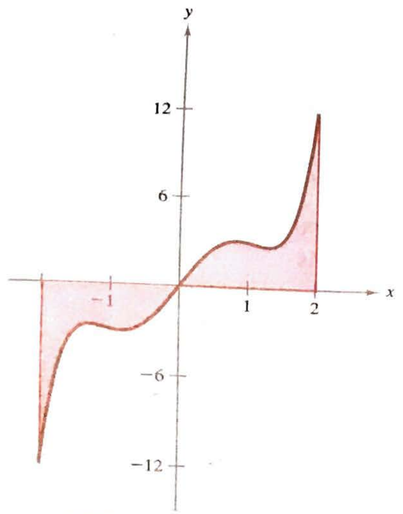

$f(x) = x^5 - 4x^3 + 6x$

FIGURE 5.32

EXAMPLE 10 Integration of an odd function

Evaluate

$$\int_{-2}^{2} (x^5 - 4x^3 + 6x)\, dx.$$

SOLUTION

By letting $f(x) = x^5 - 4x^3 + 6x$, we have

$$f(-x) = (-x)^5 - 4(-x)^3 + 6(-x) = -x^5 + 4x^3 - 6x = -f(x).$$

Thus, f is an odd function, and since $[-2, 2]$ is symmetric about the origin, we can apply Theorem 5.17 to conclude that

$$\int_{-2}^{2} (x^5 - 4x^3 + 6x)\, dx = 0.$$

REMARK From Figure 5.32, we see that the two regions on either side of the y-axis have the same area. However, since one lies below the x-axis and one lies above, integration produces a cancellation effect. (We will say more about finding the area of a region below the x-axis in Section 6.1.)

EXERCISES for Section 5.5

In Exercises 1–4, complete the table by identifying u and du for the given integral.

$\int f(g(x))g'(x)\, dx$	$u = g(x)$	$du = g'(x)\, dx$
1. $\int (5x^2 + 1)^2(10x)\, dx$		
2. $\int x^2\sqrt{x^3 + 1}\, dx$		
3. $\int \dfrac{x}{\sqrt{x^2 + 1}}\, dx$		
4. $\int (x^3 + 3)\, 3x^2\, dx$		

In Exercises 5–28, evaluate the indefinite integral and check the result by differentiation.

5. $\int (1 + 2x)^4(2)\, dx$
6. $\int (x^2 - 1)^3(2x)\, dx$
7. $\int \sqrt{9 - x^2}(-2x)\, dx$
8. $\int (1 - 2x^2)^3(-4x)\, dx$
9. $\int x^2(x^3 - 1)^4\, dx$
10. $\int x(4x^2 + 3)^3\, dx$
11. $\int 5x\sqrt[3]{1 - x^2}\, dx$
12. $\int u^3\sqrt{u^4 + 2}\, du$
13. $\int \dfrac{x^2}{(1 + x^3)^2}\, dx$
14. $\int \dfrac{x^2}{(16 - x^3)^2}\, dx$
15. $\int \dfrac{4x}{\sqrt{16 - x^2}}\, dx$
16. $\int \dfrac{10x^2}{\sqrt{1 + x^3}}\, dx$
17. $\int \dfrac{x + 1}{(x^2 + 2x - 3)^2}\, dx$
18. $\int \dfrac{x - 4}{\sqrt{x^2 - 8x + 1}}\, dx$
19. $\int \left(1 + \dfrac{1}{t}\right)^3\left(\dfrac{1}{t^2}\right)\, dt$
20. $\int \dfrac{1}{(3x)^2}\, dx$
21. $\int \dfrac{1}{\sqrt{2x}}\, dx$
22. $\int \dfrac{1}{2\sqrt{x}}\, dx$
23. $\int \dfrac{x^2 + 3x + 7}{\sqrt{x}}\, dx$
24. $\int \dfrac{t + 2t^2}{\sqrt{t}}\, dt$
25. $\int t^2\left(t - \dfrac{2}{t}\right)\, dt$
26. $\int \left(\dfrac{t^3}{3} + \dfrac{1}{4t^2}\right)\, dt$
27. $\int (9 - y)\sqrt{y}\, dy$
28. $\int 2\pi y(8 - y^{3/2})\, dy$

In Exercises 29–38, evaluate the indefinite integral by the method shown in Example 5.

