The function $A(x) = x(100 - 2x) = -2x^2 + 100x$ gives the area of the pen, so we want to find the maximum value for $A(x)$. The vertex for the parabola is (h, k), where

$$h = -\frac{b}{2a} = \frac{-100}{2(-2)} = \frac{100}{4} = 25 \qquad A(x) = -2x^2 + 100x, \text{ so } a = -2, b = 100$$

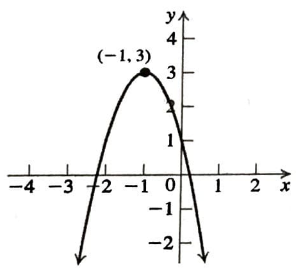
$$k = A(25) = -2(25)^2 + 100(25) = 1250. \qquad k = \text{maximum value}$$

The maximum area is 1250 square feet, which occurs when $x = h = 25$.
The length of the side perpendicular to the wall is 25 feet.
The length of the side parallel to the wall is $100 - 2(25) = 50$ feet.

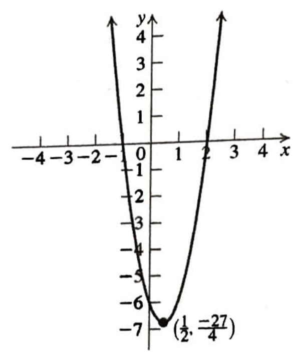
Practice Problem 7 A children's center receives a donation of 1000 ft of fence to enclose a rectangular playground. What is the maximum area that can be enclosed? What are the playground's dimensions?

Answers to Practice Problems

1. $y = 3(x - 1)^2 - 5$; f has minimum value -5.
2. The graph is a parabola. $a = -2, h = -1, k = 3$. It opens downward. Vertex $(-1, 3)$. It has maximum value 3; x-intercepts: $\pm\sqrt{\frac{3}{2}} - 1$; y-intercept: 1



3. The graph is a parabola. It opens upward. Vertex $\left(\frac{1}{2}, \frac{-27}{4}\right)$; It has a minimum value $-\frac{27}{4}$; x-intercepts: $-1, 2$; y-intercept: -6



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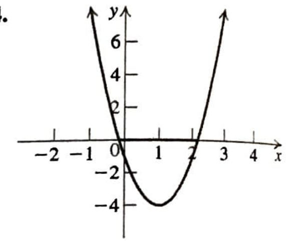


$$\left[\frac{3 - 2\sqrt{3}}{3}, \frac{3 + 2\sqrt{3}}{3}\right]$$

5. $a < 0, b < 0, c > 0$ **6. a.** $h(t) = -16t^2 + 96t + 100$ (feet) **b.** 3 seconds **7.** Maximum area is 62,500 sq ft. The playground is a square of side length 250 feet on each side.

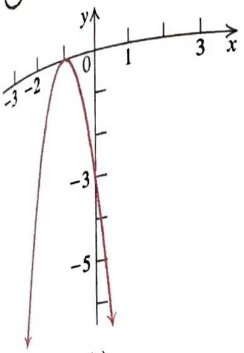
SECTION 3.1 Exercises

Basic Concepts and Skills

1. A point where the axis of the parabola meets the parabola is called the ________.
2. The vertex of the graph of $f(x) = -2(x + 3)^2 - 5$ is ________.
3. **True or False.** The graph of the function in Exercise 2 opens down.
4. **True or False.** The graph of $f(x) = -2 - x + x^2$ opens down.
5. The x-coordinate of the vertex of the parabola of Exercise 4 is ________.
6. **True or False.** The y-coordinate of the vertex of the parabola $f(x) = x^2 - 2x + 5$ is $f(1)$.
7. **True or False.** If $a > 0$, then the minimum value of $f(x) = ax^2 + bx + c$ is $f\left(-\frac{b}{2a}\right)$.
8. **True or False.** If $a < 0$, then $f(x) = ax^2 + bx + c$ has no minimum value.

In Exercises 9–16, match each quadratic function with its graph.

