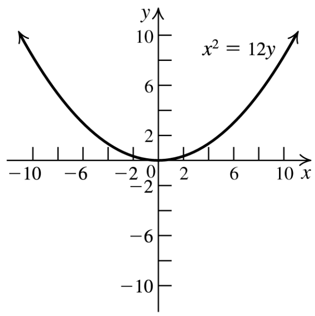
Chapter 10 Conic Sections

10.2 The Parabola

10.2 Practice Problems

1. a.



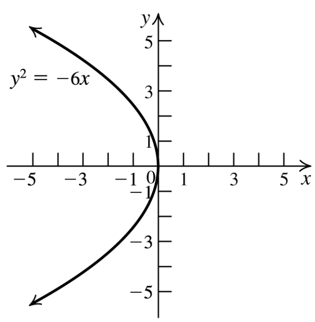
Vertex: (0, 0)

$x^2 = 12y = 4ay \Rightarrow a = 3,$ so the focus is (0, 3).

The directrix is $y = -3$.

The axis is the y-axis.

b.



Vertex: (0, 0)

$y^2 = -6x = 4ax \Rightarrow a = -\frac{3}{2},$ so the focus is $\left(-\frac{3}{2}, 0\right)$.

The directrix is $x = \frac{3}{2}$.

The axis is the x-axis.

2. a. The vertex is (0, 0) and the focus is (0, 2), so the graph opens up. The general form is $x^2 = 4ay$.

$a = 2 \Rightarrow x^2 = 4(2)y \Rightarrow x^2 = 8y$.

b. Since the vertex is (0, 0), the axis of the parabola is the x-axis, and the parabola passes through (1, 2) which is to the right of the vertex, the parabola opens to the right and the general form is $y^2 = 4ax$.

We need to solve for a:

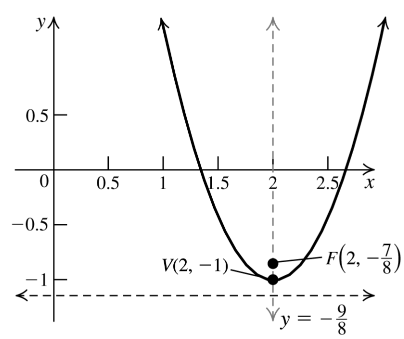
$2^2 = 4a(1) \Rightarrow 4 = 4a \Rightarrow 1 = a.$ The equation is $y^2 = 4x$.

3. Rewrite the equation as $2\left(x^2 - 4x\right) = y - 7$, then complete the square to put the equation into standard form:

$$\begin{aligned} 2x^2 - 8x - y + 7 &= 0 \\ 2x^2 - 8x &= y - 7 \\ 2\left(x^2 - 4x\right) &= y - 7 \\ 2\left(x^2 - 4x + 4\right) &= y - 7 + 2(4) \\ 2(x-2)^2 &= y + 1 \\ (x-2)^2 &= \frac{1}{2}(y+1) \end{aligned}$$

$4a = \frac{1}{2} \Rightarrow a = \frac{1}{8};$ vertex $= (h, k) = (2, -1)$

The parabola opens up, so the focus is at $(h, k + a) = \left(2, -1 + \frac{1}{8}\right) = \left(2, -\frac{7}{8}\right)$. The directrix is located at $y = k - a = -1 - \frac{1}{8} = -\frac{9}{8}$.



4. The equation is $x^2 = 4ay \Rightarrow x^2 = 4(7.3)y \Rightarrow x^2 = 29.2y$. To find the thickness y of the mirror at the edge, substitute $x = 1.5$ (half the diameter) in the equation and solve for y.

$1.5^2 = 29.2y \Rightarrow y \approx 0.077055$

The mirror is about 0.077055 in. thick at the edge.

10.2 Basic Concepts Skills

1. A parabola is the set of all points P in the plane that are equidistant from a fixed line called the directrix and a fixed point not on the line called the focus.
2. The line segment through the focus perpendicular to the directrix with endpoints on the parabola is called the latus rectum of the parabola.
3. The point at which the axis intersects the parabola is called the vertex of the parabola.
4. The graph of $y+2=3(x-4)^2$ is obtained by shifting the graph of $y=3x^2$ to the right four units and shifting down two units.
5. True
6. True
7. $x^2=2y=4ay \Rightarrow a=\frac{1}{2}$

   Focus: $\left(0,\frac{1}{2}\right)$, directrix: $y=-\frac{1}{2}$, graph (e)
8. $9x^2=4y \Rightarrow x^2=\frac{1}{9}(4a)y \Rightarrow a=\frac{1}{9}$

   Focus: $\left(0,\frac{1}{9}\right)$, directrix: $y=-\frac{1}{9}$, graph (h)
9. $16x^2=-9y \Rightarrow x^2=-\frac{9}{16}y=4ay \Rightarrow a=-\frac{9}{64}$

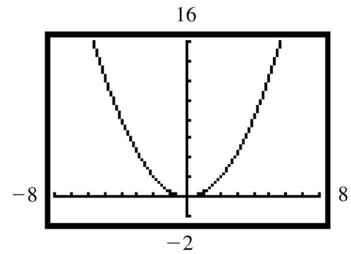
   Focus: $\left(0,-\frac{9}{64}\right)$, directrix: $y=\frac{9}{64}$, graph (d)
10. $x^2=-2y=4ay \Rightarrow a=-\frac{1}{2}$

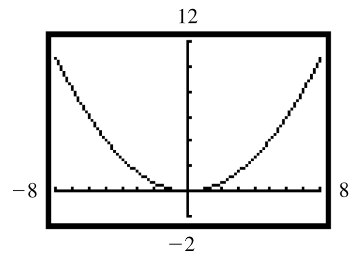
    Focus: $\left(0,-\frac{1}{2}\right)$, directrix: $y=\frac{1}{2}$, graph (a)
11. $y^2=2x=4ax \Rightarrow a=\frac{1}{2}$

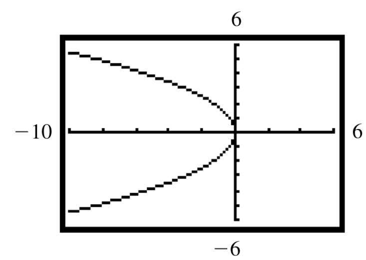
    Focus: $\left(\frac{1}{2},0\right)$, directrix: $x=-\frac{1}{2}$, graph (g)
12. $9y^2=16x \Rightarrow y^2=\frac{16}{9}x=4ax \Rightarrow a=\frac{4}{9}$

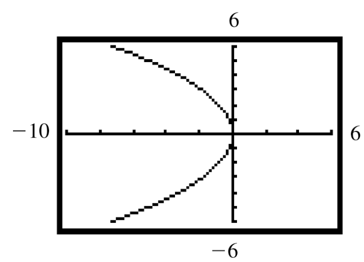
    Focus: $\left(\frac{4}{9},0\right)$, directrix: $x=-\frac{4}{9}$, graph (c)
13. $9y^2=-16x \Rightarrow y^2=-\frac{16}{9}x=4ax \Rightarrow a=-\frac{4}{9}$

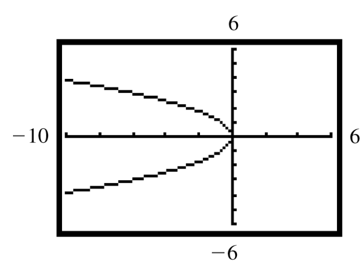
    Focus: $\left(-\frac{4}{9},0\right)$, directrix: $x=\frac{4}{9}$, graph (f)
14. $y^2=-2x=4ax \Rightarrow a=-\frac{1}{2}$

    Focus: $\left(-\frac{1}{2},0\right)$, directrix: $x=\frac{1}{2}$, graph (b)
15. 

16. 

17. 

18. 

19. 

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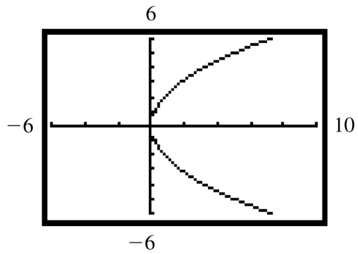


21. The focus is (0, 2) and the directrix is $y = 4$, so the vertex is (0, 3), and the graph opens down. The general form is $(x-h)^2 = -4a(y-k)$. $a = 3-2 = 1 \Rightarrow x^2 = -4(y-3)$. The length of the latus rectum = 4(1) = 4.

22. The focus is (0, 4) and the directrix is $y = -2$, so the vertex is (0, 1), and the graph opens up. The general form is $(x-h)^2 = 4a(y-k)$.
$a = 4-1 = 3 \Rightarrow x^2 = 4(3)(y-1) \Rightarrow$
$x^2 = 12(y-1)$.
The length of the latus rectum = 4(3) = 12.

23. The focus is (−2, 0) and the directrix is $x = 3$, so the vertex is $(1/2, 0)$, and the graph opens to the left. The general form is
$(y-k)^2 = -4a(x-h)$. $a = \frac{1}{2} - (-2) = \frac{5}{2} \Rightarrow$
$y^2 = -4\left(\frac{5}{2}\right)\left(x - \frac{1}{2}\right) \Rightarrow y^2 = -10\left(x - \frac{1}{2}\right)$.
The length of the latus rectum = $4\left(\frac{5}{2}\right) = 10$.

24. The focus is (−1, 0) and the directrix is $x = -2$, so the vertex is $(-3/2, 0)$, and the graph opens to the right. The general form is
$(y-k)^2 = 4a(x-h)$.
$a = -1 - \left(-\frac{3}{2}\right) = \frac{1}{2} \Rightarrow$
$y^2 = 4\left(\frac{1}{2}\right)\left(x - \left(-\frac{3}{2}\right)\right) \Rightarrow y^2 = 2\left(x + \frac{3}{2}\right)$.
The length of the latus rectum = $4\left(\frac{1}{2}\right) = 2$.

25. The vertex is (1, 1) and the directrix is $x = 3$, so the graph opens to the left. The general form is $(y-k)^2 = -4a(x-h)$.
$a = 3-1 = 2 \Rightarrow$
$(y-1)^2 = -4(2)(x-1) \Rightarrow (y-1)^2 = -8(x-1)$.
The length of the latus rectum = 4(2) = 8.

26. The vertex is (1, 1) and the directrix is $y = 2$, so the graph opens down. The general form is $(x-h)^2 = -4a(y-k)$. $a = 2-1 = 1 \Rightarrow$
$(x-1)^2 = -4(1)(y-1) \Rightarrow (x-1)^2 = -4(y-1)$.
The length of the latus rectum = 4(1) = 4.

27. The vertex is (1, 1) and the directrix is $y = -3$, so the graph opens up. The general form is $(x-h)^2 = 4a(y-k)$. $a = 1-(-3) = 4 \Rightarrow$
$(x-1)^2 = 4(4)(y-1) \Rightarrow (x-1)^2 = 16(y-1)$.
The length of the latus rectum = 4(4) = 16.

28. The vertex is (1, 1) and the directrix is $x = -2$, so the graph opens to the right. The general form is $(y-k)^2 = 4a(x-h)$.
$a = 1-(-2) = 3 \Rightarrow (y-1)^2 = 4(3)(x-1) \Rightarrow$
$(y-1)^2 = 12(x-1)$. The length of the latus rectum = 4(3) = 12.

29. The vertex is (1, 0) and the focus is (3, 0), so the graph opens to the right. The general form is $(y-k)^2 = 4a(x-h)$. $a = 3-1 = 2 \Rightarrow$
$(y-0)^2 = 4(2)(x-1) \Rightarrow y^2 = 8(x-1)$. The length of the latus rectum = 4(2) = 8.

30. The vertex is (0, 1) and the focus is (0, 2), so the graph opens up. The general form is $(x-h)^2 = 4a(y-k)$. $a = 2-1 = 1 \Rightarrow$
$(x-0)^2 = 4(1)(y-1) \Rightarrow x^2 = 4(y-1)$. The length of the latus rectum = 4(1) = 4.

31. The vertex is (0, 1) and the focus is (0, −2), so the graph opens down. The general form is $(x-h)^2 = -4a(y-k)$. $a = 1-(-2) = 3 \Rightarrow$
$(x-0)^2 = -4(3)(y-1) \Rightarrow x^2 = -12(y-1)$.
The length of the latus rectum = 4(3) = 12.

32. The vertex is (−1, 0) and the focus is (−3, 0), so the graph opens to the left. The general form is $(y-k)^2 = -4a(x-h)$.
$a = -1-(-3) = 2 \Rightarrow$
$(y-0)^2 = -4(2)(x-(-1)) \Rightarrow y^2 = -8(x+1)$.
The length of the latus rectum = 4(2) = 8.

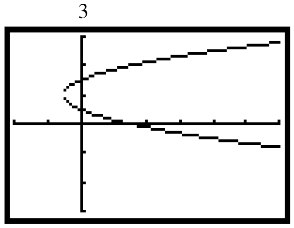
33. The vertex is (2, 3) and the directrix is $x = 4$, so the graph opens to the left. The general form is $(y-k)^2 = -4a(x-h)$. $a = 4-2 = 2 \Rightarrow$
$(y-3)^2 = -4(2)(x-2) \Rightarrow (y-3)^2 = -8(x-2)$.
The length of the latus rectum = 4(2) = 8.

34. The vertex is (2, 3) and the directrix is $y = 4$ so the graph opens down. The general form is $(x-h)^2 = -4a(y-k)$. $a = 4-3 = 1 \Rightarrow$
$(x-2)^2 = -4(1)(y-3) \Rightarrow (x-2)^2 = -4(y-3)$.
The length of the latus rectum = 4(1) = 4.

35. The vertex is (2, 3) and the directrix is $y = 1$ so the graph opens up. The general form is $(x-h)^2 = 4a(y-k)$. $a = 3-1 = 2 \Rightarrow$
$(x-2)^2 = 4(2)(y-3) \Rightarrow (x-2)^2 = 8(y-3)$.
The length of the latus rectum = 4(2) = 8.

36. The vertex is (2, 3) and the directrix is $x = 1$, so the graph opens to the right. The general form is $(y-k)^2 = 4a(x-h)$. $a = 2-1 = 1 \Rightarrow$
$(y-3)^2 = 4(1)(x-2) \Rightarrow (y-3)^2 = 4(x-2)$.
The length of the latus rectum = 4(1) = 4.

37.



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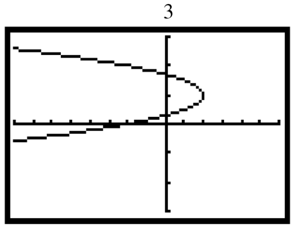

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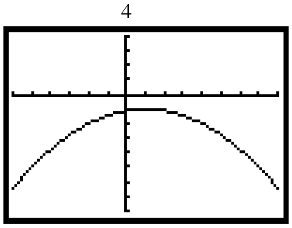

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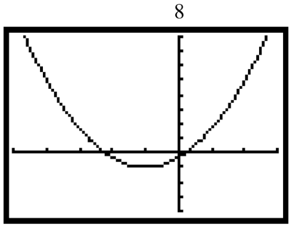
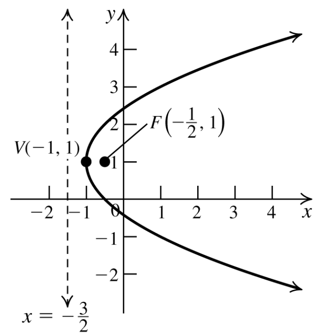
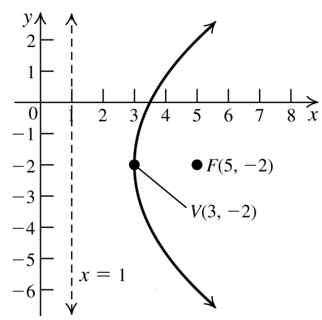


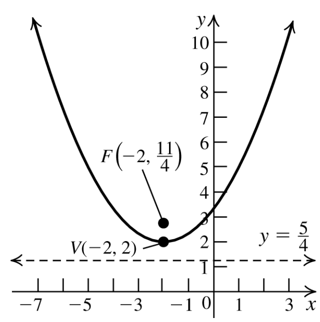
41. $4a = 2 \Rightarrow a = 1/2$. The graph opens to the right, so the focus is $(h + a, k)$, and the directrix is $x = h - a$. Vertex: (−1, 1), focus $(-1/2, 1)$, directrix: $x = -3/2$.



42. $4a = 8 \Rightarrow a = 2$. The graph opens to the right, so the focus is $(h + a, k)$, and the directrix is $x = h - a$. Vertex: (3, −2), focus (5, −2), directrix: $x = 1$.



43. $4a = 3 \Rightarrow a = 3/4$. The graph opens up, so the focus is $(h, k + a)$, and the directrix is $y = k - a$. Vertex: (−2, 2), focus $(-2, 11/4)$, directrix: $y = 5/4$.



44. $4a = 4 \Rightarrow a = 1$. The graph opens up, so the focus is $(h, k + a)$, and the directrix is $y = k - a$. Vertex: (3, −1), focus (3, 0), directrix: $y = -2$.

