REMARK In Example 8, note that the position function has the form

$$s(t) = \frac{1}{2}gt^2 + v_0t + s_0$$

where $g = -32$, v_0 is the initial velocity, and s_0 is the initial height, as presented earlier in Section 3.2.

Before you begin the exercise set for this section, be sure you realize that one of the most important steps in integration is *rewriting the integrand* in a form that fits the basic integration rules. To further illustrate this point, we list several additional examples in Table 5.1.

TABLE 5.1

Given	*Rewrite*	*Integrate*	*Simplify*
$\int \frac{2}{\sqrt{x}}\,dx$	$2\int x^{-1/2}\,dx$	$2\left(\frac{x^{1/2}}{1/2}\right) + C$	$4x^{1/2} + C$
$\int (t^2+1)^2\,dt$	$\int (t^4 + 2t^2 + 1)\,dt$	$\frac{t^5}{5} + 2\left(\frac{t^3}{3}\right) + t + C$	$\frac{1}{5}t^5 + \frac{2}{3}t^3 + t + C$
$\int \frac{x^3+3}{x^2}\,dx$	$\int (x + 3x^{-2})\,dx$	$\frac{x^2}{2} + 3\left(\frac{x^{-1}}{-1}\right) + C$	$\frac{1}{2}x^2 - \frac{3}{x} + C$
$\int \sqrt[3]{x}(x-4)\,dx$	$\int (x^{4/3} - 4x^{1/3})\,dx$	$\frac{x^{7/3}}{7/3} - 4\left(\frac{x^{4/3}}{4/3}\right) + C$	$\frac{3}{7}x^{4/3}(x-7) + C$

EXERCISES for Section 5.1

In Exercises 1–6, complete the table using Table 5.1 as a model.

Given	*Rewrite*	*Integrate*	*Simplify*
1. $\int \sqrt[3]{x}\,dx$			
2. $\int \frac{1}{x^2}\,dx$			
3. $\int \frac{1}{x\sqrt{x}}\,dx$			
4. $\int x(x^2+3)\,dx$			
5. $\int \frac{1}{2x^3}\,dx$			
6. $\int \frac{1}{(2x)^3}\,dx$			

In Exercises 7–26, evaluate the indefinite integral and check your result by differentiation.

7. $\int (x^3 + 2)\,dx$

8. $\int (x^2 - 2x + 3)\,dx$

9. $\int (x^{3/2} + 2x + 1)\,dx$

10. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)dx$

11. $\int \sqrt[3]{x^2}\,dx$

12. $\int (\sqrt[4]{x^3} + 1)\,dx$

13. $\int \frac{1}{x^3}\,dx$

14. $\int \frac{1}{x^4}\,dx$

15. $\int \frac{1}{4x^2}\,dx$

16. $\int (2x + x^{-1/2})\,dx$

17. $\int \frac{x^2+x+1}{\sqrt{x}}\,dx$

18. $\int \frac{x^2+1}{x^2}\,dx$

19. $\int (x+1)(3x-2)\,dx$

20. $\int (2t^2 - 1)^2\,dt$

21. $\int \frac{t^2 + 2}{t^2}\, dt$

22. $\int (1 - 2y + 3y^2)\, dy$

23. $\int y^2\sqrt{y}\, dy$

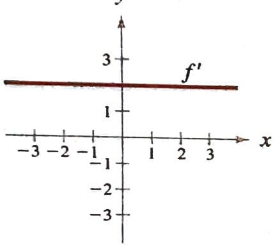
24. $\int (1 + 3t)t^2\, dt$

25. $\int dx$

26. $\int 3\, dt$

In Exercises 27–30, the graph of the derivative of a function is given. Sketch the graphs of *two* functions that have the given derivative. (There is more than one correct answer.)

27.



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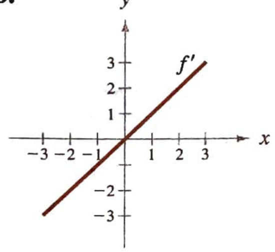

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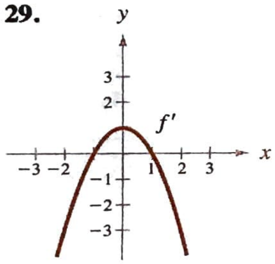

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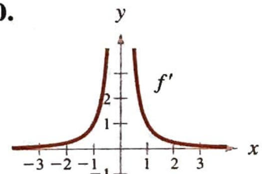
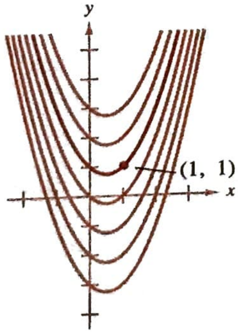
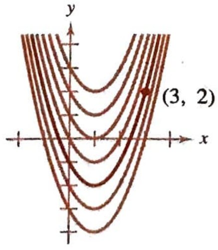


In Exercises 31–34, find the equation of the curve, given the derivative and the indicated point on the curve.

