## SECTION 2.1 Exercises

### Basic Concepts and Skills

1. A point with a negative first coordinate and a positive second coordinate lies in the ________ quadrant.
2. Any point on the $x$-axis has second coordinate ________.
3. The distance between the points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ is given by the formula $d(P, Q) =$ ________.
4. The coordinates of the midpoint $M = (x, y)$ of the line segment joining $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ are given by $(x, y) =$ ________.
5. **True or False.** For any points $(x_1, y_1)$ and $(x_2, y_2)$ $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
6. **True or False.** The point $(7, -4)$ is 4 units to the right and 2 units below the point $(3, 2)$.
7. Plot and label each of the given points in a Cartesian coordinate plane and state the quadrant, if any, in which each point is located. $(2, 2)$, $(3, -1)$, $(-1, 0)$, $(-2, -5)$, $(0, 0)$, $(-7, 4)$, $(0, 3)$, $(-4, 2)$
8. a. Write the coordinates of any five points on the $x$-axis. What do these points have in common?
   b. Plot the points $(-2, 1)$, $(0, 1)$, $(0.5, 1)$, $(1, 1)$, and $(2, 1)$. Describe the set of all points of the form $(x, 1)$, where $x$ is a real number.
9. a. If the $x$-coordinate of a point is 0, where does that point lie?
   b. Plot the points $(-1, 1)$, $(-1, 1.5)$, $(-1, 2)$, $(-1, 3)$, and $(-1, 4)$. Describe the set of all points of the form $(-1, y)$, where $y$ is a real number.
10. What figure is formed by the set of all points in a Cartesian coordinate plane that have
    a. $x$-coordinate equal to $-3$?
    b. $y$-coordinate equal to 4?
11. Let $P(x, y)$ be a point in a coordinate plane.
    a. If the point $P(x, y)$ lies above the $x$-axis, what must be true of $y$?
    b. If the point $P(x, y)$ lies below the $x$-axis, what must be true of $y$?
    c. If the point $P(x, y)$ lies to the left of the $y$-axis, what must be true of $x$?
    d. If the point $P(x, y)$ lies to the right of the $y$-axis, what must be true of $x$?
12. Let $P(x, y)$ be a point in a coordinate plane. In which quadrant does $P$ lie
    a. if $x$ and $y$ are both negative?
    b. if $x$ and $y$ are both positive?
    c. if $x$ is positive and $y$ is negative?
    d. if $x$ is negative and $y$ is positive?

**In Exercises 13–22, find (a) the distance between $P$ and $Q$ and (b) the coordinates of the midpoint of the line segment $PQ$.**

13. $P(2, 1)$, $Q(2, 5)$
14. $P(3, 5)$, $Q(-2, 5)$
15. $P(-1, -5)$, $Q(2, -3)$
16. $P(-4, 1)$, $Q(-7, -9)$
17. $P(-1, 1.5)$, $Q(3, -6.5)$
18. $P(0.5, 0.5)$, $Q(1, -1)$
19. $P(\sqrt{2}, 4)$, $Q(\sqrt{2}, 5)$
20. $P(v - w, t)$, $Q(v + w, t)$
21. $P(t, k)$, $Q(k, t)$
22. $P(m, n)$, $Q(-n, -m)$

**In Exercises 23–30, determine whether the given points are collinear. Points are *collinear* if they can be labeled $P$, $Q$, and $R$ so that $d(P, Q) + d(Q, R) = d(P, R)$.**

23. $(0, 0)$, $(1, 2)$, $(-1, -2)$
24. $(3, 4)$, $(0, 0)$, $(-3, -4)$
25. $(4, -2)$, $(-2, 8)$, $(1, 3)$
26. $(9, 6)$, $(0, -3)$, $(3, 1)$
27. $(-1, 4)$, $(3, 0)$, $(11, -8)$
28. $(-2, 3)$, $(3, 1)$, $(2, -1)$
29. $(4, -4)$, $(15, 1)$, $(1, 2)$
30. $(1, 7)$, $(-7, 8)$, $(-3, 7.5)$
31. Find the coordinates of the points that divide the line segment joining the points $P = (-4, 0)$ and $Q = (0, 8)$ into four equal parts.
32. Repeat Exercise 31 with $P = (-8, 4)$ and $Q = (16, -12)$.

**In Exercises 33–40, identify the triangle $PQR$ as *isosceles* (two sides of equal length), *equilateral* (three sides of equal length), or a *scalene* triangle (three sides of different lengths).**

33. $P(-5, 5)$, $Q(-1, 4)$, $R(-4, 1)$
34. $P(3, 2)$, $Q(6, 6)$, $R(-1, 5)$
35. $P(-4, 8)$, $Q(0, 7)$, $R(-3, 5)$
36. $P(6, 6)$, $Q(-1, -1)$, $R(-5, 3)$
37. $P(0, -1)$, $Q(9, -9)$, $R(5, 1)$
38. $P(-4, 4)$, $Q(4, 5)$, $R(0, -2)$
39. $P(1, -1)$, $Q(-1, 1)$, $R(-\sqrt{3}, -\sqrt{3})$
40. $P(-0.5, -1)$, $Q(-1.5, 1)$, $R\left(\sqrt{3} - 1, \frac{\sqrt{3}}{2}\right)$
41. Show that the points $P(7, -12)$, $Q(-1, 3)$, $R(14, 11)$, and $S(22, -4)$ are the vertices of a square. Find the length of the diagonals.

## SECTION 2.2 Exercises

### Basic Concepts and Skills

1. The graph of an equation in two variables such as $x$ and $y$ is the set of all ordered pairs $(a, b)$ ____________.
2. If $(-2, 4)$ is a point on a graph that is symmetric with respect to the $y$-axis, then the point ____________ is also on the graph.
3. If $(0, -5)$ is a point of a graph, then $-5$ is a(n) ____________ intercept of the graph.
4. An equation in standard form of a circle with center $(1, 0)$ and radius 2 is ____________.
5. **True or False.** The graph of the equation $3x^2 - 2x + y + 3 = 0$ is a circle.
6. **True or False.** If a graph is symmetric about the $x$-axis, then it must have at least one $x$-intercept.

**In Exercises 7–12, determine whether the given points are on the graph of the equation.**

| Equation | Points |
|---|---|
| 7. $y = x - 1$ | $(-3, -4), (1, 0), (4, 3), (2, 3)$ |
| 8. $2y = 3x + 5$ | $(-1, 1), (0, 2), \left(-\frac{5}{3}, 0\right), (1, 4)$ |
| 9. $y = \sqrt{x + 1}$ | $(3, 2), (0, 1), (8, -3), (8, 3)$ |
| 10. $y = \frac{1}{x}$ | $\left(-3, \frac{1}{3}\right), (1, 1), (0, 0), \left(2, \frac{1}{2}\right)$ |
| 11. $x^2 - y^2 = 1$ | $(1, 0), (0, -1), (2, \sqrt{3}), (2, -\sqrt{3})$ |
| 12. $y^2 = x$ | $(1, -1), (1, 1)\ (0, 0), (2, -\sqrt{2})$ |

**In Exercises 13–24, find**

a. the intercepts.
b. symmetries, about the $x$-axis, the $y$-axis, and the origin.

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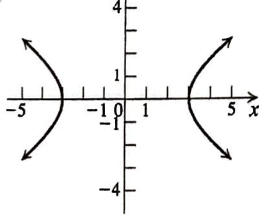

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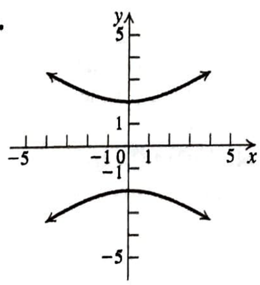

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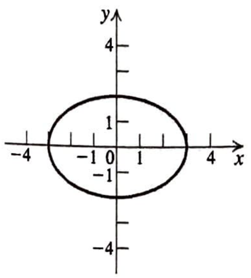

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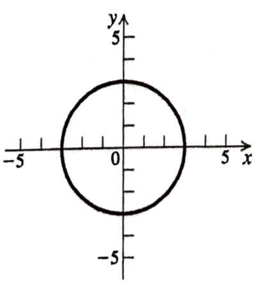

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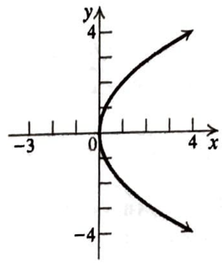

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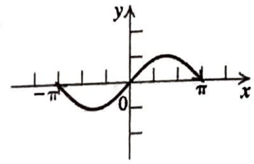

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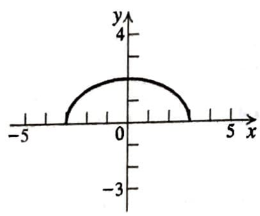

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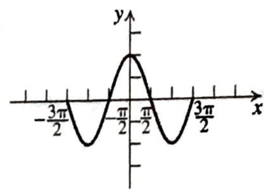

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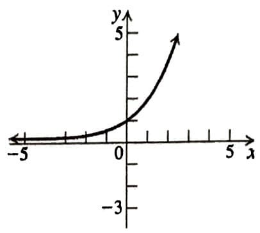

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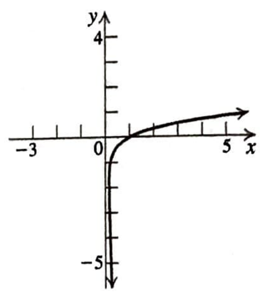

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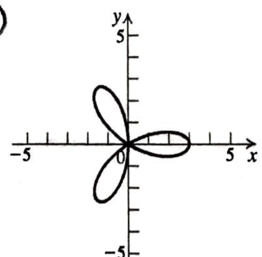

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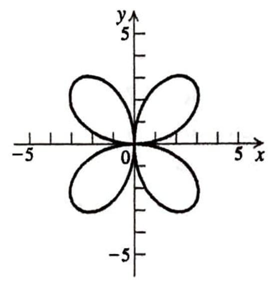