29. $\int x\sqrt{x + 2}\, dx$
30. $\int x\sqrt{2x + 1}\, dx$
31. $\int x^2\sqrt{1 - x}\, dx$
32. $\int x^3\sqrt{x + 2}\, dx$
33. $\int \dfrac{x^2 - 1}{\sqrt{2x - 1}}\, dx$
34. $\int \dfrac{2x - 1}{\sqrt{x + 3}}\, dx$

35. $\displaystyle\int \frac{-x}{(x+1)-\sqrt{x+1}}\,dx$

36. $\displaystyle\int t\sqrt[3]{t-4}\,dt$

37. $\displaystyle\int \frac{x}{\sqrt{2x+1}}\,dx$

38. $\displaystyle\int (x+1)\sqrt{2-x}\,dx$

In Exercises 39–50, evaluate the definite integral.

39. $\displaystyle\int_{-1}^{1} x(x^2+1)^3\,dx$

40. $\displaystyle\int_{0}^{1} x\sqrt{1-x^2}\,dx$

41. $\displaystyle\int_{0}^{4} \frac{1}{\sqrt{2x+1}}\,dx$

42. $\displaystyle\int_{0}^{2} \frac{x}{\sqrt{1+2x^2}}\,dx$

43. $\displaystyle\int_{1}^{9} \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$

44. $\displaystyle\int_{0}^{2} x\sqrt[3]{4+x^2}\,dx$

45. $\displaystyle\int_{1}^{2} (x-1)\sqrt{2-x}\,dx$

46. $\displaystyle\int_{0}^{4} \frac{x}{\sqrt{2x+1}}\,dx$

47. $\displaystyle\int_{3}^{7} x\sqrt{x-3}\,dx$

48. $\displaystyle\int_{0}^{1} \frac{1}{\sqrt{x}+\sqrt{x+1}}\,dx$

49. $\displaystyle\int_{0}^{7} x\sqrt[3]{x+1}\,dx$

50. $\displaystyle\int_{-2}^{6} x^2\sqrt[3]{x+2}\,dx$

51. Use the fact that

$$\int_0^2 x^2\,dx = \frac{8}{3}$$

to evaluate the following definite integrals without using the Fundamental Theorem of Calculus.

(a) $\displaystyle\int_{-2}^{0} x^2\,dx$ (b) $\displaystyle\int_{-2}^{2} x^2\,dx$

(c) $\displaystyle\int_{0}^{2} -x^2\,dx$ (d) $\displaystyle\int_{-2}^{0} 3x^2\,dx$

52. Find the equation of the function f whose graph passes through the point $\left(0, \frac{7}{3}\right)$ and whose derivative is $f'(x) = x\sqrt{1-x^2}$.

53. A lumber company is seeking a model that yields the average weight loss W per log as a function of the number of days of drying time t. The model is to be reliable up to 100 days after the log is cut. Based on the weight loss during the first 30 days, it was determined that

$$\frac{dW}{dt} = \frac{12}{\sqrt{16t+9}}.$$

(a) Find W as a function of t. Note that no weight loss occurs until the tree is cut.

(b) Find the total weight loss after 100 days.

54. The marginal cost for a certain commodity has been determined to be

$$\frac{dC}{dx} = \frac{12}{\sqrt[3]{12x+1}}.$$

(a) Find the cost function if $C = 100$ when $x = 13$.

(b) Graph the marginal cost function and the cost function on the same set of axes.

5.6 Numerical Integration

The Trapezoidal Rule ▪ Simpson's Rule

Occasionally, we encounter functions for which we cannot find antiderivatives. Of course, that may be due to a lack of cleverness on our part. On the other hand, some elementary functions simply do not possess antiderivatives that are elementary functions. For example, there is no elementary function that has either of the following functions as its derivative.

$$\sqrt[3]{x}\sqrt{1-x} \qquad \sqrt{1-x^3}$$

If we wish to evaluate a definite integral involving a function whose antiderivative we cannot find, then the Fundamental Theorem of Calculus cannot be applied, and we must resort to an approximation technique. We describe two such techniques in this section.

The Trapezoidal Rule

One way to approximate a definite integral is by the use of n trapezoids, as shown in Figure 5.33. In the development of this method, we assume that f is continuous and positive on the interval $[a, b]$, and thus the definite integral $\int_a^b f(x)\,dx$ represents the area of the region bounded by the graph of f and the x-axis, from $x = a$ to $x = b$.

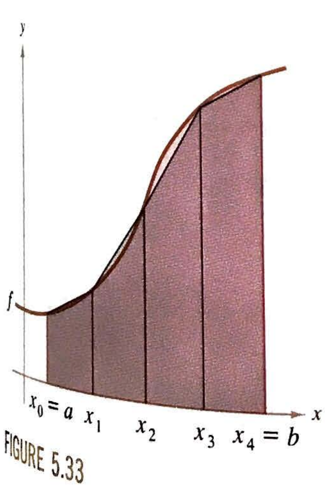

FIGURE 5.33