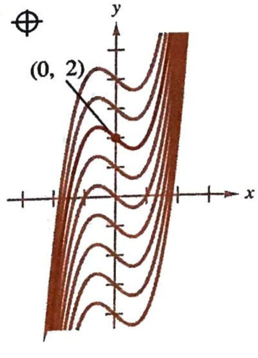
31. $\frac{dy}{dx} = 2x - 1$



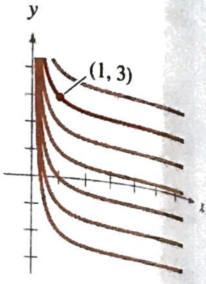
32. $\frac{dy}{dx} = 2(x - 1)$



33. $\frac{dy}{dx} = 3x^2 - 1$



34. $\frac{dy}{dx} = -\frac{1}{x^2},\ x > 0$



In Exercises 35–38, find $y = f(x)$ satisfying the given conditions.

35. $f''(x) = 2, f'(2) = 5, f(2) = 10$
36. $f''(x) = x^2, f'(0) = 6, f(0) = 3$
37. $f''(x) = x^{-3/2}, f'(4) = 2, f(0) = 0$
38. $f''(x) = x^{-3/2}, f'(1) = 2, f(9) = -4$

In Exercises 39–43, use $a(t) = -32$ ft/s^2 as the acceleration due to gravity. (Neglect air resistance.)

39. An object is dropped from a balloon that is stationary at 1600 feet above the ground. Express its height above the ground as a function of t. How long does it take the object to reach the ground?

40. A ball is thrown vertically upward from the ground with an initial velocity of 60 feet per second. How high will the ball go?

41. With what initial velocity must an object be thrown upward (from ground level) to reach a maximum height of 550 feet (approximate height of the Washington Monument)?

42. Show that the height above the ground of an object thrown upward from a point s_0 feet above the ground with an initial velocity of v_0 feet per second is given by the function

$$f(t) = -16t^2 + v_0 t + s_0.$$

43. A balloon, rising vertically with a velocity of 16 feet per second, releases a sandbag at the instant when the balloon is 64 feet above the ground.
(a) How many seconds after its release will the bag strike the ground?
(b) With what velocity will it reach the ground?

44. Assume that a fully loaded plane starting from rest has a constant acceleration while moving down the runway.

In the next example we look at a region that is bounded by the y-a (rather than the x-axis).

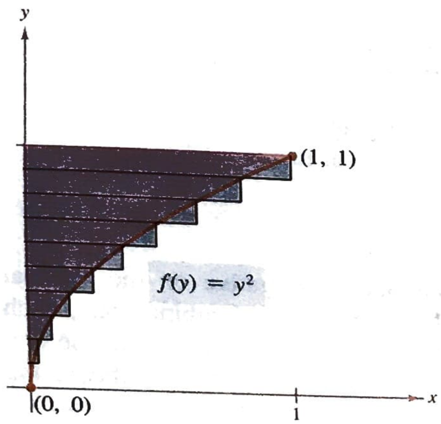


FIGURE 5.14

EXAMPLE 8 A region bounded by the y-axis

Find the area of the region bounded by the graph of $f(y) = y^2$ and the y-ax for $0 \le y \le 1$, as shown in Figure 5.14.

SOLUTION

When f is a continuous, nonnegative function of y, we still can use the sam basic procedure illustrated in Example 6. We partition the interval $[0, 1]$ in n equal subintervals, each of width $\Delta y = 1/n$. Using the upper endpoin $c_i = i/n$, we obtain the following.

$$\begin{aligned} \text{area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta y &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2\left(\frac{1}{n}\right) = \lim_{n\to\infty} \frac{1}{n^3} \sum_{i=1}^{n} i^2 \\ &= \lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6n^3} \\ &= \lim_{n\to\infty} \left(\frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right) = \frac{1}{3} \end{aligned}$$

EXERCISES for Section 5.2

In Exercises 1–8, find the given sum.

1. $\sum_{i=1}^{5} (2i + 1)$
2. $\sum_{i=1}^{6} 2i$
3. $\sum_{k=0}^{4} \frac{1}{k^2 + 1}$
4. $\sum_{j=3}^{5} \frac{1}{j}$
5. $\sum_{k=1}^{4} c$
6. $\sum_{n=1}^{10} \frac{3}{n + 1}$
7. $\sum_{i=1}^{4} [(i - 1)^2 + (i + 1)^3]$
8. $\sum_{k=2}^{5} (k + 1)(k - 3)$

In Exercises 9–18, use sigma notation to write the given sum.

9. $\frac{1}{3(1)} + \frac{1}{3(2)} + \frac{1}{3(3)} + \cdots + \frac{1}{3(9)}$
10. $\frac{5}{1 + 1} + \frac{5}{1 + 2} + \frac{5}{1 + 3} + \cdots + \frac{5}{1 + 15}$
11. $\left[2\left(\frac{1}{8}\right) + 3\right] + \left[2\left(\frac{2}{8}\right) + 3\right] + \cdots + \left[2\left(\frac{8}{8}\right) + 3\right]$
12. $\left[1 - \left(\frac{1}{4}\right)^2\right] + \left[1 - \left(\frac{2}{4}\right)^2\right] + \cdots + \left[1 - \left(\frac{4}{4}\right)^2\right]$
13. $\left[\left(\frac{1}{6}\right)^2 + 2\right]\left(\frac{1}{6}\right) + \cdots + \left[\left(\frac{6}{6}\right)^2 + 2\right]\left(\frac{1}{6}\right)$
14. $\left[\left(\frac{1}{n}\right)^2 + 2\right]\left(\frac{1}{n}\right) + \cdots + \left[\left(\frac{n}{n}\right)^2 + 2\right]\left(\frac{1}{n}\right)$
15. $\left[\left(\frac{2}{n}\right)^3 - \frac{2}{n}\right]\left(\frac{2}{n}\right) + \cdots + \left[\left(\frac{2n}{n}\right)^3 - \frac{2n}{n}\right]\left(\frac{2}{n}\right)$
16. $\left[1 - \left(\frac{2}{n} - 1\right)^2\right]\left(\frac{2}{n}\right) + \cdots + \left[1 - \left(\frac{2n}{n} - 1\right)^2\right]\left(\frac{2}{n}\right)$
17. $\left[2\left(1 + \frac{3}{n}\right)^2\right]\left(\frac{3}{n}\right) + \cdots + \left[2\left(1 + \frac{3n}{n}\right)^2\right]\left(\frac{3}{n}\right)$
18. $\left(\frac{1}{n}\right)\sqrt{1 - \left(\frac{0}{n}\right)^2} + \cdots + \left(\frac{1}{n}\right)\sqrt{1 - \left(\frac{n - 1}{n}\right)^2}$