9. $y = -\frac{1}{3}x^2$
10. $y = -3x^2$
11. $y = -3(x + 1)^2$
12. $y = 2(x + 1)^2$
13. $y = (x - 1)^2 + 2$
14. $y = (x - 1)^2 - 3$
15. $y = 2(x + 1)^2 - 3$
16. $y = -3(x + 1)^2 + 2$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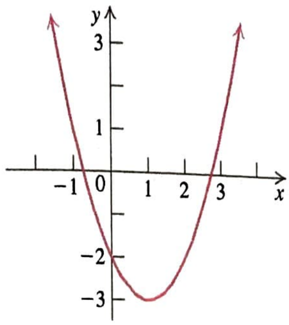
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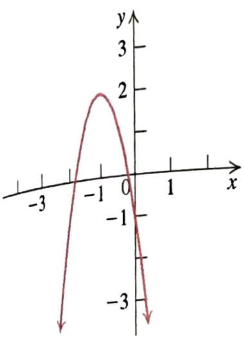
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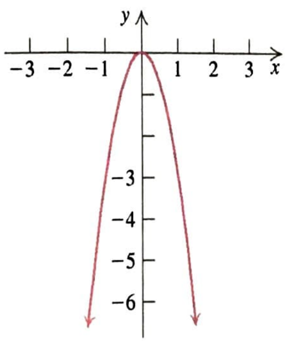
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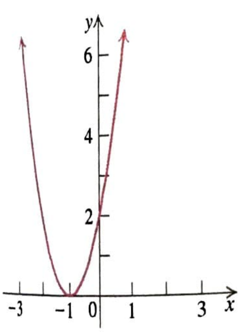
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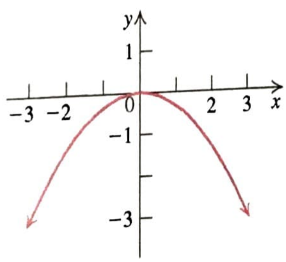
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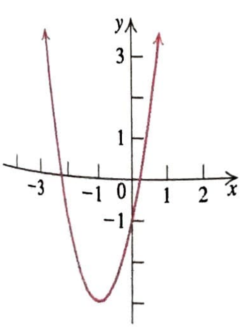
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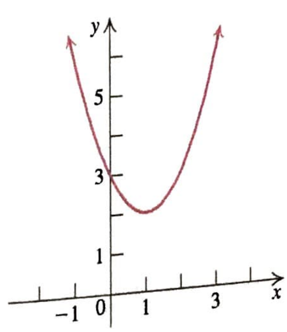

(h)

In Exercises 17–20, find a quadratic function of the form $y = ax^2$ that passes through the given point.

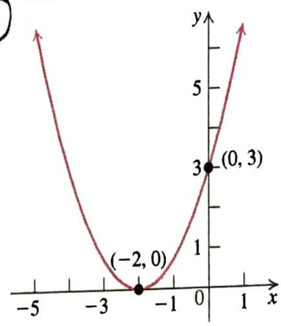
17. $(2, -8)$
18. $(-3, 3)$
19. $(2, 20)$
20. $(-3, -6)$

In Exercises 21–30, find the quadratic function $y = f(x)$ that has the given vertex and whose graph passes through the given point. Write the function in standard form.

21. Vertex $(0, 0)$; passing through $(-2, 8)$
22. Vertex $(2, 0)$; passing through $(1, 3)$
23. Vertex $(-3, 0)$; passing through $(-5, -4)$
24. Vertex $(0, 1)$; passing through $(-1, 0)$
25. Vertex $(2, 5)$; passing through $(3, 7)$
26. Vertex $(-3, 4)$; passing through $(0, 0)$
27. Vertex $(2, -3)$; passing through $(-5, 8)$
28. Vertex $(-3, -2)$; passing through $(0, -8)$
29. Vertex $\left(\frac{1}{2}, \frac{1}{2}\right)$; passing through $\left(\frac{3}{4}, -\frac{1}{4}\right)$
30. Vertex $\left(-\frac{3}{2}, -\frac{5}{2}\right)$; passing through $\left(1, \frac{55}{8}\right)$

In Exercises 31–34, the graph of a quadratic function $y = f(x)$ is given. Find the standard form of the function.

31.



