SECTION 1.1 Exercises

Basic Concepts and Skills

1. The domain of the variable in an equation is the set of all real numbers for which both sides of the equation are _____.
2. Standard form for a linear equation in x is _____.
3. Two equations with the same solutions set are called _____.
4. A conditional equation is an equation that is not true for _____ values of the variables.
5. True or False. If \$100 is invested at 5% for three years, the interest $I = (100)(5)(3)$ dollars.
6. True or False. If an object is traveling at a uniform rate of 60 ft/sec, the distance covered in 15 minutes is $d = (60)(15)$ ft.

In Exercises 7–11, determine whether the given value of the variable is a solution of the equation.

7. $x - 2 = 5x + 6$
   a. $x = 0$ b. $x = -2$
8. $8x + 3 = 14x - 1$
   a. $x = -1$ b. $x = \frac{2}{3}$
9. $\frac{2}{x} = \frac{1}{3} + \frac{1}{x+2}$
   a. $x = 4$ b. $x = 1$
10. $(x - 3)(2x + 1) = 0$
    a. $x = \frac{1}{2}$ b. $x = 3$
11. $2x + 3x = 5x$
    a. $x = 157$ b. $x = -2046$

In Exercises 12–16, find the domain of the variable in each equation. Write the answer in interval notation.

12. $(2 - x) - 4x = 7 - 3(x + 4)$
13. $\frac{y}{y-1} = \frac{3}{y+2}$
14. $\frac{1}{y} = 2 + \sqrt{y}$
15. $\frac{3x}{(x-3)(x-4)} = 2x + 9$
16. $\frac{1}{\sqrt{x}} = x^2 - 1$

In Exercises 17–20, determine whether the given equation is an identity. If the equation is not an identity, give a value that demonstrates that fact.

17. $2x + 3 = 5x + 1$
18. $3x + 4 = 6x + 2 - (3x - 2)$
19. $\frac{1}{x} + \frac{1}{2} = \frac{2+x}{2x}$
20. $\frac{1}{x+3} = \frac{1}{x} + \frac{1}{3}$

In Exercises 21–46, solve each equation.

21. $3x + 5 = 14$
22. $2x - 17 = 7$
23. $-10x + 12 = 32$
24. $-2x + 5 = 6$
25. $3 - y = -4$
26. $2 - 7y = 23$
27. $7x + 7 = 2(x + 1)$
28. $3(x + 2) = 4 - x$
29. $3(2 - y) + 5y = 3y$
30. $9y - 3(y - 1) = 6 + y$
31. $4y - 3y + 7 - y = 2 - (7 - y)$
32. $3(y - 1) = 6y - 4 + 2y - 4y$
33. $3(x - 2) + 2(3 - x) = 1$
34. $2x - 3 - (3x - 1) = 6$
35. $2x + 3(x - 4) = 7x + 10$
36. $3(2 - 3x) - 4x = 3x - 10$
37. $4[x + 2(3 - x)] = 2x + 1$
38. $3 - [x - 3(x + 2)] = 4$
39. $3(4y - 3) = 4[y - (4y - 3)]$
40. $5 - (6y + 9) + 2y = 2(y + 1)$
41. $2x - 3(2 - x) = (x - 3) + 2x + 1$
42. $5(x - 3) - 6(x - 4) = -5$
43. $\frac{2x+1}{9} - \frac{x+4}{6} = 1$
44. $\frac{2-3x}{7} + \frac{x-1}{3} = \frac{3x}{7}$
45. $\frac{1-x}{4} + \frac{5x+1}{2} = 3 - \frac{2(x+1)}{8}$
46. $\frac{x+4}{3} + 2x - \frac{1}{2} = \frac{3x+2}{6}$

In Exercises 47–58, solve each formula for the indicated variable.

47. $d = rt$ for r
48. $F = ma$ for a
49. $C = 2\pi r$ for r
50. $A = 2\pi rx + \pi r^2$ for x
51. $I = \frac{E}{R}$ for R
52. $A = P(1 + rt)$ for t
53. $A = \frac{(a+b)h}{2}$ for h
54. $T = a + (n - 1)d$ for d
55. $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ for u
56. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ for R_2
57. $y = mx + b$ for m
58. $ax + by = c$ for y

In Exercises 59–62, write an algebraic expression for the specified quantity.

59. Leroy buys in-line skates for \$327.62, including sales tax of $6\frac{1}{2}\%$. Let x = the price of in-line skates before tax. Write an algebraic expression in x for "the tax paid on the in-line skates."

In Exercises 17–22, solve each equation by factoring.

17. $x^2 - 5x = 0$
18. $x^2 - 5x + 4 = 0$
19. $x^2 + 5x = 14$
20. $x^2 - 11x = 12$
21. $x^2 = 5x + 6$
22. $x = x^2 - 12$

In Exercises 23–28, solve each equation by the square root method.

23. $3x^2 = 48$
24. $2x^2 = 50$
25. $x^2 + 1 = 5$
26. $2x^2 - 1 = 17$
27. $(x - 1)^2 = 16$
28. $(2x - 3)^2 = 25$

In Exercises 29–38, add a constant term to the expression to make it a perfect square.

29. $x^2 + 4x$
30. $y^2 + 10y$
31. $x^2 + 6x$
32. $y^2 - 8y$
33. $x^2 - 7x$
34. $x^2 - 3x$
35. $x^2 + \frac{1}{3}x$
36. $x^2 - \frac{3}{2}x$
37. $x^2 + ax$
38. $x^2 - \frac{2a}{3}x$

In Exercises 39–44, solve each equation by completing the square.