**In Exercises 25–46, graph each equation by plotting points. Let $x = -3, -2, -1, 0, 1, 2,$ and $3$ where applicable.**

25. $y = x + 1$
26. $y = x - 1$
27. $y = 2x$
28. $y = \frac{1}{2}x$
29. $y = x^2$
30. $y = -x^2$
31. $y = |x|$
32. $y = |x + 1|$
33. $y = |x| + 1$
34. $y = -|x| + 1$
35. $y = 4 - x^2$
36. $y = x^2 - 4$
37. $y = \sqrt{9 - x^2}$
38. $y = -\sqrt{9 - x^2}$
39. $y = x^3$
40. $y = -x^3$
41. $y^3 = x$
42. $y^3 = -x$

43. $x = |y|$

44. $|x| = |y|$

45. $y = |2 - x|$

46. $|x| + |y| = 1$

**In Exercises 47–56, find the x- and y-intercepts of the graph of the equation.**

47. $3x + 4y = 12$

48. $\frac{x}{5} + \frac{y}{3} = 1$

49. $y = x^2 - 6x + 8$

50. $x = y^2 - 5y + 6$

51. $x^2 + y^2 = 4$

52. $y = \sqrt{9 - x^2}$

53. $y = \sqrt{x^2 - 1}$

54. $xy = 1$

55. $y = x^2 + x + 1$

56. $x^3 + 3xy^2 + y^3 = 1$

**In Exercises 57–66, test each equation for symmetry with respect to the x-axis, the y-axis, and the origin.**

57. $y = x^2 + 1$

58. $x = y^2 + 1$

59. $y = x^3 + x$

60. $y = 2x^3 - x$

61. $y = 5x^4 + 2x^2$

62. $y = -3x^6 + 2x^4 + x^2$

63. $y = -3x^5 + 2x^3$

64. $y = 2x^2 - |x|$

65. $x^2y^2 + 2xy = 1$

66. $x^2 + y^2 = 16$

**In Exercises 67–70, specify the center and the radius of each circle.**

67. $(x - 2)^2 + (y - 3)^2 = 36$

68. $(x + 1)^2 + (y - 3)^2 = 16$

69. $(x + 2)^2 + (y + 3)^2 = 11$

70. $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{3}{4}$

**In Exercises 71–80, find the standard form of the equation of a circle that satisfies the given conditions. Graph each equation.**

71. Center $(0, 1)$; radius 2

72. Center $(1, 0)$; radius 1

73. Center $(-1, 2)$; radius $\sqrt{2}$

74. Center $(-2, -3)$; radius $\sqrt{7}$

75. Center $(3, -4)$; passing through the point $(-1, 5)$

76. Center $(-1, 1)$; passing through the point $(2, 5)$

77. Center $(1, 2)$; touching the x-axis

78. Center $(1, 2)$; touching the y-axis

79. Diameter with endpoints $(7, 4)$ and $(-3, 6)$

80. Diameter with endpoints $(2, -3)$ and $(8, 5)$

**In Exercises 81–86, find**

**a. the center and radius of each circle.**

**b. the x- and y-intercepts of the graph of each circle.**

81. $x^2 + y^2 - 2x - 2y - 4 = 0$

82. $x^2 + y^2 - 4x - 2y - 15 = 0$

83. $2x^2 + 2y^2 + 4y = 0$

84. $3x^2 + 3y^2 + 6x = 0$

85. $x^2 + y^2 - x = 0$

86. $x^2 + y^2 + 1 = 0$

## Applying the Concepts

**In Exercises 87–89, a graph is described *geometrically* as the path of a point $P(x, y)$ on the graph. Find an equation for the graph described.**

87. **Geometry.** $P(x, y)$ is on the graph if and only if the distance from $P(x, y)$ to the x-axis is equal to its distance to the y-axis.

88. **Geometry.** $P(x, y)$ is the same distance from the two points $(1, 2)$ and $(3, -4)$.

89. **Geometry.** $P(x, y)$ is the same distance from the point $(2, 0)$ and the y-axis.

90. **Spending on prescription drugs.** Americans spent \$136 billion on prescription drugs in 2000 and \$234 billion in 2008. Assuming that this trend continued, estimate the amount spent on prescription drugs during 2002, 2004, and 2006. (*Source: U.S. Census Bureau.*)

91. **Corporate profits.** The equation $P = -0.5t^2 - 3t + 8$ describes the monthly profits (in millions of dollars) of ABCD Corp. for the year 2012, with $t = 0$ representing July 2012.
   a. How much profit did the corporation make in March 2012?
   b. How much profit did the corporation make in October 2012?
   c. Sketch the graph of the equation.
   d. Find the t-intercepts. What do they represent?
   e. Find the P-intercept. What does it represent?

92. **Female students in colleges.** The equation

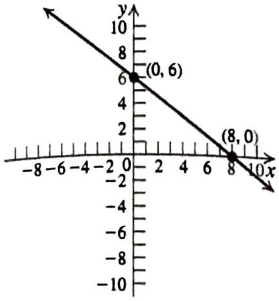
$$P = -0.002t^2 + 0.51t + 17.5$$

models the approximate number (in millions) of female college students in the United States for the academic years 2005–2009, with $t = 0$ representing 2005.
   a. Sketch the graph of the equation.
   b. Find the P-intercept. What does it represent? (*Source: Statistical Abstracts of the United States.*)

93. **Motion.** An object is thrown up from the top of a building that is 320 feet high. The equation $y = -16t^2 + 128t + 320$ gives the object's height (in feet) above the ground at any time $t$ (in seconds) after the object is thrown.
   a. What is the height of the object after 0, 1, 2, 3, 4, 5, and 6 seconds?
   b. Sketch the graph of the equation $y = -16t^2 + 128t + 320$.
   c. What part of the graph represents the physical aspects of the problem?
   d. What are the intercepts of this graph, and what do they mean?

94. **Diving for treasure.** A treasure-hunting team of divers is placed in a computer-controlled diving cage. The equation $d = \frac{40}{3}t - \frac{2}{9}t^2$ describes the depth $d$ (in feet) that the cage will descend in $t$ minutes.
   a. Sketch the graph of the equation $d = \frac{40}{3}t - \frac{2}{9}t^2$.
   b. What part of the graph represents the physical aspects of the problem?
   c. What is the total time of the entire diving experiment?

7.




8. Between 1.77 and 1.79 meters

9. a. $4x - 3y + 23 = 0$

b. $5x - 4y - 31 = 0$

10. 10.22 million

## SECTION 2.3 Exercises

### Basic Concepts and Skills

1. The number that measures the "steepness" of a line is called the ________.

2. In the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, the number $y_2 - y_1$ is called the ________.

3. If two lines have the same slope, they are ________.

4. The $y$-intercept of the line with equation $y = 3x - 5$ is ________.

5. A line perpendicular to a line with slope $-\frac{1}{5}$ has slope ________.

6. True or False. A line with undefined slope is perpendicular to every line with slope 0.

7. True or False. If $(1, 2)$ is a point on a line with slope 4, then so is the point $(2, 4)$.

8. True or False. If the points $(1, 3)$ and $(2, y)$ are on a line with negative slope, then $y > 3$.

**In Exercises 9–16, find the slope of the line through the given pair of points. Without plotting any points, state whether the line is rising, falling, horizontal, or vertical.**

9. $(1, 3), (4, 7)$
10. $(0, 4), (2, 0)$
11. $(3, -2), (6, -2)$
12. $(-3, 7), (-3, -4)$
13. $(0.5, 2), (3, -3.5)$
14. $(3, -2), (2, -3)$
15. $(\sqrt{2}, 1), (1 + \sqrt{2}, 5)$
16. $(1 - \sqrt{3}, 0), (1 + \sqrt{3}, 3\sqrt{3})$

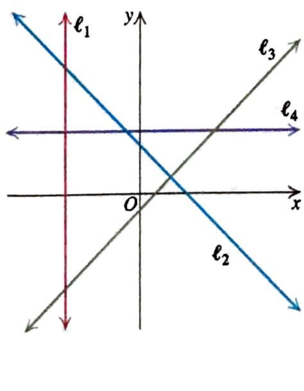
17. In the figure, identify the line with the given slope $m$.

a. $m = 1$

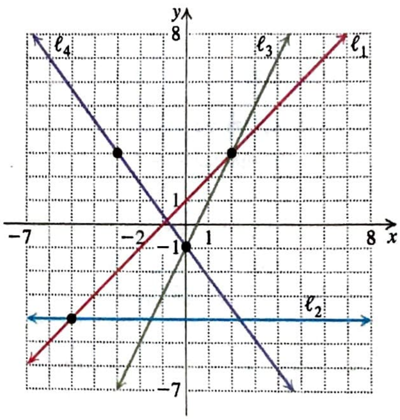
b. $m = -1$

c. $m = 0$

d. $m$ is undefined.




18. Find the slope of each line. (The scale is the same on both axes.)




**In Exercises 19 and 20, write an equation whose graph is described.**

19. a. Graph is the $x$-axis.

b. Graph is the $y$-axis.

20. a. Graph consists of all points with a $y$-coordinate of $-4$.

b. Graph consists of all points with an $x$-coordinate of 5.

**In Exercises 21–26, find an equation in slope–intercept form of the line that passes through the given point and has slope $m$. Also sketch the graph of the line by locating the second point with the rise-and-run method.**

21. $(0, 4); m = \frac{1}{2}$

22. $(0, 4); m = -\frac{1}{2}$

23. $(2, 1); m = -\frac{3}{2}$

24. $(-1, 0); m = \frac{2}{5}$

25. $(5, -4); m = 0$

26. $(5, -4)$; $m$ is undefined.