In Exercises 19–24, use the properties of sigma notation and summation formulas to evaluate the given sum.

19. $\sum_{i=1}^{20} 2i$
20. $\sum_{i=1}^{10} i(i^2 + 1)$
21. $\sum_{i=1}^{20} (i - 1)^2$
22. $\sum_{i=1}^{15} (2i - 3)$
23. $\sum_{i=1}^{15} \frac{1}{n^3}(i - 1)^2$
24. $\sum_{i=1}^{10} (i^2 - 1)$

In Exercises 25–30, find the limit of $s(n)$ as $n \to \infty$.

25. $s(n) = \left(\frac{4}{3n^3}\right)(2n^3 + 3n^2 + n)$

26. $s(n) = \left(\frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2}\right)$

27. $s(n) = \frac{81}{n^4}\left[\frac{n^2(n+1)^2}{4}\right]$

28. $s(n) = \frac{64}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$

29. $s(n) = \frac{18}{n^2}\left[\frac{n(n+1)}{2}\right]$

30. $s(n) = \frac{1}{n^2}\left[\frac{n(n+1)}{2}\right]$

In Exercises 31–36, use the properties of sigma notation to find a formula for the given sum of n terms. Then use the formula to find the limit as $n \to \infty$.

31. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n^3}(i-1)^2$

32. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{2i}{n}\right)^2\left(\frac{2}{n}\right)$

33. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{16i}{n^2}$

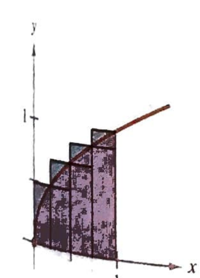
34. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)\left(\frac{2}{n}\right)$

35. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{2i}{n}\right)^3\left(\frac{2}{n}\right)$

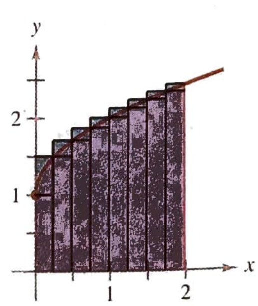
36. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{i}{n}\right)\left(\frac{2}{n}\right)$

In Exercises 37–42, use the upper and lower sums to approximate the area of the given region using the indicated number of (equal) subintervals.

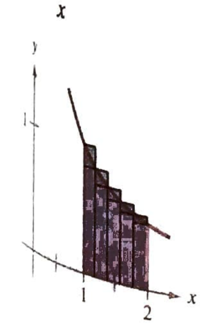
37. $y = \sqrt{x}$



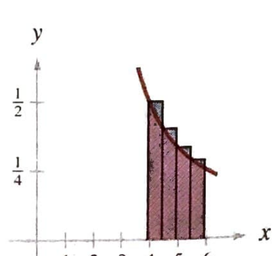
38. $y = \sqrt{x} + 1$



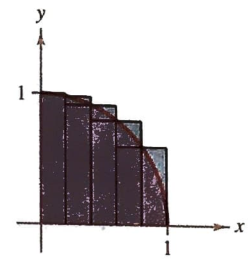
39. $y = \frac{1}{x}$



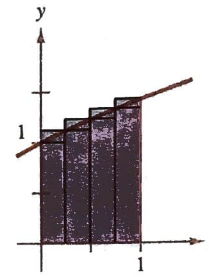
40. $y = \frac{1}{x-2}$



41. $y = \sqrt{1 - x^2}$



42. $y = \sqrt{x} + 1$



43. Consider the triangle of area 2 bounded by the graphs of $y = x$, $y = 0$, and $x = 2$.
(a) Sketch the graph of the region.
(b) Divide the interval $[0, 2]$ into n equal subintervals and show that the endpoints are

$$0 < 1\left(\frac{2}{n}\right) < \cdots < (n-1)\left(\frac{2}{n}\right) < n\left(\frac{2}{n}\right).$$

(c) Show that $s(n) = \sum_{i=1}^{n} \left[(i-1)\left(\frac{2}{n}\right)\right]\left(\frac{2}{n}\right)$.
(d) Show that $S(n) = \sum_{i=1}^{n} \left[i\left(\frac{2}{n}\right)\right]\left(\frac{2}{n}\right)$.
(e) Complete the following table.

n	5	10	50	100
$s(n)$				
$S(n)$				

(f) Show that $\lim_{n\to\infty} s(n) = \lim_{n\to\infty} S(n) = 2$.