39. $x^2 + 2x - 5 = 0$
40. $x^2 + 6x = -7$
41. $x^2 - 3x - 1 = 0$
42. $x^2 - x - 3 = 0$
43. $2r^2 + 3r = 9$
44. $3k^2 - 5k + 1 = 0$

In Exercises 45–50, solve each equation by using the quadratic formula.

45. $x^2 + 2x - 4 = 0$
46. $m^2 + 3m + 2 = 0$
47. $6x^2 = 7x + 5$
48. $t^2 - 7 = 4t$
49. $3z^2 - 2z = 7$
50. $6y^2 + 11y = 10$

In Exercises 51–76, solve each equation by any method.

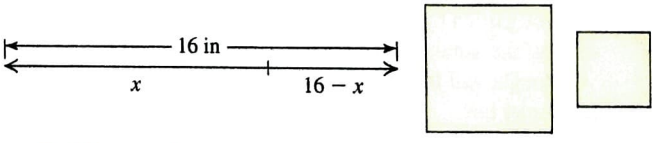
51. $2x^2 + 5x - 3 = 0$
52. $2x^2 - 9x + 10 = 0$
53. $(3x - 2)^2 - 16 = 0$
54. $(4x + 1)^2 - 25 = 0$
55. $5x^2 - 6x = 4x^2 + 6x - 3$
56. $x^2 + 7x - 5 = x - x^2$
57. $3p^2 + 8p + 4 = 0$
58. $x^2 = 5(x - 1)$
59. $3y^2 + 5y + 2 = 0$
60. $6x^2 + 11x + 4 = 0$
61. $5x^2 + 12x + 4 = 0$
62. $3x^2 - 2x - 5 = 0$
63. $5y^2 + 10y + 4 = 2y^2 + 3y + 1$
64. $3x^2 - 1 = 5x^2 - 3x - 5$
65. $2x^2 + x = 15$
66. $6x^2 = 1 - x$
67. $12x^2 - 10x = 12$
68. $-x^2 + 10x + 1200 = 0$
69. $(x + 13)(x + 5) = -2$
70. $3(x^2 + 1) = 2x^2 + 4x + 1$
71. $18x^2 - 45x = -7$
72. $18x^2 + 57x + 45 = 0$
73. $2t^2 - 5 = 0$
74. $3k^2 - 48 = 0$
75. $4x^2 - 10x - 750 = 0$
76. $12x^2 + 43x + 36 = 0$

In Exercises 77–80, write your answer rounded to two decimal places.

77. Find the length of the golden rectangle whose width is 14.72 in.
78. Find the length of the golden rectangle whose width is 18.63 ft.
79. Find the width of the golden rectangle whose length is 8.46 cm.
80. Find the width of the golden rectangle whose length is 4.68 m.

Applying the Concepts

81. **Dimensions.** The length of a rectangular plot is three times its width. Assuming that the area of the plot is 10,800 square feet, find the dimensions of the plot.
82. **Dimensions.** The diagonal of a square tile is 4 inches longer than its side. Find the length of the side.
83. **Finding integers.** Find two numbers whose sum is 28 and whose product is 147.
84. **Finding integers.** Find an integer such that the sum of twice the square of the integer and the integer itself is 55.
85. The sum of two numbers is 57, and their product is 782. Find the numbers.
86. The perimeter of a rectangle is 82 ft, and its area is 400 ft^2. Find dimensions of the rectangle.
87. **Geometry.** The length of a rectangle is 5 centimeters greater than its width. The area of the rectangle is 500 square centimeters. Find the dimensions of the rectangle.
88. **Geometry.** The sides of a rectangle are in the ratio 3:2. The area of the rectangle is 216 square centimeters. Find the dimensions of the rectangle.
89. **Cutting a wire.** A wire of length 16 inches is to be cut into two pieces; then each piece will be bent to form a square. Find the length of the two pieces assuming that the sum of the areas of the two squares is 10 square inches.



90. **Cutting a wire.** A piece of wire is 38 inches long. The wire is cut into two pieces; then each piece is bent to form a square. Find the length of each piece if three times the area of the larger square exceeds the area of the smaller square by 95.75 square inches.
91. **Manufacturing.** The surface area A of a cylinder with height h and radius r is given by the equation $A = 2\pi rh + 2\pi r^2$. A company makes soup cans by using 32π square inches of aluminum sheet for each can. Assuming that the height of the can is 6 inches, find the radius of the can.
92. **Concrete patio.** To make a rectangular concrete patio, a homeowner used 70 feet of perimeter, into which she poured 138 cubic feet of concrete to form a slab 6 inches thick. What were the dimensions of the patio?

Discriminant	Description of Solutions
$b^2 - 4ac > 0$	There are two unequal real solutions.
$b^2 - 4ac = 0$	There is one real solution.
$b^2 - 4ac < 0$	There are two nonreal complex solutions, and they are conjugates.

EXAMPLE 10 **Using the Discriminant**

Use the discriminant to determine the number and type of solutions of each quadratic equation.

a. $x^2 - 4x + 2 = 0$ **b.** $2t^2 + 2t + 19 = 0$ **c.** $4x^2 + 4x + 1 = 0$

Solution

Equation	$b^2 - 4ac$	Conclusion
a. $x^2 - 4x + 2 = 0$	$(-4)^2 - 4(1)(2) = 8 > 0$	Two unequal real solutions
b. $2t^2 + 2t + 19 = 0$	$(2)^2 - 4(2)(19) = -148 < 0$	Two nonreal complex solutions
c. $4x^2 + 4x + 1 = 0$	$(4)^2 - 4(4)(1) = 0$	Exactly one real solution

Practice Problem 10 Determine the number and type of solutions.

a. $9x^2 - 6x + 1 = 0$ **b.** $x^2 - 5x + 3 = 0$ **c.** $2x^2 - 3x + 4 = 0$.