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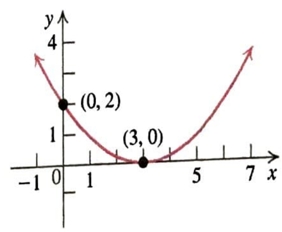
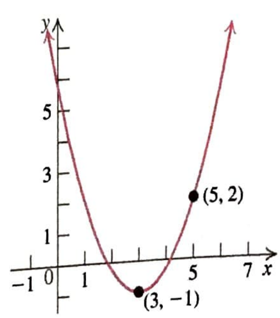
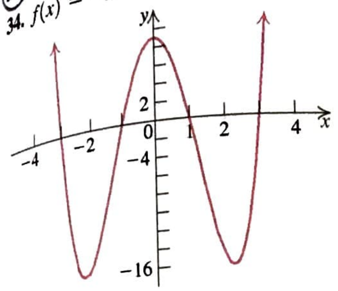


33.



In Exercises 29–34, match the polynomial function with its graph. Use the leading-term test and the y-intercept.

29. $f(x) = -x^4 + 3x^2 + 4$
30. $f(x) = x^6 - 7x^4 + 7x^2 + 15$
31. $f(x) = x^4 - 10x^2 + 9$
32. $f(x) = x^3 + x^2 - 17x + 15$
33. $f(x) = x^3 + 6x^2 + 12x + 8$
34. $f(x) = -x^3 - 2x^2 + 11x + 12$



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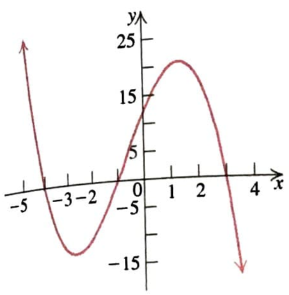

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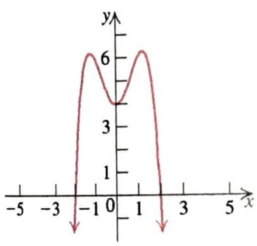

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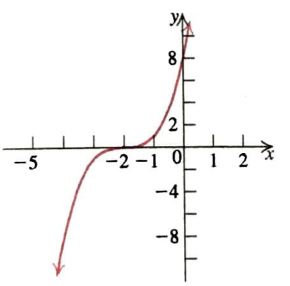

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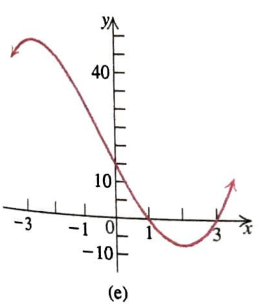

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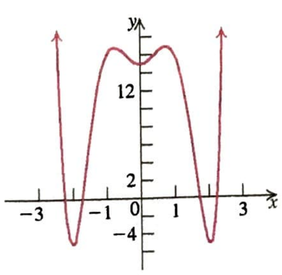


(f)

In Exercises 35–42, (a) find the real zeros of each polynomial function and state the multiplicity for each zero and (b) state whether the graph crosses or touches but does not cross the x-axis at each x-intercept. (c) What is the maximum possible number of turning points?

35. $f(x) = 5(x - 1)(x + 5)$
36. $f(x) = 3(x + 2)^2(x - 3)$
37. $f(x) = (x + 1)^2(x - 1)^3$
38. $f(x) = 2(x + 3)^2(x + 1)(x - 6)$
39. $f(x) = -5\left(x + \frac{2}{3}\right)\left(x - \frac{1}{2}\right)^2$
40. $f(x) = x^3 - 6x^2 + 8x$
41. $f(x) = -x^4 + 6x^3 - 9x^2$
42. $f(x) = x^4 - 5x^2 - 36$

In Exercises 43–46, use the Intermediate Value Theorem to show that each polynomial $P(x)$ has a real zero in the specified interval. Approximate this zero to two decimal places.