44. Consider the trapezoid of area 4 bounded by the graphs of $y = x$, $y = 0$, $x = 1$, and $x = 3$.
(a) Sketch the graph of the region.
(b) Divide the interval $[1, 3]$ into n equal subintervals and show that the endpoints are

$$1 < 1 + 1\left(\frac{2}{n}\right) < \cdots < 1 + (n-1)\left(\frac{2}{n}\right) < 1 + n\left(\frac{2}{n}\right).$$

(c) Show that $s(n) = \sum_{i=1}^{n} \left[1 + (i-1)\left(\frac{2}{n}\right)\right]\left(\frac{2}{n}\right)$.
(d) Show that $S(n) = \sum_{i=1}^{n} \left[1 + i\left(\frac{2}{n}\right)\right]\left(\frac{2}{n}\right)$.
(e) Complete the following table.

n	5	10	50	100
$s(n)$				
$S(n)$				

(f) Show that $\lim_{n\to\infty} s(n) = \lim_{n\to\infty} S(n) = 4$.

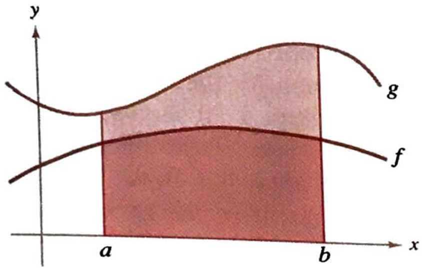


FIGURE 5.21

If f and g are continuous on the closed interval $[a, b]$ and $0 \le f(x) \le g(x)$ for $a \le x \le b$, then the following properties are true. First, the area of the region bounded by the graph of f and the x-axis (between a and b) must be nonnegative; second, this area must be less than or equal to the area of the region bounded by the graph of g and the x-axis (between a and b), as shown in Figure 5.21. These two results are generalized in the following theorem. (A proof of this theorem is given in Appendix A.)

THEOREM 5.10
PRESERVATION OF INEQUALITY

1. If f is integrable and nonnegative on the closed interval $[a, b]$, then

$$0 \le \int_a^b f(x)\,dx.$$

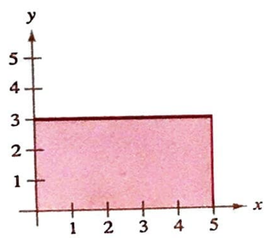
2. If f and g are integrable on the closed interval $[a, b]$ and $f(x) \le g(x)$ for every x in $[a, b]$, then

$$\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx.$$

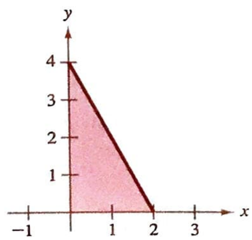
EXERCISES for Section 5.3

In Exercises 1–10, set up a definite integral that yields the area of the given region. (Do not evaluate the integral.)

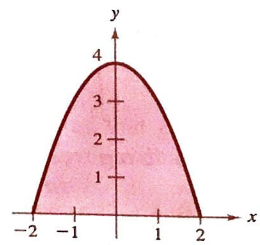
1. $f(x) = 3$



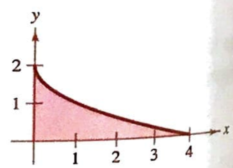
2. $f(x) = 4 - 2x$



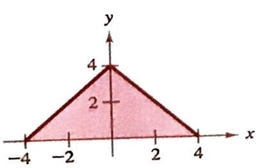
5. $f(x) = 4 - x^2$



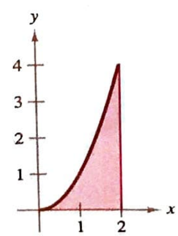
6. $f(y) = (y - 2)^2$



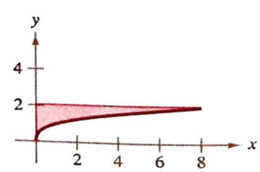
3. $f(x) = 4 - |x|$



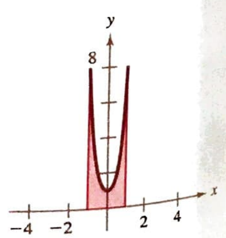
4. $f(x) = x^2$



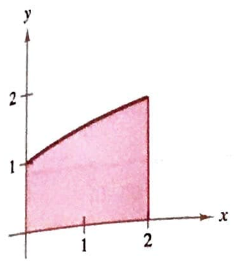
7. $g(y) = y^3$



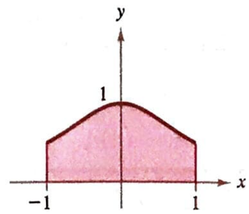
8. $f(x) = (x^2 + 1)^3$



9. $f(x) = \sqrt{x} + 1$



10. $f(x) = \dfrac{1}{x^2 + 1}$



In Exercises 11–20, sketch the region whose area is indicated by the given definite integral. Then use a geometric formula to evaluate the integral.