Answers to Practice Problems

1. a. Real $= -1$, imaginary $= 2$ **b.** Real $= -\frac{1}{3}$, imaginary $= -6$ **c.** Real $= 8$, imaginary $= 0$
2. $a = -2; b = 1$ **3. a.** $4 - 2i$ **b.** $-1 + 4i$ **c.** $-2 + 5i$
4. a. $26 + 2i$ **b.** $-15 - 21i$ **5. a.** $5 - 12i$ **b.** $16 + 14\sqrt{2}i$
6. a. 37 **b.** 4
7. a. $1 + i$ **b.** $-\frac{15}{41} - \frac{12}{41}i$ **8.** $\frac{23}{10} + \frac{11}{10}i$
9. a. $\left\{-\frac{3}{2}i, \frac{3}{2}i\right\}$ **b.** $\{2 - 3i, 2 + 3i\}$
10. a. One real **b.** Two unequal real **c.** Two nonreal complex

SECTION 1.3 Exercises

Basic Concepts and Skills

1. We define $i =$ _____ so that $i^2 =$ _____.
2. A complex number in the form $a + bi$ is said to be in _____.
3. For $b > 0$, $\sqrt{-b} =$ _____.
4. The conjugate of $a + bi$ is _____, and the conjugate of $a - bi$ is _____.
5. **True or False.** The product of a complex number and its conjugate is a real number.
6. **True or False.** Division by a nonzero complex number z is accomplished by multiplying the numerator and denominator by $\bar{z}$.

In Exercises 7–10, use the definition of equality of complex numbers to find the real numbers x and y so that the equation is true.

7. $2 + xi = y + 3i$
8. $x - 2i = 7 + yi$
9. $x - \sqrt{-16} = 2 + yi$
10. $3 + yi = x - \sqrt{-25}$

In Exercises 11–34, perform each operation and write the result in the standard form $a + bi$.

11. $(5 + 2i) + (3 + i)$
12. $(6 + i) + (1 + 2i)$
13. $(4 - 3i) - (5 + 3i)$
14. $(3 - 5i) - (3 + 2i)$

15. $(-2 - 3i) + (-3 - 2i)$
16. $(-5 - 3i) + (2 - i)$
17. $3(5 + 2i)$
18. $4(3 + 5i)$
19. $-4(2 - 3i)$
20. $-7(3 - 4i)$
21. $3i(5 + i)$
22. $2i(4 + 3i)$
23. $4i(2 - 5i)$
24. $-3i(5 - 2i)$
25. $(3 + i)(2 + 3i)$
26. $(4 + 3i)(2 + 5i)$
27. $(2 - 3i)(2 + 3i)$
28. $(4 - 3i)(4 + 3i)$
29. $(3 + 4i)(4 - 3i)$
30. $(-2 + 3i)(-3 + 10i)$
31. $(\sqrt{3} - 12i)^2$
32. $(-\sqrt{5} - 13i)^2$
33. $(2 - \sqrt{-16})(3 + 5i)$
34. $(5 - 2i)(3 + \sqrt{-25})$

In Exercises 35–40, write the conjugate $\bar{z}$ of each complex number z. Then find $z\bar{z}$.

35. $z = 2 - 3i$
36. $z = 4 + 5i$
37. $z = \frac{1}{2} - 2i$
38. $z = \frac{2}{3} + \frac{1}{2}i$
39. $z = \sqrt{2} - 3i$
40. $z = \sqrt{5} + \sqrt{3}\,i$

In Exercises 41–54, write each quotient in the standard form $a + bi$.

41. $\frac{5}{-i}$
42. $\frac{2}{-3i}$
43. $\frac{-1}{1 + i}$
44. $\frac{1}{2 - i}$
45. $\frac{5i}{2 + i}$
46. $\frac{3i}{2 - i}$
47. $\frac{2 + 3i}{1 + i}$
48. $\frac{3 + 5i}{4 + i}$
49. $\frac{2 - 5i}{4 - 7i}$
50. $\frac{3 + 5i}{1 - 3i}$
51. $\frac{2 + \sqrt{-4}}{1 + i}$
52. $\frac{5 - \sqrt{-9}}{3 + 2i}$
53. $\frac{-2 + \sqrt{-25}}{2 - 3i}$
54. $\frac{-5 - \sqrt{-4}}{5 - \sqrt{-9}}$

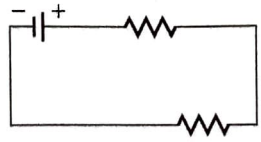
In Exercises 55–64, solve each equation.

55. $x^2 + 5 = 1$
56. $4x^2 + 9 = 0$
57. $z^2 - 2z + 2 = 0$
58. $x^2 - 6x + 11 = 0$
59. $2x^2 - 20x + 49 = -7$
60. $4y^2 + 4y + 5 = 0$
61. $8(x^2 - x) = x^2 - 3$
62. $t(t + 1) = 3t^2 + 1$
63. $9k^2 + 25 = 0$
64. $3k^2 + 4 = 0$

Applying the Concepts

65. **Series circuits.** Assuming that the impedance of a resistor in a circuit is $Z_1 = 4 + 3i$ ohms and the impedance of a second resistor is $Z_2 = 5 - 2i$ ohms, find the total impedance of the two resistors when placed in series (sum of the two impedances).

