REMARK In Example 8, note that the position function has the form

$$s(t) = \frac{1}{2}gt^2 + v_0t + s_0$$

where $g = -32$, v_0 is the initial velocity, and s_0 is the initial height, as presented earlier in Section 3.2.

Before you begin the exercise set for this section, be sure you realize that one of the most important steps in integration is *rewriting the integrand* in a form that fits the basic integration rules. To further illustrate this point, we list several additional examples in Table 5.1.

TABLE 5.1

Given	*Rewrite*	*Integrate*	*Simplify*
$\int \frac{2}{\sqrt{x}}\,dx$	$2\int x^{-1/2}\,dx$	$2\left(\frac{x^{1/2}}{1/2}\right) + C$	$4x^{1/2} + C$
$\int (t^2+1)^2\,dt$	$\int (t^4 + 2t^2 + 1)\,dt$	$\frac{t^5}{5} + 2\left(\frac{t^3}{3}\right) + t + C$	$\frac{1}{5}t^5 + \frac{2}{3}t^3 + t + C$
$\int \frac{x^3+3}{x^2}\,dx$	$\int (x + 3x^{-2})\,dx$	$\frac{x^2}{2} + 3\left(\frac{x^{-1}}{-1}\right) + C$	$\frac{1}{2}x^2 - \frac{3}{x} + C$
$\int \sqrt[3]{x}(x-4)\,dx$	$\int (x^{4/3} - 4x^{1/3})\,dx$	$\frac{x^{7/3}}{7/3} - 4\left(\frac{x^{4/3}}{4/3}\right) + C$	$\frac{3}{7}x^{4/3}(x-7) + C$

EXERCISES for Section 5.1

In Exercises 1–6, complete the table using Table 5.1 as a model.

Given	Rewrite	Integrate	Simplify
1. $\int \sqrt[3]{x}\,dx$			
2. $\int \frac{1}{x^2}\,dx$			
3. $\int \frac{1}{x\sqrt{x}}\,dx$			
4. $\int x(x^2+3)\,dx$			
5. $\int \frac{1}{2x^3}\,dx$			
6. $\int \frac{1}{(2x)^3}\,dx$			

In Exercises 7–26, evaluate the indefinite integral and check your result by differentiation.

7. $\int (x^3 + 2)\,dx$

8. $\int (x^2 - 2x + 3)\,dx$

9. $\int (x^{3/2} + 2x + 1)\,dx$

10. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)dx$

11. $\int \sqrt[3]{x^2}\,dx$

12. $\int (\sqrt[4]{x^3} + 1)\,dx$

13. $\int \frac{1}{x^3}\,dx$

14. $\int \frac{1}{x^4}\,dx$

15. $\int \frac{1}{4x^2}\,dx$

16. $\int (2x + x^{-1/2})\,dx$

17. $\int \frac{x^2+x+1}{\sqrt{x}}\,dx$

18. $\int \frac{x^2+1}{x^2}\,dx$

19. $\int (x+1)(3x-2)\,dx$

20. $\int (2t^2-1)^2\,dt$

21. $\int \frac{t^2 + 2}{t^2}\, dt$

22. $\int (1 - 2y + 3y^2)\, dy$

23. $\int y^2\sqrt{y}\, dy$

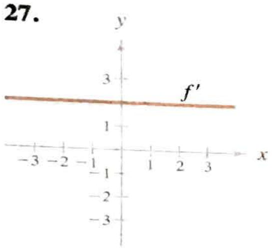
24. $\int (1 + 3t)t^2\, dt$

25. $\int dx$

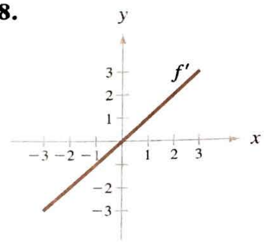
26. $\int 3\, dt$

In Exercises 27–30, the graph of the derivative of a function is given. Sketch the graphs of *two* functions that have the given derivative. (There is more than one correct answer.)

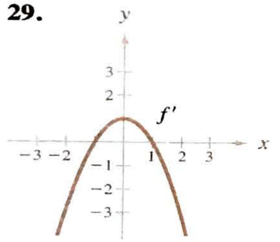
27.



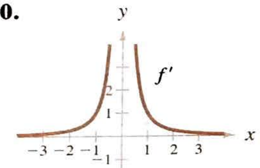
28.



29.

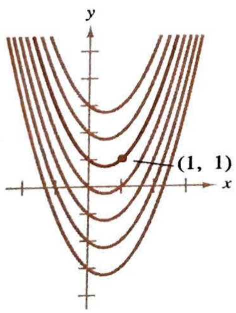


30.

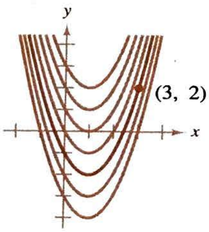


In Exercises 31–34, find the equation of the curve, given the derivative and the indicated point on the curve.

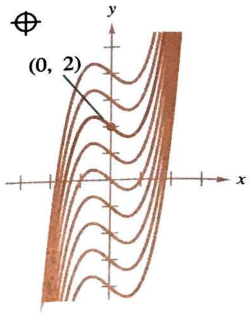
31. $\frac{dy}{dx} = 2x - 1$



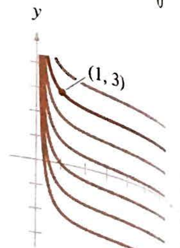
32. $\frac{dy}{dx} = 2(x - 1)$



33. $\frac{dy}{dx} = 3x^2 - 1$



34. $\frac{dy}{dx} = -\frac{1}{x^2}, x > 0$



In Exercises 35–38, find $y = f(x)$ satisfying the given conditions.

35. $f''(x) = 2, f'(2) = 5, f(2) = 10$
36. $f''(x) = x^2, f'(0) = 6, f(0) = 3$
37. $f''(x) = x^{-3/2}, f'(4) = 2, f(0) = 0$
38. $f''(x) = x^{-3/2}, f'(1) = 2, f(9) = -4$

In Exercises 39–43, use $a(t) = -32$ ft/s^2 as the acceleration due to gravity. (Neglect air resistance.)

39. An object is dropped from a balloon that is stationary at 1600 feet above the ground. Express its height above the ground as a function of t. How long does it take the object to reach the ground?

40. A ball is thrown vertically upward from the ground with an initial velocity of 60 feet per second. How high will the ball go?

41. With what initial velocity must an object be thrown upward (from ground level) to reach a maximum height of 550 feet (approximate height of the Washington Monument)?

42. Show that the height above the ground of an object thrown upward from a point s_0 feet above the ground with an initial velocity of v_0 feet per second is given by the function

$$f(t) = -16t^2 + v_0t + s_0.$$

43. A balloon, rising vertically with a velocity of 16 feet per second, releases a sandbag at the instant when the balloon is 64 feet above the ground.
(a) How many seconds after its release will the bag strike the ground?
(b) With what velocity will it reach the ground?

44. Assume that a fully loaded plane starting from rest has a constant acceleration while moving down the runway