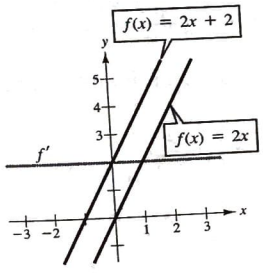
# Chapter 5

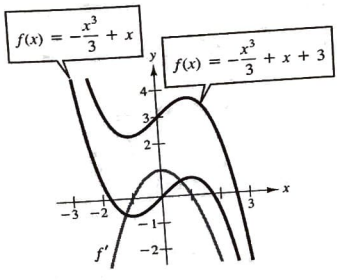
## Section 5.1

| | Given | Rewrite | Integrate | Simplify |
|---|---|---|---|---|
| 1. | $\int \sqrt[3]{x}\,dx$ | $\int x^{1/3}\,dx$ | $\frac{x^{4/3}}{4/3} + C$ | $\frac{3}{4}x^{4/3} + C$ |
| 3. | $\int \frac{1}{x\sqrt{x}}\,dx$ | $\int x^{-3/2}\,dx$ | $\frac{x^{-1/2}}{-1/2} + C$ | $-\frac{2}{\sqrt{x}} + C$ |
| 5. | $\int \frac{1}{2x^3}\,dx$ | $\frac{1}{2}\int x^{-3}\,dx$ | $\frac{1}{2}\left(\frac{x^{-2}}{-2}\right) + C$ | $-\frac{1}{4x^2} + C$ |

7. $\frac{1}{4}x^4 + 2x + C$ 9. $\frac{2}{5}x^{5/2} + x^2 + x + C$
11. $\frac{3}{5}x^{5/3} + C$ 13. $-\frac{1}{2x^2} + C$ 15. $-\frac{1}{4x} + C$
17. $\frac{2}{15}x^{1/2}(3x^2 + 5x + 15) + C$
19. $x^3 + \frac{1}{2}x^2 - 2x + C$ 21. $t - \frac{2}{t} + C$
23. $\frac{2}{7}y^{7/2} + C$ 25. $x + C$
27.




29.

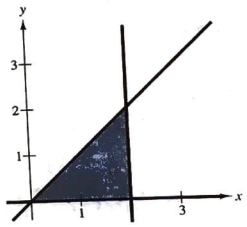



31. $y = x^2 - x + 1$ 33. $y = x^3 - x + 2$
35. $f(x) = x^2 + x + 4$
37. $f(x) = -4x^{1/2} + 3x = -4\sqrt{x} + 3x$
39. $s(t) = -16t^2 + 1600$, $t = 10$ sec
41. $v_0 \approx 187.617$ ft/sec
43. (a) $\frac{1 + \sqrt{17}}{2} \approx 2.562$ sec
(b) $-16\sqrt{17} \approx -65.970$ ft/sec
45. (a) $\frac{154}{39} \approx 3.95$ ft/sec$^2$ (b) $\frac{1859}{3} \approx 619.67$ ft
47. (a) 300 ft (b) 60 ft/sec $\approx$ 41 mi/hr

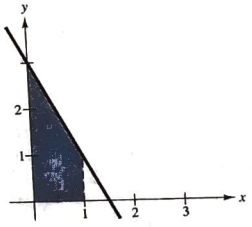
## Section 5.2

1. 35 3. $\frac{158}{85}$ 5. $4c$ 7. 238 9. $\sum_{i=1}^{9} \frac{1}{3i}$
11. $\sum_{j=1}^{8} \left[2\left(\frac{j}{8}\right) + 3\right]$ 13. $\frac{1}{6}\sum_{k=1}^{6} \left[\left(\frac{k}{6}\right)^2 + 2\right]$
15. $\frac{2}{n}\sum_{i=1}^{n} \left[\left(\frac{2i}{n}\right)^3 - \left(\frac{2i}{n}\right)\right]$ 17. $\frac{3}{n}\sum_{i=1}^{n} \left[2\left(1 + \frac{3i}{n}\right)^2\right]$
19. 420 21. 2470 23. $\frac{1015}{n^3}$
25. $\frac{8}{3}$ 27. $\frac{81}{4}$ 29. 9
31. $\lim_{n\to\infty} \frac{1}{6}\left(\frac{2n^3 - 3n^2 + n}{n^3}\right) = \frac{1}{3}$
33. $\lim_{n\to\infty} \left[8\left(\frac{n^2 + n}{n^2}\right)\right] = 8$
35. $\lim_{n\to\infty} 2\left(\frac{10n^4 + 13n^3 + 4n^2}{n^4}\right) = 20$
37. $S \approx 0.768$, $s \approx 0.518$ 39. $S \approx 0.746$, $s \approx 0.646$ 41. $S \approx 0.859$, $s \approx 0.659$
43.