Series circuit

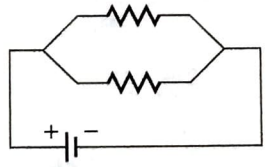


66. **Parallel circuits.** If the two resistors in Exercise 65 are connected in parallel, the total impedance is given by

$$\frac{Z_1 Z_2}{Z_1 + Z_2}.$$

Find the total impedance assuming that the resistors in Exercise 65 are connected in parallel.

Parallel circuit



As with impedance, the current I and voltage V in a circuit can be represented by complex numbers. The three quantities (voltage, V; impedance, Z; and current, I) are related by the equation $Z = \frac{V}{I}$. Thus, if two of these values are given, the value of the third can be found from the equation.

In Exercises 67–72, use the equation $V = ZI$ to find the value that is not specified.

67. **Finding impedance:** $I = 7 + 5i$ $V = 35 + 70i$
68. **Finding impedance:** $I = 7 + 4i$ $V = 45 + 88i$
69. **Finding voltage:** $Z = 5 - 7i$ $I = 2 + 5i$
70. **Finding voltage:** $Z = 7 - 8i$ $I = \frac{1}{3} + \frac{1}{6}i$
71. **Finding current:** $V = 12 + 10i$ $Z = 12 + 6i$
72. **Finding current:** $V = 29 + 18i$ $Z = 25 + 6i$

Beyond the Basics

In Exercises 73–82, find each power of i and simplify the expression.

73. i^{17}
74. i^{125}
75. i^{-7}
76. i^{-24}
77. $i^{10} + 7$
78. $9 + i^3$

Solution

Substitute $t_0 = 15$ (moving-frame time) and $t = 25$ (fixed-frame time) to obtain

$$15 = 25\sqrt{1 - \frac{v^2}{c^2}}$$

$$\frac{3}{5} = \sqrt{1 - \frac{v^2}{c^2}} \qquad \text{Divide both sides by 25; } \frac{15}{25} = \frac{3}{5}.$$

$$\frac{9}{25} = 1 - \frac{v^2}{c^2} \qquad \text{Square both sides.}$$

$$\frac{v^2}{c^2} = 1 - \frac{9}{25} \qquad \text{Add } \frac{v^2}{c^2} - \frac{9}{25} \text{ to both sides.}$$

$$\left(\frac{v}{c}\right)^2 = \frac{16}{25} \qquad \text{Simplify.}$$

$$\frac{v}{c} = \pm\frac{4}{5} \qquad \text{Square root property}$$

$$\frac{v}{c} = \frac{4}{5} \qquad \text{Reject the negative value.}$$

$$v = \frac{4}{5}c = 0.8c \qquad \text{Multiply both sides by } c.$$

Thus, the spacecraft must have been traveling at 80% of the speed of light $(0.8c)$ when your sister found the "trip of youth."

Practice Problem 12 Suppose your sister's trip took 20 years according to the clock and calendar on the spacecraft, but 25 years judging by time on Earth. Find the speed of the spacecraft.

Answers to Practice Problems

1. $\{-2, -1, 1\}$ **2.** $\{-2, 0, 2\}$ **3.** $\{-2, 2, 5\}$
4. $\{-10, 1\}$ **5.** $\varnothing$ **6.** $\{0, 3\}$ **7.** $\{10\}$
8. $\{9\}$ **9. a.** $\{3\}$ **b.** $\left\{-\frac{9}{2}, \frac{7}{2}\right\}$ **10.** $\{1, 216\}$
11. $\left\{\frac{1}{3}, 1\right\}$ **12.** $0.6c$ or 60% of the speed of light

SECTION 1.4 Exercises

Basic Concepts and Skills

1. If an apparent solution does not satisfy the equation, it is called a(n) _____ solution.
2. When solving a rational equation, we multiply both sides by the _____.
3. If $x^{3/4} = 8$, then $x =$ _____.
4. If $x^{4/3} = 16$, then $x =$ _____ or $x =$ _____.
5. **True or False.** Rational equations always have extraneous solutions.
6. **True or False.** If $x^{2/3} = k$, then $x = \pm\sqrt{k^3}$.

In Exercises 7–16, find the real solutions of each equation by factoring.

7. $x^3 = 2x^2$
8. $3x^4 - 27x^2 = 0$
9. $(\sqrt{x})^3 = \sqrt{x}$
10. $(\sqrt{x})^5 = 16\sqrt{x}$
11. $x^3 + x = 0$
12. $x^3 - 1 = 0$
13. $x^4 - x^3 = x^2 - x$
14. $x^3 - 36x = 16(x - 6)$
15. $x^4 = 27x$
16. $3x^4 = 24x$

In Exercises 17–26, solve each equation by multiplying both sides by the LCD.