43. $P(x) = x^4 - x^3 - 10$; $[2, 3]$
44. $P(x) = x^4 - x^2 - 2x - 5$; $[1, 2]$
45. $P(x) = x^5 - 9x^2 - 15$; $[2, 3]$
46. $P(x) = x^5 + 5x^4 + 8x^3 + 4x^2 - x - 5$; $[0, 1]$

In Exercises 47–60, for each polynomial function f,

a. Describe the end behavior of f.
b. Find the real zeros of f. Determine whether the graph of f crosses or touches but does not cross the x-axis at each x-intercept.
c. Use the zeros of f and test numbers to find the intervals over which the graph of f is above or below the x-axis.
d. Determine the y-intercept.
e. Find any symmetries of the graph of the function.
f. Determine the maximum possible number of turning points.
g. Sketch the graph of f.

47. $f(x) = x^2 + 3$
48. $f(x) = x^2 - 4x + 4$
49. $f(x) = x^2 + 4x - 21$
50. $f(x) = -x^2 + 4x + 12$
51. $f(x) = -2x^2(x + 1)$
52. $f(x) = x - x^3$
53. $f(x) = x^2(x - 1)^2$
54. $f(x) = x^2(x + 1)(x - 2)$
55. $f(x) = (x - 1)^2(x + 3)(x - 4)$
56. $f(x) = (x + 1)^2(x^2 + 1)$
57. $f(x) = -x^2(x^2 - 1)(x + 1)$
58. $f(x) = -x^2(x^2 - 4)(x + 2)$
59. $f(x) = x(x + 1)(x - 1)(x + 2)$
60. $f(x) = x^2(x^2 + 1)(x - 2)$

Answers to Practice Problems

1. Quotient: $3x + 4$; remainder $= -2x + 3$
2. $x^2 + x + 3 + \frac{2x - 3}{x^2 - x + 3}$
3. $2x^2 - x - 3 + \frac{-4}{x - 3}$
4. $2x^2 - 5x - 3 + \frac{2}{x + 3}$
5. 4
6. 32
7. $\left\{-2, -\frac{2}{3}, 3\right\}$
8. 18

SECTION 3.3 Exercises

Basic Concepts and Skills

1. In the division $\frac{x^4 - 2x^3 + 5x^2 - 2x + 1}{x^2 - 2x + 3} = x^2 + 2 + \frac{2x - 5}{x^2 - 2x + 3}$, the dividend is ____________, the divisor is ____________, the quotient is ____________, and the remainder is ____________.
2. If $P(x)$, $Q(x)$, and $F(x)$ are polynomials and $F(x) = P(x) \cdot Q(x)$, then the factors of $F(x)$ are ____________ and ____________.
3. The Remainder Theorem states that if a polynomial $F(x)$ is divided by $(x - a)$, then the remainder $R =$ ____________.
4. The Factor Theorem states that $(x - a)$ is a factor of a polynomial $F(x)$ if and only if ____________ $= 0$.
5. **True or False.** You cannot use synthetic division to divide a polynomial by $x^2 - 1$.
6. **True or False.** When a polynomial of degree $n + 1$ is divided by a polynomial of degree n, the remainder is a constant polynomial.

In Exercises 7–14, use long division to find the quotient and the remainder.

7. $\frac{6x^2 - x - 2}{2x + 1}$
8. $\frac{4x^3 - 2x^2 + x - 3}{2x - 3}$
9. $\frac{3x^4 - 6x^2 + 3x - 7}{x + 1}$
10. $\frac{x^6 + 5x^3 + 7x + 3}{x^2 + 2}$
11. $\frac{4x^3 - 4x^2 - 9x + 5}{2x^2 - x - 5}$
12. $\frac{y^5 + 3y^4 - 6y^2 + 2y - 7}{y^2 + 2y - 3}$
13. $\frac{z^4 - 2z^2 + 1}{z^2 - 2z + 1}$
14. $\frac{6x^4 + 13x - 11x^3 - 10 - x^2}{3x^2 - 5 - x}$

In Exercises 15–30, use synthetic division to find the quotient and the remainder when the first polynomial is divided by the second polynomial.