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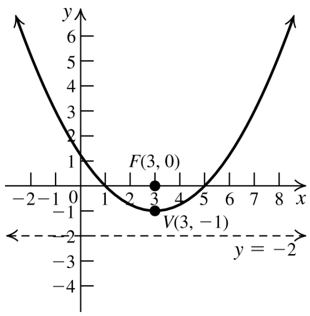
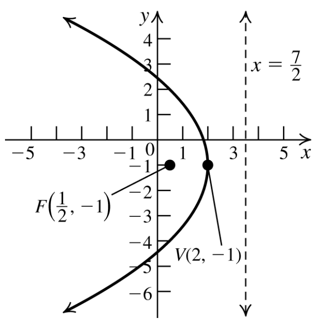
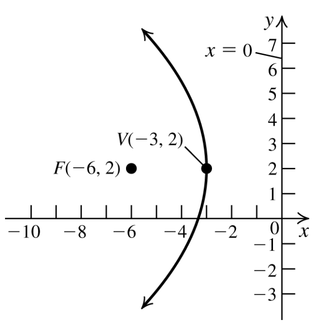


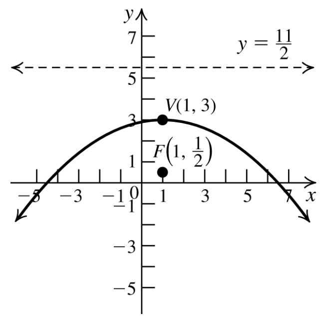
45. $4a = 6 \Rightarrow a = 3/2$. The graph opens to the left, so the focus is (*h* – *a*, *k*), and the directrix is *x* = *h* + *a*. Vertex: (2, –1), focus $(1/2, -1)$, directrix: $x = 7/2$.



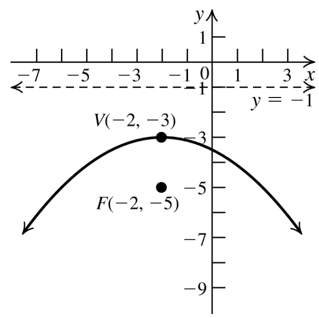
46. $4a = 12 \Rightarrow a = 3$. The graph opens to the left, so the focus is (*h* – *a*, *k*), and the directrix is *x* = *h* + *a*. Vertex: (–3, 2), focus (–6, 2), directrix: $x = 0$ (*y*-axis).



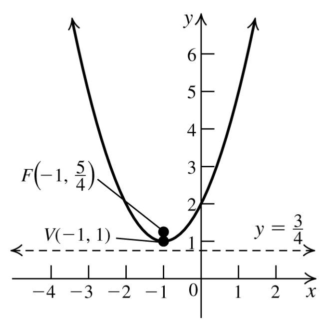
47. $4a = 10 \Rightarrow a = 5/2$. The graph opens down, so the focus is (*h*, *k* – *a*), and the directrix is *y* = *k* + *a*. Vertex: (1, 3), focus $(1, 1/2)$, directrix: $y = 11/2$.



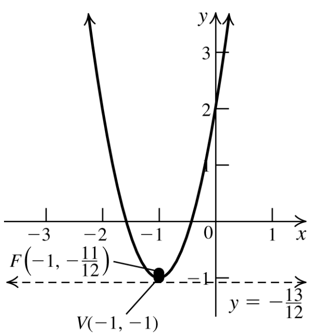
48. $4a = 8 \Rightarrow a = 2$. The graph opens down, so the focus is (*h*, *k* – *a*), and the directrix is *y* = *k* + *a*. Vertex: (–2, –3), focus (–2, –5), directrix: $y = -1$.



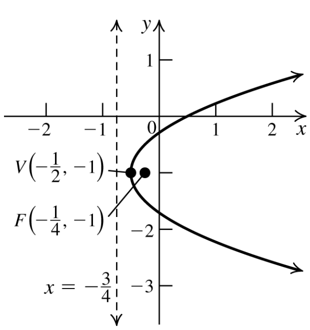
49. Rewrite the equation as $y - 2 = x^2 + 2x$, then complete the square to put the equation into standard form: $y - 2 + 1 = x^2 + 2x + 1 \Rightarrow$ $y - 1 = (x+1)^2 \Rightarrow 4a = 1 \Rightarrow a = 1/4$. The graph opens up, so the focus is (*h*, *k* + *a*), and the directrix is *y* = *k* – *a*. Vertex: (–1, 1), focus $(-1, 5/4)$, directrix: $y = 3/4$.



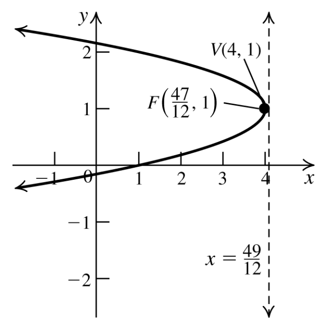
50. Rewrite the equation as $y-2=3(x^2+2x)$, then complete the square to put the equation into standard form:
$y-2+3=3(x^2+2x+1)\Rightarrow$
$y+1=3(x+1)^2\Rightarrow\frac{1}{3}(y+1)=(x+1)^2\Rightarrow$
$4a=\frac{1}{3}\Rightarrow a=\frac{1}{12}$. The graph opens up, so the focus is $(h, k+a)$, and the directrix is $y=k-a$. Vertex: $(-1,-1)$, focus $\left(-1,-\frac{11}{2}\right)$, directrix: $y=-\frac{13}{12}$.



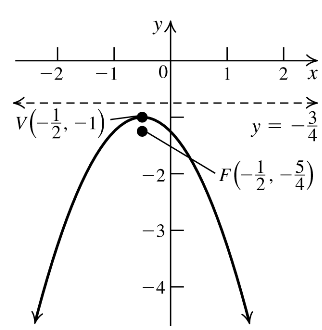
51. Rewrite the equation as $2(y^2+2y)=2x-1$, then complete the square to put the equation into standard form:
$2(y^2+2y+1)=2x-1+2\Rightarrow$
$2(y+1)^2=2\left(x+\frac{1}{2}\right)\Rightarrow(y+1)^2=x+\frac{1}{2}\Rightarrow$
$4a=1\Rightarrow a=\frac{1}{4}$. The graph opens to the right, so the focus is $(h+a, k)$, and the directrix is $x=h-a$. Vertex: $(-1/2,-1)$, focus $(-1/4,-1)$, directrix: $x=-3/4$.



52. Rewrite the equation as $3(y^2-2y)=-x+1$, then complete the square to put the equation into standard form:
$3(y^2-2y+1)=-x+1+3\Rightarrow$
$3(y-1)^2=-(x-4)\Rightarrow(y-1)^2=-\frac{1}{3}(x-4)\Rightarrow$
$4a=\frac{1}{3}\Rightarrow a=\frac{1}{12}$. The graph opens to the left, so the focus is $(h-a, k)$, and the directrix is $x=h+a$. Vertex: $(4, 1)$, focus $\left(\frac{47}{12},1\right)$, directrix: $x=\frac{49}{12}$.



53. Rewrite the equation as $x^2+x=-y-\frac{5}{4}$, then complete the square to put the equation into standard form: $x^2+x+\frac{1}{4}=-y-\frac{5}{4}+\frac{1}{4}\Rightarrow$
$\left(x+\frac{1}{2}\right)^2=-(y+1)\Rightarrow 4a=1\Rightarrow a=\frac{1}{4}$. The graph opens down, so the focus is $(h, k-a)$, and the directrix is $y=k+a$. Vertex: $(-1/2,-1)$, focus $(-1/2,-5/4)$, directrix: $y=-3/4$.



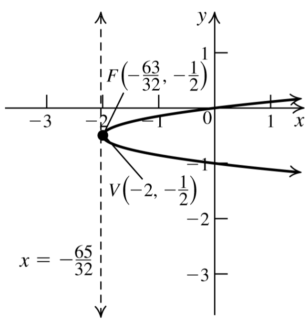
54. Rewrite the equation as $x = 8(y^2 + y)$, then complete the square to put the equation into standard form: $x + 2 = 8\left(y^2 + y + \frac{1}{4}\right) \Rightarrow$

$x + 2 = 8\left(y + \frac{1}{2}\right)^2 \Rightarrow \frac{1}{8}(x+2) = \left(y + \frac{1}{2}\right)^2 \Rightarrow$

$4a = \frac{1}{8} \Rightarrow a = \frac{1}{32}$. The graph opens to the right, so the focus is $(h + a, k)$, and the directrix is

$x = h - a$. Vertex: $\left(-2, -\frac{1}{2}\right)$, focus

$\left(-\frac{63}{32}, -\frac{1}{2}\right)$, directrix: $x = -\frac{65}{32}$.



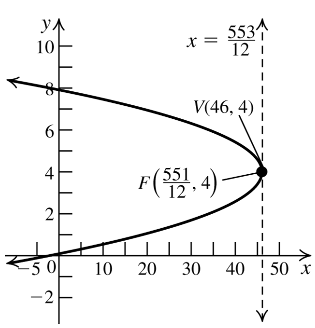
55. Complete the square to put the equation into standard form:

$x + 2 - 48 = -3\left(y^2 - 8y + 16\right) \Rightarrow$

$x - 46 = -3(y-4)^2 \Rightarrow -\frac{1}{3}(x - 46) = (y-4)^2 \Rightarrow$

$4a = \frac{1}{3} \Rightarrow a = \frac{1}{12}$. The graph opens to the left, so the focus is $(h - a, k)$, and the directrix is $x = h + a$. Vertex: (46, 4), focus $\left(\frac{551}{12}, 4\right)$,

directrix: $x = \frac{553}{12}$.



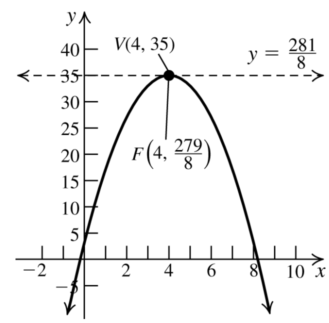
56. Complete the square to put the equation into standard form:

$y - 3 - 32 = -2(x^2 - 8x + 16) \Rightarrow$

$y - 35 = -2(x-4)^2 \Rightarrow -\frac{1}{2}(y - 35) = (x-4)^2 \Rightarrow$

$4a = \frac{1}{2} \Rightarrow a = \frac{1}{8}$. The graph opens to the left, so the focus is $(h - a, k)$, and the directrix is $y = h + a$. Vertex: (4, 35), focus $\left(4, \frac{279}{8}\right)$,

directrix: $y = \frac{281}{8}$.



57. If the axis of symmetry is the y-axis, point P is above the vertex, and the parabola opens up. Substitute the coordinates of the vertex and P into the standard equation to find a:

$(x-h)^2 = 4a(y-k) \Rightarrow (1-0)^2 = 4a(2-0) \Rightarrow$

$a = \frac{1}{8} \Rightarrow$ the equation is $x^2 = \frac{1}{2}y$.

If the axis of symmetry is the x-axis, point P is to the right of the vertex, and the parabola opens to the right. Substitute the coordinates of the vertex and P into the standard equation to find a: $(y-k)^2 = 4a(x-h) \Rightarrow$

$(2-0)^2 = 4a(1-0) \Rightarrow a = 1 \Rightarrow y^2 = 4x$.

58. If the axis of symmetry is the y-axis, point P is above the vertex, and the parabola opens up. Substitute the coordinates of the vertex and P into the standard equation to find a:

$(x-h)^2 = 4a(y-k) \Rightarrow (-3-0)^2 = 4a(2-0) \Rightarrow$

$a = \frac{9}{8} \Rightarrow$ the equation is $x^2 = \frac{9}{2}y$.

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(*continued*)

If the axis of symmetry is the x-axis, point P is to the left of the vertex, and the parabola opens to the left. Substitute the coordinates of the vertex and P into the standard equation to find a: $(y-k)^2 = -4a(x-h) \Rightarrow$

$(2-0)^2 = -4a(-3-0) \Rightarrow a = \dfrac{1}{3} \Rightarrow$

the equation is $y^2 = -\dfrac{4}{3}x$.

59. If the axis of symmetry is parallel to the y-axis, point P is above the vertex, and the parabola opens up. Substitute the coordinates of the vertex and P into the standard equation to find a: $(x-h)^2 = 4a(y-k) \Rightarrow$

$(2-0)^2 = 4a(3-1) \Rightarrow a = 1/2 \Rightarrow$ the equation is $x^2 = 2(y-1)$.

If the axis of symmetry is parallel to the x-axis, point P is to the right of the vertex, and the parabola opens to the right. Substitute the coordinates of the vertex and P into the standard equation to find a:

$(y-k)^2 = 4a(x-h) \Rightarrow$

$(3-1)^2 = 4a(2-0) \Rightarrow a = 1/2 \Rightarrow$ the equation is $(y-1)^2 = 2x$.

60. If the axis of symmetry is parallel to the y-axis, point P is below the vertex, and the parabola opens down. Substitute the coordinates of the vertex and P into the standard equation to find a:

$(x-h)^2 = -4a(y-k) \Rightarrow$

$(2-1)^2 = -4a(1-2) \Rightarrow a = 1/4 \Rightarrow$ the equation is $(x-1)^2 = -(y-2)$.

If the axis of symmetry is parallel to the x-axis, point P is to the right of the vertex, and the parabola opens to the right. Substitute the coordinates of the vertex and P into the standard equation to find a:

$(y-k)^2 = 4a(x-h) \Rightarrow$

$(1-2)^2 = 4a(2-1) \Rightarrow a = \dfrac{1}{4} \Rightarrow$ the equation is $(y-2)^2 = x-1$.

61. If the axis of symmetry is parallel to the y-axis, point P is below the vertex, and the parabola opens down. Substitute the coordinates of the vertex and P into the standard equation to find a:

$(x-h)^2 = -4a(y-k) \Rightarrow$

$(-3-(-2))^2 = -4a(0-1) \Rightarrow a = \dfrac{1}{4} \Rightarrow$ the equation is $(x+2)^2 = -(y-1)$.

If the axis of symmetry is parallel to the x-axis, point P is to the left of the vertex, and the parabola opens to the left. Substitute the coordinates of the vertex and P into the standard equation to find a:

$(y-k)^2 = -4a(x-h) \Rightarrow$

$(0-1)^2 = -4a(-3-(-2)) \Rightarrow a = \dfrac{1}{4} \Rightarrow$ the equation is $(y-1)^2 = -(x+2)$.

62. If the axis of symmetry is parallel to the y-axis, point P is above the vertex, and the parabola opens up. Substitute the coordinates of the vertex and P into the standard equation to find a: $(x-h)^2 = 4a(y-k) \Rightarrow$

$(0-1)^2 = 4a(0-(-1)) \Rightarrow a = \dfrac{1}{4} \Rightarrow$ the equation is $(x-1)^2 = y+1$. If the axis of symmetry is parallel to the x-axis, point P is to the left of the vertex, and the parabola opens to the left. Substitute the coordinates of the vertex and P into the standard equation to find a: $(y-k)^2 = -4a(x-h) \Rightarrow$

$(0-(-1))^2 = -4a(0-1) \Rightarrow a = \dfrac{1}{4} \Rightarrow$ the equation is $(y+1)^2 = -(x-1)$.

63. If the axis of symmetry is parallel to the y-axis, point P is above the vertex, and the parabola opens up. Substitute the coordinates of the vertex and P into the standard equation to find a: $(x-h)^2 = 4a(y-k) \Rightarrow$

$(0-(-1))^2 = 4a(2-1) \Rightarrow a = 1/4 \Rightarrow$ the equation is $(x+1)^2 = y-1$.

If the axis of symmetry is parallel to the x-axis, point P is to the right of the vertex, and the parabola opens to the right. Substitute the coordinates of the vertex and P into the standard equation to find a:

$(y-k)^2 = 4a(x-h) \Rightarrow$

$(2-1)^2 = 4a(0-(-1)) \Rightarrow a = \dfrac{1}{4} \Rightarrow$ the equation is $(y-1)^2 = x+1$.

64. If the axis of symmetry is parallel to the y-axis, point P is below the vertex, and the parabola opens down. Substitute the coordinates of the vertex and P into the standard equation to find a:

$(x-h)^2 = -4a(y-k) \Rightarrow$

$(3-2)^2 = -4a(-1-3) \Rightarrow a = 1/16 \Rightarrow$ the equation is $(x-2)^2 = -\frac{1}{4}(y-3)$.

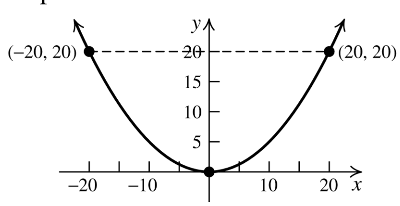
If the axis of symmetry is parallel to the x-axis, point P is to the right of the vertex, and the parabola opens to the right. Substitute the coordinates of the vertex and P into the standard equation to find a:

$(y-k)^2 = 4a(x-h) \Rightarrow$

$(-1-3)^2 = 4a(3-2) \Rightarrow a = 4 \Rightarrow$ the equation is $(y-3)^2 = 16(x-2)$.

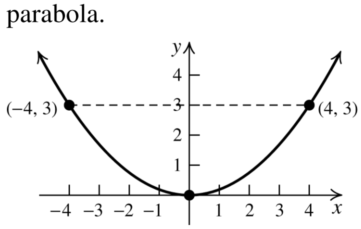
10.2 Applying the Concepts

65. If we sketch the parabola so that the focus is on the y-axis, and the vertex is at (0, 0), then the points (20, 20) and (−20, 20) must lie on the parabola.



