbf{v} = \langle -1, -1, -1 \rangle$

74. $\mathbf{u} = \langle \cos\theta, \sin\theta, -1 \rangle$
$\mathbf{v} = \langle \sin\theta, -\cos\theta, 0 \rangle$

In Exercises 75–80, (a) find the projection of $\mathbf{u}$ onto $\mathbf{v}$, and (b) find the vector component of $\mathbf{u}$ orthogonal to $\mathbf{v}$.

75. $\mathbf{u} = \langle 2, 1, 2 \rangle$, $\mathbf{v} = \langle 0, 3, 4 \rangle$

76. $\mathbf{u} = \langle 0, 4, 1 \rangle$, $\mathbf{v} = \langle 0, 2, 3 \rangle$

77. $\mathbf{u} = \langle 1, 1, 1 \rangle$, $\mathbf{v} = \langle -2, -1, 1 \rangle$

78. $\mathbf{u} = \langle -2, -1, 1 \rangle$, $\mathbf{v} = \langle 1, 1, 1 \rangle$

79. $\mathbf{u} = \langle 5, -4, 3 \rangle$, $\mathbf{v} = \langle 1, 0, 0 \rangle$

80. $\mathbf{u} = \langle 5, -4, 3 \rangle$, $\mathbf{v} = \langle 0, 1, 0 \rangle$

In Exercises 81–84, find the direction cosines of $\mathbf{u}$ and demonstrate that the sum of the squares of the direction cosines is 1.

81. $\mathbf{u} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

82. $\mathbf{u} = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}$

83. $\mathbf{u} = \langle 0, 6, -4 \rangle$

84. $\mathbf{u} = \langle a, b, c \rangle$

85. Let $\mathbf{u} = \mathbf{i} + \mathbf{j}$, $\mathbf{v} = \mathbf{j} + \mathbf{k}$, and $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$.
(a) Sketch $\mathbf{u}$ and $\mathbf{v}$.
(b) If $\mathbf{w} = \mathbf{0}$, show that a and b must both be zero.
(c) Find a and b such that $\mathbf{w} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$.
(d) Show that no choice of a and b yields $\mathbf{w} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

86. The initial and terminal points of the vector $\mathbf{v}$ are (x_1, y_1, z_1) and (x, y, z), respectively. Describe the set of all points (x, y, z) such that $\|\mathbf{v}\| = 4$.

87. Find the component form of the unit vector $\mathbf{v}$ representing the diagonal of the cube (see figure).

FIGURE FOR 87

88. The guy wire to a 100-foot tower has a tension of 550 pounds. Using the distances shown in the accompanying figure, write the component form of the vector $\mathbf{F}$ representing the tension in the wire.

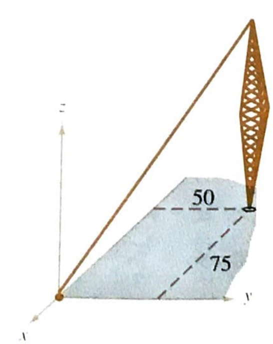


FIGURE FOR 88

Therefore, the volume is

$$\begin{aligned} V = \text{(height)(area of base)} &= \|\text{proj}_{\mathbf{v}\times\mathbf{w}}\mathbf{u}\| \, \|\mathbf{v} \times \mathbf{w}\| \\ &= \left| \frac{\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})}{\|\mathbf{v} \times \mathbf{w}\|} \right| \|\mathbf{v} \times \mathbf{w}\| \\ &= |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})|. \end{aligned}$$

EXAMPLE 5 Volume by the triple scalar product

Find the volume of the parallelepiped having $\mathbf{u} = 3\mathbf{i} - 5\mathbf{j} + \mathbf{k}$, $\mathbf{v} = 2\mathbf{j} - 2\mathbf{k}$, and $\mathbf{w} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$ as adjacent edges.

SOLUTION

By Theorem 14.4, we have

$$V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = \begin{vmatrix} 3 & -5 & 1 \\ 0 & 2 & -2 \\ 3 & 1 & 1 \end{vmatrix} = 36. \quad \square$$

A natural consequence of Theorem 14.4 is that the volume of the parallelepiped is zero if and only if the three vectors are coplanar. This gives us the following test.

Test for Coplanar Vectors: If the vectors $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$, $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$, and $\mathbf{w} = \langle w_1, w_2, w_3 \rangle$ have the same initial point, then they lie in the same plane if and only if

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = 0.$$

EXERCISES for Section 14.2

In Exercises 1–6, find the cross product of the given unit vectors and sketch your result.

1. $\mathbf{j} \times \mathbf{i}$
2. $\mathbf{i} \times \mathbf{j}$
3. $\mathbf{j} \times \mathbf{k}$
4. $\mathbf{k} \times \mathbf{j}$
5. $\mathbf{i} \times \mathbf{k}$
6. $\mathbf{k} \times \mathbf{i}$

In Exercises 7–14, find $\mathbf{u} \times \mathbf{v}$ and show that it is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

7. $\mathbf{u} = \langle 2, -3, 1 \rangle$, $\mathbf{v} = \langle 1, -2, 1 \rangle$
8. $\mathbf{u} = \langle -1, 1, 2 \rangle$, $\mathbf{v} = \langle 0, 1, 0 \rangle$
9. $\mathbf{u} = \langle 12, -3, 0 \rangle$, $\mathbf{v} = \langle -2, 5, 0 \rangle$

10. $\mathbf{u} = \langle -10, 0, 6\rangle$, $\mathbf{v} = \langle 7, 0, 0\rangle$
11. $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$
12. $\mathbf{u} = \mathbf{j} + 6\mathbf{k}$, $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$
13. $\mathbf{u} = -3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$, $\mathbf{v} = \frac{1}{2}\mathbf{i} - \frac{3}{4}\mathbf{j} + \frac{1}{10}\mathbf{k}$
14. $\mathbf{u} = \frac{2}{3}\mathbf{k}$, $\mathbf{v} = \frac{1}{2}\mathbf{i} + 6\mathbf{k}$

In Exercises 15–18, find the area of the parallelogram that has the given vectors as adjacent sides.

15. $\mathbf{u} = \mathbf{j}$, $\mathbf{v} = \mathbf{j} + \mathbf{k}$
16. $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{v} = \mathbf{j} + \mathbf{k}$
17. $\mathbf{u} = \langle 3, 2, -1\rangle$, $\mathbf{v} = \langle 1, 2, 3\rangle$
18. $\mathbf{u} = \langle 2, -1, 0\rangle$, $\mathbf{v} = \langle -1, 2, 0\rangle$

In Exercises 19 and 20, find the area of the parallelogram with the given vertices.

19. $(1, 1, 1), (2, 3, 4), (6, 5, 2), (7, 7, 5)$
20. $(2, -1, 1), (5, 1, 4), (0, 1, 1), (3, 3, 4)$

In Exercises 21–24, find the area of the triangle with the given vertices. ($\frac{1}{2}\|\mathbf{u} \times \mathbf{v}\|$ is the area of the triangle having $\mathbf{u}$ and $\mathbf{v}$ as adjacent sides.)