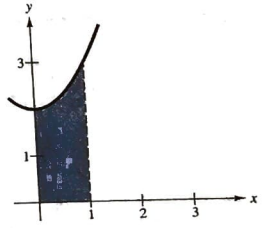
| $n$ | 5 | 10 | 50 | 100 |
|---|---|---|---|---|
| $s(n)$ | 1.6 | 1.8 | 1.96 | 1.98 |
| $S(n)$ | 2.4 | 2.2 | 2.04 | 2.02 |



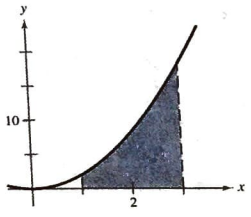
45. $A = 2$



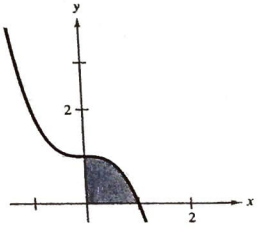
47. $A = \frac{7}{3}$



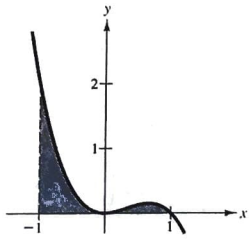
49. $A = \frac{52}{3}$



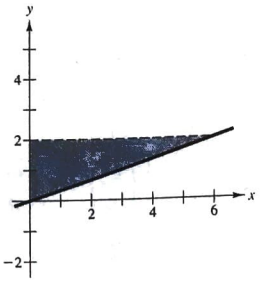
51. $A = \frac{3}{4}$



**53.** $A = \frac{2}{3}$



**55.** $A = 6$



**57.** $\frac{69}{8}$ **59.** 0.673

**61.**

| $n$ | 4 | 8 | 12 | 16 | 20 |
|---|---|---|---|---|---|
| *Approximate area* | 5.3838 | 5.3523 | 5.3439 | 5.3403 | 5.3384 |

**63.** $N(5) \approx 167$, $N(10) \approx 250$, $N(25) \approx 400$

$$\lim_{t \to \infty} \frac{10(5 + 3t)}{1 + 0.04t} = 750$$

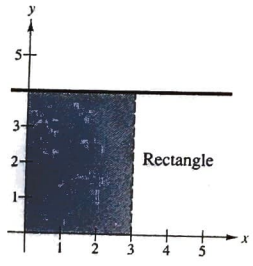
## Section 5.3

**1.** $\int_0^5 3\,dx$ **3.** $\int_{-4}^{4} (4 - |x|)\,dx$

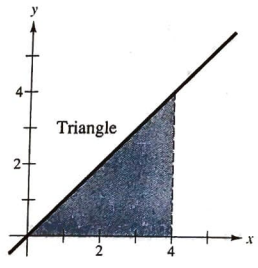
**5.** $\int_{-2}^{2} (4 - x^2)\,dx$ **7.** $\int_0^2 y^3\,dy$

**9.** $\int_0^2 \sqrt{x + 1}\,dx$

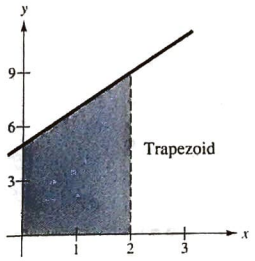
**11.** $A = 12$



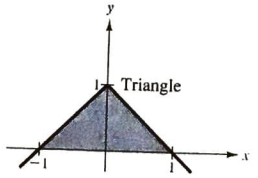

**13.** $A = 8$



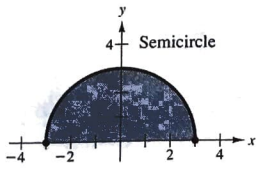

**15.** $A = 14$




**17.** $A = 1$




**19.** $A = \frac{9\pi}{2}$




**21.** (a) 13 (b) −10 (c) 0 (d) 30

**23.** (a) 8 (b) −12 (c) −4 (d) 30

**25.** 36 **27.** 0 **29.** $\frac{10}{3}$ **31.** $\frac{4\sqrt{2}}{3}$

**33.** $\int_{-1}^{5} (3x + 10)\,dx$

**35.**

| $n$ | 4 | 8 | 12 | 16 | 20 |
|---|---|---|---|---|---|
| $L(n)$ | 3.6830 | 3.9956 | 4.0707 | 4.1016 | 4.1177 |
| $M(n)$ | 4.3082 | 4.2076 | 4.1838 | 4.1740 | 4.1690 |
| $R(n)$ | 3.6830 | 3.9956 | 4.0707 | 4.1016 | 4.1177 |