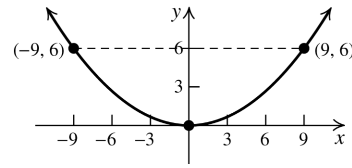
$x^2 = 4ay \Rightarrow 20^2 = 4a(20) \Rightarrow a = 5 \Rightarrow$ the receptor should be placed at the focus 5 inches from the vertex.

66. If we sketch the parabola so that the focus is on the y-axis, and the vertex is at (0, 0), then the points (4, 3) and (−4, 3) must lie on the parabola.



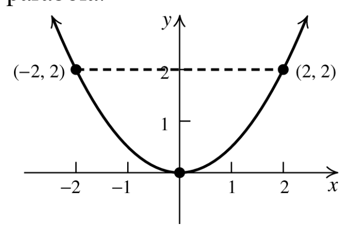
$x^2 = 4ay \Rightarrow 4^2 = 4a(3) \Rightarrow a = \frac{4}{3} \Rightarrow$ the receptor should be placed at the focus $\frac{4}{3}$ feet from the vertex.

67. If we sketch the parabola so that the focus is on the y-axis, and the vertex is at (0, 0), then the points (9, 6) and (−9, 6) must lie on the parabola.



$x^2 = 4ay \Rightarrow 9^2 = 4a(6) \Rightarrow a = \frac{81}{24} = \frac{27}{8} \Rightarrow$ the heating element should be placed at the focus $3\frac{3}{8}$ feet from the vertex.

68. If we sketch the parabola so that the focus is on the y-axis, and the vertex is at (0, 0), then the points (2, 2) and (−2, 2) must lie on the parabola.



$x^2 = 4ay \Rightarrow 2^2 = 4a(2) \Rightarrow a = \frac{1}{2} \Rightarrow$ the light bulb should be placed at the focus $1/2$ inch from the vertex.

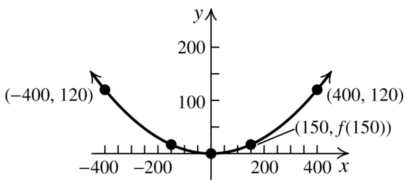
69. $x = 4y^2 \Rightarrow \frac{x}{4} = y^2 \Rightarrow 4a = \frac{1}{4} \Rightarrow a = \frac{1}{16} \Rightarrow$ the bulb should be placed at the focus, $\left(\frac{1}{16}, 0\right)$.

70. $x = \frac{1}{2}y^2 \Rightarrow 2x = y^2 \Rightarrow 4a = 2 \Rightarrow a = \frac{1}{2} \Rightarrow$ the bulb should be placed at the focus, $\left(\frac{1}{2}, 0\right)$.

71. $y = 4x^2 \Rightarrow \frac{1}{4}y = x^2 \Rightarrow 4a = \frac{1}{4} \Rightarrow a = \frac{1}{16} \Rightarrow$ the microphone should be placed at the focus, $\left(0, \frac{1}{16}\right)$.

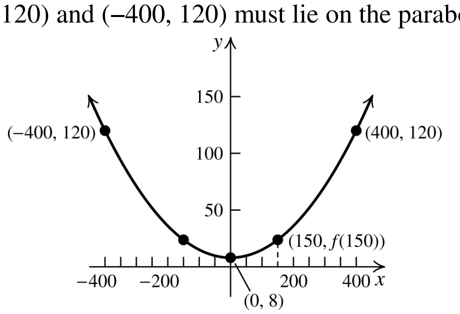
72. $y = x^2 \Rightarrow 4a = 1 \Rightarrow a = \frac{1}{4} \Rightarrow$ the microphone should be placed at the focus, $\left(0, \frac{1}{4}\right)$.

73. If we sketch the parabola so that the focus is on the y-axis, the roadbed is the x-axis, and the vertex is at (0, 0), then the points (400, 120) and (−400, 120) must lie on the parabola.



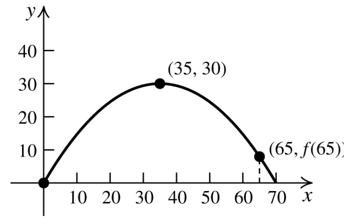
$x^2 = 4ay \Rightarrow 400^2 = 4a(120) \Rightarrow a = \frac{1000}{3} \Rightarrow$ the equation of the parabola is $x^2 = \frac{4000}{3}y \Rightarrow y = \frac{3}{4000}x^2$. The point on the cable 250 feet from the tower has coordinates $(150, f(150))$. The length of the cable is $f(150) = \frac{135}{8} = 16.875$ feet.

74. If we sketch the parabola so that the focus is on the y-axis, the roadbed is the x-axis, and the vertex is at (0, 8), then the points (400, 120) and (−400, 120) must lie on the parabola.



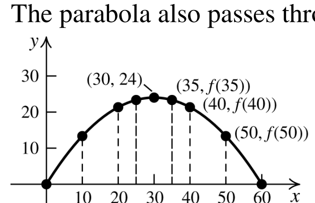
$x^2 = 4a(y-8) \Rightarrow 400^2 = 4a(120-8) \Rightarrow$ $a = \frac{2500}{7} \Rightarrow$ the equation of the parabola is $x^2 = \frac{10,000}{7}(y-8) \Rightarrow y = \frac{7}{10,000}x^2 + 8$.
The point on the cable 250 feet from the tower has coordinates $(150, f(150))$. The height of the cable is $f(150) = \frac{95}{4} = 23.75$ feet.

75. The vertex of the parabola occurs at (35, 30). The parabola also passes through the origin.



The standard form of the equation is $(x-35)^2 = -4a(y-30)$. Substitute (0, 0) for (x, y) and solve for a:
$(0-35)^2 = -4a(0-30) \Rightarrow a = \frac{245}{24}$. The equation of the parabola is
$(x-35)^2 = -\frac{245}{6}(y-30) \Rightarrow$
$y = -\frac{6}{245}(x-35)^2 + 30$.
Now find $f(65) = \frac{390}{49} \approx 7.96$ yards.

76. The vertex of the parabola occurs at (30, 24). The parabola also passes through the origin.



The standard form of the equation is $(x-30)^2 = -4a(y-24)$. Substitute (0, 0) for (x, y) and solve for a:
$(0-30)^2 = -4a(0-24) \Rightarrow a = \frac{75}{8}$. The equation of the parabola is
$(x-30)^2 = -4\left(\frac{75}{8}\right)(y-24) \Rightarrow$
$y = -\frac{2}{75}(x-30)^2 + 24$. The height at 5 feet from the center is $f(35) = \frac{70}{3} \approx 23.3$ feet, the height at 10 feet from the center is $f(40) = \frac{64}{3} \approx 21.3$ feet, and the height at 20 feet from the center is $f(50) = \frac{40}{3} \approx 13.3$ feet.

77. Rewrite the equation as $10y - 500 = x^2 - 30x$, then complete the square to put the equation in standard form:
$10y - 500 + 225 = x^2 - 30x + 225 \Rightarrow$
$10(y - 27.5) = (x - 15)^2$ So, the vertex is (15, 27.5). The output is 15 tons at a cost of \$27.50.

78. Rewrite the equation as $8y - 480 = x^2 - 16x$, then complete the square to put the equation in standard form:
$8y - 480 + 64 = x^2 - 16x + 64 \Rightarrow$
$8(y - 52) = (x - 8)^2$. So the vertex is (8, 52). The output is 8 tons at a cost of \$52.

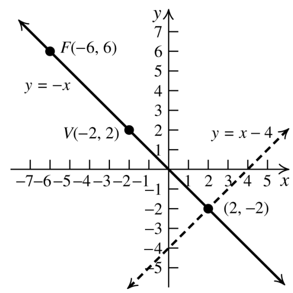
10.2 Beyond the Basics

79. Solve the system $\begin{cases} 2x - 3y = -16 \\ y^2 = 16x \end{cases}$ using substitution: $2x - 3y = -16 \Rightarrow x = \frac{3}{2}y - 8 \Rightarrow$
$y^2 = 16\left(\frac{3}{2}y - 8\right) \Rightarrow y^2 = 24y - 128 \Rightarrow$
$y^2 - 24y + 128 = 0 \Rightarrow (y - 8)(y - 16) = 0 \Rightarrow$
$y = 8$ or $y = 16$.
$2x - 3(8) + 16 = 0 \Rightarrow x = 4$
$2x - 3(16) + 16 = 0 \Rightarrow x = 16$. The line and the parabola intersect at (16, 16) and (4, 8). Verify using a graphing calculator.

80. Solve the system $\begin{cases} y = 2x + 3 \\ y^2 = 24x \end{cases}$ using substitution: $(2x + 3)^2 = 24x \Rightarrow$
$4x^2 - 12x + 9 = 0 \Rightarrow (2x - 3)^2 = 0 \Rightarrow x = \frac{3}{2}$
$y = 2\left(\frac{3}{2}\right) + 3 = 6.$ There is only one point of intersection, $(3/2, 6)$.

81. The vertex is the midpoint of the perpendicular segment connecting the directrix with the focus. The slope of the line connecting the vertex and the focus is $\frac{6 - 2}{-6 - (-2)} = \frac{4}{-4} = -1.$ The equation of that line is $y - 2 = -1(x + 2) \Rightarrow y = -x.$

The distance between the focus and the vertex is $\sqrt{(-6 - (-2))^2 + (6 - 2)^2} = \sqrt{32} = 4\sqrt{2},$ so the distance from the vertex to the directrix is also $4\sqrt{2}.$ Let (*x*, *y*) be the point on the directrix on the line $y = -x - 4$. Then
$\sqrt{(x - (-2))^2 + (y - 2)^2} = 4\sqrt{2} \Rightarrow$
$(x + 2)^2 + (y - 2)^2 = 32$. Substituting $y = -x$, we have
$(x + 2)^2 + (-x - 2)^2 = 2x^2 + 8x + 8 = 32 \Rightarrow$
$2(x^2 + 4x - 12) = 0 \Rightarrow x = -6$ or $x = 2.$ The point we are looking for is (2, −2). The slope of the directrix is 1. The equation of the directrix is $y + 2 = x - 2 \Rightarrow y = x - 4.$

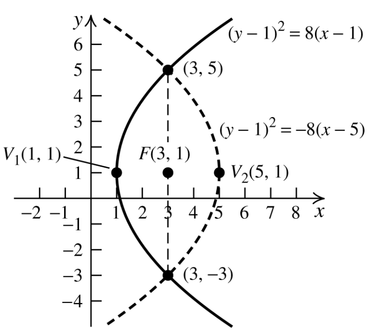


82. a. The axis of the parabola is perpendicular to the directrix and passes through the focus. The slope of the directrix is $\frac{3}{4},$ so the slope of the axis is $-\frac{4}{3}.$ The equation of the axis is
$y - 5 = -\frac{4}{3}(x - (-4)) \Rightarrow y = -\frac{4}{3}x - \frac{1}{3}.$

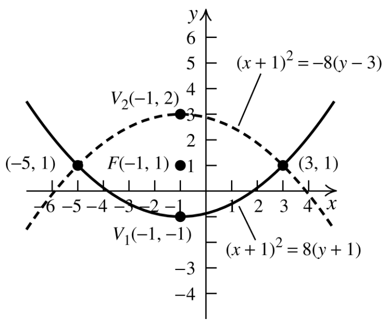
b. To find the point of intersection of axis and the directrix, solve the system
$\begin{cases} y = -\frac{4}{3}x - \frac{1}{3} \\ 3x - 4y = 18 \end{cases} \Rightarrow 3x - 4\left(-\frac{4}{3}x - \frac{1}{3}\right) = 18 \Rightarrow$
$x = 2, y = -\frac{4}{3}(2) - \frac{1}{3} = -3$. The point of intersection is (2, −3).

c. The vertex is the midpoint of the segment joining the point found in part (b) and the focus. $V = \left(\frac{2 - 4}{2}, \frac{-3 + 5}{2}\right) = (-1, 1).$

83. The latus rectum passes through the focus and is perpendicular to the axis of the parabola. The focus is the midpoint of the latus rectum, so the focus is (3, 1). The parabola opens to the right or to the left. Then (h, 1) = the coordinates of the vertex, and $a = 3 - h$. Substituting the coordinates of the point (3, 5) into the general form of the equation of a parabola, we have $(5-1)^2 = 4(3-h)(3-h) \Rightarrow h = 1$ or $h = 5$. If $h = 1$, the vertex of the parabola is (1, 1), $a = 2$, and the parabola opens to the right. Its equation is $(y-1)^2 = 8(x-1)$.
If $h = 5$, the vertex of the parabola is (5, 1), $a = -2$, and the parabola opens to the left. Its equation is $(y-1)^2 = -8(x-5)$.



84. The latus rectum passes through the focus and is perpendicular to the axis of the parabola. The focus is the midpoint of the latus rectum, so the focus is (−1, 1). The parabola opens up or down. Then (−1, k) = the coordinates of the vertex, and $a = 1 - k$. Substituting the coordinates of the point (3, 1) into the general form of the equation of a parabola, we have $(3-(-1))^2 = 4(1-k)(1-k) \Rightarrow k = -1$ or $k = 3$. If $k = -1$, the vertex of the parabola is (−1, −1), $a = 2$, and the parabola opens up. Its equation is $(x+1)^2 = 8(y+1)$. If $k = 3$, the vertex of the parabola is (−1, 3), $a = -2$, and the parabola opens down. Its equation is $(x+1)^2 = -8(y-3)$.



85. The parabola opens up or down, so the equation is of the form $(x-h)^2 = \pm 4a(y-k)$. Substitute the coordinates of the given points into the equation $ax^2 + bx + c = y$, and solve the system

$$\begin{cases} a(0^2) + b(0) + c = 5 \\ a(1^2) + b(1) + c = 4 \\ a(2^2) + b(2) + c = 7 \end{cases} \Rightarrow a = 2, b = -3, c = 5.$$

Now rewrite the equation $2x^2 - 3x + 5 = y$ as $2\left(x^2 - \frac{3}{2}x\right) = y - 5$ and complete the square:
$2\left(x^2 - \frac{3}{2}x + \frac{9}{16}\right) = y - 5 + \frac{9}{8} \Rightarrow$
$\left(x - \frac{3}{4}\right)^2 = \frac{1}{2}\left(y - \frac{31}{8}\right)$.

86. The parabola opens right or left, so the equation is of the form $(y-k)^2 = \pm 4a(x-h)$. Substitute the coordinates of the given points into the equation $ay^2 + by + c = x$, and solve the system

$$\begin{cases} a(-1)^2 + b(-1) + c = -1 \\ a(1^2) \quad + b(1) \quad + c = 3 \\ a(0^2) \quad + b(0) \quad + c = 4 \end{cases} \Rightarrow$$
$a = -3, b = 2, c = 4$

Rewrite the equation $-3y^2 + 2y + 4 = x$ as $-3\left(y^2 - \frac{2}{3}y\right) = x - 4$ and complete the square: $-3\left(y^2 - \frac{2}{3}y + \frac{1}{9}\right) = x - 4 - \frac{1}{3} \Rightarrow$
$\left(y - \frac{1}{3}\right)^2 = -\frac{1}{3}\left(x - \frac{13}{3}\right)$

87. Rewrite the equation as $x^2 - 8x = -2y - 4$, and then complete the square to put the equation in standard form:
$x^2 - 8x + 16 = -2y - 4 + 16 \Rightarrow$
$(x-4)^2 = -2(y-6)$. The vertex is (4, 6).
$4a = -2 \Rightarrow a = -\frac{1}{2}$, so the focus is $\left(4, \frac{11}{2}\right)$.
The directrix is $y = \frac{13}{2}$. The axis of the parabola is $x = 4$.

88. Rewrite the equation as $Ax^2 + Dx = -Ey - F$, and then complete the square to put the equation in standard form:

$$A\left(x^2 + \frac{D}{A}x + \left(\frac{D/A}{2}\right)^2\right) = -Ey - F + A\left(\frac{D/A}{2}\right)^2 \Rightarrow$$

$$A\left(x^2 + \frac{D}{A}x + \frac{D^2}{4A^2}\right) = -Ey - F + \frac{D^2}{4A} \Rightarrow$$

$$A\left(x + \frac{D}{2A}\right)^2 = -E\left(y + \left(\frac{F}{E} - \frac{D^2}{4EA}\right)\right) \Rightarrow$$

$$\left(x + \frac{D}{2A}\right)^2 = -\frac{E}{A}\left(y - \left(\frac{D^2}{4EA} - \frac{F}{E}\right)\right)$$

The vertex is $\left(-\frac{D}{2A}, \left(\frac{D^2}{4EA} - \frac{F}{E}\right)\right)$.

$4a = -\frac{E}{A} \Rightarrow a = -\frac{E}{4A}$, so the focus is $\left(-\frac{D}{2A}, \left(\frac{D^2}{4EA} - \frac{F}{E} - \frac{E}{4A}\right)\right)$, and the directrix is $y = \frac{D^2}{4EA} - \frac{F}{E} + \frac{E}{4A}$. The axis is $x = -\frac{D}{2A}$.