15. $x^3 - x^2 - 7x + 2; x - 1$
16. $2x^3 - 3x^2 - x + 2; x + 2$
17. $x^3 + 4x^2 - 7x - 10; x + 2$
18. $x^3 + x^2 - 13x + 2; x - 3$
19. $x^4 - 3x^3 + 2x^2 + 4x + 5; x - 2$
20. $x^4 - 5x^3 - 3x^2 + 10; x - 1$
21. $2x^3 + 4x^2 - 3x + 1; x - \frac{1}{2}$
22. $3x^3 + 8x^2 + x + 1; x - \frac{1}{3}$
23. $2x^3 - 5x^2 + 3x + 2; x + \frac{1}{2}$
24. $3x^3 - 2x^2 + 8x + 2; x + \frac{1}{3}$
25. $x^5 + x^4 - 7x^3 + 2x^2 + x - 1; x - 1$
26. $2x^5 + 4x^4 - 3x^3 - 7x^2 + 3x - 2; x + 2$
27. $x^5 + 1; x + 1$
28. $x^5 + 1; x - 1$
29. $x^6 + 2x^4 - x^3 + 5; x + 1$
30. $3x^5; x - 1$

In Exercises 31–34, use synthetic division and the Remainder Theorem to find each function value. Check your answer by evaluating the function at the given x-value.

31. $f(x) = x^3 + 3x^2 + 1$
    a. $f(1)$ b. $f(-1)$ c. $f\left(\frac{1}{2}\right)$ d. $f(10)$
32. $g(x) = 2x^3 - 3x^2 + 1$
    a. $g(-2)$ b. $g(-1)$ c. $g\left(-\frac{1}{2}\right)$ d. $g(7)$
33. $h(x) = x^4 + 5x^3 - 3x^2 - 20$
    a. $h(1)$ b. $h(-1)$ c. $h(-2)$ d. $h(2)$
34. $f(x) = x^4 + 0.5x^3 - 0.3x^2 - 20$
    a. $f(0.1)$ b. $f(0.5)$ c. $f(1.7)$ d. $f(-2.3)$

17. As $x \to -2^+, f(x) \to$ ________.

18. As $x \to -2^-, f(x) \to$ ________.

19. As $x \to \infty, f(x) \to$ ________.

20. As $x \to -\infty, f(x) \to$ ________.

21. The domain of $f(x)$ is ________.

22. There are ________ vertical asymptotes.

23. The equations of the vertical asymptotes of the graph are ________ and ________.

24. The equation of the horizontal asymptote of the graph is ________.

In Exercises 25–32, graph each rational function as a translation of $f(x) = \frac{1}{x}$. Identify the vertical and horizontal asymptotes and state the domain and rage.

25. $f(x) = \frac{3}{x-4}$ 26. $g(x) = \frac{-4}{x+3}$

27. $f(x) = \frac{-x}{3x+1}$ 28. $g(x) = \frac{2x}{4x-2}$

29. $g(x) = \frac{-3x+2}{x+2}$ 30. $f(x) = \frac{-x+1}{x-3}$

31. $g(x) = \frac{5x-3}{x-4}$ 32. $f(x) = \frac{2x+12}{x+5}$

In Exercises 33–42, find the vertical asymptotes, if any, of the graph of each rational function.

33. $f(x) = \frac{x}{x-1}$ 34. $f(x) = \frac{x+3}{x-2}$

35. $g(x) = \frac{(x+1)(2x-2)}{(x-3)(x+4)}$

36. $g(x) = \frac{(2x-1)(x+2)}{(2x+3)(3x-4)}$

37. $h(x) = \frac{x^2-1}{x^2+x-6}$ 38. $h(x) = \frac{x^2-4}{3x^2+x-4}$

39. $f(x) = \frac{x^2-6x+8}{x^2-x-12}$ 40. $f(x) = \frac{x^2-9}{x^3-4x}$

41. $g(x) = \frac{2x+1}{x^2+x+1}$ 42. $g(x) = \frac{x^2-36}{x^2+5x+9}$

In Exercises 43–50, find the horizontal asymptote, if any, of the graph of each rational function.