21. $(0, 0, 0), (1, 2, 3), (-3, 0, 0)$
22. $(2, -3, 4), (0, 1, 2), (-1, 2, 0)$
23. $(1, 3, 5), (3, 3, 0), (-2, 0, 5)$
24. $(1, 2, 0), (-2, 1, 0), (0, 0, 0)$

In Exercises 25–28, find $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$.

25. $\mathbf{u} = \mathbf{i}$, $\mathbf{v} = \mathbf{j}$, $\mathbf{w} = \mathbf{k}$
26. $\mathbf{u} = \langle 1, 1, 1\rangle$, $\mathbf{v} = \langle 2, 1, 0\rangle$, $\mathbf{w} = \langle 0, 0, 1\rangle$
27. $\mathbf{u} = \langle 2, 0, 1\rangle$, $\mathbf{v} = \langle 0, 3, 0\rangle$, $\mathbf{w} = \langle 0, 0, 1\rangle$
28. $\mathbf{u} = \langle 2, 0, 0\rangle$, $\mathbf{v} = \langle 1, 1, 1\rangle$, $\mathbf{w} = \langle 0, 2, 2\rangle$

In Exercises 29 and 30, use the triple scalar product to find the volume of the parallelepiped having adjacent edges $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

29. $\mathbf{u} = \mathbf{i} + \mathbf{j}$, $\mathbf{v} = \mathbf{j} + \mathbf{k}$, $\mathbf{w} = \mathbf{i} + \mathbf{k}$
(See figure.)
30. $\mathbf{u} = \langle 1, 3, 1\rangle$, $\mathbf{v} = \langle 0, 5, 5\rangle$, $\mathbf{w} = \langle 4, 0, 4\rangle$
(See figure.)

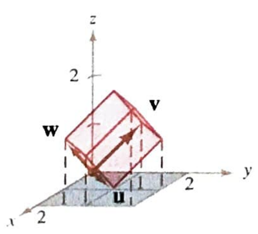


FIGURE FOR 29

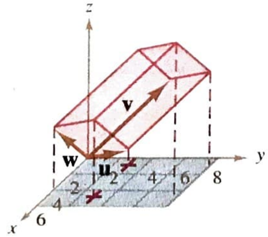


FIGURE FOR 30

31. Find the volume of the parallelepiped (see figure) with vertices $(0, 0, 0)$, $(3, 0, 0)$, $(0, 5, 1)$, $(3, 5, 1)$, $(2, 0, 5)$, $(5, 0, 5)$, $(2, 5, 6)$, $(5, 5, 6)$.
32. A force of 60 pounds acts on the pipe wrench shown in the figure.
(a) Find the magnitude of the moment about O by evaluating $\|\overrightarrow{OA} \times \mathbf{F}\|$. (The result will be a function of θ.)
(b) Use the result of part (a) to determine the magnitude of the moment when $\theta = 45°$.
(c) Use the result of part (a) to determine the angle θ when the magnitude of the moment is maximum. Is the answer what you expected? Why or why not?

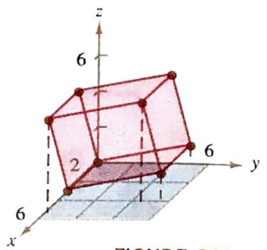


FIGURE FOR 31

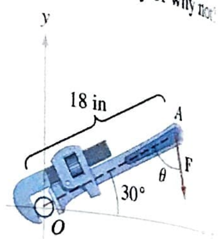


FIGURE FOR 32

33. A child applies the brakes on a bicycle by applying a downward force of 20 pounds on the pedal when the crank makes a 40° angle with the horizontal (see figure). Find the torque at P if the crank is 6 inches in length.
34. Both the magnitude and direction of the force on a crankshaft change as the crankshaft rotates. Find the torque on the crankshaft using the position and data shown in the accompanying figure.

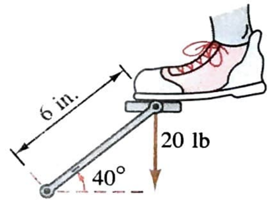


FIGURE FOR 33

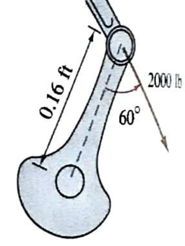
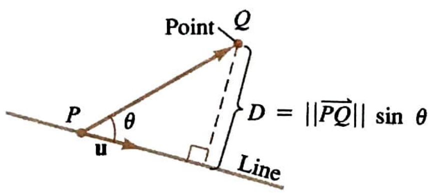


FIGURE FOR 34

In Exercises 35–43, prove the property of the cross product.

35. $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$
36. $(c\mathbf{u}) \times \mathbf{v} = c(\mathbf{u} \times \mathbf{v})$
37. $\mathbf{u} \times \mathbf{0} = \mathbf{0} \times \mathbf{u} = \mathbf{0}$
38. $\mathbf{u} \times \mathbf{u} = \mathbf{0}$
39. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$
40. $\mathbf{u} \times \mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$
41. $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ if and only if $\mathbf{u}$ and $\mathbf{v}$ are scalar multiples of each other.
42. $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\,\|\mathbf{v}\|$ if $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.
43. $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}$
44. Prove Theorem 14.3.



Distance Between a Point and a Line

FIGURE 14.30

where θ is the angle between $\mathbf{u}$ and $\overrightarrow{PQ}$. By Theorem 14.2, we have

$$\|\mathbf{u}\|\,\|\overrightarrow{PQ}\| \sin\theta = \|\mathbf{u} \times \overrightarrow{PQ}\| = \|\overrightarrow{PQ} \times \mathbf{u}\|.$$

Consequently,

$$D = \|\overrightarrow{PQ}\| \sin\theta = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|}.$$

EXAMPLE 7 Finding the distance between a point and a line

Find the distance between the point $Q = (3, -1, 4)$ and the line given by

$$x = -2 + 3t, \qquad y = -2t, \qquad \text{and} \qquad z = 1 + 4t.$$

SOLUTION

Using the direction numbers 3, -2, and 4, we find the direction vector for the line to be

$$\mathbf{u} = \langle 3, -2, 4 \rangle. \qquad \text{Direction vector for line}$$

To find a point on the line, we let $t = 0$ and obtain

$$P = (-2, 0, 1). \qquad \text{Point on the line}$$

Thus,

$$\overrightarrow{PQ} = \langle 3 - (-2), -1 - 0, 4 - 1 \rangle = \langle 5, -1, 3 \rangle$$

and we form the cross product

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -1 & 3 \\ 3 & -2 & 4 \end{vmatrix} = 2\mathbf{i} - 11\mathbf{j} - 7\mathbf{k} = \langle 2, -11, -7 \rangle.$$

Finally, using Theorem 14.8, we find the distance to be

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{174}}{\sqrt{29}} = \sqrt{6}.$$

EXERCISES for Section 14.3

1. The figure shows the line given by

$$x = 1 + 3t, \quad y = 2 - t, \quad \text{and} \quad z = 2 + 5t.$$

(a) Draw an arrow on the line to indicate its orientation.