89. Rewrite the equation as $y^2 - 6y = -3x - 15$, and then complete the square to put the equation in standard form:

$y^2 - 6y + 9 = -3x - 15 + 9 \Rightarrow$

$(y-3)^2 = -3(x+2)$. The vertex is $(-2, 3)$.

$4a = -3 \Rightarrow a = -\frac{3}{4}$, so the focus is $\left(-\frac{11}{4}, 3\right)$, and the directrix is $x = -\frac{5}{4}$. The axis is $y = 3$.

90. Rewrite the equation as $C\left(y^2 + \frac{E}{C}y\right) = -Dx - F$, and then complete the square to put the equation in standard form:

$$C\left(y^2 + \frac{E}{C}y + \left(\frac{E/C}{2}\right)^2\right) = -Dx - F + C\left(\frac{E/C}{2}\right)^2 \Rightarrow$$

$$C\left(y + \frac{E}{2C}\right)^2 = -D\left(x + \left(\frac{F}{D} - \frac{E^2}{4CD}\right)\right) \Rightarrow$$

$$\left(y + \frac{E}{2C}\right)^2 = -\frac{D}{C}\left(x - \left(\frac{E^2}{4CD} - \frac{F}{D}\right)\right)$$

The vertex is $\left(\left(\frac{E^2}{4CD} - \frac{F}{D}\right), -\frac{E}{2C}\right)$.

$4a = -\frac{D}{C} \Rightarrow a = -\frac{D}{4C}$, so the focus is $\left(\left(\frac{E^2}{4CD} - \frac{F}{D} - \frac{D}{4C}\right), -\frac{E}{2C}\right)$, and the directrix is $x = \frac{E^2}{4CD} - \frac{F}{D} + \frac{D}{4C}$. The axis is $y = -\frac{E}{2C}$.

91. Step 1: Let m = the slope of the tangent line. Then the equation of the tangent line is $y - 3 = m(x-1) \Rightarrow y = mx - m + 3$.

Step 2: $y = 3x^2 \Rightarrow mx - m + 3 = 3x^2$.

Step 3:

$mx - m + 3 = 3x^2 \Rightarrow 3x^2 - mx + m - 3 = 0 \Rightarrow$

$a = 3,\ b = -m,\ c = m - 3$

$b^2 - 4ac = (-m)^2 - 4(3)(m-3)$, so

$b^2 - 4ac = 0 \Rightarrow (-m)^2 - 4(3)(m-3) = 0 \Rightarrow$

$m^2 - 12m + 36 = 0 \Rightarrow (m-6)^2 = 0 \Rightarrow m = 6$

Step 4: The equation of the tangent line is $y - 3 = 6(x-1) \Rightarrow y = 6x - 3$.

92. Step 1: Let m = the slope of the tangent line. Then the equation of the tangent line is $y - y_1 = m(x - x_1) \Rightarrow y = mx - mx_1 + y_1$.

Step 2: $y = 4ax^2 \Rightarrow mx - mx_1 + y_1 = 4ax^2$.

Step 3:

$mx - mx_1 + y_1 = 4ax^2 \Rightarrow 4ax^2 - mx + mx_1 - y_1$

$a = 4a,\ b = -m,\ c = mx_1 - y_1$

$b^2 - 4ac = (-m)^2 - 4(4a)(mx_1 - y_1)$, so

$b^2 - 4ac = 0 \Rightarrow (-m)^2 - 4(4a)(mx_1 - y_1) = 0 \Rightarrow$

$m^2 - 16amx_1 + 16ay_1 = 0$

$y_1 = 4ax_1^2$, so $m^2 - 16amx_1 + 16ay_1 = 0 \Rightarrow$

$m^2 - 16amx_1 + 16a(4ax_1^2) = 0 \Rightarrow$

$m^2 - 16amx_1 + 64a^2x_1^2 = 0 \Rightarrow$

$(m - 8ax_1)^2 = 0 \Rightarrow m = 8ax_1$

(*continued on next page*)

(*continued*)

Step 4: The equation of the tangent line is

$y - y_1 = 8ax_1(x - x_1) \Rightarrow$

$y - y_1 = 8ax_1x - 8ax_1^2 \Rightarrow$

$y - y_1 = 8ax_1x - 2y_1 \Rightarrow y = 8ax_1x - y_1$

93. Step 1: Let m = the slope of the tangent line. Then the equation of the tangent line is

$y - y_1 = m(x - x_1) \Rightarrow \frac{y - y_1}{m} + x_1 = x.$

Step 2: $x = 4ay^2 \Rightarrow \frac{y - y_1}{m} + x_1 = 4ay^2.$

$x_1 = 4ay_1^2 \Rightarrow \frac{y - y_1}{m} + x_1 = 4ay^2 \Rightarrow$

$\frac{y - y_1}{m} + 4ay_1^2 = 4ay^2$

Step 3:

$\frac{y - y_1}{m} + 4ay_1^2 = 4ay^2 \Rightarrow$

$4ay^2 - \frac{y}{m} + \left(\frac{y_1}{m} - 4ay_1^2\right) = 0 \Rightarrow$

$4ay^2 - \frac{y}{m} + \left(\frac{y_1 - 4may_1^2}{m}\right) = 0$

$a = 4a,\ b = -\frac{1}{m},\ c = \frac{y_1}{m} - 4ay_1^2$

$b^2 - 4ac = \left(-\frac{1}{m}\right)^2 - 4(4a)\left(\frac{y_1}{m} - 4ay_1^2\right)$, so

$b^2 - 4ac = 0 \Rightarrow \frac{1}{m^2} - \frac{16ay_1}{m} + 64a^2y_1^2 = 0 \Rightarrow$

$\left(\frac{1}{m} - 8ay_1\right)^2 = 0 \Rightarrow \frac{1}{m} = 8ay_1 \Rightarrow m = \frac{1}{8ay_1}$

Step 4: The equation of the tangent line is

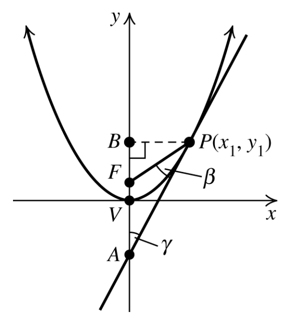
$y - y_1 = \frac{1}{8ay_1}(x - x_1) \Rightarrow$

$y - y_1 = \frac{1}{8ay_1}x - \frac{1}{8ay_1}x_1 \Rightarrow$

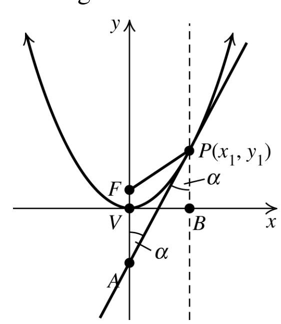
$y - y_1 = \frac{1}{8ay_1}x - \frac{1}{8ay_1}\left(4ay_1^2\right) \Rightarrow$

$y - y_1 = \frac{1}{8ay_1}x - \frac{1}{2}y_1 \Rightarrow y = \frac{1}{8ay_1}x + \frac{1}{2}y_1$

94. a. From exercise 92, we know that the equation of the tangent line is $y = 8ax_1x - y_1$. At point A, $x_1 = 0$, so $y = y_1$. The distance from F to V is a, and the distance from V to A, is y_1, so $d(F, A) = a + y_1$. A lies on the directrix $y = a$ because $d(F, V) = a$ and $d(V, A) = a$. By the definition of a parabola, $d(F, A) = a + y_1 = d(P, F)$. Because $\triangle PFA$ is isosceles, $m\angle\beta = m\angle\gamma$.



b. Since the line $x = x_1$ is parallel to the y-axis, the tangent line, PA is a transversal and therefore, $m\angle VAP = m\angle APB$, since the angles are alternate interior angles.

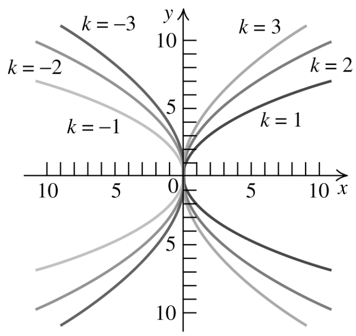


10.2 Critical Thinking/Discussion/Writing

95. a. No. A parabola that opens to the right or left is not the graph of a function because it does not pass the vertical line test.

b. A parabola is always a function if the directrix is parallel to the x-axis, so the slope of the directrix is 0.

96.



The parabola widens as $|k|$ increases.

97. False. Its axis is parallel to the x-axis.

98. For a point $P(x_1, y_1)$ on a parabola, the distance from P to the focus is the same as the distance from P to the directrix. Using the given formula, the distance from P to the directrix is $\frac{|3x_1+4y_1-7|}{\sqrt{3^2+4^2}} = \frac{|3x_1+4y_1-7|}{5}$.

The distance from P to the focus is $\sqrt{(x_1-4)^2+(y_1-5)^2}$.

Find the equation of the parabola by setting $\frac{|3x_1+4y_1-7|}{5} = \sqrt{(x_1-4)^2+(y_1-5)^2}$ and simplifying:

$16x^2+9y^2-24xy-158x-194y+976=0$

The axis is perpendicular to the directrix and passes through the focus. The slope of the directrix is $-\frac{3}{4}$, so the slope of the axis is $\frac{4}{3}$.

The equation of the line through (4, 5) with slope $\frac{4}{3}$ is $y-5=\frac{4}{3}(x-4) \Rightarrow y=\frac{4}{3}x-\frac{1}{3}$.

99. Using the formula given in group project 1, the distance from the vertex to the directrix is $\frac{|3(6)-5(-3)+1|}{\sqrt{3^2+(-5)^2}} = \frac{34}{\sqrt{34}} = \sqrt{34}$. The distance from the focus (x, y) to the vertex is

(1) $\sqrt{(x-6)^2+(y+3)^2}=\sqrt{34}$. The distance from the focus (x, y) to the directrix is twice the distance from the focus to the vertex:

(2) $\frac{|3x-5y+1|}{\sqrt{3^2+(-5)^2}} = 2\sqrt{34} \Rightarrow |3x-5y+1| = 68.$

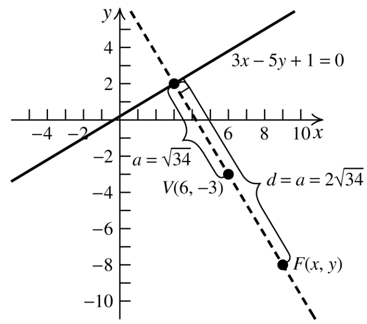
Solve the system consisting of equations (1) and (2) to find the coordinates of the focus:

$$\begin{cases}(x-6)^2+(y+3)^2=34\\3x-5y=67\end{cases} \Rightarrow$$

$$\begin{cases}x^2+y^2-12x+6y+11=0\\x=\frac{5}{3}y+\frac{67}{3}\end{cases} \Rightarrow$$

$y^2+16y+64=0 \Rightarrow y=-8$

$3x-5(-8)=67 \Rightarrow x=9$



The focus is at (9, −8).

10.2 Maintaining Skills

100. $d=\sqrt{(6-(-1))^2+(2-3)^2}=\sqrt{7^2+(-1)^2}$
$=\sqrt{50}=5\sqrt{2}$

101. $d=\sqrt{(-1-(-1))^2+(-5-2)^2}=\sqrt{0+(-7)^2}$
$=\sqrt{49}=7$

102. $(x-5)^2=x^2-10x+25$

103. $(2x+3)^2=4x^2+12x+9$

104. To find the x-intercepts, let $y = 0$ and solve for x.

$x^2+x-2(0)=2 \Rightarrow x^2+x-2=0 \Rightarrow$
$(x+2)(x-1)=0 \Rightarrow x=-2,\ x=1$

To find the y-intercept, let $x = 0$ and solve for y.

$0^2+0-2y=2 \Rightarrow y=-1$

The x-intercepts are −2 and 1. The y-intercept is −1.

105. To find the x-intercepts, let $y = 0$ and solve for x.

$x^2+8x+0^2-6(0)+16=0 \Rightarrow$
$x^2+8x+16=0 \Rightarrow (x+4)^2=0 \Rightarrow x=4$

(*continued on next page*)

(*continued*)

To find the y-intercept, let $x = 0$ and solve for y.

$0^2 + 8(0) + y^2 - 6y + 16 = 0 \Rightarrow$

$y^2 - 6y + 16 = 0$

Use the quadratic formula to solve for y.

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(16)}}{2(1)} = \frac{6 \pm \sqrt{36-64}}{2} = \frac{6 \pm \sqrt{-28}}{2},$$ which is not a real number.

The x-intercept is 4. There are no y-intercepts.

In exercises 106 and 107, be sure to check your solution(s) in the original equation.

106. $\sqrt{6x+7} = x + 2 \Rightarrow 6x + 7 = (x+2)^2 \Rightarrow$
$6x + 7 = x^2 + 4x + 4 \Rightarrow 0 = x^2 - 2x - 3 \Rightarrow$
$0 = (x+1)(x-3) \Rightarrow x = -1,\ x = 3$
Both solutions are valid in the original equation, so the solution set is $\{-1, 3\}$.

107. $2 + \sqrt{3x-5} = x - 1 \Rightarrow \sqrt{3x-5} = x - 3 \Rightarrow$
$3x - 5 = (x-3)^2 \Rightarrow 3x - 5 = x^2 - 6x + 9 \Rightarrow$
$0 = x^2 - 9x + 14 \Rightarrow 0 = (x-2)(x-7) \Rightarrow$
$x = 2,\ x = 7$
When we check $x = 2$ in the original equation, we have
$2 + \sqrt{3(2)-5} = 2 - 1 \Rightarrow 2 + \sqrt{1} = 2 - 1 \Rightarrow$
$3 = 1$, which is false. Thus the solution set is $\{7\}$.

108. $$x^2 - 6x + 7 = (x^2 - 6x) + 7 = (x^2 - 6x + 9) + (7 - 9) = (x-3)^2 - 2$$

109. $$4x^2 - 8x + 14 = 4(x^2 - 2x) + 14 = 4(x^2 - 2x + 1) + (14 - 4) = 4(x-1)^2 + 10$$

110. The vertex of $y = 4x^2$ is (0, 0). The vertex of the new parabola is (1, −2), so the new parabola is shifted one unit right and two units down. Thus, the equation of the new parabola is $y = 4(x-1)^2 - 2$.

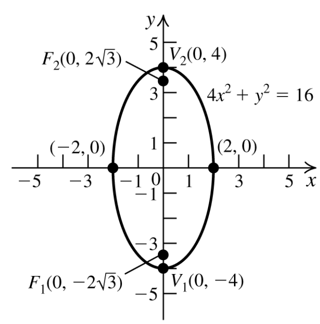
111. The vertex of $y = (x+2)^2 + 3$ is (−2, 3). The vertex of the new parabola is (1, −2), so the new parabola is shifted one unit to the left and one unit down. The equation of the new parabola is $y = (x+1)^2 + 2$.

10.3 The Ellipse

10.3 Practice Problems

1. Since the foci are (0, −8) and (0, 8), the major axis is on the y-axis, and $c = 8$. One vertex is (0, 10), so the other vertex is (0, −10), and $a = 10$.
$b^2 = a^2 - c^2 \Rightarrow b^2 = 10^2 - 8^2 \Rightarrow b = 6.$
Thus, the equation is $\frac{x^2}{36} + \frac{y^2}{100} = 1.$

2. $4x^2 + y^2 = 16 \Rightarrow \frac{x^2}{4} + \frac{y^2}{16} = 1 \Rightarrow$
$a^2 = 16 \Rightarrow a = 4;\quad b^2 = 4 \Rightarrow b = 2$



The length of the major axis = $2a = 8$.
The length of the minor axis = $2b = 4$.

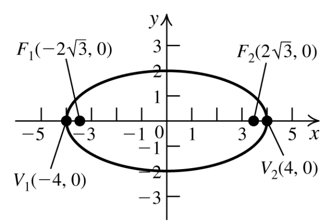
3. Since the foci (2, −3) and (2, 5) lie on the vertical line $x = 2$, the ellipse is a vertical ellipse. The center of the ellipse is at $\left(2, \frac{-3+5}{2}\right) = (2, 1) = (h, k)$. Since the major axis has length 10, the vertices are 5 units from the center, so $a = 5$. The foci are 4 units from the center, so $c = 4$.
$b^2 = a^2 - c^2 \Rightarrow b^2 = 5^2 - 4^2 \Rightarrow b^2 = 9.$
Thus, the equation is $\frac{(x-2)^2}{9} + \frac{(y-1)^2}{25} = 1.$

4. Rewrite the equation as $\left(x^2-6x\right)+4\left(y^2+2y\right)=29$, then complete both squares and write the equation in standard form:
$$\left(x^2-6x\right)+4\left(y^2+2y\right)=29$$
$$\left(x^2-6x+9\right)+4\left(y^2+2y+1\right)=29+9+4$$
$$(x-3)^2+4(y+1)^2=42$$
$$\frac{(x-3)^2}{42}+\frac{2(y+1)^2}{21}=1$$
$$\frac{(x-3)^2}{42}+\frac{(y+1)^2}{21/2}=1$$
The center is (3, −1).
$a^2=42\Rightarrow a=\sqrt{42}$, so the vertices are $\left(3-\sqrt{42},\,-1\right)$ and $\left(3+\sqrt{42},\,-1\right)$.
$$b^2=a^2-c^2\Rightarrow\frac{21}{2}=42-c^2\Rightarrow c^2=\frac{63}{2}\Rightarrow$$
$c=\sqrt{\frac{63}{2}}=\frac{\sqrt{126}}{2}=\frac{3\sqrt{14}}{2}$, the foci are $\left(3-\frac{3\sqrt{14}}{2},\,-1\right)$ and $\left(3+\frac{3\sqrt{14}}{2},\,-1\right)$.