43. $f(x) = \frac{x+1}{x^2+5}$ 44. $f(x) = \frac{2x-1}{x^2-4}$

45. $g(x) = \frac{2x-3}{3x+5}$ 46. $g(x) = \frac{3x+4}{-4x+5}$

47. $h(x) = \frac{x^2-49}{x+7}$ 48. $h(x) = \frac{x+3}{x^2-9}$

49. $f(x) = \frac{2x^2-3x+7}{3x^3+5x+11}$ 50. $f(x) = \frac{3x^3+2}{x^2+5x+11}$

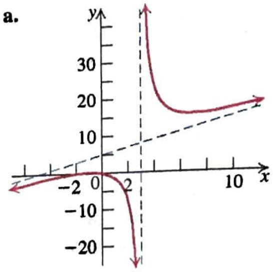
In Exercises 51–56, match the rational function with its graph.

51. $f(x) = \frac{2}{x-3}$ 52. $f(x) = \frac{x-2}{x+3}$

53. $f(x) = \frac{1}{x^2-2x}$ 54. $f(x) = \frac{x}{x^2+1}$

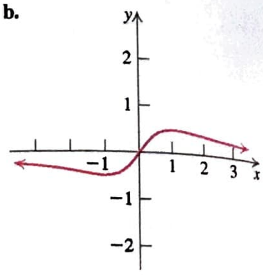
55. $f(x) = \frac{x^2+2x}{x-3}$ 56. $f(x) = \frac{x^2}{x^2-4}$

a.



(a)

b.



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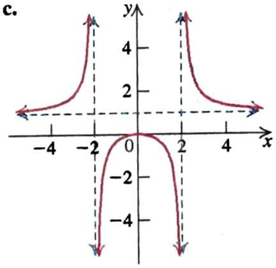

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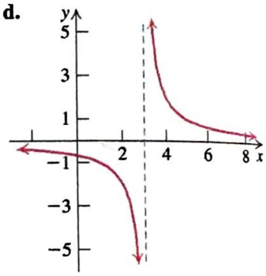
d.



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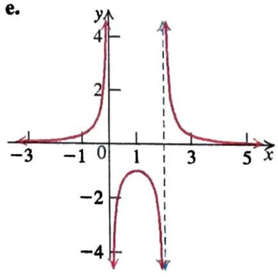

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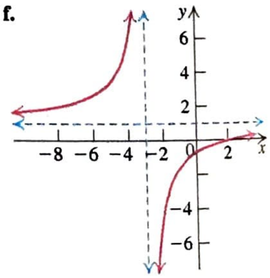
f.



(f)

In Exercises 57–72, use the six-step procedure on pages 363–364 to graph each rational function.

57. $f(x) = \frac{2x}{x-3}$ 58. $f(x) = \frac{-x}{x-1}$

59. $f(x) = \frac{x}{x^2-4}$ 60. $f(x) = \frac{x}{1-x^2}$

61. $h(x) = \frac{-2x^2}{x^2-9}$ 62. $h(x) = \frac{4-x^2}{x^2}$

63. $f(x) = \frac{2}{x^2-2}$

65. $g(x) = \frac{x+1}{(x-2)(x+3}$

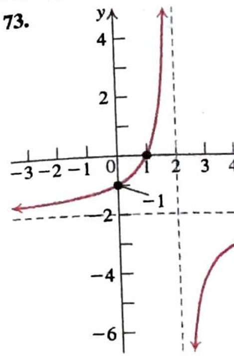
67. $h(x) = \frac{x^2}{x^2+1}$

69. $f(x) = \frac{x^3-4x}{x^3-9x}$

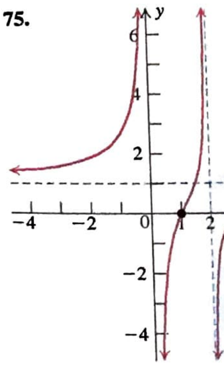
71. $g(x) = \frac{(x-2)^2}{x-2}$

In Exercises 73–76, find a[n]
having the given asymptot[es]

73.



75.



In Exercises 77–84, fin[d]
graph of each rational

77. $f(x) = \frac{2x^2+1}{x}$

79. $g(x) = \frac{x^3-1}{x^2}$

81. $h(x) = \frac{x^2-x+}{x+1}$

83. $f(x) = \frac{x^3-2x^2}{x^2-1}$