(b) Find the coordinates of two points P and Q on the line. Determine the vector $\overrightarrow{PQ}$. What is the relationship of the components of the vector with the coefficients of t in the parametric equations? Why is this true?

(c) Determine the coordinates of the point where the line intersects the xz-plane.

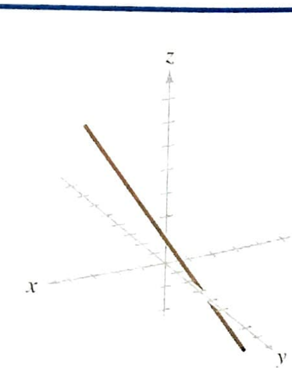


FIGURE FOR 1

FIGURE FOR 2

2. The figure shows the line given by

$$x = 2 - 3t, \quad y = 2, \quad \text{and} \quad z = 1 - t.$$

(a) Draw an arrow on the line to indicate its orientation.
(b) Find the coordinates of two points P and Q on the line. Determine the vector $\overrightarrow{PQ}$. What is the relationship of the components of the vector with the coefficients of t in the parametric equations? Why is this true?
(c) Determine the coordinates of the point where the line intersects the xy-plane.
(d) Does the line intersect the xz-coordinate plane? Explain why or why not.

In Exercises 3–8, find a set of (a) parametric equations and (b) symmetric equations of the line through the point and parallel to the specified vector or line. (For each line, express the direction numbers as integers.)

Point	Parallel to
3. $(0, 0, 0)$	$\mathbf{v} = \langle 1, 2, 3 \rangle$
4. $(0, 0, 0)$	$\mathbf{v} = \left\langle -2, \frac{5}{2}, 1 \right\rangle$
5. $(-2, 0, 3)$	$\mathbf{v} = 2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$
6. $(-2, 0, 3)$	$\mathbf{v} = 6\mathbf{i} + 3\mathbf{j}$
7. $(1, 0, 1)$	$x = 3 + 3t$, $y = 5 - 2t$, $z = -7 + t$
8. $(-3, 5, 4)$	$\dfrac{x-1}{3} = \dfrac{y+1}{-2} = z - 3$

In Exercises 9 and 10, find a set of (a) parametric equations and (b) symmetric equations of the line through the two points. (For each line, express the direction numbers as integers.)

9. $(5, -3, -2), \left(-\frac{2}{3}, \frac{2}{3}, 1\right)$
10. $(1, 0, 1), (1, 3, -2)$

In Exercises 11 and 12, find a set of parametric equations of the line.

11. The line passes through the point $(2, 3, 4)$ and is parallel to the xz-plane and the yz-plane.
12. The line passes through the point $(2, 3, 4)$ and is perpendicular to the plane given by $3x + 2y - z = 6$.

In Exercises 13 and 14, determine which of the points lie on the line L.

13. The line L passes through the point $(-2, 3, 1)$ and is parallel to the vector $\mathbf{v} = 4\mathbf{i} - \mathbf{k}$.
(a) $(2, 3, 0)$ (b) $(-6, 3, 2)$
(c) $(2, 1, 0)$ (d) $(6, 3, -2)$

14. The line L passes through the points $(2, 0, -3)$ and $(4, 2, -2)$.
(a) $(4, 1, -2)$ (b) $\left(\frac{5}{2}, \frac{1}{2}, -\frac{11}{4}\right)$
(c) $(-1, -3, -4)$ (d) $(0, -2, -4)$

In Exercises 15–18, determine whether the lines intersect, and if so, find the point of intersection and the cosine of the angle of intersection.

15. $x = 4t + 2$, $y = 3$, $z = -t + 1$; $\quad x = 2s + 2$, $y = 2s + 3$, $z = s + 1$
16. $x = -3t + 1$, $y = 4t + 1$, $z = 2t + 4$; $\quad x = 3s + 1$, $y = 2s + 4$, $z = -s + 1$
17. $\dfrac{x}{3} = \dfrac{y-2}{-1} = z + 1$
$\dfrac{x-1}{4} = y + 2 = \dfrac{z+3}{-3}$
18. $\dfrac{x-2}{-3} = \dfrac{y-2}{6} = z - 3$
$\dfrac{x-3}{2} = y + 5 = \dfrac{z+2}{4}$

In Exercises 19–32, find the equation of the specified plane.

19. The plane passes through the point $(2, 1, 2)$ and has normal vector $\mathbf{n} = \mathbf{i}$.
20. The plane passes through the point $(1, 0, -3)$ and has normal vector $\mathbf{n} = \mathbf{k}$.
21. The plane passes through the point $(3, 2, 2)$ and has normal vector $\mathbf{n} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$.
22. The plane passes through the point $(3, 2, 2)$ and is perpendicular to the line given by

$$\frac{x-1}{4} = y + 2 = \frac{z+3}{-3}.$$

23. The plane passes through the points $(0, 0, 0)$, $(1, 2, 3)$, and $(-2, 3, 3)$.
24. The plane passes through the points $(1, 2, -3)$, $(2, 3, 1)$, and $(0, -2, -1)$.
25. The plane passes through the points $(1, 2, 3)$, $(3, 2, 1)$, and $(-1, -2, 2)$.
26. The plane passes through the point $(1, 2, 3)$ and is parallel to the yz-plane.
27. The plane passes through the point $(1, 2, 3)$ and is parallel to the xy-plane.
28. The plane contains the y-axis and makes an angle of $\pi/6$ with the positive x-axis.
29. The plane contains lines given by

$$\frac{x-1}{-2} = y - 4 = z$$

$$\frac{x-2}{-3} = \frac{y-1}{4} = \frac{z-2}{-1}.$$

30. The plane passes through the point (2, 2, 1) and contains the line given by $\frac{x}{2} = \frac{y-4}{-1} = z$.

31. The plane passes through the points (2, 2, 1) and (−1, 1, −1) and is perpendicular to the plane $2x - 3y + z = 3$.

32. The plane passes through the points (4, 2, 1) and (−3, 5, 7) and is parallel to the z-axis.

In Exercises 33–38, determine whether the planes are parallel, orthogonal, or neither. If they are neither parallel nor orthogonal, find the angle of intersection.

33. $5x - 3y + z = 4,\ x + 4y + 7z = 1$
34. $3x + y - 4z = 3,\ -9x - 3y + 12z = 4$
35. $x - 3y + 6z = 4,\ 5x + y - z = 4$
36. $3x + 2y - z = 7,\ x - 4y + 2z = 0$
37. $x - 5y - z = 1,\ 5x - 25y - 5z = -3$
38. $2x - z = 1,\ 4x + y + 8z = 10$

In Exercises 39–46, mark the intercepts and sketch the graph of the plane.