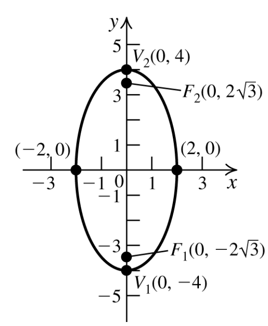
5. Since the length of the major axis of the ellipse is 8 feet, $a = 4$. Since the length of the minor axis is 4 feet, $b = 2$.
$2^2=4^2-c^2\Rightarrow c^2=12\Rightarrow c=2\sqrt{3}$. If we position the center of the ellipse at (0, 0) and the major axis along the x-axis, the foci of the ellipse are $\left(-2\sqrt{3},0\right)$ and $\left(2\sqrt{3},0\right)$. The distance between the two foci is $4\sqrt{3}\approx 6.9282$ feet. Thus the stone should be positioned 6.9282 feet from the source.

10.3 Basic Concepts Skills

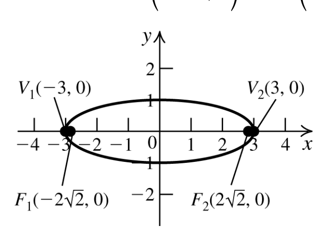
1. An ellipse is the set of all points in the plane, the <u>sum</u> of whose distances from two fixed points is a constant.
2. The points of intersection of the ellipse with the line through the foci are called the <u>vertices</u> of the ellipse.
3. The standard equation of an ellipse with center $(0,0)$, vertices $(\pm a,0)$, foci $(\pm c,0)$ is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $b^2=\underline{a^2-c^2}$.
4. In the equation $\frac{x^2}{m^2}+\frac{y^2}{n^2}=1$,
(*i*) if $m^2>n^2$, the graph is a <u>horizontal</u> ellipse;
(*ii*) if $m^2<n^2$, the graph is a <u>vertical</u> ellipse;
(*iii*) if $m^2=n^2$, the graph is a <u>circle</u>.
5. True
6. True (assuming a circle is a special kind of ellipse.)
7. $a^2=16\Rightarrow$ the vertices are (4, 0) and (−4, 0).
$b^2=a^2-c^2\Rightarrow 4=16-c^2\Rightarrow c=2\sqrt{3}\Rightarrow$ the foci are $\left(2\sqrt{3},0\right)$ and $\left(-2\sqrt{3},0\right)$.



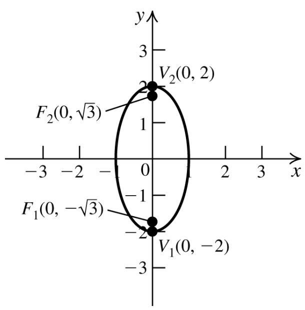
8. $a^2=16\Rightarrow$ the vertices are (0, 4) and (0, −4).
$b^2=a^2-c^2\Rightarrow 4=16-c^2\Rightarrow c=2\sqrt{3}\Rightarrow$ the foci are $\left(0,2\sqrt{3}\right)$ and $\left(0,-2\sqrt{3}\right)$.



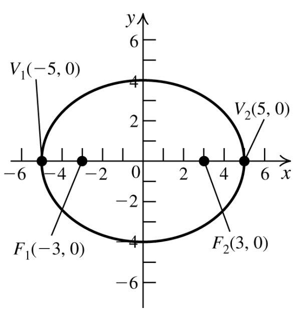
9. $a^2=9\Rightarrow$ the vertices are (3, 0) and (−3, 0).
$b^2=a^2-c^2\Rightarrow 1=9-c^2\Rightarrow c=2\sqrt{2}\Rightarrow$ the foci are $\left(2\sqrt{2},0\right)$ and $\left(-2\sqrt{2},0\right)$.



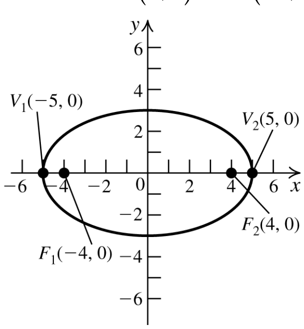
10. $a^2 = 4 \Rightarrow$ the vertices are (0, 2) and (0, −2).
$b^2 = a^2 - c^2 \Rightarrow 1 = 4 - c^2 \Rightarrow c = \sqrt{3} \Rightarrow$
the foci are $\left(0, \sqrt{3}\right)$ and $\left(0, -\sqrt{3}\right)$.



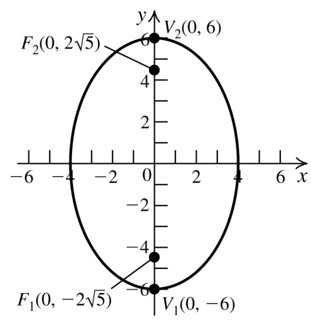
11. $a^2 = 25 \Rightarrow$ the vertices are (5, 0) and (−5, 0).
$b^2 = a^2 - c^2 \Rightarrow 16 = 25 - c^2 \Rightarrow c = 3 \Rightarrow$
the foci are $(3, 0)$ and $(-3, 0)$.



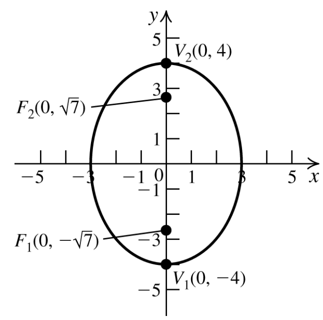
12. $a^2 = 25 \Rightarrow$ the vertices are (5, 0) and (−5, 0).
$b^2 = a^2 - c^2 \Rightarrow 9 = 25 - c^2 \Rightarrow c = 4 \Rightarrow$
the foci are $(4, 0)$ and $(-4, 0)$.



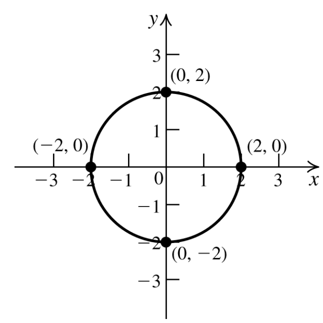
13. $a^2 = 36 \Rightarrow$ the vertices are (0, 6) and (0, −6).
$b^2 = a^2 - c^2 \Rightarrow 16 = 36 - c^2 \Rightarrow c = 2\sqrt{5} \Rightarrow$
the foci are $\left(0, 2\sqrt{5}\right)$ and $\left(0, -2\sqrt{5}\right)$.



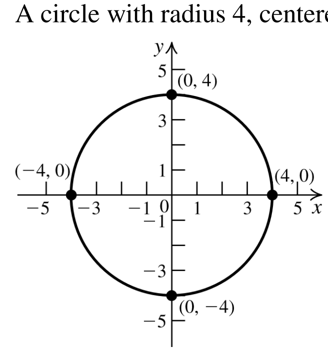
14. $a^2 = 16 \Rightarrow$ the vertices are (0, 4) and (0, −4).
$b^2 = a^2 - c^2 \Rightarrow 9 = 16 - c^2 \Rightarrow c = \sqrt{7} \Rightarrow$
the foci are $\left(0, \sqrt{7}\right)$ and $\left(0, -\sqrt{7}\right)$.



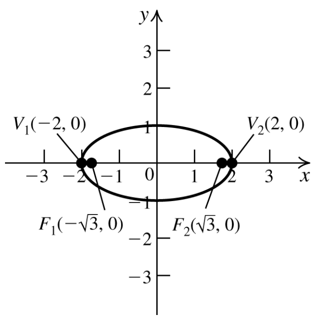
15. A circle with radius 2, centered at the origin.



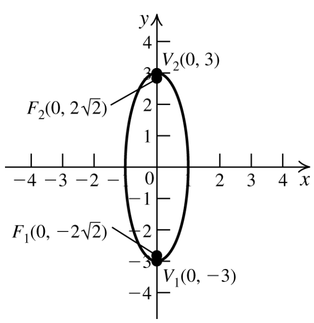
16. A circle with radius 4, centered at the origin.



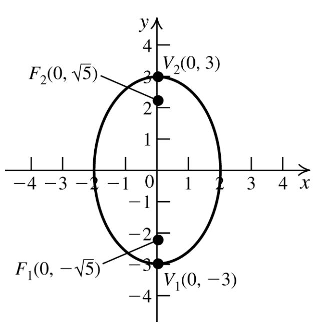
17. $x^2+4y^2=4 \Rightarrow \frac{x^2}{4}+y^2=1$. $a^2=4 \Rightarrow$ the vertices are (2, 0) and (−2, 0).
$b^2=a^2-c^2 \Rightarrow 1=4-c^2 \Rightarrow c=\sqrt{3} \Rightarrow$ the foci are $\left(\sqrt{3},0\right)$ and $\left(-\sqrt{3},0\right)$.



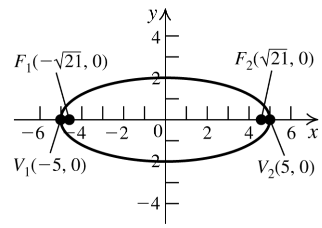
18. $9x^2+y^2=9 \Rightarrow x^2+\frac{y^2}{9}=1$. $a^2=9 \Rightarrow$ the vertices are (0, 3) and (0, −3).
$b^2=a^2-c^2 \Rightarrow 1=9-c^2 \Rightarrow c=2\sqrt{2} \Rightarrow$ the foci are $\left(0,2\sqrt{2}\right)$ and $\left(0,-2\sqrt{2}\right)$.



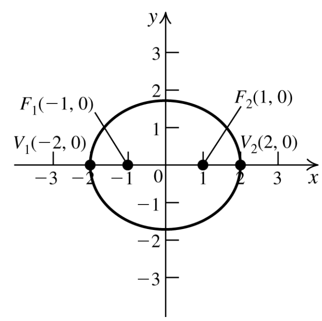
19. $9x^2+4y^2=36 \Rightarrow \frac{x^2}{4}+\frac{y^2}{9}=1$. $a^2=9 \Rightarrow$ the vertices are (0, 3) and (0, −3).
$b^2=a^2-c^2 \Rightarrow 4=9-c^2 \Rightarrow c=\pm\sqrt{5} \Rightarrow$ the foci are $\left(0,\sqrt{5}\right)$ and $\left(0,-\sqrt{5}\right)$.



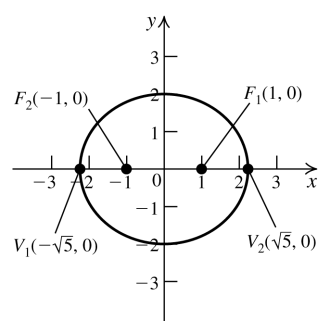
20. $4x^2+25y^2=100 \Rightarrow \frac{x^2}{25}+\frac{y^2}{4}=1$.
$a^2=25 \Rightarrow$ the vertices are (5, 0) and (−5, 0).
$b^2=a^2-c^2 \Rightarrow 4=25-c^2 \Rightarrow c=\sqrt{21} \Rightarrow$ the foci are $\left(\sqrt{21},0\right)$ and $\left(-\sqrt{21},0\right)$.



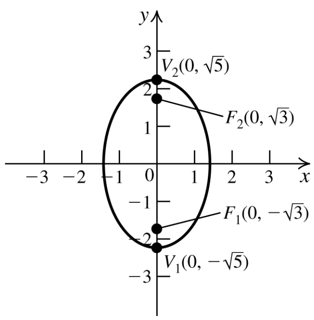
21. $3x^2+4y^2=12 \Rightarrow \frac{x^2}{4}+\frac{y^2}{3}=1$. $a^2=4 \Rightarrow$ the vertices are (2, 0) and (−2, 0).
$b^2=a^2-c^2 \Rightarrow 3=4-c^2 \Rightarrow c=1 \Rightarrow$ the foci are $(1,0)$ and $(-1,0)$.



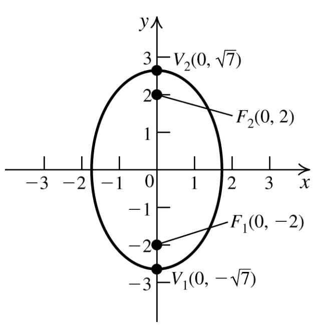
22. $4x^2+5y^2=20 \Rightarrow \frac{x^2}{5}+\frac{y^2}{4}=1$. $a^2=5 \Rightarrow$ the vertices are $\left(\sqrt{5},0\right)$ and $\left(-\sqrt{5},0\right)$.
$b^2=a^2-c^2 \Rightarrow 4=5-c^2 \Rightarrow c=1 \Rightarrow$ the foci are $(1,0)$ and $(-1,0)$.



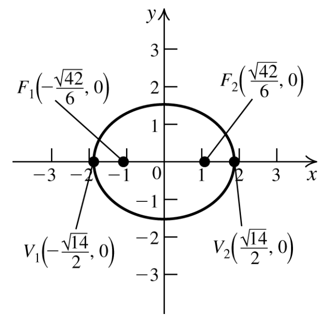
23. $5x^2 - 10 = -2y^2 \Rightarrow 5x^2 + 2y^2 = 10 \Rightarrow$
$\dfrac{x^2}{2} + \dfrac{y^2}{5} = 1.$ $a^2 = 5 \Rightarrow$ the vertices are $\left(0, \sqrt{5}\right)$ and $\left(0, -\sqrt{5}\right)$.
$b^2 = a^2 - c^2 \Rightarrow 2 = 5 - c^2 \Rightarrow c = \sqrt{3} \Rightarrow$ the foci are $\left(0, \sqrt{3}\right)$ and $\left(0, -\sqrt{3}\right)$.



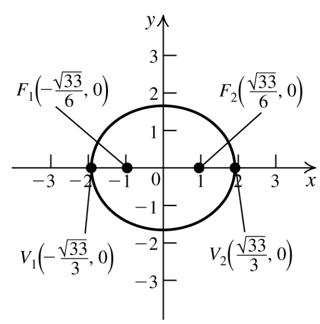
24. $3y^2 = 21 - 7x^2 \Rightarrow 7x^2 + 3y^2 = 21 \Rightarrow$
$\dfrac{x^2}{3} + \dfrac{y^2}{7} = 1.$ $a^2 = 7 \Rightarrow$ the vertices are $\left(0, \sqrt{7}\right)$ and $\left(0, -\sqrt{7}\right)$.
$b^2 = a^2 - c^2 \Rightarrow 3 = 7 - c^2 \Rightarrow c = 2 \Rightarrow$ the foci are $(0, 2)$ and $(0, -2)$.



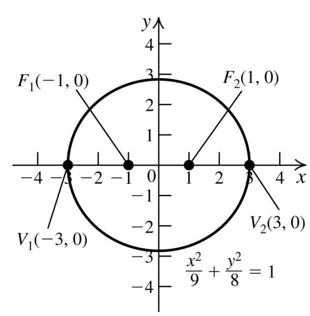
25. $2x^2 + 3y^2 = 7 \Rightarrow \dfrac{x^2}{7/2} + \dfrac{y^2}{7/3} = 1 \Rightarrow a^2 = \dfrac{7}{2} \Rightarrow$
the vertices are $\left(\dfrac{\sqrt{14}}{2}, 0\right)$ and $\left(-\dfrac{\sqrt{14}}{2}, 0\right)$.
$b^2 = a^2 - c^2 \Rightarrow \dfrac{7}{3} = \dfrac{7}{2} - c^2 \Rightarrow c = \dfrac{\sqrt{42}}{6} \Rightarrow$
the foci are $\left(\dfrac{\sqrt{42}}{6}, 0\right)$ and $\left(-\dfrac{\sqrt{42}}{6}, 0\right)$.



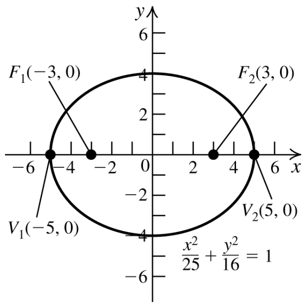
26. $3x^2 + 4y^2 = 11 \Rightarrow \dfrac{x^2}{11/3} + \dfrac{y^2}{11/4} = 1 \Rightarrow$
$a^2 = \dfrac{11}{3} \Rightarrow$ the vertices are $\left(\dfrac{\sqrt{33}}{3}, 0\right)$ and $\left(-\dfrac{\sqrt{33}}{3}, 0\right)$. $b^2 = a^2 - c^2 \Rightarrow \dfrac{11}{4} = \dfrac{11}{3} - c^2 \Rightarrow$
$c = \dfrac{\sqrt{33}}{6} \Rightarrow$ the foci are $\left(\dfrac{\sqrt{33}}{6}, 0\right)$ and $\left(-\dfrac{\sqrt{33}}{6}, 0\right)$.