11. $\displaystyle\int_0^3 4\,dx$

12. $\displaystyle\int_{-a}^{a} 4\,dx$

13. $\displaystyle\int_0^4 x\,dx$

14. $\displaystyle\int_0^4 \frac{x}{2}\,dx$

15. $\displaystyle\int_0^2 (2x + 5)\,dx$

16. $\displaystyle\int_0^5 (5 - x)\,dx$

17. $\displaystyle\int_{-1}^{1} (1 - |x|)\,dx$

18. $\displaystyle\int_{-a}^{a} (a - |x|)\,dx$

19. $\displaystyle\int_{-3}^{3} \sqrt{9 - x^2}\,dx$

20. $\displaystyle\int_{-r}^{r} \sqrt{r^2 - x^2}\,dx$

21. Given $\int_0^5 f(x)\,dx = 10$ and $\int_5^7 f(x)\,dx = 3$, find the following.

(a) $\displaystyle\int_0^7 f(x)\,dx$ (b) $\displaystyle\int_5^0 f(x)\,dx$

(c) $\displaystyle\int_5^5 f(x)\,dx$ (d) $\displaystyle\int_0^5 3f(x)\,dx$

22. Given $\int_0^3 f(x)\,dx = 4$ and $\int_3^6 f(x)\,dx = -1$, find the following.

(a) $\displaystyle\int_0^6 f(x)\,dx$ (b) $\displaystyle\int_6^3 f(x)\,dx$

(c) $\displaystyle\int_4^4 f(x)\,dx$ (d) $\displaystyle\int_3^6 -5f(x)\,dx$

23. Given $\int_2^6 f(x)\,dx = 10$ and $\int_2^6 g(x)\,dx = -2$, find the following.

(a) $\displaystyle\int_2^6 [f(x) + g(x)]\,dx$ (b) $\displaystyle\int_2^6 [g(x) - f(x)]\,dx$

(c) $\displaystyle\int_2^6 2g(x)\,dx$ (d) $\displaystyle\int_2^6 3f(x)\,dx$

24. Given $\int_{-1}^{1} f(x)\,dx = 0$ and $\int_0^1 f(x)\,dx = 5$, find the following.

(a) $\displaystyle\int_{-1}^{0} f(x)\,dx$ (b) $\displaystyle\int_0^1 f(x)\,dx - \int_{-1}^{0} f(x)\,dx$

(c) $\displaystyle\int_{-1}^{1} 3f(x)\,dx$ (d) $\displaystyle\int_0^1 3f(x)\,dx$

In Exercises 25–30, evaluate the definite integral by the limit definition.

25. $\displaystyle\int_4^{10} 6\,dx$

26. $\displaystyle\int_{-2}^{3} x\,dx$

27. $\displaystyle\int_{-1}^{1} x^3\,dx$

28. $\displaystyle\int_0^1 x^3\,dx$

29. $\displaystyle\int_1^2 (x^2 + 1)\,dx$

30. $\displaystyle\int_1^2 4x^2\,dx$

In Exercises 31 and 32, use Example 1 as a model to evaluate the limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

over the region bounded by the graphs of the equations.

31. $f(x) = \sqrt{x}$, $y = 0$, $x = 0$, $x = 2$
[Hint: Let $c_i = 2i^2/n^2$.]

32. $f(x) = \sqrt[3]{x}$, $y = 0$, $x = 0$, $x = 1$
[Hint: Let $c_i = i^3/n^3$.]

In Exercises 33 and 34, express the given limit as a definite integral on the interval $[a, b]$ where c_i is any point in the ith subinterval.

	Limit	*Interval*
33.	$\displaystyle\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} (3c_i + 10)\Delta x_i$	$[-1, 5]$
34.	$\displaystyle\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} 6c_i(4 - c_i)^2\Delta x_i$	$[0, 4]$

Write a computer program or use a spreadsheet to approximate a definite integral by using the Riemann sum

$$\sum_{i=1}^{n} f(c_i)\Delta x_i$$

where the subintervals are of equal width. The output should give three approximations of the integral where c_i is the left-hand endpoint, midpoint, and right-hand endpoint of each subinterval. Denote these by $L(n)$, $M(n)$, and $R(n)$, respectively. In Exercises 35 and 36, use the program or spreadsheet to approximate the definite integral and complete the table.

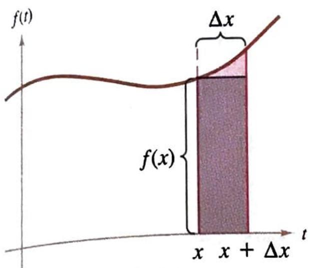
n	4	8	12	16	20
$L(n)$					
$M(n)$					
$R(n)$					

35. $\displaystyle\int_0^3 x\sqrt{3 - x}\,dx$

36. $\displaystyle\int_0^3 \frac{5}{x^2 + 1}\,dx$

Now, from the Mean Value Theorem for Integrals, we know there exists a number c in the interval $[x, x + \Delta x]$ such that the integral in the above expression is equal to $f(c)\Delta x$. Moreover, since $x \leq c \leq x + \Delta x$, it follows that $c \rightarrow x$ as $\Delta x \rightarrow 0$. Thus, we have

$$F'(x) = \lim_{\Delta x \to 0} \left[\frac{1}{\Delta x} f(c)\Delta x\right] = \lim_{\Delta x \to 0} f(c) = f(x).$$



$f(x)\Delta x \approx \int_x^{x+\Delta x} f(t)\,dt$

FIGURE 5.29

REMARK Using the area model for definite integrals, we can view the approximation

$$f(x)\Delta x \approx \int_x^{x+\Delta x} f(t)\,dt$$

as saying that the area of the rectangle of height $f(x)$ and width Δx is approximately equal to the area of the region lying between the graph of f and the x-axis on the interval $[x, x + \Delta x]$, as shown in Figure 5.29.