39. $4x + 2y + 6z = 12$
40. $3x + 6y + 2z = 6$
41. $2x - y + 3z = 4$
42. $2x - y + z = 4$
43. $y + z = 5$
44. $x + 2y = 4$
45. $2x + y - z = 6$
46. $x - 3z = 3$

In Exercises 47 and 48, find a set of parametric equations for the line of intersection of the planes.

47. $3x + 2y - z = 7,\ x - 4y + 2z = 0$
48. $x - 3y + 6z = 4,\ 5x + y - z = 4$

In Exercises 49 and 50, find the point of intersection (if any) of the plane and the line. Also determine whether the line lies in the plane.

49. $2x - 2y + z = 12$
$$x - \frac{1}{2} = \frac{y + (3/2)}{-1} = \frac{z+1}{2}$$

50. $2x + 3y = -5$
$$\frac{x-1}{4} = \frac{y}{2} = \frac{z-3}{6}$$

In Exercises 51 and 52, find the distance between the point and the line.

51. $(10, 3, -2);\ x = 4t - 2,\ y = 3,\ z = -t + 1$
52. $(4, 1, -2);\ x = 2t + 2,\ y = 2t,\ z = t - 3$

In Exercises 53 and 54, find the distance between the planes.

53. $x - 3y + 4z = 10,\ x - 3y + 4z = 6$
54. $2x - 4z = 4,\ 2x - 4z = 10$

In Exercises 55 and 56, find the distance between the two skew lines (lines that are neither parallel nor intersecting).

55. $x = \frac{y}{2} = \frac{z}{3}$, $\quad \frac{x-1}{-1} = y - 4 = z + 1$

56. $x = 3t$, $y = -t + 2$, $z = t - 1$; $\quad x = 4s + 1$, $y = s - 2$, $z = -3s - 3$

57. A chute at the top of the grain elevator of a combine has the purpose of funneling the grain into a bin (see figure). Find the angle between two adjacent sides.

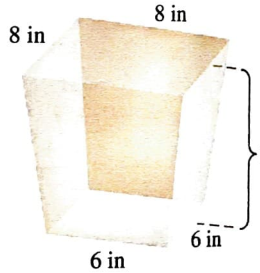


FIGURE FOR 57

58. Consider the two nonzero vectors **u** and **v**. Describe the geometric figure generated by the terminal points of the following vectors where s and t represent all real numbers.
(a) $t\mathbf{v}$ (b) $\mathbf{u} + t\mathbf{v}$ (c) $s\mathbf{u} + t\mathbf{v}$

59. If a_1, b_1, c_1 and a_2, b_2, c_2 are two sets of direction numbers for the same line, show that there exists a scalar d such that $a_1 = a_2 d$, $b_1 = b_2 d$, and $c_1 = c_2 d$.

14.4 Surfaces in Space

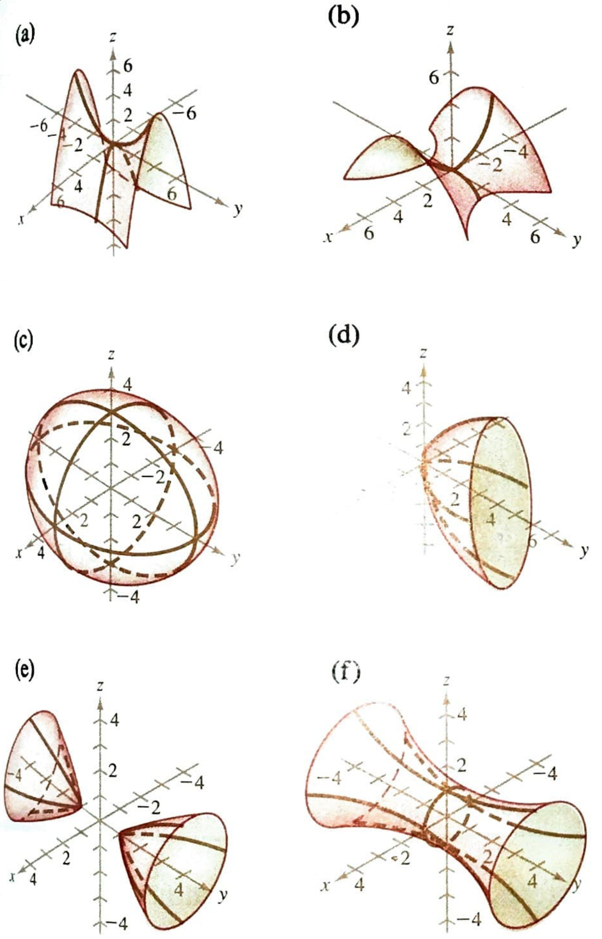
Cylindrical surfaces ▪ Quadric surfaces ▪ Surfaces of revolution

In the first five sections of this chapter, we introduced the vector portion of the preliminary work necessary to study vector calculus and the calculus of space. In this and the next section we complete this preliminary development.

EXERCISES for Section 14.4

In Exercises 1–6, match the equation with its graph. [The graphs are labeled (a)–(f).]

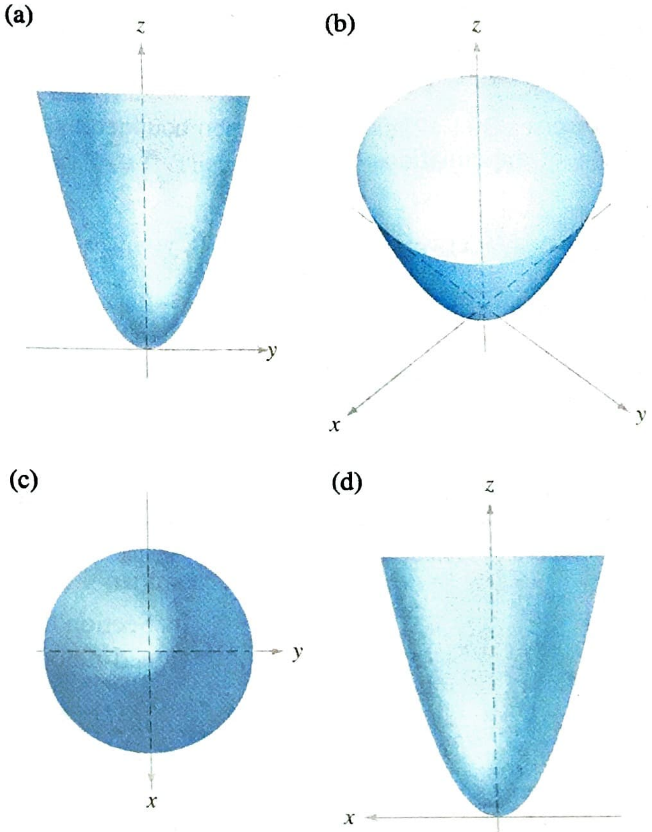
1. $\dfrac{x^2}{9} + \dfrac{y^2}{16} + \dfrac{z^2}{9} = 1$
2. $15x^2 - 4y^2 + 15z^2 = -4$
3. $4x^2 - y^2 + 4z^2 = 4$
4. $12z = -3y^2 + 4x^2$
5. $4x^2 - 4y + z^2 = 0$
6. $4x^2 - y^2 + 4z = 0$



In Exercises 7–16, describe and sketch each surface.