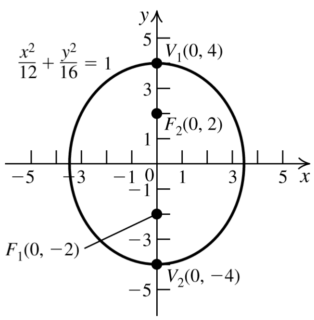
27. The major axis is on the *x*-axis.
$b^2 = a^2 - c^2 \Rightarrow b^2 = 3^2 - 1^2 = 8$. The equation is $\dfrac{x^2}{9} + \dfrac{y^2}{8} = 1$.



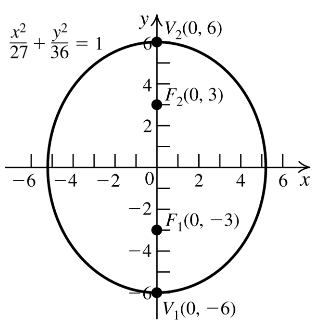
28. The major axis is on the x-axis. $b^2 = a^2 - c^2 \Rightarrow b^2 = 5^2 - 3^2 = 16$. The equation is $\frac{x^2}{25} + \frac{y^2}{16} = 1$.



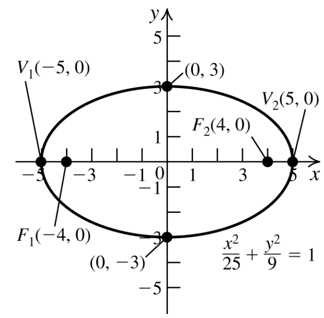
29. The major axis is on the y-axis. $b^2 = a^2 - c^2 \Rightarrow b^2 = 4^2 - 2^2 = 12$. The equation is $\frac{x^2}{12} + \frac{y^2}{16} = 1$.



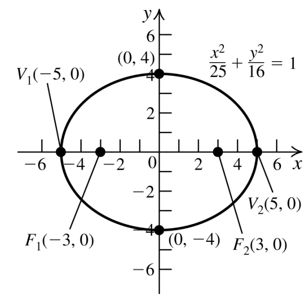
30. The major axis is on the y-axis. $b^2 = a^2 - c^2 \Rightarrow b^2 = 6^2 - 3^2 = 27$. The equation is $\frac{x^2}{27} + \frac{y^2}{36} = 1$.



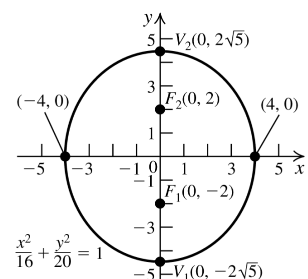
31. The major axis is on the x-axis. $b^2 = a^2 - c^2 \Rightarrow 3^2 = a^2 - 4^2 \Rightarrow a^2 = 25$. The equation is $\frac{x^2}{25} + \frac{y^2}{9} = 1$.



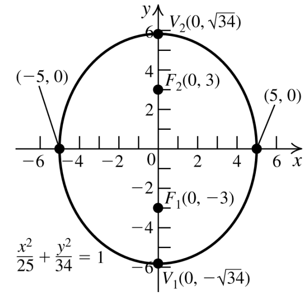
32. The major axis is on the x-axis. $b^2 = a^2 - c^2 \Rightarrow 4^2 = a^2 - 3^2 \Rightarrow a^2 = 25$. The equation is $\frac{x^2}{25} + \frac{y^2}{16} = 1$.



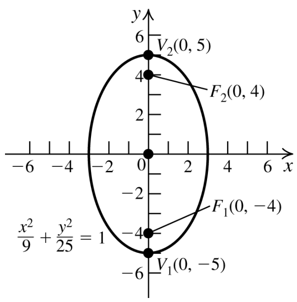
33. The major axis is on the y-axis. $b^2 = a^2 - c^2 \Rightarrow 4^2 = a^2 - 2^2 \Rightarrow a^2 = 20$. The equation is $\frac{x^2}{16} + \frac{y^2}{20} = 1$.



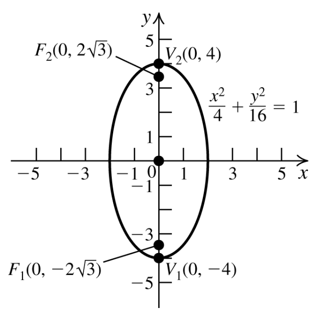
34. The major axis is on the y-axis. $b^2 = a^2 - c^2 \Rightarrow 5^2 = a^2 - 3^2 \Rightarrow a^2 = 34$. The equation is $\frac{x^2}{25} + \frac{y^2}{34} = 1$.



35. The major axis is on the y-axis. Major axis length = 10 $\Rightarrow a = 5$ and minor axis length = 6 $\Rightarrow b = 3$. The equation is $\frac{x^2}{9} + \frac{y^2}{25} = 1$.

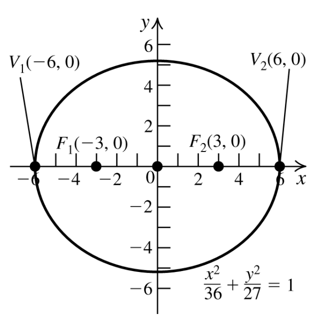


36. The major axis is on the y-axis. Major axis length = 8 $\Rightarrow a = 4$ and minor axis length = 4 $\Rightarrow b = 2$. The equation is $\frac{x^2}{4} + \frac{y^2}{16} = 1$.



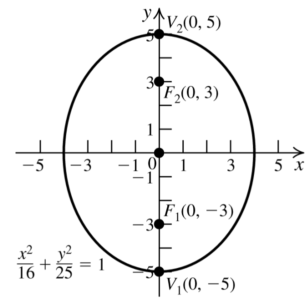
37. The major axis is on the x-axis. $a = 6$. $b^2 = a^2 - c^2 \Rightarrow b^2 = 6^2 - 3^2 = 27$.

The equation is $\frac{x^2}{36} + \frac{y^2}{27} = 1$.



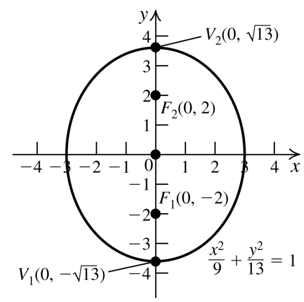
38. The major axis is on the y-axis. $a = 5$. $b^2 = a^2 - c^2 \Rightarrow b^2 = 5^2 - 3^2 = 16$.

The equation is $\frac{x^2}{16} + \frac{y^2}{25} = 1$.



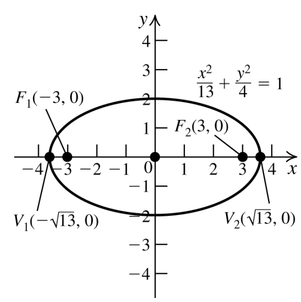
39. The major axis is on the y-axis. $c = 2$. $b^2 = a^2 - c^2 \Rightarrow 3^2 = a^2 - 2^2 \Rightarrow a^2 = 13$.

The equation is $\frac{x^2}{9} + \frac{y^2}{13} = 1$.

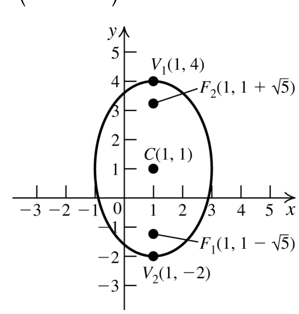


40. The major axis is on the x-axis. $c = 3$. $b^2 = a^2 - c^2 \Rightarrow 2^2 = a^2 - 3^2 \Rightarrow a^2 = 13$.

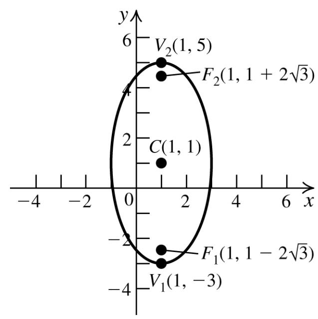
The equation is $\frac{x^2}{13} + \frac{y^2}{4} = 1$.



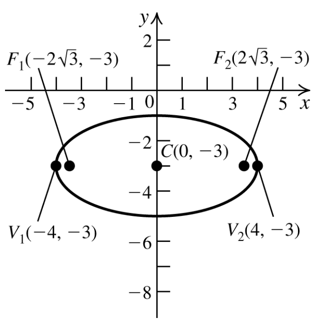
41. Center: (1, 1); $a = 3 \Rightarrow$ vertices: (1, 4) and (1, −2). $b^2 = a^2 - c^2 \Rightarrow 4 = 9 - c^2 \Rightarrow c = \sqrt{5}$. The foci are $\left(1, 1 + \sqrt{5}\right)$ and $\left(1, 1 - \sqrt{5}\right)$.



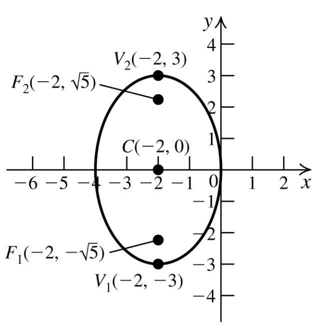
42. Center: (1, 1); $a = 4 \Rightarrow$ vertices: (1, 5) and (1, −3). $b^2 = a^2 - c^2 \Rightarrow 4 = 16 - c^2 \Rightarrow c = 2\sqrt{3}$. The foci are $\left(1, 1+2\sqrt{3}\right)$ and $\left(1, 1-2\sqrt{3}\right)$.



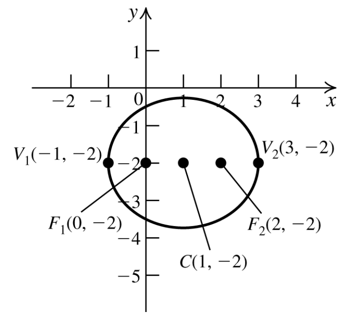
43. Center: (0, −3); $a = 4 \Rightarrow$ vertices: (4, −3) and (−4, −3). $b^2 = a^2 - c^2 \Rightarrow 4 = 16 - c^2 \Rightarrow c = 2\sqrt{3}$. The foci are $\left(2\sqrt{3}, -3\right)$ and $\left(-2\sqrt{3}, -3\right)$.



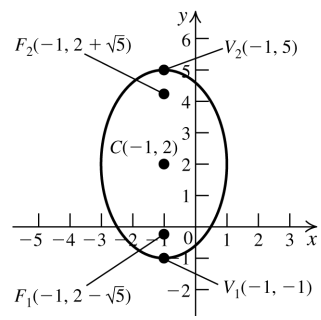
44. Center: (−2, 0); $a = 3 \Rightarrow$ vertices: (−2, 3), (−2, −3). $b^2 = a^2 - c^2 \Rightarrow 4 = 9 - c^2 \Rightarrow c = \sqrt{5}$. The foci are $\left(-2, \sqrt{5}\right)$ and $\left(-2, -\sqrt{5}\right)$.



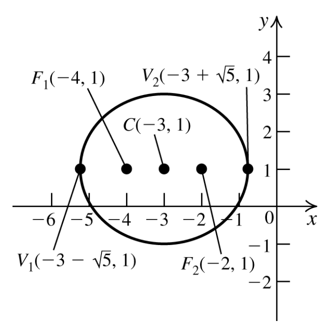
45. $3(x-1)^2 + 4(y+2)^2 = 12 \Rightarrow \dfrac{(x-1)^2}{4} + \dfrac{(y+2)^2}{3} = 1 \Rightarrow$ center: $(1, -2)$. $a = 2 \Rightarrow$ vertices: (3, −2) and (−1, −2). $b^2 = a^2 - c^2 \Rightarrow 3 = 4 - c^2 \Rightarrow c = 1$. The foci are (2, −2), (0, −2).



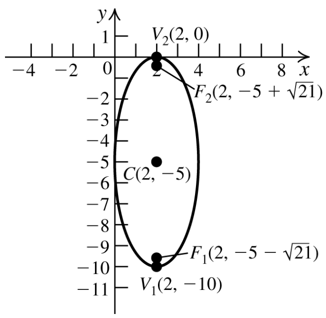
46. $9(x+1)^2 + 4(y-2)^2 = 36 \Rightarrow \dfrac{(x+1)^2}{4} + \dfrac{(y-2)^2}{9} = 1 \Rightarrow$ center: $(-1, 2)$. $a = 3 \Rightarrow$ vertices: (−1, 5) and (−1, −1). $b^2 = a^2 - c^2 \Rightarrow 4 = 9 - c^2 \Rightarrow c = \sqrt{5}$. The foci are $\left(-1, 2+\sqrt{5}\right)$ and $\left(-1, 2-\sqrt{5}\right)$.



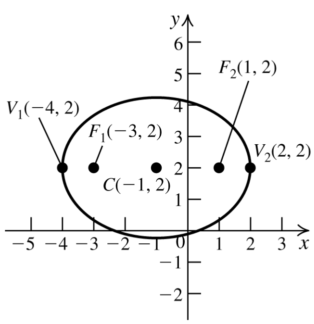
47. $4(x+3)^2 + 5(y-1)^2 = 20 \Rightarrow \dfrac{(x+3)^2}{5} + \dfrac{(y-1)^2}{4} = 1 \Rightarrow$ center: $(-3, 1)$. $a = \sqrt{5} \Rightarrow$ vertices: $\left(-3+\sqrt{5}, 1\right)$ and $\left(-3-\sqrt{5}, 1\right)$. $b^2 = a^2 - c^2 \Rightarrow 4 = 5 - c^2 \Rightarrow c = \sqrt{5}$. The foci are $(-2, 1)$ and $(-4, 1)$.



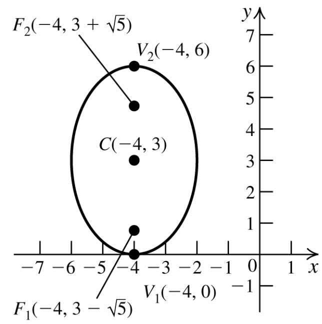
48. $25(x-2)^2+4(y+5)^2=100 \Rightarrow$
$\dfrac{(x-2)^2}{4}+\dfrac{(y+5)^2}{25}=1 \Rightarrow$ center: $(2,-5)$.
$a=5 \Rightarrow$ vertices: $(2,0)$ and $(2,-10)$.
$b^2=a^2-c^2 \Rightarrow 25=4-c^2 \Rightarrow c=\sqrt{21}$.
The foci are $\left(2,-5+\sqrt{21}\right)$ and $\left(2,-5-\sqrt{21}\right)$.



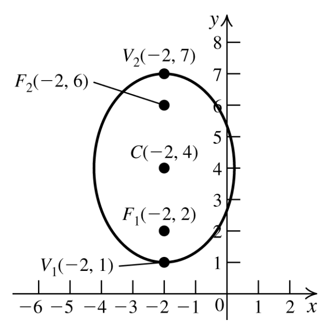
49. Rewrite the equation as
$5(x^2+2x)+9(y^2-4y)=4$, then complete both squares:
$5(x^2+2x+1)+9(y^2-4y+4)=4+5+36 \Rightarrow$
$5(x+1)^2+9(y-2)^2=45 \Rightarrow$
$\dfrac{(x+1)^2}{9}+\dfrac{(y-2)^2}{5}=1 \Rightarrow$ center: $(-1,2)$.
$a=3 \Rightarrow$ vertices: $(2,2)$ and $(-4,2)$.
$b^2=a^2-c^2 \Rightarrow 5=9-c^2 \Rightarrow c=2$.
The foci are $(1, 2)$ and $(-3, 2)$.



50. Rewrite the equation as
$9(x^2+8x)+4(y^2-6y)=-144$, then complete both squares:
$9(x^2+8x+16)+4(y^2-6y+9)$
$=-144+144+36 \Rightarrow$
$9(x+4)^2+4(y-3)^2=36 \Rightarrow$
$\dfrac{(x+4)^2}{4}+\dfrac{(y-3)^2}{9}=1 \Rightarrow$ center: $(-4,3)$.
$a=3 \Rightarrow$ vertices: $(-4,6)$ and $(-4,0)$.
$b^2=a^2-c^2 \Rightarrow 4=9-c^2 \Rightarrow c=\sqrt{5}$. The foci are $\left(-4,3+\sqrt{5}\right)$ and $\left(-4,3-\sqrt{5}\right)$.



51. Rewrite the equation as
$9(x^2+4x)+5(y^2-8y)=-71$, then complete both squares: $9(x^2+4x+4)+5(y^2-8y+16)$
$=-71+36+80 \Rightarrow$
$9(x+2)^2+5(y-4)^2=45 \Rightarrow$
$\dfrac{(x+2)^2}{5}+\dfrac{(y-4)^2}{9}=1 \Rightarrow$ center: $(-2,4)$.
$a=3 \Rightarrow$ vertices: $(-2,7)$ and $(-2,1)$.
$b^2=a^2-c^2 \Rightarrow 5=9-c^2 \Rightarrow c=2$. The foci are $(-2, 2)$ and $(-2, 6)$.