**In Exercises 27–48, use the given conditions to find an equation in slope–intercept form of each nonvertical line. Write vertical lines in the form $x = h$.**

27. Passing through $(0, 1)$ and $(1, 0)$
28. Passing through $(0, 1)$ and $(1, 3)$
29. Passing through $(-1, 3)$ and $(3, 3)$
30. Passing through $(-5, 1)$ and $(2, 7)$
31. Passing through $(-2, -1)$ and $(1, 1)$
32. Passing through $(-1, -3)$ and $(6, -9)$
33. Passing through $\left(\frac{1}{2}, \frac{1}{4}\right)$ and $(0, 2)$
34. Passing through $(4, -7)$ and $(4, 3)$
35. A vertical line through $(5, 1.7)$
36. A horizontal line through $(1.4, 1.5)$
37. A horizontal line through $(0, 0)$
38. A vertical line through $(0, 0)$
39. $m = 0$; $y$-intercept $= 14$
40. $m = 2$; $y$-intercept $= 5$
41. $m = -\frac{2}{3}$; $y$-intercept $= -4$
42. $m = -6$; $y$-intercept $= -3$
43. $x$-intercept $= -3$; $y$-intercept $= 4$
44. $x$-intercept $= -5$; $y$-intercept $= -2$
45. Parallel to $y = 5$; passing through $(4, 7)$
46. Parallel to $x = 5$; passing through $(4, 7)$
47. Perpendicular to $x = -4$; passing through $(-3, -5)$
48. Perpendicular to $y = -4$; passing through $(-3, -5)$
49. Let $\ell_1$ be a line with slope $m = -2$. Determine whether the given line $\ell_2$ is parallel to $\ell_1$, perpendicular to $\ell_1$, or neither.
    - **a.** $\ell_2$ is the line through the points $(1, 5)$ and $(3, 1)$.
    - **b.** $\ell_2$ is the line through the points $(7, 3)$ and $(5, 4)$.
    - **c.** $\ell_2$ is the line through the points $(2, 3)$ and $(4, 4)$.
50. Find equations of the lines $\ell_1$, $\ell_2$, $\ell_3$ and $\ell_4$ of Exercise 18.

**In Exercises 51–58, find the slope and intercepts from the equation of the line. Sketch the graph of each equation.**

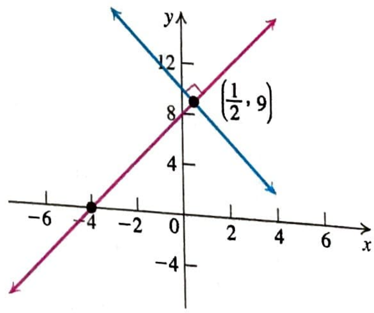
51. $x + 2y - 4 = 0$
52. $x = 3y - 9$
53. $3x - 2y + 6 = 0$
54. $2x = -4y + 15$
55. $x - 5 = 0$
56. $2y + 5 = 0$
57. $x = 0$
58. $y = 0$
59. Show that an equation of a line through the points $(a, 0)$ and $(0, b)$, with $a$ and $b$ nonzero, can be written in the form

$$\frac{x}{a} + \frac{y}{b} = 1.$$

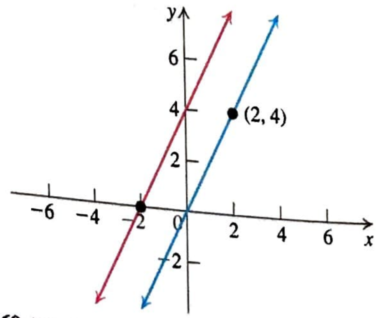
This form is called the **two-intercept form** of the equation of a line.

**In Exercises 60–64, use the two-intercept form of the equation of a line given in Exercise 59.**

60. Find an equation of the line whose $x$-intercept is 4 and $y$-intercept is 3.
61. Find the $x$- and $y$-intercepts of the graph of the equation $2x + 3y = 6$.
62. Repeat Exercise 61 for the equation $3x - 4y + 12 = 0$.
63. Find an equation of the line that has equal intercepts and passes through the point $(3, -5)$.
64. Find an equation of the line making intercepts on the axes equal in magnitude but opposite in sign and passing through the point $(-5, -8)$.
65. Find an equation of the line passing through the points $(2, 4)$ and $(7, 9)$. Use the equation to show that the three points $(2, 4)$, $(7, 9)$, and $(-1, 1)$ are on the same line.
66. Use the technique of Exercise 65 to check whether the points $(7, 2)$, $(2, -3)$, and $(5, 1)$ are on the same line.
67. Write the slope–intercept form of the equation of the blue line shown in the figure that is perpendicular to the red line with $x$-intercept $-4$.




68. Write the point–slope form of the equation of the red line shown in the figure parallel to the blue line through the origin.




**In Exercises 69–76, determine whether each pair of lines are parallel, perpendicular, or neither.**

69. $x = -1$ and $2x + 7 = 9$
70. $x = 0$ and $y = 0$
71. $2x + 3y = 7$ and $y = 2$
72. $y = 3x + 1$ and $6y + 2x = 0$
73. $x = 4y + 8$ and $y = -4x + 1$

74. $4x + 3y = 1$ and $3 + y = 2x$

75. $3x + 8y = 7$ and $5x - 7y = 0$

76. $10x + 2y = 3$ and $y + 1 = -5x$

**In Exercises 77–86, find the equation of the line in slope–intercept form satisfying the given conditions.**

77. a. Passing through $(2, -3)$ and parallel to a line with slope 3
    b. Passing through $(-1, 2)$ and perpendicular to a line with slope $-\frac{1}{2}$

78. a. Passing through $(-2, -5)$ and parallel to the line joining the points $(1, -2)$ and $(-3, 2)$
    b. Passing through $(1, -2)$ and perpendicular to the line joining the points $(-3, 2)$ and $(-4, -1)$

79. Parallel to $x + y = 1$; passing through $(1, 1)$

80. Parallel to $y = 6x + 5$; $y$-intercept of $-2$

81. Perpendicular to $3x - 9y = 18$; passing through $(-2, 4)$

82. Perpendicular to $-2x + y = 14$; passing through $(0, 0)$

83. Perpendicular to $y = 6x + 5$; $y$-intercept of 4

84. Parallel to $-2x + 3y - 7 = 0$; passing through $(1, 0)$

85. Perpendicular bisector of the line segment with endpoints $A = (2, 3)$ and $B(-1, 5)$

86. Show that $y = x$ is the perpendicular bisector of the line segment with endpoints $A(a, b)$ and $B(b, a)$.

## Applying the Concepts

87. **Jet takeoff.** Upon takeoff, a jet climbs to 4 miles as it passes over 40 miles of land below it. Find the slope of the jet's climb.

88. **Road gradient.** Driving down the Smoky Mountains in Tennessee, Samantha descends 2000 feet in elevation while moving 4 miles horizontally away from a high point on the straight road. Find the slope (gradient) of the road (1 mile = 5280 feet).

**In Exercises 89–94, write an equation of a line in slope–intercept form. To describe this situation, interpret (a) the variables $x$ and $y$ and (b) the meaning of the slope and the $y$-intercept.**

89. **Christmas savings account.** You currently have \$130 in a Christmas savings account. You deposit \$7 per week into this account. (Ignore interest.)

90. **Golf club charges.** Your golf club charges a yearly membership fee of \$1000 and \$35 per session of golf.

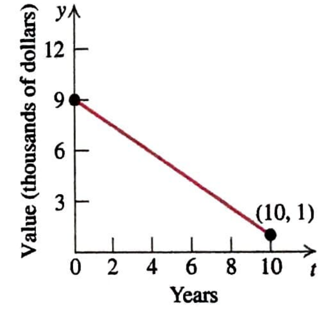
91. **Wages.** Judy's job at a warehouse pays \$11 per hour up to 40 hours per week. If she works more than 40 hours in a week, she is paid 1.5 times her usual wage for each hour after 40. [*Hint:* You will need two equations.]

92. **Paying off a refrigerator.** You bought a new refrigerator for \$700. You made a down payment of \$100 and promised to pay \$15 per month. (Ignore interest.)

93. **Converting currency.** In February 2013, according to the International Monetary Fund, 1 U.S. dollar equaled 53.87 Indian rupees. Convert currency from rupees to dollars.

94. **Life expectancy.** In 2010, the life expectancy of a female born in the United States was 80.8 years and was increasing at a rate of 0.17 years per year. (Assume that this rate of increase remains constant.)

95. **Machine Value.** The graph represents the value of a piece of factory machinery over a ten-year period. Each point $(t, y)$ gives the machine's value $y$ after $t$ years.
    a. What does the $y$-intercept represent?
    b. What is the value of the machine after ten years?
    c. How much does the machine depreciate in value each year?
    d. Write a linear equation for the value of the machine after $t$ years, for $0 \le t \le 10$.
    e. What does the slope found in part d represent?



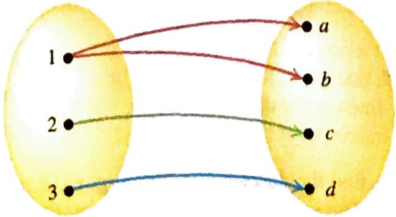

96. **Depreciating a tractor.** The value $v$ of a tractor purchased for \$14,000 and depreciated linearly at the rate of \$1400 per year is given by $v = -1400t + 14{,}000$, where $t$ represents the number of years since the purchase. Find the value of the tractor after (a) two years and (b) six years. When will the tractor have no value?

97. **Playing for a concert.** A famous band is considering playing a concert and charging \$40,000 plus \$5 per person attending the concert. Write an equation relating the income $y$ of the band to the number $x$ of people attending the concert.

98. **Car rental.** The U Drive car rental agency charges \$30.00 per day and \$0.25 per mile.
    a. Find an equation relating the cost $C$ of renting a car from U Drive for a one-day trip covering $x$ miles.
    b. Draw the graph of the equation in part (a).
    c. How much does it cost to rent the car for one day covering 60 miles?
    d. How many miles were driven if the one-day rental cost was \$47.75?

99. **Female prisoners.** The number of females in a state's prisons rose from 2425 in 2005 to 4026 in 2011.
    a. Find a linear equation relating the number $y$ of women prisoners to the year $t$. (Take $t = 0$ to represent 2005.)
    b. Draw the graph of the equation from part (a).
    c. How many women prisoners were there in 2008?
    d. Predict the number of women prisoners in 2017.

12.




13.

| $x$ | 0 | 3 | 8 | 0 | 3 | 8 |
|---|---|---|---|---|---|---|
| $y$ | −1 | −2 | −3 | 1 | 2 | 2 |

14.