17. $\frac{x+1}{3x-2} = \frac{5x-4}{3x+2}$

18. $\frac{x}{2x+1} = \frac{3x+2}{4x+3}$

19. $\frac{1}{x} + \frac{2}{x+1} = 1$

20. $\frac{x}{x+1} + \frac{x+1}{x+2} = 1$

21. $\frac{6x-7}{x} - \frac{1}{x^2} = 5$

22. $\frac{1}{x} + \frac{2}{x+1} + \frac{3}{x+2} = 0$

23. $\frac{1}{x} + \frac{1}{x-3} = \frac{7}{3x-5}$

24. $\frac{x+3}{x-1} + \frac{x+4}{x+1} = \frac{8x+5}{x^2-1}$

25. $\frac{5}{x+1} - \frac{4}{2x+2} + \frac{2}{2x-1} = \frac{13}{18}$

26. $\frac{6}{2x-2} - \frac{1}{x+1} = \frac{2}{2x+2} + 1$

In Exercises 27–34, solve each equation. Check your answers.

27. $\frac{x}{x-5} - \frac{5}{x+5} = \frac{10x}{x^2-25}$

28. $\frac{x}{x-4} - \frac{4}{x+4} = \frac{8x}{x^2-16}$

29. $\frac{x}{x-3} + \frac{3}{x+3} = \frac{6x}{x^2-9}$

30. $\frac{x}{x+7} + \frac{7}{x-7} = \frac{14x}{x^2-49}$

31. $\frac{1}{x-1} + \frac{x}{x+3} = \frac{4}{x^2+2x-3}$

32. $\frac{x}{x-2} + \frac{2}{x+3} = -\frac{10}{x^2+x-6}$

33. $\frac{2x}{x+3} - \frac{x}{x-1} = \frac{14}{x^2+2x-3}$

34. $\frac{2(x+1)}{x-2} - \frac{x}{x+1} = \frac{9}{x^2-x-2}$

In Exercises 35–56, solve each equation. Check your answers.

35. $\sqrt[3]{3x-1} = 2$

36. $\sqrt[3]{2x+3} = 3$

37. $\sqrt{x-1} = -2$

38. $\sqrt{3x+4} = -1$

39. $x + \sqrt{x+6} = 0$

*40. $x - \sqrt{6x+7} = 0$ one real

41. $\sqrt{y+6} = y$

42. $r + 11 = 6\sqrt{r+3}$

43. $\sqrt{6y-11} = 2y - 7$

44. $\sqrt{3y+1} = y - 1$

45. $t - \sqrt{3t+6} = -2$

46. $\sqrt{5x^2-10x+9} = 2x - 1$

47. $\sqrt{x-3} = \sqrt{2x-5} - 1$

48. $x + \sqrt{x+1} = 5$

49. $\sqrt{2y+9} = 2 + \sqrt{y+1}$

50. $\sqrt{m} - 1 = \sqrt{m-5}$

51. $\sqrt{7z+1} - \sqrt{5z+4} = 1$

52. $\sqrt{3q+1} - \sqrt{q-1} = 2$

53. $\sqrt{2x+5} + \sqrt{x+6} = 3$

54. $\sqrt{5x-9} - \sqrt{x+4} = 1$

55. $\sqrt{2x-5} - \sqrt{x-3} = 1$

56. $\sqrt{3x+5} + \sqrt{6x+3} = 3$

In Exercises 57–60, solve each equation. Check your answers.

57. $(x-4)^{3/2} = 27$

58. $(x+7)^{3/2} = 64$

59. $(5x-3)^{2/3} - 5 = 4$

60. $(2x-6)^{2/3} + 9 = 13$

In Exercises 61–84, solve each equation by using an appropriate substitution. Check your answers.

61. $x - 5\sqrt{x} + 6 = 0$

62. $x - 3\sqrt{x} + 2 = 0$

63. $2y - 15\sqrt{y} = -7$

64. $y + 44 = 15\sqrt{y}$

65. $x^{-2} - x^{-1} - 42 = 0$

66. $x^{1/2} + 3 - 4x^{-1/2} = 0$

67. $x^{2/3} - 6x^{1/3} + 8 = 0$

68. $x^{2/5} + x^{1/5} - 2 = 0$

69. $2x^{1/2} + 3x^{1/4} - 2 = 0$

70. $2x^{1/2} - x^{1/4} - 1 = 0$

71. $x^4 - 13x^2 + 36 = 0$

72. $x^4 - 7x^2 + 12 = 0$

73. $2t^4 + t^2 - 1 = 0$

74. $81y^4 + 1 = 18y^2$

75. $p^2 - 3 + 4\sqrt{p^2-3} - 5 = 0$

76. $x^2 - 3 - 4\sqrt{x^2-3} - 12 = 0$

77. $(3t+1)^2 - 3(3t+1) + 2 = 0$

78. $(7z+5)^2 + 2(7z+5) - 15 = 0$

79. $(\sqrt{y}+5)^2 - 9(\sqrt{y}+5) + 20 = 0$

80. $(2\sqrt{t}+1)^2 - 2(2\sqrt{t}+1) - 3 = 0$

81. $(x^2-4)^2 - 3(x^2-4) - 4 = 0$

82. $(x^2+2)^2 - 5(x^2+2) - 6 = 0$

83. $(x^2-3x)^2 - 2(x^2-3x) - 8 = 0$

84. $(x^2-4x)^2 + 7(x^2-4x) + 12 = 0$

Applying the Concepts

85. **Fractions.** The numerator of a fraction is two less than the denominator. The sum of the fraction and its reciprocal is $\frac{25}{12}$. Find the numerator and denominator assuming that each is a positive integer.

86. **Fractions.** The numerator of a fraction is five less than the denominator. The sum of the fraction and six times its reciprocal is $\frac{25}{2}$. Find the numerator and denominator assuming that each is an integer.