52. Rewrite the equation as
$3(x^2+4x)+4(y^2-4y)=32$, then complete both squares: $3(x^2+4x+4)+4(y^2-4y+4)$
$=32+12+16 \Rightarrow$
$3(x+2)^2+4(y-2)^2=60 \Rightarrow$
$\dfrac{(x+2)^2}{20}+\dfrac{(y-2)^2}{15}=1 \Rightarrow$ center: $(-2,2)$.
$a=2\sqrt{5} \Rightarrow$ vertices: $(-2+2\sqrt{5},2)$ and $(-2-2\sqrt{5},2)$.
$b^2=a^2-c^2 \Rightarrow 15=20-c^2 \Rightarrow c=\sqrt{5}$.
The foci are $(-2+\sqrt{5},2)$ and $(-2-\sqrt{5},2)$.

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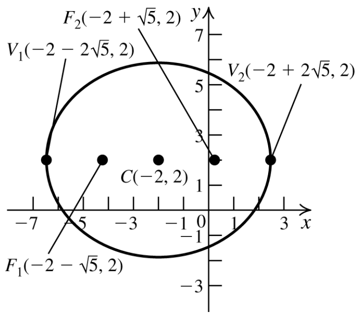
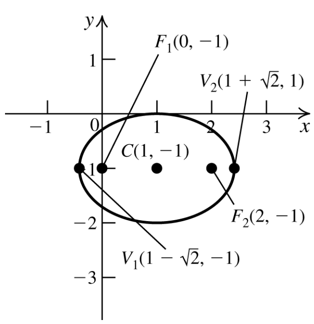
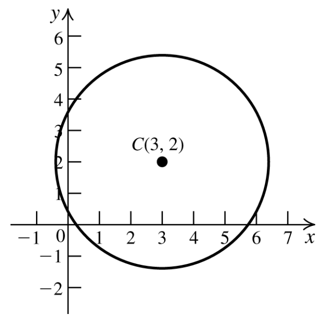


53. Rewrite the equation as $(x^2 - 2x) + 2(y^2 + 2y) = -1$, then complete both squares:
$(x^2 - 2x + 1) + 2(y^2 + 2y + 1) = -1 + 1 + 2 \Rightarrow$
$(x-1)^2 + 2(y+1)^2 = 2 \Rightarrow$
$\frac{(x-1)^2}{2} + (y+1)^2 = 1 \Rightarrow$ center: $(1, -1)$.
$a = \sqrt{2} \Rightarrow$ vertices: $\left(1+\sqrt{2}, -1\right)$ and $\left(1-\sqrt{2}, -1\right)$.
$b^2 = a^2 - c^2 \Rightarrow 1 = 2 - c^2 \Rightarrow c = 1$. The foci are $(2, -1)$ and $(0, -1)$.



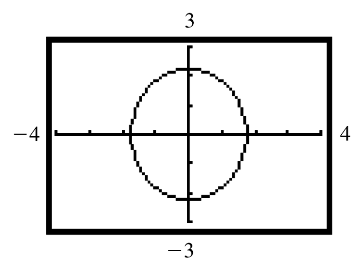
54. Rewrite the equation as $2(x^2 - 6x) + 2(y^2 - 4y) = -3$, then complete both squares:
$2(x^2 - 6x + 9) + 2(y^2 - 4y + 4) = -3 + 18 + 8 \Rightarrow$
$2(x-3)^2 + 2(y-2)^2 = 23 \Rightarrow$
$(x-3)^2 + (y-2)^2 = \frac{23}{2} \Rightarrow$ the curve is a circle centered at (3, 2) with radius $\sqrt{46}/2$.



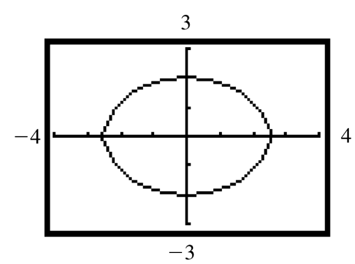
55. Rewrite the equation as $2(x^2 - 2x) + 9(y^2 + 2y) = -12$, then complete both squares:
$2(x^2 - 2x + 1) + 9(y^2 + 2y + 1) = -12 + 2 + 9 \Rightarrow$
$2(x-1)^2 + 9(y+1)^2 = -1 \Rightarrow$ there is no graph.

56. Rewrite the equation as $3(x^2 + 4x) + 2(y^2 - 2y) = -15$, then complete both squares:
$3(x^2 + 4x + 4) + 2(y^2 - 2y + 1) = -15 + 12 + 2 \Rightarrow$
$3(x+2)^2 + 2(y-1)^2 = -1 \Rightarrow$ there is no graph.

57.



58.



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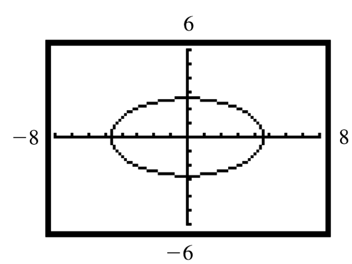

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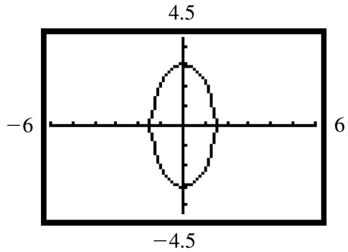
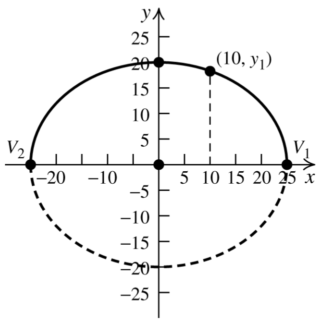


10.3 Applying the Concepts

61. Sketch the ellipse so that its center is at the origin and the major axis lies on the x-axis.

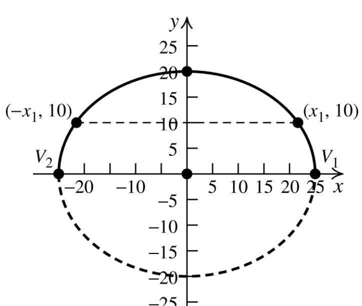


Then $a = 25$ and $b = 20$. The equation of the ellipse is $\frac{x^2}{25^2}+\frac{y^2}{20^2}=1.$ Let $x = 10$, then solve for y:

$$\frac{10^2}{25^2}+\frac{y^2}{20^2}=1 \Rightarrow y = 4\sqrt{21} \approx 18.3 \text{ ft.}$$

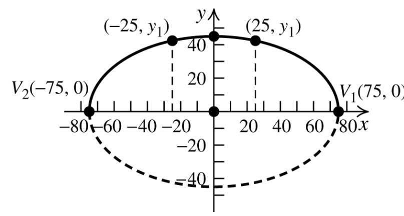
62. From exercise 61, the equation of the ellipse is $\frac{x^2}{25^2}+\frac{y^2}{20^2}=1.$ Let $y = 10$, then solve for x:

$$\frac{x^2}{25^2}+\frac{10^2}{20^2}=1 \Rightarrow x = \frac{25\sqrt{3}}{2} \approx 21.7 \text{ ft.}$$



The distance between the two points is $21.65 \times 2 = 43.3$ feet.

63. Sketch the ellipse so that its center is at the origin and the major axis lies on the x-axis.



Then $a = 75$ and $b = 45$. The equation of the ellipse is $\frac{x^2}{75^2}+\frac{y^2}{45^2}=1.$

Let $x = 25$, then solve for y:

$$\frac{25^2}{75^2}+\frac{y^2}{45^2}=1 \Rightarrow y = 30\sqrt{2} \approx 42.4 \text{ m.}$$

64. $a = 3$ and $b = 2$, so $b^2 = a^2 - c^2 \Rightarrow$ $4 = 9 - c^2 \Rightarrow c = \sqrt{5}$. The foci are approximately 2.2 feet from the center, along the major axis.

65. The bet should not be accepted. The pool shark can hit the ball from any point straight into the pocket, or he can shoot it through the other focus and it will fall into the pocket because of the reflecting property.

66. $a = 48$ and $b = 23$, so $b^2 = a^2 - c^2 \Rightarrow$ $23^2 = 48^2 - c^2 \Rightarrow c = 5\sqrt{71} \approx 42.13$ feet from the center or $48 - 42.13 \approx 5.9$ feet from the wall on the opposite side.

67. $a = 125$ and $b = 80$, so $b^2 = a^2 - c^2 \Rightarrow$ $80^2 = 125^2 - c^2 \Rightarrow c = 15\sqrt{41} \approx 96.05$ feet from the center or $125 - 96.05 \approx 29$ feet from the wall on the opposite side.

68. Using the hints given, we have

$$a = \frac{128.49+154.83}{2} = 141.66 \Rightarrow$$

$a^2 = 20{,}067.5556$ and

$c = 141.66 - 128.49 = 13.17$.

$b^2 = 141.66^2 - 13.17^2 = 19{,}894.1067$. The equation is $\frac{x^2}{20{,}067.5556}+\frac{y^2}{19{,}894.1067}=1$.

69. Using the hints given in exercise 58, we have

$$a = \frac{91.38+94.54}{2} = 92.96 \Rightarrow a^2 = 8641.5616$$

and $c = 92.96 - 91.38 = 1.58$.

$b^2 = 92.96^2 - 1.58^2 = 8639.0652$. The equation is $\frac{x^2}{8641.5616}+\frac{y^2}{8639.0652}=1$.

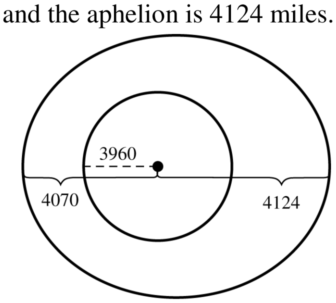
70. Using the hints given in exercise 58, we have
$a = \dfrac{28.56 + 43.88}{2} = 36.22 \Rightarrow a^2 = 1311.8884$
and $c = 36.22 - 28.56 = 7.66$.
$b = 36.22^2 - 7.66^2 = 1253.2128$. The equation is $\dfrac{x^2}{1311.8884} + \dfrac{y^2}{1253.2128} = 1$.

71. Using the hints given in exercise 58, we have
$a = \dfrac{837.05 + 936.37}{2} = 886.71 \Rightarrow$
$a^2 = 786,254.6241$ and
$c = 886.71 - 837.05 = 49.66$.
$b^2 = 886.71^2 - 49.66^2 = 783,788.5085$.
The equation is
$\dfrac{x^2}{786,254.6241} + \dfrac{y^2}{783,788.5085} = 1$.

72. $a = \dfrac{768,806}{2} = 384,403$ and
$b = \dfrac{767,746}{2} = 383,873$. So $b^2 = a^2 - c^2 \Rightarrow$
$383,873^2 = 384,403^2 - c^2 \Rightarrow c \approx 20,179$.
The perihelion = $a - c \approx 364,224$ km. The aphelion = $a + c \approx 404,582$ km.

73. $a = \dfrac{5.39 \times 10^9}{2} = 2.695 \times 10^9$ and
$b = \dfrac{1.36 \times 10^9}{2} = 6.8 \times 10^8$. So
$b^2 = a^2 - c^2 \Rightarrow$
$(6.8 \times 10^8)^2 = (2.695 \times 10^9)^2 - c^2 \Rightarrow$
$c \approx 2.6078 \times 10^9$. The perihelion = $a - c$
$\approx 8.72 \times 10^7$ km. The aphelion =
$a + c \approx 5.3028 \times 10^9$ km.

74. The perihelion of the satellite is 4070 miles and the aphelion is 4124 miles.



Not drawn to scale

$a = \dfrac{4070 + 4124}{2} = 4097 \Rightarrow a^2 = 16,785,409$
and $c = 4097 - 4070 = 27$. Then
$b^2 = 16,784,680$. The equation is
$\dfrac{x^2}{16,785,409} + \dfrac{y^2}{16,784,680} = 1$.

10.3 Beyond the Basics

75. The center is (0, 0) and $a = 5$, so the equation is either $\dfrac{x^2}{5^2} + \dfrac{y^2}{b^2} = 1$ or $\dfrac{x^2}{b^2} + \dfrac{y^2}{5^2} = 1$.
Substitute the coordinates of the point for x and y, then solve for b:
$\dfrac{(-3)^2}{5^2} + \dfrac{(16/5)^2}{b^2} = 1 \Rightarrow 9b^2 + 256 = 25b^2 \Rightarrow$
$b^2 = 16 \Rightarrow b = 4.$
$\dfrac{(-3)^2}{b^2} + \dfrac{(16/5)^2}{5^2} = 1 \Rightarrow$
$5625 + 256b^2 = 625b^2 \Rightarrow b^2 = \dfrac{625}{41}.$
The equations are either $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ or
$\dfrac{x^2}{625/41} + \dfrac{y^2}{25} = 1$.

76. The center is (0, 0) and $a = 3$, so the equation is either $\dfrac{x^2}{3^2} + \dfrac{y^2}{b^2} = 1$ or $\dfrac{x^2}{b^2} + \dfrac{y^2}{3^2} = 1$.
Substitute the coordinates of the point for x and y, then solve for b:
$\dfrac{(1)^2}{5^2} + \dfrac{\left(3\sqrt{5}/2\right)^2}{b^2} = 1 \Rightarrow \dfrac{1}{25} + \dfrac{45}{4b^2} = 1 \Rightarrow$
$b^2 = \dfrac{1125}{96} \Rightarrow b \approx 3.4$.
Because the exercise states that the length of the major axis is 6 there is no solution.

77. Substitute the coordinates of the points into the equation of the ellipse and then solve the system: $\begin{cases} \dfrac{2^2}{b^2} + \dfrac{1^2}{a^2} = 1 \\ \dfrac{1^2}{b^2} + \dfrac{(-3)^2}{a^2} = 1 \end{cases}$. Let $u = \dfrac{1}{a^2}$ and
$v = \dfrac{1}{b^2}$.

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Then

$$\begin{cases} \dfrac{2^2}{b^2} + \dfrac{1^2}{a^2} = 1 \\ \dfrac{1^2}{b^2} + \dfrac{(-3)^2}{a^2} = 1 \end{cases} \Rightarrow \begin{cases} 4v + u = 1 \\ v + 9u = 1 \end{cases} \Rightarrow$$

$u = \dfrac{3}{35}, v = \dfrac{8}{35} \Rightarrow \dfrac{1}{a^2} = \dfrac{3}{35} \Rightarrow a = \dfrac{\sqrt{105}}{3}$ and

$\dfrac{1}{b^2} = \dfrac{8}{35} \Rightarrow b = \dfrac{\sqrt{70}}{4}$.

The length of the major axis is $\dfrac{2\sqrt{105}}{3}$. The length of the minor axis is $\dfrac{\sqrt{70}}{2}$.

78. The general form of the equation is $\dfrac{(x-2)^2}{a^2} + \dfrac{(y-1)^2}{b^2} = 1$. Substitute the coordinates of the points into the equation of the ellipse and then solve the system:

$$\begin{cases} \dfrac{(10-2)^2}{a^2} + \dfrac{(1-1)^2}{b^2} = 1 \\ \dfrac{(6-2)^2}{a^2} + \dfrac{(2-1)^2}{b^2} = 1 \end{cases} \Rightarrow \begin{cases} \dfrac{8^2}{a^2} + \dfrac{0}{b^2} = 1 \\ \dfrac{16}{a^2} + \dfrac{1}{b^2} = 1 \end{cases} \Rightarrow$$

$a^2 = 64, b^2 = 4/3$. Then the equation is

$\dfrac{(x-2)^2}{64} + \dfrac{(y-1)^2}{4/3} = 1 \Rightarrow$.

$(x-2)^2 + 48(y-1)^2 = 64$

79. When $e = 0$, the ellipse becomes a circle.

80. $a = 4, b^2 = a^2 - c^2 \Rightarrow 9 = 16 - c^2 \Rightarrow c = \sqrt{7}$

$e = \dfrac{\sqrt{7}}{4}$

81. $20x^2 + 36y^2 = 720 \Rightarrow \dfrac{x^2}{36} + \dfrac{y^2}{20} = 1$

$a = 6, b^2 = a^2 - c^2 \Rightarrow 20 = 36 - c^2 \Rightarrow c = 4$

$e = \dfrac{4}{6} = \dfrac{2}{3}$

82. $x^2 + 4y^2 = 1 \Rightarrow x^2 + \dfrac{y^2}{1/4} = 1$

$a = 1, b^2 = a^2 - c^2 \Rightarrow \dfrac{1}{4} = 1 - c^2 \Rightarrow c = \dfrac{\sqrt{3}}{2}$

$e = \dfrac{\sqrt{3}}{2}$

83. $a = 5, b^2 = a^2 - c^2 \Rightarrow 9 = 25 - c^2 \Rightarrow c = 4$

$e = \dfrac{4}{5}$

84. Rewrite the equation as $(x^2 - 2x) + 2(y^2 + 2y) = -1$, then complete both squares:

$(x^2 - 2x + 1) + 2(y^2 + 2y + 1) = -1 + 1 + 2 \Rightarrow$

$(x+1)^2 + 2(y+1)^2 = 2 \Rightarrow \dfrac{(x+1)^2}{2} + (y+1)^2 = 1$

$a = \sqrt{2}, b^2 = a^2 - c^2 \Rightarrow 1 = 2 - c^2 \Rightarrow c = 1$

$e = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

85. $2c = 4 \Rightarrow c = 2; e = \dfrac{c}{a} \Rightarrow \dfrac{1}{2} = \dfrac{2}{a} \Rightarrow a = 4$

$b^2 = a^2 - c^2 \Rightarrow b^2 = 16 - 4 = 12$

The equation is $\dfrac{x^2}{16} + \dfrac{y^2}{12} = 1$.