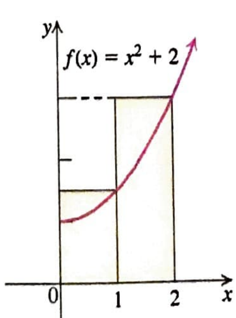
| $x$ | −3 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | −8 | 0 | 1 | 0 | −3 | −8 |

**In Exercises 15–28, determine whether each equation defines $y$ as a function of $x$.**

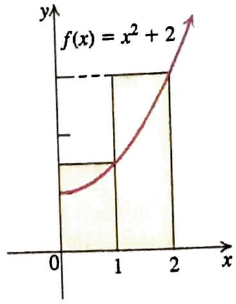
15. $x + y = 2$
16. $x = y - 1$
17. $y = \frac{1}{x}$
18. $xy = -1$
19. $y^2 = x^2$
20. $x = |y|$
21. $y = \frac{1}{\sqrt{2x - 5}}$
22. $y = \frac{1}{\sqrt{x^2 - 1}}$
23. $2 - y = 3x$
24. $3x - 5y = 15$
25. $x + y^2 = 8$
26. $x = y^2$
27. $x^2 + y^3 = 5$
28. $x + y^3 = 8$

**In Exercises 29–32, let $f(x) = x^2 - 3x + 1, g(x) = \frac{2}{\sqrt{x}}$, and $h(x) = \sqrt{2 - x}$.**

29. Find $f(0), g(0), h(0), f(a)$, and $f(-x)$.
30. Find $f(1), g(1), h(1), g(a)$, and $g(x^2)$.
31. Find $f(-1), g(-1), h(-1), h(c)$, and $h(-x)$.
32. Find $f(4), g(4), h(4), g(2 + k)$, and $f(a + k)$.
33. Let $f(x) = \frac{2x}{\sqrt{4 - x^2}}$. Find each function value.
   a. $f(0)$
   b. $f(1)$
   c. $f(2)$
   d. $f(-2)$
   e. $f(-x)$
34. Let $g(x) = 2x + \sqrt{x^2 - 4}$. Find each function value.
   a. $g(0)$
   b. $g(1)$
   c. $g(2)$
   d. $g(-3)$
   e. $g(-x)$
35. In the figure for Exercise 35, find the sum of the areas of the shaded rectangles.




36. Repeat Exercise 35 in the figure for Exercise 36.

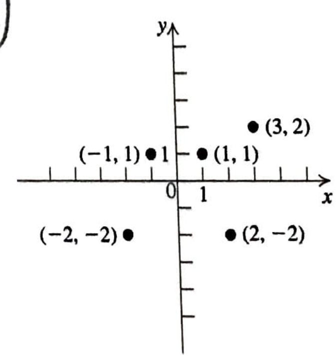



**In Exercises 37–48, find the domain of each function.**

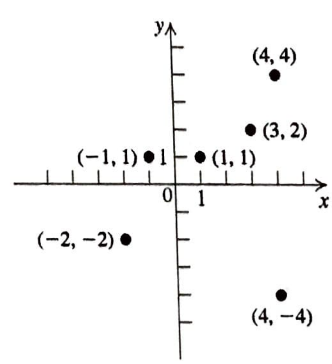
37. $f(x) = -8x + 7$
38. $f(x) = 2x^2 - 11$
39. $f(x) = \frac{1}{x - 9}$
40. $f(x) = \frac{1}{x + 9}$
41. $h(x) = \frac{2x}{x^2 - 1}$
42. $h(x) = \frac{x - 3}{x^2 - 4}$
43. $G(x) = \frac{\sqrt{x - 3}}{x + 2}$
44. $f(x) = \frac{3}{\sqrt{4 - x}}$
45. $F(x) = \frac{x + 4}{x^2 + 3x + 2}$
46. $F(x) = \frac{1 - x}{x^2 + 5x + 6}$
47. $g(x) = \frac{\sqrt{x^2 + 1}}{x}$
48. $g(x) = \frac{1}{x^2 + 1}$

**In Exercises 49–54, use the vertical-line test to determine whether the given graph represents a function.**

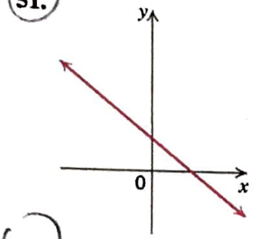
49.



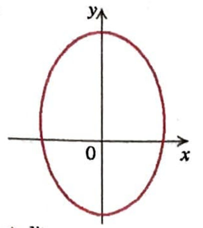

50.



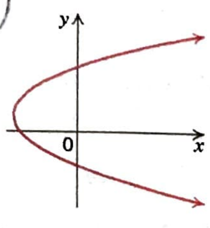
**51.**



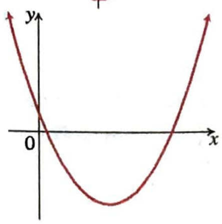

**52.**




**53.**

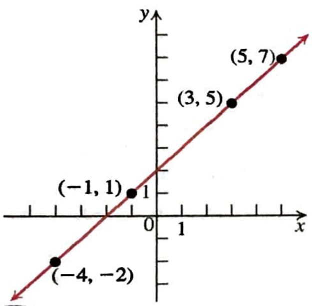



**54.**

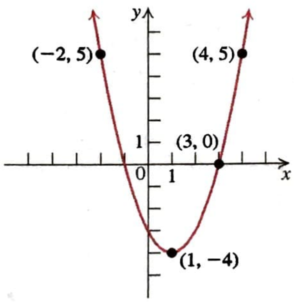



**In Exercises 55–58, the graph of a function is given. Find the indicated function values.**

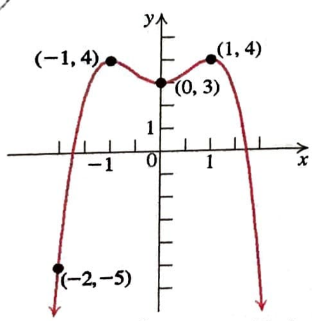
**55.** $f(-4)$, $f(-1)$, $f(3)$, $f(5)$



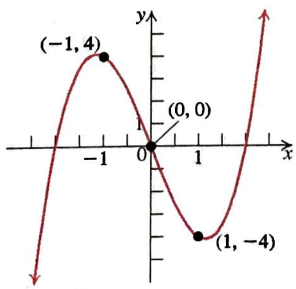

**56.** $g(-2)$, $g(1)$, $g(3)$, $g(4)$




**57.** $h(-2)$, $h(-1)$, $h(0)$, $h(1)$




**58.** $f(-1)$, $f(0)$, $f(1)$




**59.** Let $h(x) = x^2 - x + 1$. Find $x$ such that $(x, 7)$ is on the graph of $h$.

**60.** Let $H(x) = x^2 + x + 8$. Find $x$ such that $(x, 7)$ is on the graph of $H$.

**61.** Let $f(x) = -2(x + 1)^2 + 7$.

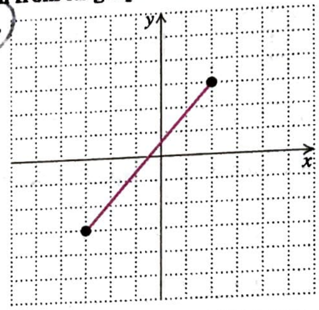
**a.** Is $(1, 1)$ a point of the graph of $f$?
**b.** Find all $x$ such that $(x, 1)$ is on the graph of $f$.
**c.** Find all $y$-intercepts of the graph of $f$.
**d.** Find all $x$-intercepts of the graph of $f$.

**62.** Let $f(x) = -3x^2 - 12x$.

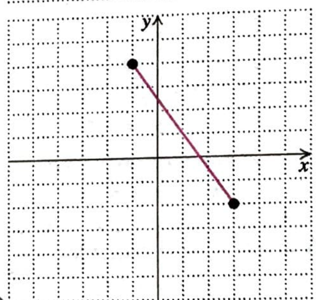
**a.** Is $(-2, 10)$ a point of the graph of $f$?
**b.** Find $x$ such that $(x, 12)$ is on the graph of $g$.
**c.** Find the $y$-intercept of the graph of $f$.
**d.** Find all $x$-intercepts of the graph of $f$.

**In Exercises 63–70, find the domain and the range of each function from its graph. The axes are marked in one-unit intervals.**

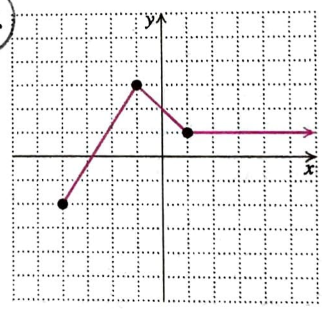
**63.**



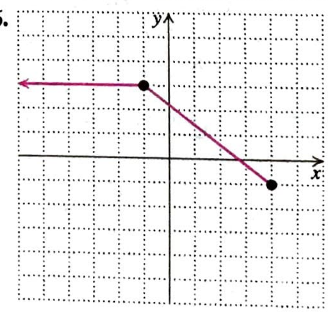

**64.**




**65.**

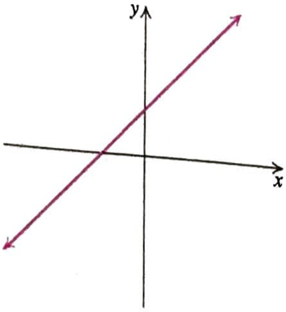



**66.**

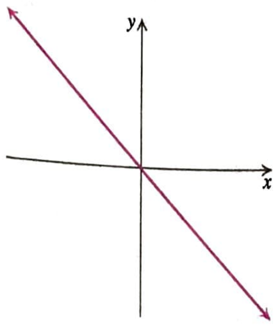


**In Exercises 7–14, the graph of a function is given. For each function, determine the intervals over which the function is increasing, decreasing, or constant.**