7. $z = 3$
8. $x = 4$
9. $y^2 + z^2 = 9$
10. $x^2 + z^2 = 16$
11. $x^2 - y = 0$
12. $y^2 + z = 4$
13. $4x^2 + y^2 = 4$
14. $y^2 - z^2 = 4$
15. $z - \sin y = 0$
16. $z - e^y = 0$

17. The four figures are graphs of the quadric surface
$$z = x^2 + y^2.$$
Match the four graphs with the point in space from which the paraboloid is viewed. The four points are (0, 0, 20), (0, 20, 0), (20, 0, 0), and (10, 10, 20).



18. Use a graphing utility to sketch three views of the equation $y^2 + z^2 = 4$, with views from the points (a) (10, 0, 0), (b) (0, 10, 0), and (c) (10, 10, 10).

In Exercises 19–30, identify and sketch the given quadric surface.

19. $x^2 + \dfrac{y^2}{4} + z^2 = 1$
20. $\dfrac{x^2}{9} + \dfrac{y^2}{16} + \dfrac{z^2}{16} = 1$
21. $16x^2 - y^2 + 16z^2 = 4$
22. $z^2 - x^2 - \dfrac{y^2}{4} = 1$
23. $x^2 - y + z^2 = 0$
24. $z = 4x^2 + y^2$
25. $x^2 - y^2 + z = 0$
26. $3z = -y^2 + x^2$
27. $z^2 = x^2 + \dfrac{y^2}{4}$
28. $x^2 = 2y^2 + 2z^2$
29. $16x^2 + 9y^2 + 16z^2 - 32x - 36y + 36 = 0$
30. $4x^2 + y^2 - 4z^2 - 16x - 6y - 16z + 9 = 0$

In Exercises 31–38, use a computer algebra system to obtain a graph of the surface. Identify the surface as a cylinder, quadric surface, or surface of revolution. (Hint: It may be necessary to solve for z and acquire two equations to graph.)

31. $z = 2 \sin x$

32. $z = x^2 + 0.5y^2$

33. $x^2 + z^2 = \left(\frac{2}{y}\right)^2$

34. $x^2 + y = e^{-z}$

35. $z^2 = x^2 + 4y^2$

36. $4y = x^2 + z^2$

37. $4x^2 - y^2 + 4z^2 = -16$

38. $9x^2 + 4y^2 - 8z^2 = 72$

In Exercises 39–42, sketch the region bounded by the graphs of the equations.

39. $z = 2\sqrt{x^2 + y^2}$, $z = 2$

40. $z = \sqrt{4 - x^2}$, $y = \sqrt{4 - x^2}$, $x = 0$, $y = 0$, $z = 0$

41. $x^2 + y^2 = 1$, $x + z = 2$, $z = 0$

42. $z = \sqrt{4 - x^2 - y^2}$, $y = 2z$, $z = 0$

In Exercises 43–48, find an equation for the surface of revolution generated by revolving the curve in the given coordinate plane about the specified axis.

	Equation of Curve	*Coordinate Plane*	*Axis of Revolution*
43.	$z^2 = 4y$	yz-plane	y-axis
44.	$z = 2y$	yz-plane	y-axis
45.	$z = 2y$	yz-plane	z-axis
46.	$2z = \sqrt{4 - x^2}$	xz-plane	x-axis
47.	$xy = 2$	xy-plane	x-axis
48.	$z = \ln y$	yz-plane	z-axis

In Exercises 49 and 50, find an equation of a generating curve given the equation of its surface revolution.

49. $x^2 + y^2 - 2z = 0$

50. $x^2 + z^2 = \sin^2 y$

14.5 Curves and Vector-Valued Functions in Space

Space curves ▪ Vector-valued functions in space ▪ Arc length of a space curve

In Section 13.3, we showed how to represent a *plane curve* by a vector-valued function of the form

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$$

where f and g are continuous functions of t on an interval I. This definition can be extended to three-dimensional space in a natural way. That is, we can represent a curve in space by a vector-valued function of the form

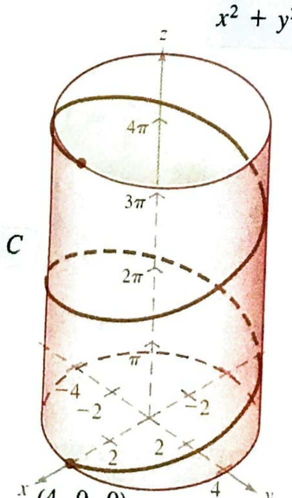
$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$$

where f, g, and h are continuous functions of t on an interval I.

EXAMPLE 1 A space curve represented by a vector-valued function

Sketch the space curve represented by the vector-valued function

$$\mathbf{r}(t) = 4 \cos t\,\mathbf{i} + 4 \sin t\,\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 4\pi.$$



Curve C is a helix.

FIGURE 14.42

SOLUTION

From the first two parametric equations, $x = 4 \cos t$ and $y = 4 \sin t$, we obtain

$$x^2 + y^2 = 16.$$

This means that the curve lies on a right circular cylinder of radius 4, centered about the z-axis. To locate the curve on this cylinder, we use the third parametric equation $z = t$. Then, as t increases from 0 to 4π, the point (x, y, z) spirals up the cylinder to produce the **helix** shown in Figure 14.42. ▭

In the next example, we look at a space curve that is defined as the intersection of two surfaces in space.