87. **Investment.** Latasha bought some stock for \$1800. If the price of each share of the stock had been \$18 less, she could have bought five more shares for the same \$1800.
   a. How many shares of the stock did she buy?
   b. How much did she pay for each share?

88. **Bus charter.** A civic club charters a bus for one day at a cost of \$575. When two more people join the group, each person's cost decreases by \$2. How many people were in the original group?

89. **Depth of a well.** A stone is dropped into a well, and the time it takes to hear the splash is 4 seconds. Find the depth of the well (to the nearest foot). Assume that the speed of sound is 1100 feet per second and that $d = 16t^2$ is the distance d an object falls in t seconds.

90. **Estimating the horizon.** You are in a hot air balloon 2 miles above the ocean. (See the accompanying diagram.)

SECTION 1.5 Exercises

Basic Concepts and Skills

In Exercises 1–10, fill in the blank with the correct inequality symbol using the rules for producing equivalent inequalities.

1. If $x < 8$, then $x - 8$ _____ 0.
2. If $x \le -3$, then $x + 3$ _____ 0.
3. If $x \ge 5$, then $x - 5$ _____ 0.
4. If $x > 7$, then $x - 7$, _____ 0.
5. If $\frac{x}{2} < 6$, then x _____ 12.
6. If $\frac{x}{3} < 2$, then x _____ 6.
7. If $-2x \le 4$, then x _____ -2.
8. If $-3x > 12$, then x _____ -4.
9. If $x + 3 < 2$, then x _____ -1.
10. If $x - 5 \le 3$, then x _____ 8.

In Exercises 11–18, graph the solution set of each inequality and write it in interval notation.

11. $-2 < x < 5$
12. $-5 \le x \le 0$
13. $0 < x \le 4$
14. $1 \le x < 7$
15. $x \ge -1$
16. $x > 2$
17. $-5x \ge 10$
18. $-2x < 2$

In Exercises 19–40, solve each inequality. Write the solution in interval notation and graph the solution set.

19. $x + 3 < 6$
20. $x - 2 < 3$
21. $1 - x \le 4$
22. $7 - x > 3$
23. $2x + 5 < 9$
24. $3x + 2 \ge 7$
25. $3 - 3x > 15$
26. $8 - 4x \ge 12$
27. $3(x + 2) < 2x + 5$
28. $4(x - 1) \ge 3x - 1$
29. $3(x - 3) \le 3 - x$
30. $-x - 2 \ge x - 10$
31. $6x + 4 > 3x + 10$
32. $4(x - 4) > 3(x - 5)$
33. $8(x - 1) - x \le 7x - 12$
34. $3(x + 2) + 2x \ge 5x + 18$
35. $5(x + 2) \le 3(x + 1) + 10$
36. $x - 4 > 2(x + 8)$
37. $2(x + 1) + 3 \ge 2(x + 2) - 1$
38. $5(2 - x) + 4x \le 12 - x$
39. $2(x + 1) - 2 \le 3(2 - x) + 9$
40. $4(1 - x) + 2x > 5(2 - x) + 4x$

In Exercises 41–50, solve each rational inequality.

41. $9x - 6 \ge \frac{3}{2}x + 9$
42. $\frac{7x - 3}{2} < 3x - 4$
43. $\frac{x - 3}{3} \le 2 + \frac{x}{2}$
44. $\frac{2x - 3}{4} \ge 3 - \frac{x}{2}$
45. $\frac{3x + 1}{2} < x - 1 + \frac{x}{2}$
46. $\frac{2x - 1}{3} \ge \frac{x + 1}{4} + \frac{x}{12}$
47. $\frac{x - 3}{2} \ge \frac{x}{3} + 1$
48. $\frac{2x + 1}{3} < \frac{x - 1}{2} + \frac{1}{6}$
49. $\frac{3x + 1}{3} - \frac{x}{2} \le \frac{x + 2}{2}$
50. $\frac{x - 1}{3} + \frac{x + 1}{4} \le \frac{x}{2} + \frac{x}{12}$

In Exercises 51–58, solve each compound *or* inequality.

51. $2x + 5 < 1$ or $2 + x > 4$
52. $3x - 2 > 7$ or $2(1 - x) > 1$
53. $\frac{2x - 3}{4} \le 2$ or $\frac{4 - 3x}{2} \ge 2$
54. $\frac{5 - 3x}{3} \ge \frac{1}{6}$ or $\frac{x - 1}{3} \le 1$
55. $\frac{2x + 1}{3} \ge x + 1$ or $\frac{x}{2} - 1 > \frac{x}{3}$
56. $\frac{x + 2}{2} < \frac{x}{3} + 1$ or $\frac{x - 1}{3} > \frac{x + 1}{5}$
57. $\frac{x - 1}{2} > \frac{x}{3} - 1$ or $\frac{2x + 5}{3} \le \frac{x + 1}{6}$
58. $\frac{2x + 1}{3} \le \frac{x}{4} + 1$ or $\frac{3 - x}{2} > \frac{x}{3} - 1$

In Exercises 59–66, solve each compound *and* inequality.

59. $3 - 2x \le 7$ and $2x - 3 \le 7$
60. $6 - x \le 3x + 10$ and $7x - 14 \le 3x + 14$
61. $2(x + 1) + 3 \ge 1$ and $2(2 - x) > -6$
62. $3(x + 1) - 2 \ge 4$ and $3(1 - x) + 13 > 4$
63. $2(x + 1) - 3 > 7$ and $3(2x + 1) + 1 < 10$
64. $5(x + 2) + 7 < 2$ and $2(5 - 3x) + 1 < 17$
65. $5 + 3(x - 1) < 3 + 3(x + 1)$ and $3x - 7 \le 8$
66. $2x - 3 > 11$ and $5(2x + 1) < 3(4x + 1) - 2(x - 1)$

In Exercises 67–78, solve each compound inequality.