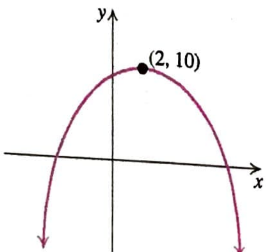
**7.**



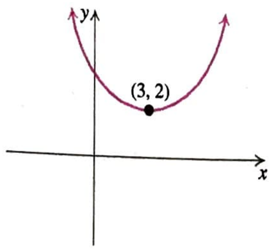

**8.**



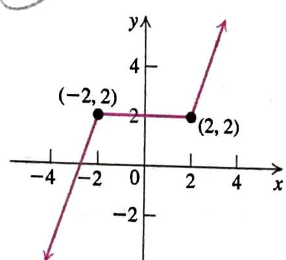

**9.**



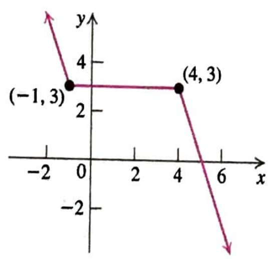

**10.**



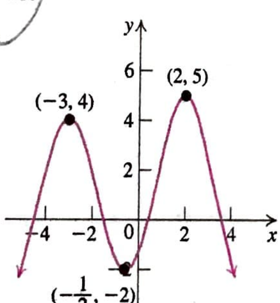

**11.**



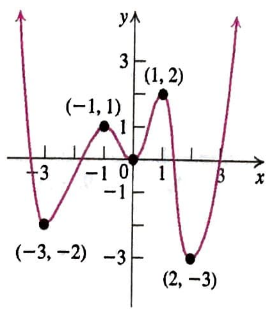

**12.**




**13.**




**14.**




**In Exercises 15–22, locate relative maximum and relative minimum points on the graph. State whether each relative extremum point is a turning point.**

**15.** The graph of Exercise 7

**16.** The graph of Exercise 8

**17.** The graph of Exercise 9

**18.** The graph of Exercise 10

**19.** The graph of Exercise 11

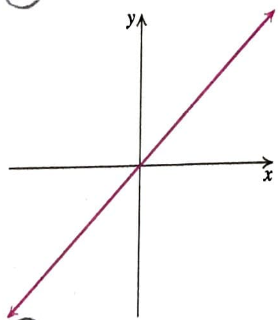
**20.** The graph of Exercise 12

**21.** The graph of Exercise 13

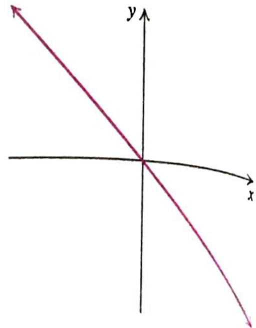
**22.** The graph of Exercise 14

**In Exercises 23–32, the graph of a function is given. State whether the function is odd, even, or neither.**

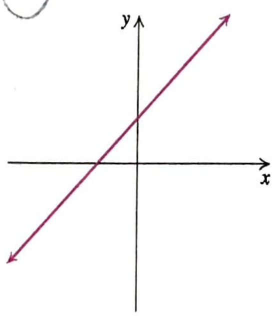
**23.**



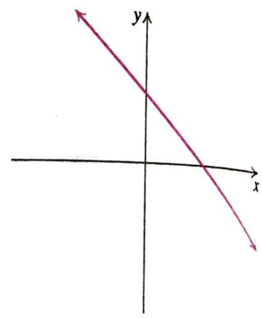

**24.**



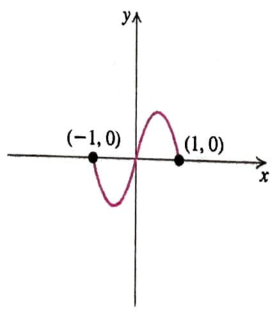

**25.**



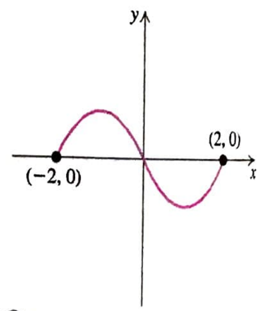

**26.**



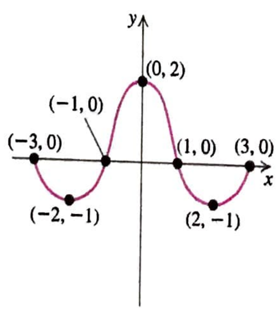

**27.**



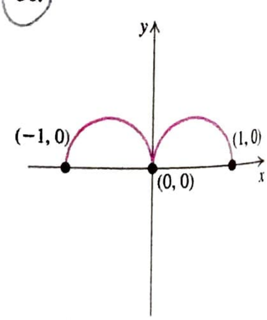

**28.**




**29.**



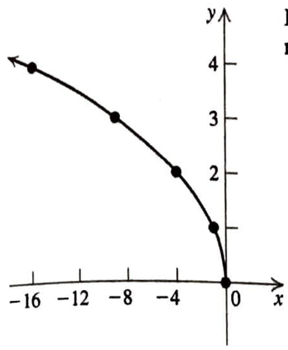

**30.**



## Answers to Practice Problems

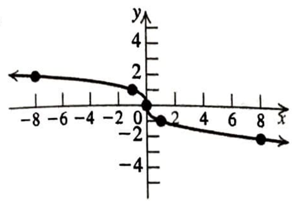
**1.** $g(x) = 2x + 6$ **2.** 15.524 m

**3.**




Domain: $(-\infty, 0]$; range: $[0, \infty)$

**4.**

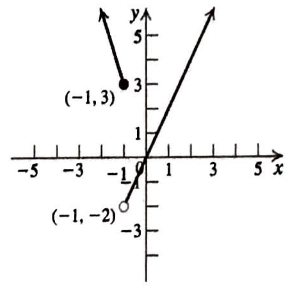



Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

**5.** $f(-2) = 4, f(3) = 6$

**6. a.** $f(x) = \begin{cases} 50 + 4(x - 55), & 56 \le x < 75 \\ 200 + 5(x - 75), & x \ge 75 \end{cases}$ **b.** \$70

**c.** \$275 **7.** $f(x) = -3x, x \le -1$ $\quad f(x) = 2x, x > -1$




**8.** $f(x) = \begin{cases} -x - 4 & \text{if } x \le -2 \\ \frac{4}{5}x - \frac{2}{5} & \text{if } -2 < x < 3 \\ -2x + 8 & \text{if } x \ge 3 \end{cases}$

**9.** $[\![-3.4]\!] = -4$; $[\![4.7]\!] = 4$

# SECTION 2.6 Exercises

## Basic Concepts and Skills

**1.** The graph of the linear function $f(x) = b$ is a(n) ____________ line.

**2.** The absolute value function can be expressed as a piecewise function by writing $f(x) = |x| =$ ____________.

**3.** The graph of the function $f(x) = \begin{cases} x^2 + 2 & \text{if } x \le 1 \\ ax & \text{if } x > 1 \end{cases}$ will have a break at $x = 1$ unless $a =$ ____________.

**4.** **True or False.** The function $f(x) = [\![x]\!]$ is increasing on the interval $(0, 3)$.

**In Exercises 5–14, write a linear function $f$ that has the indicated values. Sketch the graph of $f$.**

**5.** $f(0) = 1, f(-1) = 0$

**6.** $f(1) = 0, f(2) = 1$

**7.** $f(-1) = 1, f(2) = 7$

**8.** $f(-1) = -5, f(2) = 4$

**9.** $f(1) = 1, f(2) = -2$

**10.** $f(1) = -1, f(3) = 5$

**11.** $f(-2) = 2, f(2) = 4$

**12.** $f(2) = 2, f(4) = 5$

**13.** $f(0) = -1, f(3) = -3$

**14.** $f(1) = \frac{1}{4}, f(4) = -2$

**15.** Let

$$f(x) = \begin{cases} x & \text{if } x \ge 2 \\ 2 & \text{if } x < 2 \end{cases}.$$

**a.** Find $f(1), f(2)$, and $f(3)$.
**b.** Sketch the graph of $y = f(x)$.

**16.** Let

$$g(x) = \begin{cases} 2x & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}.$$

**a.** Find $g(-1), g(0)$, and $g(1)$.
**b.** Sketch the graph of $y = g(x)$.

**17.** Let

$$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}.$$

**a.** Find $f(-15)$ and $f(12)$.
**b.** Sketch the graph of $y = f(x)$.
**c.** Find the domain and the range of $f$.

**18.** Let

$$g(x) = \begin{cases} 2x + 4 & \text{if } x > 1 \\ x + 2 & \text{if } x \le 1 \end{cases}.$$

**a.** Find $g(-3), g(1)$, and $g(3)$.
**b.** Sketch the graph of $y = g(x)$.
**c.** Find the domain and the range of $g$.

**In Exercises 7–20, describe the transformations that produce the graphs of *g* and *h* from the graph of *f*.**

7. $f(x) = \sqrt{x}$
   a. $g(x) = \sqrt{x} + 2$
   b. $h(x) = \sqrt{x} - 1$

8. $f(x) = |x|$
   a. $g(x) = |x| + 1$
   b. $h(x) = |x| - 2$

9. $f(x) = x^2$
   a. $g(x) = (x + 1)^2$
   b. $h(x) = (x - 2)^2$

10. $f(x) = \dfrac{1}{x}$
   a. $g(x) = \dfrac{1}{x + 2}$
   b. $h(x) = \dfrac{1}{x - 3}$

11. $f(x) = \sqrt{x}$
   a. $g(x) = \sqrt{x + 1} - 2$
   b. $h(x) = \sqrt{x - 1} + 3$

12. $f(x) = x^2$
   a. $g(x) = -x^2$
   b. $h(x) = (-x)^2$

13. $f(x) = |x|$
   a. $g(x) = -|x|$
   b. $h(x) = |-x|$

14. $f(x) = \sqrt{x}$
   a. $g(x) = 2\sqrt{x}$
   b. $h(x) = \sqrt{2x}$

15. $f(x) = \dfrac{1}{x}$
   a. $g(x) = \dfrac{2}{x}$
   b. $h(x) = \dfrac{1}{2x}$

16. $f(x) = x^3$
   a. $g(x) = (x - 2)^3 + 1$
   b. $h(x) = -(x + 1)^3 + 2$

17. $f(x) = \sqrt{x}$
   a. $g(x) = -\sqrt{x} + 1$
   b. $h(x) = \sqrt{-x} + 1$

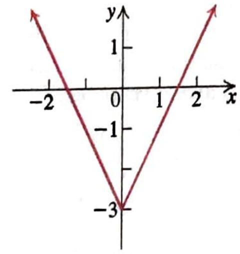
18. $f(x) = [x]$
   a. $g(x) = [x - 1] + 2$
   b. $h(x) = 3[x] - 1$

19. $f(x) = \sqrt[3]{x}$
   a. $g(x) = \sqrt[3]{x} + 1$
   b. $h(x) = \sqrt[3]{x + 1}$