17. (a) $3\mathbf{i} + \mathbf{j}$ (b) $\left(\dfrac{1}{2\sqrt{t}} - 6\right)\mathbf{i} + 2\mathbf{j}$

(c) $\dfrac{9}{2}\sqrt{t} + 8t - 4$ (d) $\dfrac{10t - 1}{\sqrt{10t^2 - 2t + 1}}$

19. $\sin t\,\mathbf{i} + (t \sin t + \cos t)\,\mathbf{j} + \mathbf{C}$

21. $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$
$\|\mathbf{v}\| = 5$
$\mathbf{a} = 0$
$\mathbf{a} \cdot \mathbf{T} = 0$
$\mathbf{a} \cdot \mathbf{N} = 0$
$K = 0$

23. $\mathbf{v} = 2\mathbf{i} - \dfrac{2}{(t+1)^2}\mathbf{j}$

$\|\mathbf{v}\| = \dfrac{2\sqrt{(t+1)^4 + 1}}{(t+1)^2}$

$\mathbf{a} = \dfrac{4}{(t+1)^3}\mathbf{j}$

$\mathbf{a} \cdot \mathbf{T} = \dfrac{-4}{(t+1)^3\sqrt{(t+1)^4 + 1}}$

$\mathbf{a} \cdot \mathbf{N} = \dfrac{4}{(t+1)\sqrt{(t+1)^4 + 1}}$

$K = \dfrac{(t+1)^3}{[(t+1)^4 + 1]^{3/2}}$

25. $\mathbf{v} = e^t\,\mathbf{i} - e^{-t}\,\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{e^{2t} + e^{-2t}}$

$\mathbf{a} = e^t\,\mathbf{i} + e^{-t}\,\mathbf{j}$

$\mathbf{a} \cdot \mathbf{T} = \dfrac{e^{2t} - e^{-2t}}{\sqrt{e^{2t} + e^{-2t}}}$

$\mathbf{a} \cdot \mathbf{N} = \dfrac{2}{\sqrt{e^{2t} + e^{-2t}}}$

$K = \dfrac{2}{(e^{2t} + e^{-2t})^{3/2}}$

27. $2(20 + 9 \ln 3) \approx 59.775$

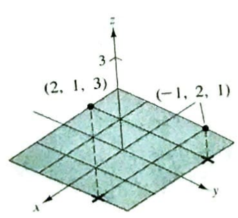
29. $\dfrac{250\sqrt{2}}{3}(3 + \sqrt{3}) \approx 557.68$ lb

31. 152 ft **33.** 4.56 mi/sec

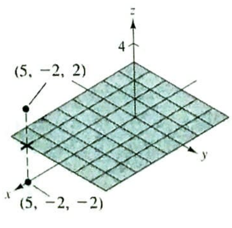
Chapter 14

Section 14.1

1.



3.



5. $3, 3\sqrt{5}, 6$, Right triangle

7. $6, 6, 2\sqrt{10}$, Isosceles triangle **9.** $\left(\frac{3}{2}, -3, 5\right)$

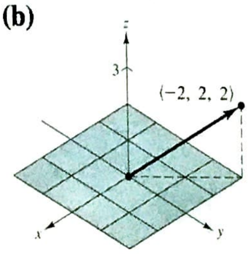
11. $x^2 + y^2 + z^2 - 4y - 10z + 25 = 0$

13. $x^2 + y^2 + z^2 - 2x - 6y = 0$

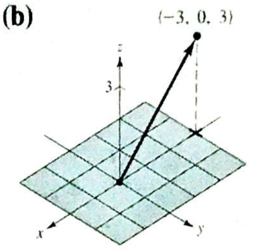
15. Center: $(1, -3, -4)$ Radius: 5

17. Center: $\left(\frac{1}{3}, -1, 0\right)$ Radius: 1

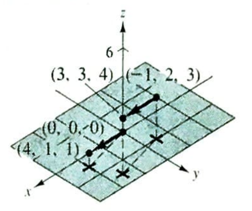
19. (a) $\langle -2, 2, 2\rangle$
(b)



21. (a) $\langle -3, 0, 3\rangle$
(b)

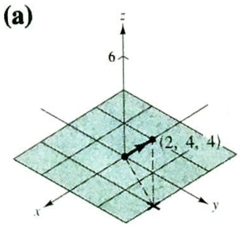


23. (a), (c)

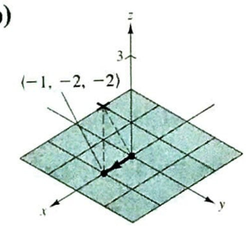


(b) $\langle 4, 1, 1\rangle$

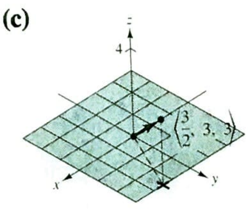
25. (a)



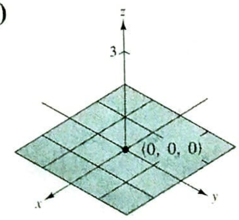
(b)



(c)



(d)



27. $\langle -1, 0, 4\rangle$ **29.** $\langle 6, 12, 6\rangle$

31. $\left\langle \frac{7}{2}, 3, \frac{5}{2}\right\rangle$ **33.** a and b **35.** a

37. Collinear **39.** Not collinear **43.** $(3, 1, 8)$

45. 0 **47.** $\sqrt{34}$ **49.** $\sqrt{14}$

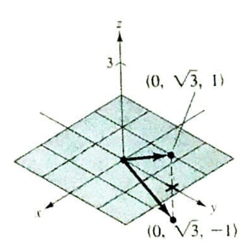
51. (a) $\frac{1}{3}\langle 2, -1, 2\rangle$ (b) $-\frac{1}{3}\langle 2, -1, 2\rangle$

53. (a) $\dfrac{1}{\sqrt{38}}\langle 3, 2, -5\rangle$ (b) $-\dfrac{1}{\sqrt{38}}\langle 3, 2, -5\rangle$

55. $\pm\dfrac{\sqrt{14}}{14}$ **57.** $\pm\dfrac{5}{3}$ **59.** $(2, -1, 2)$

61. $\left(\frac{13}{3}, 6, 3\right)$

63. $\langle 0, \sqrt{3}, \pm 1\rangle$



65. (a) 1 (b) 6 (c) 6 (d) $\mathbf{i} - \mathbf{k}$ (e) 2

67. $\arccos \dfrac{\sqrt{2}}{3} \approx 61.9°$

69. $\arccos\left(-\dfrac{8\sqrt{13}}{65}\right) \approx 116.3°$

71. Neither **73.** Orthogonal

75. (a) $\langle 0, \frac{33}{25}, \frac{44}{25} \rangle$ (b) $\langle 2, -\frac{8}{25}, \frac{6}{25} \rangle$

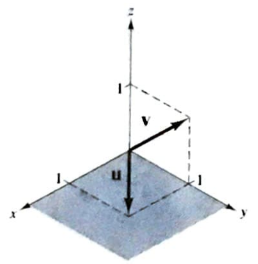
77. (a) $\langle \frac{2}{3}, \frac{1}{3}, -\frac{1}{3} \rangle$ (b) $\langle \frac{1}{3}, \frac{2}{3}, \frac{4}{3} \rangle$

79. (a) $\langle 5, 0, 0 \rangle$ (b) $\langle 0, -4, 3 \rangle$

81. $\cos \alpha = \frac{1}{3}$, $\cos \beta = \frac{2}{3}$, $\cos \gamma = \frac{2}{3}$

83. $\cos \alpha = 0$, $\cos \beta = \dfrac{3}{\sqrt{13}}$, $\cos \gamma = -\dfrac{2}{\sqrt{13}}$