Note that the Second Fundamental Theorem of Calculus tells us that if a function is continuous, then we can be sure that it has an antiderivative. This antiderivative need not, however, be an elementary function. (Recall the discussion of elementary functions in Section 1.5.)

EXAMPLE 6 Applying the Second Fundamental Theorem of Calculus

Evaluate

$$\frac{d}{dx}\int_0^x \sqrt{t^2 + 1}\,dt.$$

SOLUTION

Note that $f(t) = \sqrt{t^2 + 1}$ is continuous on the entire real line. Thus, using the Second Fundamental Theorem of Calculus, we can write

$$\frac{d}{dx}\int_0^x \sqrt{t^2 + 1}\,dt = \sqrt{x^2 + 1}.$$

EXERCISES for Section 5.4

In Exercises 1–24, evaluate the definite integral.

1. $\int_0^1 2x\,dx$

2. $\int_2^7 3\,dv$

3. $\int_{-1}^0 (x - 2)\,dx$

4. $\int_2^5 (-3v + 4)\,dv$

5. $\int_{-1}^1 (t^2 - 2)\,dt$

6. $\int_0^3 (3x^2 + x - 2)\,dx$

7. $\int_0^1 (2t - 1)^2\,dt$

8. $\int_{-1}^1 (t^3 - 9t)\,dt$

9. $\int_1^2 \left(\frac{3}{x^2} - 1\right) dx$

10. $\int_0^1 (3x^3 - 9x + 7)\,dx$

11. $\int_1^2 (5x^4 + 5)\,dx$

12. $\int_{-3}^3 v^{1/3}\,dv$

13. $\int_{-1}^1 (\sqrt[3]{t} - 2)\,dt$

14. $\int_1^2 \sqrt{\frac{2}{x}}\,dx$

15. $\int_1^4 \frac{u - 2}{\sqrt{u}}\,du$

16. $\int_{-2}^{-1} \left(u - \frac{1}{u^2}\right) du$

17. $\int_0^1 \frac{x - \sqrt{x}}{3}\,dx$

18. $\int_0^2 (2 - t)\sqrt{t}\,dt$

19. $\int_{-1}^{0} (t^{1/3} - t^{2/3})\, dt$

20. $\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}}\, dx$

21. $\int_{-1}^{1} |x|\, dx$

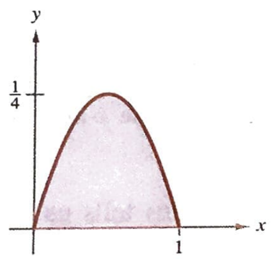
22. $\int_{0}^{3} |2x - 3|\, dx$

23. $\int_{0}^{4} |x^2 - 4x + 3|\, dx$

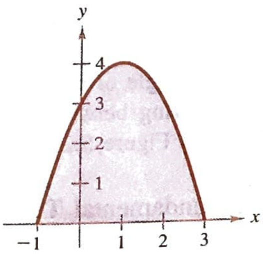
24. $\int_{-1}^{1} |x^3|\, dx$

In Exercises 25–30, determine the area of the indicated region.

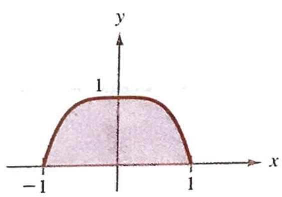
25. $y = x - x^2$



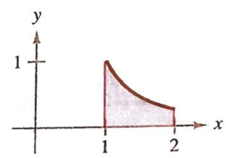
26. $y = -x^2 + 2x + 3$



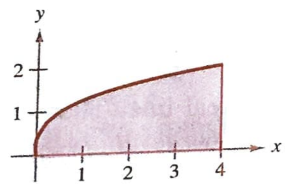
27. $y = 1 - x^4$



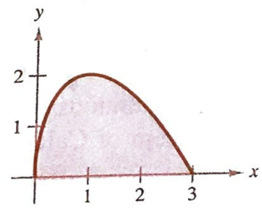
28. $y = \frac{1}{x^2}$



29. $y = \sqrt[3]{2x}$



30. $y = (3 - x)\sqrt{x}$



In Exercises 31–34, find the area of the region bounded by the graphs of the given equations.

31. $y = 3x^2 + 1,\ x = 0,\ x = 2,\ y = 0$

32. $y = 1 + \sqrt{x},\ x = 0,\ x = 4,\ y = 0$

33. $y = x^3 + x,\ x = 2,\ y = 0$

34. $y = -x^2 + 3x,\ y = 0$

In Exercises 35–38, find the values of c guaranteed by the *Mean Value Theorem for Integrals* for the given function over the specified interval.

	Function	*Interval*
35.	$f(x) = x^3$	$[0, 2]$
36.	$f(x) = \frac{9}{x^3}$	$[1, 3]$
37.	$f(x) = -x^2 + 4x$	$[0, 3]$
38.	$f(x) = \sqrt{x}$	$[1, 9]$

In Exercises 39–42, sketch the graph of the given function over the specified interval. Find the average value of the function over the interval and all values of x where the function equals its average value.