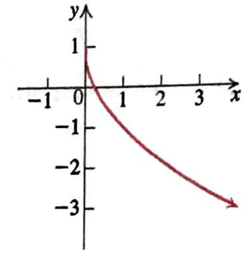
20. $f(x) = \sqrt[3]{x}$
   a. $g(x) = 2\sqrt[3]{1 - x} + 4$
   b. $h(x) = -\sqrt[3]{x - 1} + 3$

**In Exercises 21–32, match each function with its graph (a)–(l).**

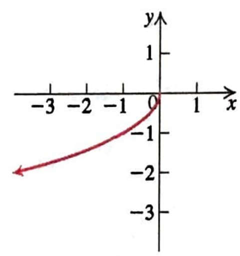
21. $y = -|x| + 1$
22. $y = -\sqrt{-x}$
23. $y = \sqrt{x^2}$
24. $y = \dfrac{1}{2}|x|$
25. $y = \sqrt{x + 1}$
26. $y = 2|x| - 3$
27. $y = 1 - 2\sqrt{x}$
28. $y = -|x - 1| + 1$
29. $y = (x - 1)^2$
30. $y = -x^2 + 3$
31. $y = -2(x - 3)^2 - 1$
32. $y = 3 - \sqrt{1 - x}$



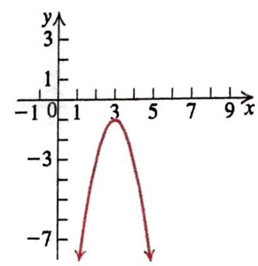
(a)



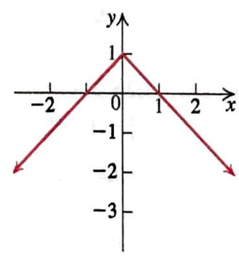
(b)



(c)



(d)



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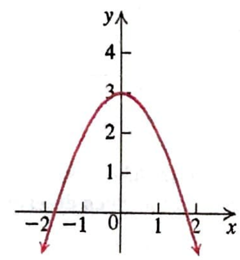
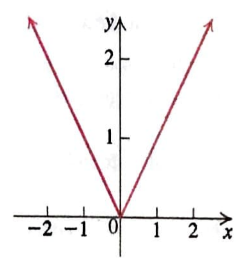
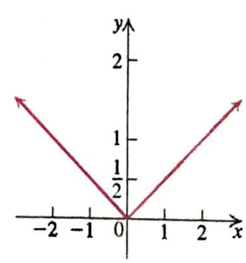


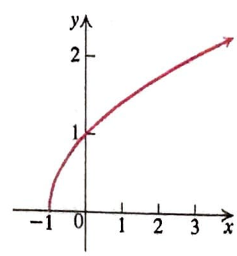
(f)



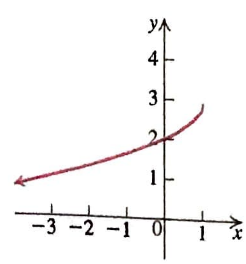
(g)



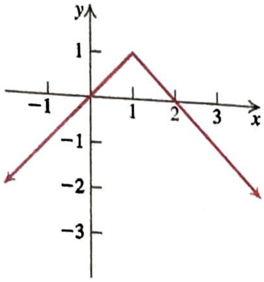
(h)



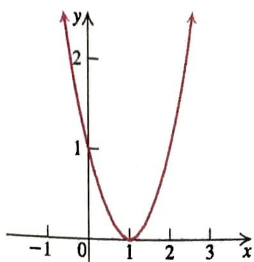
(i)



(j)




(k)




(l)

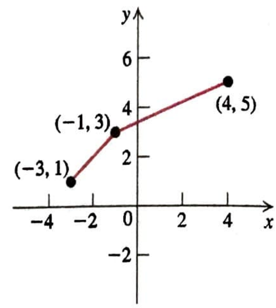
**In Exercises 33–62, graph each function by starting with a function from the library of functions and then using the techniques of shifting, compressing, stretching, and/or reflecting.**

33. $f(x) = x^2 - 2$
34. $f(x) = x^2 + 3$
35. $g(x) = \sqrt{x} + 1$
36. $g(x) = \sqrt{x} - 4$
37. $h(x) = |x + 1|$
38. $h(x) = |x - 2|$
39. $f(x) = (x - 3)^3$
40. $f(x) = (x + 2)^3$
41. $g(x) = (x - 2)^2 + 1$
42. $g(x) = (x + 3)^2 - 5$
43. $h(x) = -\sqrt{x}$
44. $h(x) = \sqrt{-x}$
45. $f(x) = -\dfrac{1}{x}$
46. $f(x) = -\dfrac{1}{2x}$
47. $g(x) = \dfrac{1}{2}|x|$
48. $g(x) = 4|x|$
49. $h(x) = -x^3 + 1$
50. $h(x) = -(x + 1)^3$
51. $f(x) = 2(x + 1)^2 - 1$
52. $f(x) = -(x - 1)^2$
53. $g(x) = 5 - x^2$
54. $g(x) = 2 - (x + 3)^2$
55. $h(x) = |1 - x|$
56. $h(x) = -2\sqrt{x - 1}$
57. $f(x) = -|x + 3| + 1$
58. $f(x) = 2 - \sqrt{x}$
59. $g(x) = -\sqrt{-x} + 2$
60. $g(x) = 3\sqrt{2 - x}$
61. $h(x) = 2[x + 1]$
62. $h(x) = [-x] + 1$

**In Exercises 63–74, write an equation for a function whose graph fits the given description.**

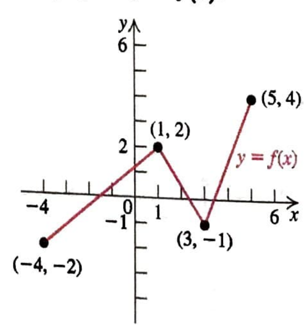
63. The graph of $f(x) = x^3$ is shifted 2 units up.
64. The graph of $f(x) = \sqrt{x}$ is shifted 3 units left.
65. The graph of $f(x) = |x|$ is reflected about the $x$-axis.
66. $f(x) = \sqrt{x}$ is reflected about the $y$-axis.
67. The graph of $f(x) = x^2$ is shifted 3 units right and 2 units up.
68. The graph of $f(x) = x^2$ is shifted 2 units left and reflected about the $x$-axis.
69. The graph of $f(x) = \sqrt{x}$ is shifted 3 units left, reflected about the $x$-axis, and shifted 2 units down.
70. The graph of $f(x) = \sqrt{x}$ is shifted 2 units down, reflected about the $x$-axis, and compressed vertically by a factor of $\frac{1}{2}$.
71. The graph of $f(x) = x^3$ is shifted 4 units left, stretched vertically by a factor of 3, reflected about the $y$-axis, and shifted 2 units up.
72. The graph of $f(x) = x^3$ is reflected about the $x$-axis, shifted 1 unit up, shifted 1 unit left, and reflected about the $y$-axis.
73. The graph of $f(x) = |x|$ is shifted 4 units right, compressed horizontally by a factor of 2, reflected about the $x$-axis, and shifted 3 units down.
74. The graph of $f(x) = |x|$ is shifted two units right, reflected about the $y$-axis, stretched horizontally by a factor of 2 and shifted three units down.

**In Exercises 75–88, graph the function $y = g(x)$ given the following graph of $y = f(x)$.**




75. $g(x) = f(x) - 1$
76. $g(x) = f(x) + 3$
77. $g(x) = -f(x)$
78. $g(x) = f(-x)$
79. $g(x) = f(2x)$
80. $g(x) = f\left(\dfrac{1}{2}x\right)$
81. $g(x) = f(x + 1)$
82. $g(x) = f(x - 2)$
83. $g(x) = 2f(x)$
84. $g(x) = \dfrac{1}{2}f(x)$
85. $g(x) = f(x - 1) + 2$
86. $g(x) = -f(-2x) - 1$
87. $g(x) = f(1 - 2x)$
88. $g(x) = f(3 - 2x)$

**In Exercises 89–96, graph (a) $y = g(x)$ and (b) $y = |g(x)|$, given the following graph of $y = f(x)$.**




89. $g(x) = f(x) + 1$
90. $g(x) = -2f(x)$
91. $g(x) = f\left(\dfrac{1}{2}x\right)$
92. $g(x) = f(-2x)$
93. $g(x) = f(x - 1)$
94. $g(x) = f(2 - x)$
95. $g(x) = -2f(x + 1) + 3$
96. $g(x) = -f(-x + 1) - 2$
97. $g(x) = 2f(1 - 2x) - 3$
98. $g(x) = -f(-2 - 2x) + 1$

## Applying the Concepts

**In Exercises 99–102, let $f$ be the function that associates the employee number $x$ of each employee of the ABC Corporation with his or her annual salary $f(x)$ in dollars.**

99. **Across-the-board raise.** Each employee was awarded an across-the-board raise of \$800 per year. Write a function $g(x)$ to describe the new salary.

**d.** $(d \circ r)(x) - (r \circ d)(x) = (0.92x - 3680) - (0.92x - 4000)$ Subtract function values.

$= \$320$

This equation shows that any car, regardless of its price, will cost \$320 more if you apply the rebate first and then the discount.