67. $3 < x + 5 < 4$
68. $9 \le x + 7 \le 12$
69. $-4 \le x - 2 < 2$
70. $-3 < x + 5 < 4$
71. $-9 \le 2x + 3 \le 5$
72. $-2 \le 3x + 1 \le 7$
73. $0 \le 1 - \frac{x}{3} < 2$
74. $0 < 5 - \frac{x}{2} \le 3$
75. $-1 < \frac{2x - 3}{5} \le 0$
76. $-4 \le \frac{5x - 2}{3} \le 0$
77. $5x \le 3x + 1 < 4x - 2$
78. $3x + 2 < 2x + 3 < 4x - 1$

In Exercises 79–84, find a and b.

79. If $-2 < x < 1$, then $a < x + 7 < b$.

80. If $1 < x < 5$, then $a < 2x + 3 < b$.

81. If $-1 < x < 1$, then $a < 2 - x < b$.

82. If $3 < x < 7$, then $a < 1 - 3x < b$.

83. If $0 < x < 4$, then $a < 5x - 1 < b$.

84. If $-4 < x < 0$, then $a < 3x + 4 < b$.

In Exercises 85–100, use the test-point method to solve each polynomial inequality.

85. $x^2 + 4x - 12 \le 0$

86. $x^2 - 8x + 7 > 0$

87. $6x^2 + 7x - 3 \ge 0$

88. $4x^2 - 2x - 2 < 0$

89. $(x + 3)(x + 1)(x - 1) \ge 0$

90. $(x + 4)(x - 1)(x + 2) \le 0$

91. $x^3 - 4x^2 - 12x > 0$

92. $x^3 + 8x^2 + 15x > 0$

93. $x^2 + 2x < -1$

94. $4x^2 + 12x < -9$

95. $x^3 - x^2 \ge 0$

96. $x^3 - 9x^2 \ge 0$

97. $x^2 \ge 1$

98. $x^4 \le 16$

99. $x^3 < -8$

100. $x^4 > 9$

In Exercises 101–120, solve each rational inequality.

101. $\dfrac{x + 2}{x - 5} < 0$

102. $\dfrac{x - 3}{x + 1} > 0$

103. $\dfrac{x + 4}{x} < 0$

104. $\dfrac{x}{x - 2} > 0$

105. $\dfrac{x + 1}{x + 2} \le 3$

106. $\dfrac{x - 1}{x - 2} \ge 3$

107. $\dfrac{(x - 2)(x + 2)}{x} > 0$

108. $\dfrac{(x - 1)(x + 3)}{x - 2} < 0$

109. $\dfrac{(x - 2)(x + 1)}{(x - 3)(x + 5)} \ge 0$

110. $\dfrac{(x - 1)(x - 3)}{(x + 2)(x + 4)} \ge 0$

111. $\dfrac{x^2 - 1}{x^2 - 4} \le 0$

112. $\dfrac{x^2 - 9}{x^2 - 64} \le 0$

113. $\dfrac{x + 4}{3x - 2} \ge 1$

114. $\dfrac{2x - 3}{x + 3} \le 1$

115. $3 \le \dfrac{2x + 6}{2x + 1}$

116. $\dfrac{x - 2}{2x + 1} < -1$

117. $\dfrac{x + 2}{x - 3} \ge \dfrac{x - 1}{x + 3}$

118. $\dfrac{x + 1}{x - 2} \ge \dfrac{x}{x - 1}$

119. $\dfrac{x - 1}{x + 1} \le \dfrac{x + 2}{x - 3}$

120. $\dfrac{x + 3}{x + 1} \le \dfrac{x - 1}{x - 2}$

Applying the Concepts

121. Appliance markup. The markup over the dealer's cost on a new refrigerator ranges from 15% to 20%. If the dealer's cost is \$1750, over what range will the selling price vary?

122. Return on investment. An investor has \$5000 to invest for a period of one year. Find the range of per annum simple interest rates required to generate interest between \$200 and \$275 inclusive.

123. Hybrid car trip. Sometime after passing a truck stop 300 miles from the start of her trip, Cora's hybrid car ran out of gas. Assuming that the tank could hold 12 gallons of gasoline and the hybrid car averaged 40 miles per gallon, find the range of gasoline (in gallons) that could have been in the tank at the start of the trip.

124. Average grade. Sean has taken three exams and earned scores of 85, 72, and 77 out of a possible 100 points. He needs an average of at least 80 to earn a B in the course. What range of scores on the fourth (and last) 100-point test will guarantee a B in the course?

125. Butterfat content. How much cream that is 30% butterfat must be added to milk that is 3% butterfat to have 270 quarts that are at least 4.5% butterfat?

126. Pedometer cost. A company produces a pedometer at a cost of \$3 each and sells the pedometer for \$5 each. If the company has to recover an initial expense of \$4000 before any profit is earned, how many pedometers must be sold to earn a profit in excess of \$3000?

127. Car sales. A car dealer has three times as many SUVs and twice as many convertibles as four-door sedans. How many four-door sedans does the dealer have if she has at least 48 cars of these three types?

128. Temperature conversion. The formula for converting Fahrenheit temperature F to Celsius temperature C is $C = \frac{5}{9}(F - 32)$. What range in Celsius degrees corresponds to a range of 68° to 86° Fahrenheit?