**37.** Not integrable because there are an infinite number of discontinuities.

## Section 5.4

**1.** 1 **3.** $-\frac{5}{2}$ **5.** $-\frac{10}{3}$ **7.** $\frac{1}{3}$ **9.** $\frac{1}{2}$

**11.** 36 **13.** −4 **15.** $\frac{2}{3}$ **17.** $-\frac{1}{18}$

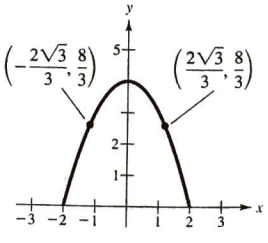
**19.** $-\frac{27}{20}$ **21.** 1 **23.** 4 **25.** $\frac{1}{6}$ **27.** $\frac{8}{5}$

**29.** 6 **31.** 10 **33.** 6 **35.** $\sqrt[3]{2}$

**37.** 1, 3

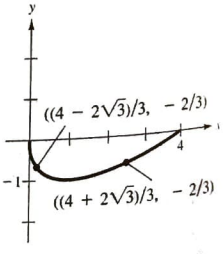
**39.** Average $= \frac{8}{3}$

$x = \pm\frac{2\sqrt{3}}{3} \approx \pm 1.155$




**41.** Average $= -\frac{2}{3}$

$x = \left(1 + \frac{1}{\sqrt{3}}\right)^2$




**43.** $\frac{1}{2}x^2 + 2x$ **45.** $\frac{3}{4}x^{4/3} - 12$ **47.** $1 - \frac{1}{x}$

**49.** $x^2 - 2x + 5$ **51.** $\sqrt{x^4 + 1}$

**53.** 0.5318 liter **55.** (a) 8 (b) $\frac{4}{3}$ (c) 20, $\frac{10}{3}$

## Section 5.5

| | $\int f(g(x))g'(x)\,dx$ | $u = g(x)$ | $du = g'(x)\,dx$ |
|---|---|---|---|
| 1. | $\int (5x^2 + 1)^2(10x)\,dx$ | $5x^2 + 1$ | $10x\,dx$ |
| 3. | $\int \frac{x}{\sqrt{x^2 + 1}}\,dx$ | $x^2 + 1$ | $2x\,dx$ |

5. $\frac{(1 + 2x)^5}{5} + C$ 7. $\frac{2}{3}(9 - x^2)^{3/2} + C$

9. $\frac{(x^3 - 1)^5}{15} + C$ 11. $-\frac{15}{8}(1 - x^2)^{4/3} + C$

13. $-\frac{1}{3(1 + x^3)} + C$ 15. $-4\sqrt{16 - x^2} + C$

17. $-\frac{1}{2(x^2 + 2x - 3)} + C$ 19. $-\frac{1}{4}\left(1 + \frac{1}{t}\right)^4 + C$

21. $\sqrt{2x} + C$ 23. $\frac{2}{5}\sqrt{x}(x^2 + 5x + 35) + C$

25. $\frac{1}{4}t^4 - t^2 + C$ 27. $\frac{2}{5}y^{3/2}(15 - y) + C$

29. $2\left[\frac{1}{5}(x + 2)^{5/2} - \frac{2}{3}(x + 2)^{3/2}\right] + C$
$= \frac{2}{15}(x + 2)^{3/2}(3x - 4) + C$

31. $-2\left[\frac{1}{3}(1 - x)^{3/2} - \frac{2}{5}(1 - x)^{5/2} + \frac{1}{7}(1 - x)^{7/2}\right] + C$
$= -\frac{2}{105}(1 - x)^{3/2}(15x^2 + 12x + 8) + C$

33. $\frac{1}{4}\left[\frac{1}{5}(2x - 1)^{5/2} + \frac{2}{3}(2x - 1)^{3/2} - 3(2x - 1)^{1/2}\right] + C$
$= \frac{\sqrt{2x - 1}}{15}(3x^2 + 2x - 13) + C$