**Practice Problem 9** Repeat Example 9 if the dealer offers a 6% discount and the manufacturer offers a \$4500 rebate.

### Answers to Practice Problems

**1.** $(f+g)(x) = x^2 + 3x + 1$
$(f-g)(x) = -x^2 + 3x - 3$
$(fg)(x) = 3x^3 - x^2 + 6x - 2$
$\left(\frac{f}{g}\right)(x) = \frac{3x-1}{x^2+2}$

**2.** Domains: $fg = [1,3]; \frac{f}{g} = [1,3); \frac{g}{f} = (1,3]$

**3. a.** $(f \circ g)(0) = -5$ **b.** $(g \circ f)(0) = 1$

**4. a.** $(g \circ f)(x) = 2x^2 - 8x + 9$
**b.** $(f \circ g)(x) = 1 - 2x^2$
**c.** $(g \circ g)(x) = 8x^4 + 8x^2 + 3$

**5.** $(-\infty, 1] \cup (3, \infty)$ **6. a.** $[-\sqrt{3}, \sqrt{3}]$ **b.** $[1, 5]$

**7.** $f(x) = \frac{1}{x}$ **8. a.** $A = 9\pi t^2$ **b.** $A = 324\pi$ square miles $\approx 1018$ square miles

**9. a.** $r(x) = x - 4500$ **b.** $d(x) = 0.94x$
**c.** (i) $(r \circ d)(x) = 0.94x - 4500$
(ii) $(d \circ r)(x) = 0.94x - 4230$
**d.** $(d \circ r)(x) - (r \circ d)(x) = 270$

## SECTION 2.8 Exercises

### Basic Concepts and Skills

**1.** If $f(2) = 12$ and $g(2) = 4$, then $(g - f)(2) =$ ________.

**2.** If $f(x) = 2x - 1$ and $f(x) + g(x) = 0$, then $g(x) =$ ________.

**3.** If $f(x) = x$ and $g(x) = 1$, then $(f \circ g)(x) =$ ________.

**4.** If $f(1) = 4$ and $g(4) = 7$, then $(g \circ f)(1) =$ ________.

**5. True or False.** If $f(x) = 2x$ and $g(x) = \frac{1}{2x}$, then $(f \circ g)(x) = x$.

**6. True or False.** It cannot happen that $f \circ g = g \circ f$.

**In Exercises 7–10, functions $f$ and $g$ are given. Find each of the given values.**

**a.** $(f+g)(-1)$ **b.** $(f-g)(0)$
**c.** $(f \cdot g)(2)$ **d.** $\left(\frac{f}{g}\right)(1)$

**7.** $f(x) = 2x; g(x) = -x$

**8.** $f(x) = 1 - x^2; g(x) = x + 1$

**9.** $f(x) = \frac{1}{\sqrt{x+2}}; g(x) = 2x + 1$

**10.** $f(x) = \frac{x}{x^2 - 6x + 8}; g(x) = 3 - x$

**In Exercises 11–22, functions $f$ and $g$ are given. Find each of the following functions and state its domain.**

**a.** $f + g$ **b.** $f - g$ **c.** $f \cdot g$
**d.** $\frac{f}{g}$ **e.** $\frac{g}{f}$

**11.** $f(x) = x - 3; g(x) = x^2$

**12.** $f(x) = 2x - 1; g(x) = x^2$

**13.** $f(x) = x^3 - 1; g(x) = 2x^2 + 5$

**14.** $f(x) = x^2 - 4; g(x) = x^2 - 6x + 8$

**15.** $f(x) = 2x - 1; g(x) = \sqrt{x}$

**16.** $f(x) = 1 - \frac{1}{x}; g(x) = \frac{1}{x}$

**17.** $f(x) = \frac{2}{x+1}; g(x) = \frac{x}{x+1}$

**18.** $f(x) = \frac{5x-1}{x+1}; g(x) = \frac{4x+10}{x+1}$

**19.** $f(x) = \frac{x^2}{x+1}; g(x) = \frac{2x}{x^2-1}$

**20.** $f(x) = \frac{x-3}{x^2-25}; g(x) = \frac{x-3}{x^2+9x+20}$

**21.** $f(x) = \frac{2x}{x^2-16}; g(x) = \frac{2x-7}{x^2-7x+12}$

**22.** $f(x) = \frac{x^2+3x+2}{x^3+4x}; g(x) = \frac{2x^3+8x}{x^2+x-2}$

**In Exercises 23–26, find the domain of each function.**

a. $f \cdot g$ b. $\dfrac{f}{g}$

23. $f(x) = \sqrt{x-1};\quad g(x) = \sqrt{5-x}$
24. $f(x) = \sqrt{x-2};\quad g(x) = \sqrt{x+2}$
25. $f(x) = \sqrt{x+2};\quad g(x) = \sqrt{9-x^2}$
26. $f(x) = \sqrt{x^2-4};\quad g(x) = \sqrt{25-x^2}$

**In Exercises 27 and 28, use each diagram to evaluate $(g \circ f)(x)$. Then evaluate $(g \circ f)(2)$ and $(g \circ f)(-3)$.**

27. $x \rightarrow$ $f(x) = x^2 - 1$ $\rightarrow$ $g(x) = 2x + 3$ $\rightarrow$

28. $x \rightarrow$ $f(x) = |x + 1|$ $\rightarrow$ $g(x) = 3x^2 - 1$ $\rightarrow$

**In Exercises 29–36, let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 3$. Evaluate each expression.**

29. $(f \circ g)(2)$
30. $(g \circ f)(2)$
31. $(f \circ g)(-3)$
32. $(g \circ f)(-5)$
33. $(g \circ f)(a)$
34. $(g \circ f)(-a)$
35. $(f \circ f)(1)$
36. $(g \circ g)(-1)$

**In Exercises 37–42, the functions $f$ and $g$ are given. Find $f \circ g$ and its domain.**

37. $f(x) = \dfrac{2}{x+1};\ g(x) = \dfrac{1}{x}$
38. $f(x) = \dfrac{1}{x-1};\ g(x) = \dfrac{2}{x+3}$
39. $f(x) = \sqrt{x-3};\ g(x) = 2 - 3x$
40. $f(x) = \dfrac{x}{x-1};\ g(x) = 2 + 5x$
41. $f(x) = |x|;\ g(x) = x^2 - 1$
42. $f(x) = 3x - 2;\ g(x) = |x - 1|$

**In Exercises 43–56, the functions $f$ and $g$ are given. Find each composite function and describe its domain.**

a. $f \circ g$ b. $g \circ f$ c. $f \circ f$ d. $g \circ g$

43. $f(x) = 2x - 3;\ g(x) = x + 4$
44. $f(x) = x - 3;\ g(x) = 3x - 5$
45. $f(x) = 1 - 2x;\ g(x) = 1 + x^2$
46. $f(x) = 2x - 3;\ g(x) = 2x^2$
47. $f(x) = x^2;\ g(x) = \sqrt{x}$
48. $f(x) = x^2 + 2x;\ g(x) = \sqrt{x+2}$
49. $f(x) = \dfrac{1}{2x-1};\ g(x) = \dfrac{1}{x^2}$
50. $f(x) = x - 1;\ g(x) = \dfrac{x}{x+1}$
51. $f(x) = \sqrt{x-1};\ g(x) = \sqrt{4-x}$
52. $f(x) = x^2 - 4;\ g(x) = \sqrt{4-x^2}$
53. $f(x) = \sqrt{x^2-1};\ g(x) = \sqrt{4-x^2}$
54. $f(x) = \sqrt{x^2-9};\ g(x) = \sqrt{9-x^2}$
55. $f(x) = 1 + \dfrac{1}{x};\ g(x) = \dfrac{1+x}{1-x}$
56. $f(x) = \sqrt[3]{x+1};\ g(x) = x^3 + 1$

**In Exercises 57–66, express the given function $H$ as a composition of two functions $f$ and $g$ such that $H(x) = (f \circ g)(x)$.**

57. $H(x) = \sqrt{x+2}$
58. $H(x) = |3x + 2|$
59. $H(x) = (x^2 - 3)^{10}$
60. $H(x) = \sqrt{3x^2 + 5}$
61. $H(x) = \dfrac{1}{3x-5}$
62. $H(x) = \dfrac{5}{2x+3}$
63. $H(x) = \sqrt[3]{x^2 - 7}$
64. $H(x) = \sqrt[4]{x^2 + x + 1}$
65. $H(x) = \dfrac{1}{|x^3 - 1|}$
66. $H(x) = \sqrt[3]{1 + \sqrt{x}}$

## Applying the Concepts

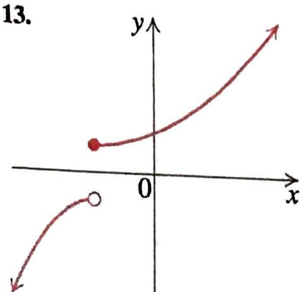
67. **Cost, revenue, and profit.** A retailer purchases $x$ shirts from a wholesaler at a price of \$12 per shirt. Her selling price for each shirt is \$22. The state has 7% sales tax. Interpret each of the following functions.
 a. $f(x) = 12x$
 b. $g(x) = 22x$
 c. $h(x) = g(x) + 0.07g(x)$
 d. $P(x) = g(x) - f(x)$

68. **Cost, revenue, and profit.** The demand equation for a product is given by $x = 5000 - 5p$, where $x$ is the number of units produced and sold at price $p$ (in dollars) per unit. The cost (in dollars) of producing $x$ units is given by $C(x) = 4x + 12{,}000$. Express each of the following as a function of price.
 a. Cost
 b. Revenue
 c. Profit

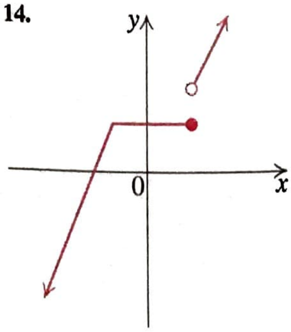
69. **Cost and revenue.** A manufacturer of radios estimates that his daily cost of producing $x$ radios is given by the equation $C = 350 + 5x$. The equation $R = 25x$ represents the revenue in dollars from selling $x$ radios.
 a. Write and simplify the profit function
 $$P(x) = R(x) - C(x).$$
 b. Find $P(20)$. What does the number $P(20)$ represent?
 c. How many radios should the manufacturer produce and sell to have a daily profit of \$500?
 d. Find the composite function $(R \circ x)(C)$. What does this function represent?
 [*Hint:* To find $x(C)$, solve $C = 350 + 5x$ for $x$.]

70. **Mail order.** You order merchandise worth $x$ dollars from D-Bay Manufacturers. The company charges you sales tax of 4% of the purchase price plus shipping and handling fees of \$3 plus 2% of the after-tax purchase price.
 a. Write a function $g(x)$ that represents the sales tax.
 b. What does the function $h(x) = x + g(x)$ represent?
 c. Write a function $f(x)$ that represents the shipping and handling fees.
 d. What does the function $T(x) = h(x) + f(x)$ represent?

13.




14.