129. Temperature. The number N of water mites in a water sample depends on the temperature t in degrees Fahrenheit and is given by $N = 132t - t^2$. At what temperature will the number of mites exceed 3200?

SECTION 1.6 Exercises

Basic Concepts and Skills

In Exercises 1–6, assume that $a > 0$.

1. The solution set of the equation $|x| = a$ is _____.
2. The solution set of the inequality $|x| < a$ is _____.
3. The solution set of the inequality $|x| \geq a$ is _____.
4. The equation $|u| = |v|$ is equivalent to $u =$ _____ or $u =$ _____.
5. **True or False.** The solution set of $|3x - 2| < a$ is the same as the solution set of $|2 - 3x| < a$.
6. **True or False.** If $a > 0$, the solution set of $|3x - 2| \leq -a$ is $\varnothing$.

In Exercises 7–34, solve each equation.

7. $|3x| = 9$
8. $|4x| = 24$
9. $|-2x| = 6$
10. $|-x| = 3$
11. $|x + 3| = 2$
12. $|x - 4| = 1$
13. $|6 - 2x| = 8$
14. $|6 - 3x| = 9$
15. $|6x - 2| = 9$
16. $|6x - 3| = 9$
17. $|2x + 3| - 1 = 0$
18. $|2x - 3| - 1 = 0$
19. $\frac{1}{2}|x| = 3$
20. $\frac{3}{5}|x| = 6$
21. $\left|\frac{1}{4}x + 2\right| = 3$
22. $\left|\frac{3}{2}x - 1\right| = 3$
23. $6|1 - 2x| - 8 = 10$
24. $5|1 - 4x| + 10 = 15$
25. $2|3x - 4| + 9 = 7$
26. $9|2x - 3| + 2 = -7$
27. $|2x + 1| = -1$
28. $|3x + 7| = -2$
29. $|x^2 - 4| = 0$
30. $|9 - x^2| = 0$
31. $|1 - 2x| = 3$
32. $|4 - 3x| = 5$
33. $\left|\frac{1}{3} - x\right| = \frac{2}{3}$
34. $\left|\frac{2}{5} - x\right| = \frac{1}{5}$

In Exercises 35–44, solve each equation.

35. $|x + 3| = |x + 5|$
36. $|x + 4| = |x - 8|$
37. $|3x - 2| = |6x + 7|$
38. $|2x - 4| = |4x + 6|$
39. $|2x - 1| = x + 1$
40. $|3x - 4| = 2(x - 1)$
41. $|4 - 3x| = x - 1$
42. $|2 - 3x| = 2x - 1$
43. $|3x + 2| = 2(x - 1)$
44. $|4x + 7| = x + 1$

In Exercises 45–60, solve each inequality.

45. $|3x| < 12$
46. $|2x| \leq 6$
47. $|4x| > 16$
48. $|3x| > 15$
49. $|x + 1| < 3$
50. $|x - 4| < 1$
51. $|x| + 2 \geq -1$
52. $|x| + 2 > -7$
53. $|2x - 3| < 4$
54. $|4x - 6| \leq 6$
55. $|5 - 2x| > 3$
56. $|3x - 3| \geq 15$
57. $|3x + 4| \leq 19$
58. $|9 - 7x| < 23$
59. $|2x - 15| < 0$
60. $|x + 5| \leq -3$

In Exercises 61–70, solve each inequality.

61. $\left|\frac{x - 2}{x + 3}\right| < 1$
62. $\left|\frac{x + 3}{x - 1}\right| < 2$
63. $\left|\frac{2x - 3}{x + 1}\right| \leq 3$
64. $\left|\frac{2x - 1}{3x + 2}\right| \leq 1$
65. $\left|\frac{x - 1}{x + 2}\right| \geq 2$
66. $\left|\frac{x + 3}{x - 2}\right| \geq 3$
67. $\left|\frac{2x + 1}{x - 1}\right| > 4$
68. $\left|\frac{2x - 1}{3x + 2}\right| > 5$
69. $|x - 1| \leq 2|2x - 5|$
70. $2|x - 5| \leq |2x - 3|$

Applying the Concepts

71. **Varying temperatures.** The inequality $|T - 75| \leq 20$, where T is in degrees Fahrenheit, describes the daily temperature in Tampa during December. Give an interpretation for this inequality assuming that the high and low temperatures in Tampa during December satisfy the equation $|T - 75| = 20$.
72. **Scale error.** A butcher's scale is accurate to within ± 0.05 pound. A sirloin steak weighs 1.14 pounds on this scale. Let $x =$ actual weight of the steak. Write an absolute value inequality in x whose solution is the range of possible values for the actual weight of the steak.
73. **Company budget.** A company budgets \$700 for office supplies. The actual expense for budget supplies must be within $\pm$ \$50 of this figure. Let $x =$ actual expense for the office supplies. Write an absolute value inequality in x whose solution is the range of possible amounts for the expense of the office supplies.
74. **National achievement scores.** Suppose 68% of the scores on a national achievement exam will be within ± 90 points of a score of 480. Let $x =$ a score among the 68% just described. Write an absolute value inequality in x whose solution is the range of possible scores within ± 90 points of 480.
75. **Blood pressure.** Suppose 60% of Americans have a systolic blood pressure reading of 120, plus or minus 6.75. Let $x =$ a blood pressure reading among the 60% just described. Write an absolute value inequality in x whose solution set is