85. (c) $a = b = 1$

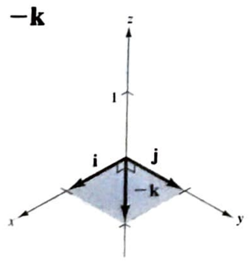


87. $\dfrac{\sqrt{3}}{3}\langle 1, 1, 1 \rangle$ **89.** 8.99 lb

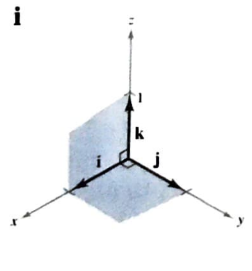
91. $T_2 = 157.5$
$T_3 = 3692.5$

Section 14.2

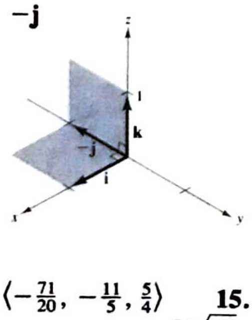
1. $-\mathbf{k}$



3. $\mathbf{i}$



5. $-\mathbf{j}$



7. $\langle -1, -1, -1 \rangle$

9. $\langle 0, 0, 54 \rangle$

11. $\langle -2, 3, -1 \rangle$

13. $\langle -\frac{71}{20}, -\frac{11}{5}, \frac{5}{4} \rangle$ **15.** 1 **17.** $6\sqrt{5}$

19. $2\sqrt{83}$ **21.** $\dfrac{3\sqrt{13}}{2}$ **23.** $\dfrac{9\sqrt{6}}{2}$ **25.** 1

27. 6 **29.** 2 **31.** 75

33. $10 \cos 40° \approx 7.66$ ft · lb

Section 14.3

1. (a)

(b) $P = (1, 2, 2)$, $Q = (10, -1, 17)$,
$\overrightarrow{PQ} = \langle 9, -3, 15 \rangle$
(There are many correct answers.) The components of the vector and the coefficients of t are proportional since the line is parallel to $\overrightarrow{PQ}$.

(c) $(7, 0, 12)$

	Parametric Equations	Symmetric Equations	Direction Numbers
3.	$x = t$, $y = 2t$, $z = 3t$	$x = \dfrac{y}{2} = \dfrac{z}{3}$	1, 2, 3
5.	$x = -2 + 2t$, $y = 4t$, $z = 3 - 2t$	$\dfrac{x+2}{2} = \dfrac{y}{4} = \dfrac{z-3}{-2}$	2, 4, −2
7.	$x = 1 + 3t$, $y = -2t$, $z = 1 + t$	$\dfrac{x-1}{3} = \dfrac{y}{-2} = \dfrac{z-1}{1}$	3, −2, 1
9.	$x = 5 + 17t$, $y = -3 - 11t$, $z = -2 - 9t$	$\dfrac{x-5}{17} = \dfrac{y+3}{-11} = \dfrac{z+2}{-9}$	17, −11, −9

11. $x = 2$
$y = 3$
$z = 4 + t$

13. a, b **15.** $(2, 3, 1)$, $\cos \theta = \dfrac{7\sqrt{17}}{51}$

17. Nonintersecting **19.** $x - 2 = 0$

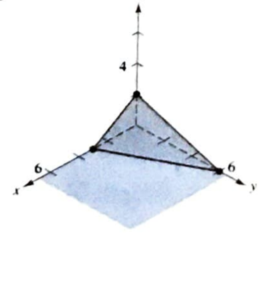
21. $2x + 3y - z = 10$ **23.** $3x + 9y - 7z = 0$

25. $4x - 3y + 4z = 10$ **27.** $z = 3$

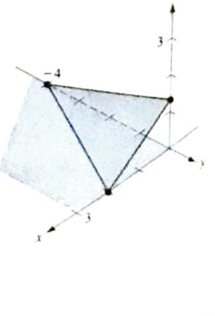
29. $x + y + z = 5$ **31.** $7x + y - 11z = 5$

33. Orthogonal **35.** 83.5° **37.** Parallel

39.



41.



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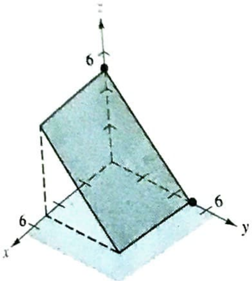

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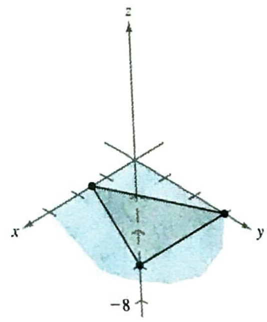


47. $x = 2$
$y = 1 + t$
$z = 1 + 2t$

49. $(2, -3, 2)$

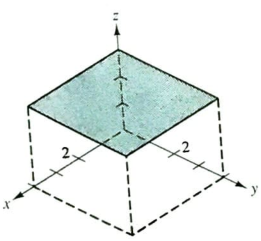
51. 0 **53.** $\frac{2\sqrt{26}}{13}$ **55.** $\frac{10\sqrt{26}}{13}$