**In Exercises 15–28, assume that the function $f$ is one-to-one with domain: $(-\infty, \infty)$.**

15. If $f(2) = 7$, find $f^{-1}(7)$.
16. If $f^{-1}(4) = -7$, find $f(-7)$.
17. If $f(-1) = 2$, find $f^{-1}(2)$.
18. If $f^{-1}(-3) = 5$, find $f(5)$.
19. If $f(a) = b$, find $f^{-1}(b)$.
20. If $f^{-1}(c) = d$, find $f(d)$.
21. Find $(f^{-1} \circ f)(337)$.
22. Find $(f \circ f^{-1})(25\pi)$.
23. Find $(f \circ f^{-1})(-1580)$.
24. Find $(f^{-1} \circ f)(9728)$.
25. For $f(x) = 2x - 3$, find each of the following.
   a. $f(3)$
   b. $f^{-1}(3)$
   c. $(f \circ f^{-1})(19)$
   d. $(f \circ f^{-1})(5)$
26. For $f(x) = x^3$, find each of the following.
   a. $f(2)$
   b. $f^{-1}(8)$
   c. $(f \circ f^{-1})(15)$
   d. $(f^{-1} \circ f)(27)$
27. For $f(x) = x^3 + 1$, find each of the following.
   a. $f(1)$
   b. $f^{-1}(2)$
   c. $(f \circ f^{-1})(269)$
28. For $g(x) = \sqrt[3]{2x^3 - 1}$, find each of the following.
   a. $g(1)$
   b. $g^{-1}(1)$
   c. $(g^{-1} \circ g)(135)$

**In Exercises 29–34, show that $f$ and $g$ are inverses of each other by verifying that $f(g(x)) = x = g(f(x))$.**

29. $f(x) = 3x + 1;\ g(x) = \dfrac{x-1}{3}$

30. $f(x) = 2 - 3x;\ g(x) = \dfrac{2-x}{3}$

31. $f(x) = x^3;\ g(x) = \sqrt[3]{x}$

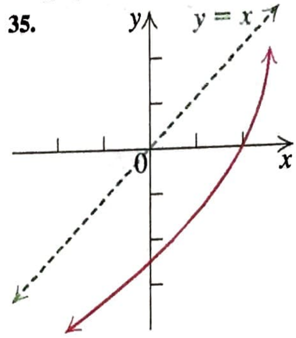
32. $f(x) = \dfrac{1}{x};\ g(x) = \dfrac{1}{x}$

33. $f(x) = \dfrac{x-1}{x+2};\ g(x) = \dfrac{1+2x}{1-x}$

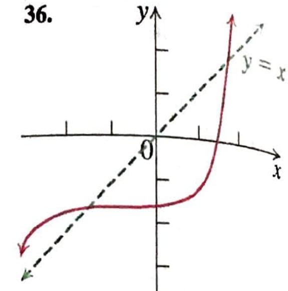
34. $f(x) = \dfrac{3x+2}{x-1};\ g(x) = \dfrac{x+2}{x-3}$

**In Exercises 35–40, the graph of a function $f$ is given. Sketch the graph of $f^{-1}$.**

35.




36.

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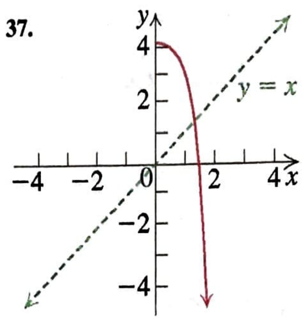


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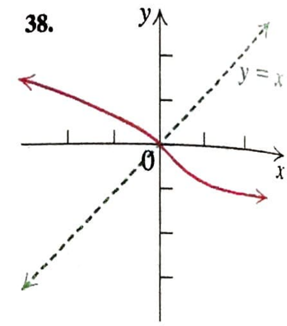


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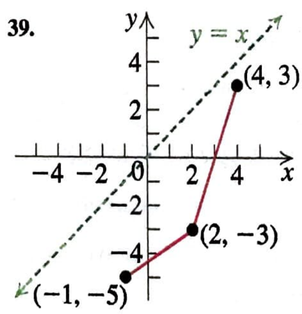
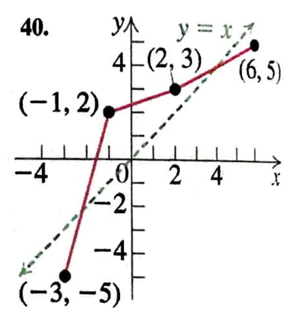



40.




**In Exercises 41–52,**

a. Determine algebraically whether the given function is a one-to-one function.
b. If the function is one-to-one, find its inverse.
c. Sketch the graph of the function and its inverse on the same coordinate axes.
d. Give the domain and intercepts of each one-to-one function and its inverse function.

41. $f(x) = 15 - 3x$
42. $g(x) = 2x + 5$
43. $f(x) = \sqrt{4 - x^2}$
44. $f(x) = -\sqrt{9 - x^2}$
45. $f(x) = \sqrt{x} + 3$
46. $f(x) = 4 - \sqrt{x}$
47. $g(x) = \sqrt[3]{x + 1}$
48. $h(x) = \sqrt[3]{1 - x}$
49. $f(x) = \dfrac{1}{x-1}, x \neq 1$
50. $g(x) = 1 - \dfrac{1}{x}, x \neq 0$
51. $f(x) = 2 + \sqrt{x + 1}$
52. $f(x) = -1 + \sqrt{x + 2}$
53. Find the domain and range of the function $f$ of Exercise 33.
54. Find the domain and range of the function $f$ of Exercise 34.

**In Exercises 55–58, assume that the given function is one-to-one. Find the inverse of the function. Also find the domain and the range of the given function.**

55. $f(x) = \frac{x+1}{x-2}, x \neq 2$
56. $g(x) = \frac{x+2}{x+1}, x \neq -1$
57. $f(x) = \frac{1-2x}{1+x}, x \neq -1$
58. $h(x) = \frac{x-1}{x-3}, x \neq 3$

**In Exercises 59–66, find the inverse of each function and sketch the graph of the function and its inverse on the same coordinate axes.**

59. $f(x) = -x^2, x \geq 0$
60. $g(x) = -x^2, x \leq 0$
61. $f(x) = |x|, x \geq 0$
62. $g(x) = |x|, x \leq 0$
63. $f(x) = x^2 + 1, x \leq 0$
64. $g(x) = x^2 + 5, x \geq 0$
65. $f(x) = -x^2 + 2, x \leq 0$
66. $g(x) = -x^2 - 1, x \geq 0$

## Applying the Concepts

67. **Temperature scales.** Scientists use the **Kelvin** temperature scale, in which the lowest possible temperature (called absolute zero) is 0 K. (K denotes degrees Kelvin.)

The function $K(C) = C + 273$ gives the relationship between the Kelvin temperature (K) and Celsius temperature (C).

a. Find the inverse function of $K(C) = C + 273$. What does it represent?
b. Use the inverse function from (a) to find the Celsius temperature corresponding to 300 K.
c. A comfortable room temperature is 22°C. What is the corresponding Kelvin temperature?

68. **Temperature scales.** The boiling point of water is 373 K, or 212°F; the freezing point of water is 273 K, or 32°F. The relationship between Kelvin and Fahrenheit temperatures is linear.

a. Write a linear function expressing $K(F)$ in terms of $F$.
b. Find the inverse of the function in part (a). What does it mean?
c. A normal human body temperature is 98.6°F. What is the corresponding Kelvin temperature?

69. **Composition of functions.** Use Exercises 67 and 68 and the composition of functions to

a. Write a function that expresses $F$ in terms of $C$.
b. Write a function that expresses $C$ in terms of $F$.

70. **Celsius and Fahrenheit temperatures.** Show that the functions in (a) and (b) of Exercise 69 are inverse functions.

71. **Currency exchange.** Alisha went to Europe last summer. She discovered that when she exchanged her U.S. dollars for euros, she received 25% fewer euros than the number of dollars she exchanged. (She got 75 euros for every 100 U.S. dollars.) When she returned to the United States, she got 25% more dollars than the number of euros she exchanged.

a. Write each conversion function.
b. Show that in part (a), the two functions are not inverse functions.
c. Does Alisha gain or lose money after converting both ways?

72. **Hourly wages.** Anwar is a short-order cook in a diner. He is paid \$4 per hour plus 5% of all food sales per hour. His average hourly wage $w$ in terms of the food sales of $x$ dollars is $w = 4 + 0.05x$.

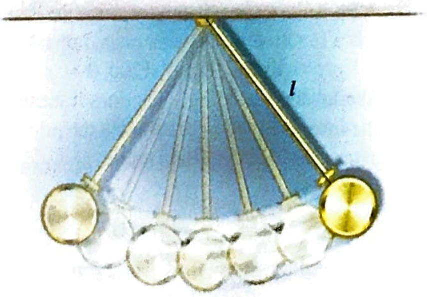
a. Write the inverse function. What does it mean?
b. Use the inverse function to estimate the hourly sales at the diner if Anwar averages \$12 per hour.

73. **Hourly wages.** In Exercise 72, suppose in addition that Anwar is guaranteed a minimum wage of \$7 per hour.

a. Write a function expressing his hourly wage $w$ in terms of food sales per hour. [*Hint:* Use a piecewise function.]
b. Does the function in part (a) have an inverse? Explain.
c. If the answer in part (b) is yes, find the inverse function. If the answer is no, restrict the domain so that the new function has an inverse.

74. **Simple pendulum.** If a pendulum is released at a certain point, the **period** is the time the pendulum takes to swing along its path and return to the point from which it was released. The period $T$ (in seconds) of a simple pendulum is a function of its length $l$ (in feet) and is given by $T = 1.11\sqrt{l}$.

a. Find the inverse function. What does it mean?
b. Use the inverse function to calculate the length of the pendulum assuming that its period is two seconds.
c. The convention center in Portland, Oregon, has the longest pendulum in the United States. The pendulum's length is 70 feet. Find the period.




75. **Water supply.** Suppose $x$ is the height of the water above the opening at the base of a water tank. The velocity $V$ of water that flows from the opening at the base is a function of $x$ and is given by $V(x) = 8\sqrt{x}$.

a. Find the inverse function. What does it mean?
b. Use the inverse function to calculate the height of the water in the tank when the flow is (i) 30 feet per second and (ii) 20 feet per second.

76. **Physics.** A projectile is fired from the origin over horizontal ground. Its altitude $y$ (in feet) is a function of its horizontal distance $x$ (in feet) and is given by

$$y = 64x - 2x^2.$$

a. Find the inverse function where the function is increasing.