	Function	*Interval*
39.	$f(x) = 4 - x^2$	$[-2, 2]$
40.	$f(x) = \frac{x^2 + 1}{x^2}$	$\left[\frac{1}{2}, 2\right]$
41.	$f(x) = x - 2\sqrt{x}$	$[0, 4]$
42.	$f(x) = \frac{1}{(x - 3)^2}$	$[0, 2]$

In Exercises 43–48, (a) integrate to find F as a function of x and (b) demonstrate the Second Fundamental Theorem of Calculus by differentiating the result of part (a).

43. $F(x) = \int_{0}^{x} (t + 2)\, dt$

44. $F(x) = \int_{0}^{x} t(t^2 + 1)\, dt$

45. $F(x) = \int_{8}^{x} \sqrt[3]{t}\, dt$

46. $F(x) = \int_{4}^{x} \sqrt{t}\, dt$

47. $F(x) = \int_{1}^{x} \frac{1}{t^2}\, dt$

48. $F(x) = \int_{0}^{x} t^{3/2}\, dt$

In Exercises 49–52, use the Second Fundamental Theorem of Calculus to find $F'(x)$.

49. $F(x) = \int_{-2}^{x} (t^2 - 2t + 5)\, dt$

50. $F(x) = \int_{1}^{x} \sqrt[4]{t}\, dt$

51. $F(x) = \int_{-1}^{x} \sqrt{t^4 + 1}\, dt$

52. $F(x) = \int_{1}^{x} \frac{t^2}{t^2 + 1}\, dt$

53. The volume V in liters of air in the lungs during a 5-second respiratory cycle is approximated by the model

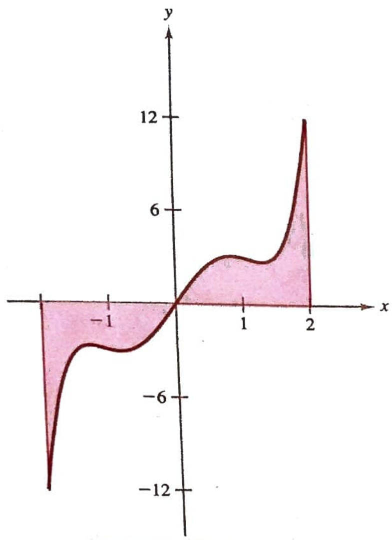
$$V = 0.1729t + 0.1522t^2 - 0.0374t^3$$

where t is the time in seconds. Approximate the average volume of air in the lungs during one cycle.

54. The velocity v of the flow of blood at a distance r from the central axis of an artery of radius R is given by

$$v = k(R^2 - r^2)$$

where k is the constant of proportionality. Find the average rate of flow of blood along a radius of the artery. (Use zero and R as the limits of integration.)



$f(x) = x^5 - 4x^3 + 6x$

FIGURE 5.32

EXAMPLE 10 Integration of an odd function

Evaluate

$$\int_{-2}^{2} (x^5 - 4x^3 + 6x)\,dx.$$

SOLUTION

By letting $f(x) = x^5 - 4x^3 + 6x$, we have

$$f(-x) = (-x)^5 - 4(-x)^3 + 6(-x) = -x^5 + 4x^3 - 6x = -f(x).$$

Thus, f is an odd function, and since $[-2, 2]$ is symmetric about the origin, we can apply Theorem 5.17 to conclude that

$$\int_{-2}^{2} (x^5 - 4x^3 + 6x)\,dx = 0.$$

REMARK From Figure 5.32, we see that the two regions on either side of the y-axis have the same area. However, since one lies below the x-axis and one lies above, integration produces a cancellation effect. (We will say more about finding the area of a region below the x-axis in Section 6.1.)

EXERCISES for Section 5.5

In Exercises 1–4, complete the table by identifying u and du for the given integral.

$\int f(g(x))g'(x)\,dx$	$u = g(x)$	$du = g'(x)\,dx$
1. $\int (5x^2 + 1)^2(10x)\,dx$		
2. $\int x^2\sqrt{x^3 + 1}\,dx$		
3. $\int \frac{x}{\sqrt{x^2 + 1}}\,dx$		
4. $\int (x^3 + 3)\,3x^2\,dx$		

In Exercises 5–28, evaluate the indefinite integral and check the result by differentiation.

5. $\int (1 + 2x)^4(2)\,dx$
6. $\int (x^2 - 1)^3(2x)\,dx$
7. $\int \sqrt{9 - x^2}(-2x)\,dx$
8. $\int (1 - 2x^2)^3(-4x)\,dx$
9. $\int x^2(x^3 - 1)^4\,dx$
10. $\int x(4x^2 + 3)^3\,dx$
11. $\int 5x\sqrt[3]{1 - x^2}\,dx$
12. $\int u^3\sqrt{u^4 + 2}\,du$
13. $\int \frac{x^2}{(1 + x^3)^2}\,dx$
14. $\int \frac{x^2}{(16 - x^3)^2}\,dx$
15. $\int \frac{4x}{\sqrt{16 - x^2}}\,dx$
16. $\int \frac{10x^2}{\sqrt{1 + x^3}}\,dx$
17. $\int \frac{x + 1}{(x^2 + 2x - 3)^2}\,dx$
18. $\int \frac{x - 4}{\sqrt{x^2 - 8x + 1}}\,dx$
19. $\int \left(1 + \frac{1}{t}\right)^3\left(\frac{1}{t^2}\right)\,dt$
20. $\int \frac{1}{(3x)^2}\,dx$
21. $\int \frac{1}{\sqrt{2x}}\,dx$
22. $\int \frac{1}{2\sqrt{x}}\,dx$
23. $\int \frac{x^2 + 3x + 7}{\sqrt{x}}\,dx$
24. $\int \frac{t + 2t^2}{\sqrt{t}}\,dt$
25. $\int t^2\left(t - \frac{2}{t}\right)\,dt$
26. $\int \left(\frac{t^3}{3} + \frac{1}{4t^2}\right)\,dt$
27. $\int (9 - y)\sqrt{y}\,dy$
28. $\int 2\pi y(8 - y^{3/2})\,dy$

In Exercises 29–38, evaluate the indefinite integral by the method shown in Example 5.