57. $\arccos \frac{1}{65} \approx 89.1°$

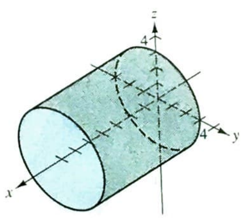
Section 14.4

1. c **2.** e **3.** f **4.** b **5.** d **6.** a

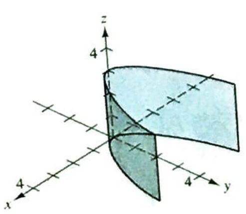
7. Plane



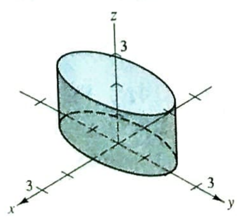
9. Right circular cylinder



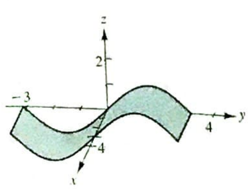
11. Parabolic cylinder



13. Elliptic cylinder

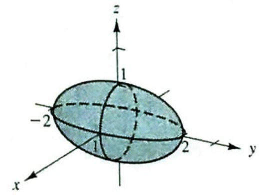


15. Cylinder

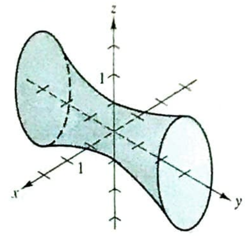


17. (a) $(20, 0, 0)$
(b) $(10, 10, 20)$
(c) $(0, 0, 20)$
(d) $(0, 20, 0)$

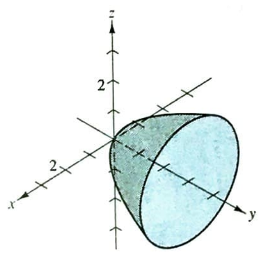
19. Ellipsoid



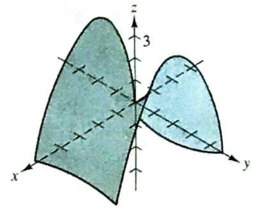
21. Hyperboloid of one sheet



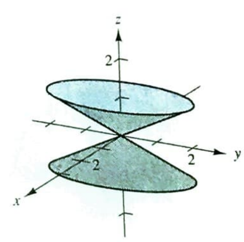
23. Elliptic paraboloid



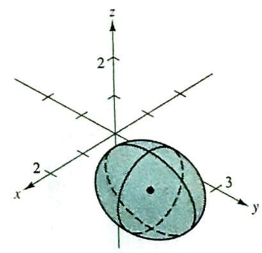
25. Hyperbolic paraboloid



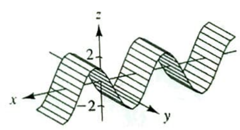
27. Cone



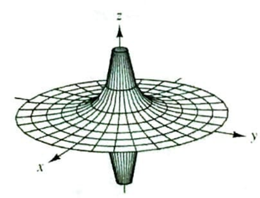
29. Ellipsoid



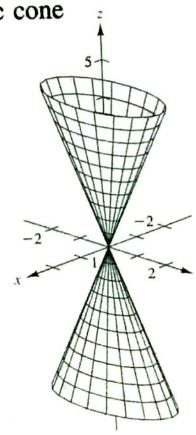
31. Cylinder



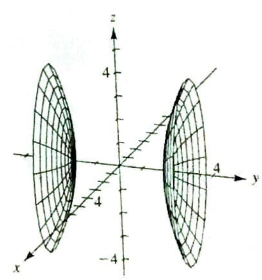
33. Surface of revolution



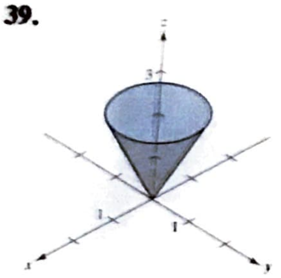
35. Elliptic cone



37. Hyperboloid of two sheets



39.



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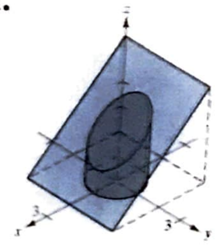


43. $x^2 + z^2 = 4y$ **45.** $4x^2 + 4y^2 = z^2$

47. $y^2 + z^2 = \dfrac{4}{x^2}$ **49.** $y = \sqrt{2z}$

Section 14.5

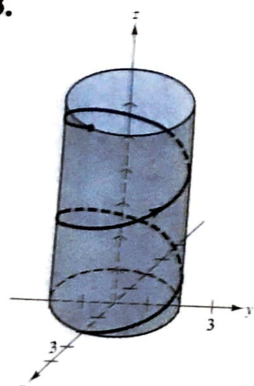
1. $(-\infty, 0), (0, \infty)$ **3.** $(0, \infty)$ **5.** $[0, \infty)$

7. $(-\infty, \infty)$ **9.** $\sqrt{1 + t^2}$

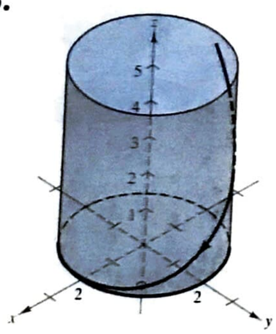
11. **(a)** $(-20, 0, 0)$ **(b)** $(10, 20, 10)$

(c) $(0, 0, 20)$ **(d)** $(20, 0, 0)$

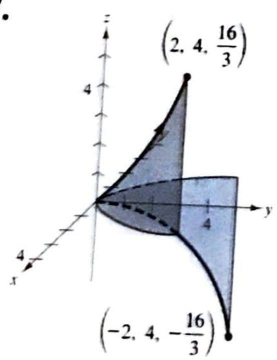
13.



15.

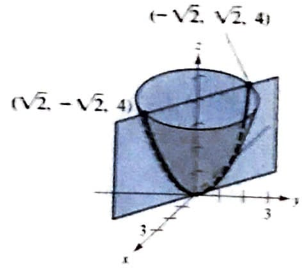


17.

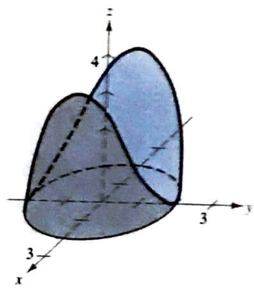


9. Parabola **21.** Helix

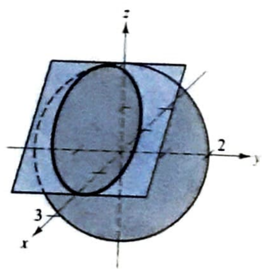
23. $x = t,\ y = -t,\ z = 2t^2$



25. $x = 2 \sin t,\ y = 2 \cos t,\ z = 4 \sin^2 t$



27. $x = 1 + \sin t,\ y = \pm\sqrt{2} \cos t,\ z = 1 - \sin t$



29. $2\mathbf{i} + 2\mathbf{j} + \frac{1}{2}\mathbf{k}$ **31.** $\mathbf{0}$

33. Limit does not exist. **35.** $[-1, 1]$

37. $\left(-\dfrac{\pi}{2} + n\pi, \dfrac{\pi}{2} + n\pi\right)$

39. $6\mathbf{i} - 14t\,\mathbf{j} + 3t^2\,\mathbf{k}$

41. $-3a \cos^2 t \sin t\,\mathbf{i} + 3a \sin^2 t \cos t\,\mathbf{j}$

43. $-e^{-t}\,\mathbf{i}$ **45.** $\langle \sin t + t \cos t, \cos t - t \sin t, 1\rangle$

47. **(a)** $\mathbf{i} + 3\mathbf{j} + 2t\,\mathbf{k}$ **(b)** $2\mathbf{k}$

(c) $8t + 9t^2 + 5t^4$

(d) $-\mathbf{i} + (9 - 2t)\,\mathbf{j} + (6t - 3t^2)\,\mathbf{k}$

(e) $8t^3\mathbf{i} + (12t^2 - 4t^3)\,\mathbf{j} + (3t^2 - 24t)\,\mathbf{k}$

(f) $\dfrac{10 + 2t^2}{\sqrt{10 + t^2}}$

49. $t^2\,\mathbf{i} + t\,\mathbf{j} + t\,\mathbf{k} + \mathbf{C}$

51. $\ln t\,\mathbf{i} + t\,\mathbf{j} - \frac{2}{5}t^{5/2}\,\mathbf{k} + \mathbf{C}$

53. $e^t\,\mathbf{i} - \cos t\,\mathbf{j} + \sin t\,\mathbf{k} + \mathbf{C}$

55. $\tan t\,\mathbf{i} + \arctan t\,\mathbf{j} + \mathbf{C}$