35. $-x - 1 - 2\sqrt{x + 1} + C$
or $-(x + 2\sqrt{x + 1}) + C_1$

37. $\frac{1}{2}\left[\frac{1}{3}(2x + 1)^{3/2} - (2x + 1)^{1/2}\right] + C$
$= \frac{1}{3}\sqrt{2x + 1}(x - 1) + C$

39. 0 41. 2 43. $\frac{1}{2}$ 45. $\frac{4}{15}$ 47. $\frac{144}{5}$

49. $\frac{1209}{28}$ 51. (a) $\frac{8}{3}$ (b) $\frac{16}{3}$ (c) $-\frac{8}{3}$ (d) 8

53. (a) $\frac{3}{2}(\sqrt{16t + 9} - 3)$
(b) $\frac{3}{2}(\sqrt{1609} - 3) \approx 55.67$ lb

## Section 5.6

| | Exact | Trapezoidal | Simpson's |
|---|---|---|---|
| 1. | 2.6667 | 2.7500 | 2.6667 |
| 3. | 4.0000 | 4.2500 | 4.0000 |
| 5. | 4.0000 | 4.0625 | 4.0000 |
| 7. | 12.6667 | 12.6640 | 12.6667 |
| 9. | 0.1667 | 0.1676 | 0.1667 |

| | Trapezoidal | Simpson's |
|---|---|---|
| 11. | 1.683 | 1.622 |
| 13. | 3.41 | 3.22 |
| 15. | 0.342 | 0.372 |
| 17. | 2.208 | 2.210 |
| 19. | 2.352 | 2.438 |
| 21. | 0.500 | 0.000 |

| | Trapezoidal | Simpson's |
|---|---|---|
| 23. | 0.010 | 0.001 |
| 25. | $n = 366$ | $n = 26$ |
| 27. | $n = 130$ | $n = 12$ |

29. 3.14159 31. 89,250 ft$^2$

33. (a) 12.5175 (b) 12.5917

35.

| $n$ | $L(n)$ | $M(n)$ | $R(n)$ | $T(n)$ | $S(n)$ |
|---|---|---|---|---|---|
| 4 | 12.7771 | 15.3965 | 18.4340 | 15.6055 | 15.4845 |
| 8 | 14.0868 | 15.4480 | 16.9152 | 15.5010 | 15.4662 |
| 10 | 14.3569 | 15.4544 | 16.6197 | 15.4883 | 15.4658 |
| 12 | 14.5386 | 15.4578 | 16.4242 | 15.4814 | 15.4657 |
| 16 | 14.7674 | 15.4613 | 16.1816 | 15.4745 | 15.4657 |
| 20 | 14.9056 | 15.4628 | 16.0370 | 15.4713 | 15.4657 |

37. 10,233.58 ft · lb

## Review Exercises for Chapter 5

1. $x^{2/3} + C$ 3. $\frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C$

5. $\frac{2\sqrt{x}}{15}(15 + 10x + 3x^2) + C$

7. $\frac{2}{3}\sqrt{x^3 + 3} + C$ 9. $\frac{1}{7}x^7 + \frac{3}{5}x^5 + x^3 + x + C$

11. $\frac{1}{8}(x^2 + 1)^4 + C$ 13. $-\frac{1}{4(x^2 + 1)^2} + C$

15. $2\left[\frac{1}{7}(x + 5)^{7/2} - 2(x + 5)^{5/2} + \frac{25}{3}(x + 5)^{3/2}\right] + C$
$= \frac{2(x + 5)^{3/2}}{21}(3x^2 - 12x + 40) + C$

17. $\frac{1}{2}x^2 - \frac{1}{x} + C$

19. (a) $\sum_{i=1}^{10}(2i - 1)$ (b) $\sum_{i=1}^{n} i^3$ (c) $\sum_{i=1}^{10}(4i + 2)$

21. 16 23. 0 25. 2 27. $\frac{422}{5}$

29. $\frac{28\pi}{15}$ 31. $y = 2 - x^2$ 33. 240 ft/sec

35. (a) 3 sec (b) 144 ft (c) $\frac{3}{2}$ sec (d) 108 ft

37. (a) $S = \frac{5mb^2}{8}$, $s = \frac{3mb^2}{8}$

(b) $S(n) = \frac{mb^2(n + 1)}{2n}$

$s(n) = \frac{mb^2(n - 1)}{2n}$

(c) $\frac{1}{2}mb^2$ (d) $\frac{1}{2}mb^2$