29. $\int x\sqrt{x + 2}\,dx$
30. $\int x\sqrt{2x + 1}\,dx$
31. $\int x^2\sqrt{1 - x}\,dx$
32. $\int x^3\sqrt{x + 2}\,dx$
33. $\int \frac{x^2 - 1}{\sqrt{2x - 1}}\,dx$
34. $\int \frac{2x - 1}{\sqrt{x + 3}}\,dx$

35. $\int \frac{-x}{(x+1) - \sqrt{x+1}}\,dx$

36. $\int t\sqrt[3]{t-4}\,dt$

37. $\int \frac{x}{\sqrt{2x+1}}\,dx$

38. $\int (x+1)\sqrt{2-x}\,dx$

In Exercises 39–50, evaluate the definite integral.

39. $\int_{-1}^{1} x(x^2+1)^3\,dx$

40. $\int_{0}^{1} x\sqrt{1-x^2}\,dx$

41. $\int_{0}^{4} \frac{1}{\sqrt{2x+1}}\,dx$

42. $\int_{0}^{2} \frac{x}{\sqrt{1+2x^2}}\,dx$

43. $\int_{1}^{9} \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$

44. $\int_{0}^{2} x\sqrt[3]{4+x^2}\,dx$

45. $\int_{1}^{2} (x-1)\sqrt{2-x}\,dx$

46. $\int_{0}^{4} \frac{x}{\sqrt{2x+1}}\,dx$

47. $\int_{3}^{7} x\sqrt{x-3}\,dx$

48. $\int_{0}^{1} \frac{1}{\sqrt{x}+\sqrt{x+1}}\,dx$

49. $\int_{0}^{7} x\sqrt[3]{x+1}\,dx$

50. $\int_{-2}^{6} x^2\sqrt[3]{x+2}\,dx$

51. Use the fact that

$$\int_0^2 x^2\,dx = \frac{8}{3}$$

to evaluate the following definite integrals without using the Fundamental Theorem of Calculus.

(a) $\int_{-2}^{0} x^2\,dx$ (b) $\int_{-2}^{2} x^2\,dx$

(c) $\int_{0}^{2} -x^2\,dx$ (d) $\int_{-2}^{0} 3x^2\,dx$

52. Find the equation of the function f whose graph passes through the point $\left(0, \frac{7}{3}\right)$ and whose derivative is $f'(x) = x\sqrt{1-x^2}$.

53. A lumber company is seeking a model that yields the average weight loss W per log as a function of the number of days of drying time t. The model is to be reliable up to 100 days after the log is cut. Based on the weight loss during the first 30 days, it was determined that

$$\frac{dW}{dt} = \frac{12}{\sqrt{16t+9}}.$$

(a) Find W as a function of t. Note that no weight loss occurs until the tree is cut.
(b) Find the total weight loss after 100 days.

54. The marginal cost for a certain commodity has been determined to be

$$\frac{dC}{dx} = \frac{12}{\sqrt[3]{12x+1}}.$$

(a) Find the cost function if $C = 100$ when $x = 13$.
(b) Graph the marginal cost function and the cost function on the same set of axes.

5.6 Numerical Integration

The Trapezoidal Rule ▪ Simpson's Rule

Occasionally, we encounter functions for which we cannot find antiderivatives. Of course, that may be due to a lack of cleverness on our part. On the other hand, some elementary functions simply do not possess antiderivatives that are elementary functions. For example, there is no elementary function that has either of the following functions as its derivative.

$$\sqrt[3]{x}\sqrt{1-x} \qquad \sqrt{1-x^3}$$

If we wish to evaluate a definite integral involving a function whose antiderivative we cannot find, then the Fundamental Theorem of Calculus cannot be applied, and we must resort to an approximation technique. We describe two such techniques in this section.

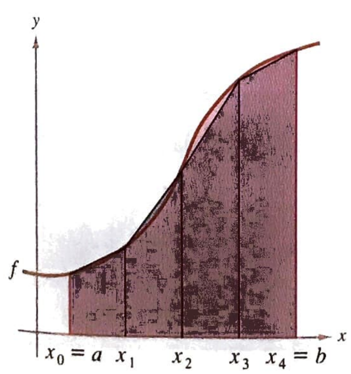


FIGURE 5.33

The Trapezoidal Rule

One way to approximate a definite integral is by the use of n trapezoids, as shown in Figure 5.33. In the development of this method, we assume that f is continuous and positive on the interval $[a, b]$, and thus the definite integral $\int_a^b f(x)\,dx$ represents the area of the region bounded by the graph of f and the x-axis, from $x = a$ to $x = b$.