86. $a = 3; e = \dfrac{c}{a} \Rightarrow \dfrac{1}{3} = \dfrac{c}{3} \Rightarrow c = 1$

$b^2 = a^2 - c^2 \Rightarrow b^2 = 9 - 1 \Rightarrow b^2 = 8$

The equation is $\dfrac{x^2}{9} + \dfrac{y^2}{8} = 1$.

87. The distance from P to the point (4, 0) is $\sqrt{(x-4)^2 + y^2}$, and the distance from P to the line is $|x - 16|$. So, the equation of the path of P is $\sqrt{(x-4)^2 + y^2} = \dfrac{1}{2}|x - 16| \Rightarrow$

$(x-4)^2 + y^2 = \dfrac{1}{4}(x-16)^2 \Rightarrow$

$\dfrac{3x^2}{4} + y^2 = 48 \Rightarrow \dfrac{x^2}{64} + \dfrac{y^2}{48} = 1 \Rightarrow$

$a = 8, 48 = 64 - c^2 \Rightarrow c = 4 \Rightarrow e = \dfrac{1}{2}$.

88. The distance from P to the point (0, 2) is $\sqrt{x^2 + (y-2)^2}$, and the distance from P to the line is $|y - 10|$. The equation of the path of P is $\sqrt{x^2 + (y-2)^2} = \dfrac{1}{3}|y - 10| \Rightarrow$

$x^2 + (y-2)^2 = \dfrac{1}{9}(y-10)^2 \Rightarrow$

$9x^2 + 8(y^2 - 2y) = 64 \Rightarrow$

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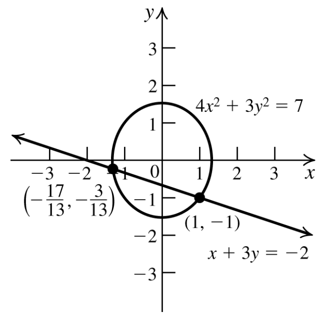
$$9x^2 + 8(y^2 - 2y + 1) = 64 + 8 \Rightarrow$$
$$9x^2 + 8(y-1)^2 = 72 \Rightarrow \frac{x^2}{8} + \frac{(y-1)^2}{9} = 1 \Rightarrow$$
$$a = 3, 8 = 9 - c^2 \Rightarrow c = 1 \Rightarrow e = \frac{1}{3}.$$

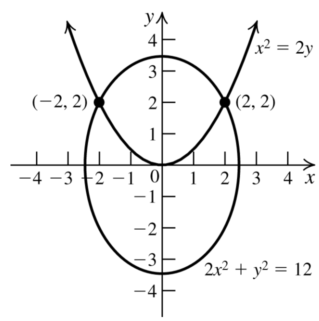
89. Assume that the center of the ellipse is at the origin, the major axis is on the x-axis, and the minor axis is on the y-axis. So the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The length of the latus rectum is the difference in the y-coordinates of the two points so the ellipse with x-coordinate c. Letting $x = c$,
$$\frac{c^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{a^2 - b^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow$$
$y = \pm\frac{b^2}{a}$. So the length of the latus rectum is $\frac{2b^2}{a}$.

90. Because the center is (0, 0) and the major axis is on the x-axis, the general form of the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. $e = \frac{c}{a} = \frac{1}{\sqrt{2}} \Rightarrow a = c\sqrt{2} \Rightarrow$
$a^2 = 2c^2$. $c^2 = a^2 - b^2 \Rightarrow c^2 = 2c^2 - b^2 \Rightarrow$
$b^2 = c^2$. Using the result of exercise 79, we know that $3 = \frac{2b^2}{a} \Rightarrow a = \frac{2b^2}{3} = \frac{2c^2}{3}$.
$c\sqrt{2} = a = \frac{2c^2}{3} \Rightarrow c = \frac{3}{\sqrt{2}}$ or $c = 0$ (reject this).
$$e = \frac{c}{a} \Rightarrow \frac{1}{\sqrt{2}} = \frac{3/\sqrt{2}}{a} \Rightarrow a = 3$$
$$\left(\frac{3}{\sqrt{2}}\right)^2 = 9 - b^2 \Rightarrow b^2 = \frac{9}{2}$$
The equation is $\frac{x^2}{9} + \frac{y^2}{9/2} = 1$.

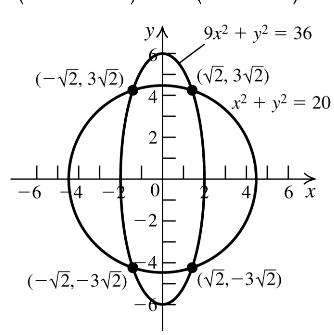
91. Using substitution, we have
$$\begin{cases} x + 3y = -2 \\ 4x^2 + 3y^2 = 7 \end{cases} \Rightarrow \begin{cases} x = -3y - 2 \\ 4x^2 + 3y^2 = 7 \end{cases} \Rightarrow$$
$$4(-3y-2)^2 + 3y^2 = 7 \Rightarrow$$
$$39y^2 + 48y + 9 = 0 \Rightarrow y = -1 \text{ or } y = -\frac{3}{13}$$
$$x + 3(-1) = -2 \Rightarrow x = 1$$
$$x + 3\left(-\frac{3}{13}\right) = -2 \Rightarrow x = -\frac{17}{13}$$
The points of intersection are $\left(-\frac{17}{13}, -\frac{3}{13}\right)$ and (1, −1).



92. Using substitution, we have
$$\begin{cases} x^2 = 2y \\ 2x^2 + y^2 = 12 \end{cases} \Rightarrow y^2 + 4y = 12 \Rightarrow$$
$y = 2$ or $y = -6$ (reject this – $x^2 = 2(-6) \Rightarrow$ $x = 2i\sqrt{3}$, which is not possible). $x^2 = 4 \Rightarrow$ $x = \pm 2$. The points of intersection are (−2, 2) and (2, 2).

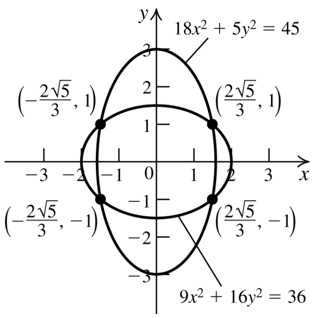


93. Using elimination, we have
$$\begin{cases} x^2 + y^2 = 20 \\ 9x^2 + y^2 = 36 \end{cases} \Rightarrow 8x^2 = 16 \Rightarrow x = \pm\sqrt{2}$$
$2 + y^2 = 20 \Rightarrow y = \pm 3\sqrt{2}$. The points of intersection are $(-\sqrt{2}, -3\sqrt{2}), (-\sqrt{2}, 3\sqrt{2}),$ $(\sqrt{2}, -3\sqrt{2})$, and $(\sqrt{2}, 3\sqrt{2})$.



94. Using elimination, we have $\begin{cases} 9x^2+16y^2=36 \\ 18x^2+\ 5y^2=45 \end{cases} \Rightarrow 27y^2=27 \Rightarrow y=\pm 1;\quad 9x^2+16=36 \Rightarrow x=\pm\frac{2\sqrt{5}}{3}$.

The points of intersection are $\left(-\frac{2\sqrt{5}}{3},-1\right),\left(-\frac{2\sqrt{5}}{3},1\right),\left(\frac{2\sqrt{5}}{3},-1\right)$, and $\left(\frac{2\sqrt{5}}{3},1\right)$.



95. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{(5/2)^2}+\frac{y^2}{5^2}=1 \Rightarrow \frac{4x^2}{25}+\frac{y^2}{25}=1 \Rightarrow 4x^2+y^2=25 \Rightarrow y^2=25-4x^2$ The tangent line equation is $y=m(x-2)+3$. Solve $25-4x^2=[m(x-2)+3]^2$ to find m. $25-4x^2=m^2(x-2)^2+6m(x-2)+9 \Rightarrow$

$(4+m^2)x^2+(6m-4m^2)x+(4m^2-12m-16)=0.$ For a unique solution, the discriminant must be zero.

$(6m-4m^2)^2-4(4+m^2)(4m^2-12m-16)=0 \Rightarrow 36m^2+192m+256=0 \Rightarrow$

$9m^2+48m+64=0 \Rightarrow (3m+8)^2=0 \Rightarrow m=-\frac{8}{3}$

The tangent line as equation $y=-\frac{8}{3}(x-2)+3=-\frac{8}{3}x+\frac{25}{3}$.

96. Let $p=x_1$ and $q=y_1$. We need to find the equation of the tangent line to the ellipse with equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point (p, q). First notice that $y^2=b^2-\frac{b^2x^2}{a^2}$, and the tangent line equation is $y=m(x-p)+q$. Now solve $b^2-\frac{b^2x^2}{a^2}=\left[m(x-p)+q\right]^2$. First multiply both sides by a^2, then expand the right side:

$a^2b^2-b^2x^2=a^2\left[m(x-p)+q\right]^2=a^2m^2x^2-2a^2m^2xp+2a^2mxq+a^2m^2p^2-2a^2mpq+a^2q^2$ Now combine like terms, factoring where possible, and set the equation equal to 0:

$\left(-b^2-a^2m^2\right)x^2+\left(2a^2m^2p-2a^2mq\right)x-a^2m^2p^2+2a^2mpq-a^2q^2+a^2b^2=0$

Since the roots of the equation must be equal for the line to be tangent to the ellipse, set the discriminant equal to 0, then solve for m.

$$\left(2a^2m^2p-2a^2mq\right)^2-4\left(-b^2-a^2m^2\right)\left(-a^2m^2p^2+2a^2mpq-a^2q^2+a^2b^2\right)=0$$
$$4a^4m^4p^2-8a^4m^3pq+4a^4m^2q^2-4\left(-b^2-a^2m^2\right)\left(-a^2m^2p^2+2a^2mpq-a^2q^2+a^2b^2\right)=0$$
$$a^4m^4p^2-2a^4m^3pq+a^4m^2q^2-\left(-b^2-a^2m^2\right)\left(-a^2m^2p^2+2a^2mpq-a^2q^2+a^2b^2\right)=0$$
$$a^2m^4p^2-2a^2m^3pq+a^2m^2q^2+\left(b^2+a^2m^2\right)\left(-m^2p^2+2mpq-q^2+b^2\right)=0$$

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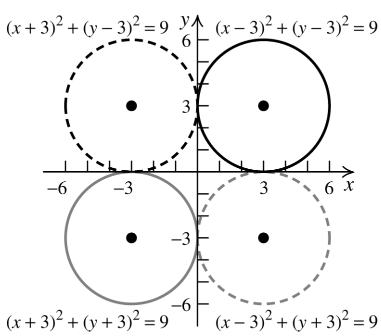
$$a^2m^4p^2 - 2a^2m^3pq + a^2m^2q^2 - b^2m^2p^2 + 2b^2mpq - b^2q^2 + b^4$$
$$-a^2m^4p^2 + 2a^2m^3pq - a^2m^2q^2 + a^2m^2b^2 = 0$$
$$-b^2m^2p^2 + 2b^2mpq - b^2q^2 + b^4 + a^2m^2b^2 = 0$$
$$-m^2p^2 + 2mpq - q^2 + b^2 + a^2m^2 = 0$$
$$-m^2p^2 + 2mpq - b^2 + \frac{b^2p^2}{a^2} + b^2 + a^2m^2 = 0$$
$$-m^2p^2 + 2mpq + \frac{b^2p^2}{a^2} + a^2m^2 = 0$$
$$-a^2m^2p^2 + 2a^2mpq + b^2p^2 + a^4m^2 = 0$$
$$a^2\left(a^2 - p^2\right)m^2 + 2a^2mpq + b^2p^2 = 0$$
$$a^2\left(a^2 - \left[a^2 - \frac{a^2q^2}{b^2}\right]\right)m^2 + 2a^2mpq + b^2p^2 = 0$$
$$a^2\left(\frac{a^2q^2}{b^2}\right)m^2 + 2a^2mpq + b^2p^2 = 0$$
$$a^4q^2m^2 + 2a^2b^2mpq + b^4p^2 = 0$$
$$\left(a^2qm + b^2p\right)^2 = 0$$

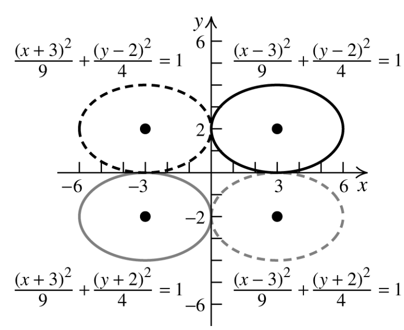
Thus, $m = -\frac{b^2p}{a^2q}$, and the equation for the tangent line is $y = -\frac{b^2p}{a^2q}(x-p) + q$ or $y = -\frac{b^2p}{a^2q}x + \frac{b^2p^2}{a^2q} + q$.

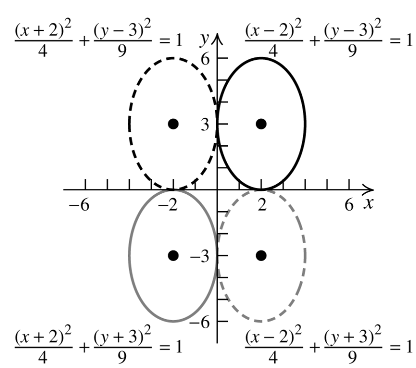
10.3 Critical Thinking/Discussion/Writing

97. There are four circles, one in each quadrant:
$(x-3)^2 + (y-3)^2 = 9, (x+3)^2 + (y-3)^2 = 9,$
$(x-3)^2 + (y+3)^2 = 9,$ and
$(x+3)^2 + (y+3)^2 = 9$.



98. There are eight ellipses, two in each quadrant:
$\frac{(x-3)^2}{9} + \frac{(y-2)^2}{4} = 1, \frac{(x-2)^2}{4} + \frac{(y-3)^2}{9} = 1,$
$\frac{(x+3)^2}{9} + \frac{(y-2)^2}{4} = 1, \frac{(x+2)^2}{4} + \frac{(y-3)^2}{9} = 1,$
$\frac{(x+3)^2}{9} + \frac{(y+2)^2}{4} = 1, \frac{(x+2)^2}{4} + \frac{(y+3)^2}{9} = 1,$
$\frac{(x-3)^2}{9} + \frac{(y+2)^2}{4} = 1, \frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1$





10.3 Maintaining Skills

99. $$x^2+6x+y^2-2y-5=0$$
$$\left(x^2+6x\right)+\left(y^2-2y\right)=5$$
$$\left(x^2+6x+9\right)+\left(y^2-2y+1\right)=5+9+1$$
$$(x+3)^2+(y-1)^2=15$$
Center: (−3, 1)
Radius: $\sqrt{15}$

100. $$x^2+4x+y^2+6y+5=0$$
$$\left(x^2+4x\right)+\left(y^2+6y\right)=-5$$
$$\left(x^2+4x+4\right)+\left(y^2+6y+9\right)=-5+4+9$$
$$(x+2)^2+(y+3)^2=8$$
Center: (−2, −3)
Radius: $\sqrt{8}=2\sqrt{2}$

101. To find the x-intercepts, let $y = 0$ and solve for x.
$x^2+0^2=25 \Rightarrow x^2=25 \Rightarrow x=\pm 5$
To find the y-intercept, let $x = 0$ and solve for y.
$0^2+y^2=25 \Rightarrow y^2=25 \Rightarrow y=\pm 25$
The x-intercepts are −5 and 5. The y-intercepts are −5 and 5.

102. To find the x-intercepts, let $y = 0$ and solve for x.
$(x-2)^2+(0+3)^2=8 \Rightarrow (x-2)^2+9=8 \Rightarrow$
$(x-2)^2=-1$ which does not have a real solution.
To find the y-intercept, let $x = 0$ and solve for y.
$(0-2)^2+(y+3)^2=8 \Rightarrow 4+(y+3)^2=8 \Rightarrow$
$(y+3)^2=4 \Rightarrow y+3=-2$ or $y+3=2 \Rightarrow$
$y=-5$ or $y=-1$
There are no x-intercepts. The y-intercepts are −5 and −1.

103. $y=-4x+11$

104. $y=7$

105. $m=\dfrac{-5-7}{-1-(-3)}=\dfrac{-12}{2}=-6$
$y-7=-6(x-(-3)) \Rightarrow y-7=-6(x+3) \Rightarrow$
$y=-6x-18+7=-6x-11$

106. Since the line we are seeking is parallel to the given line, the slopes are the same.
$6x+2y=7 \Rightarrow 2y=-6x+7 \Rightarrow y=-3x+\dfrac{7}{2}$
$m=-3$.
$y-(-4)=-3(x-2) \Rightarrow y=-3x+6-4 \Rightarrow$
$y=-3x+2$

Review Section 3.6 for the procedures to find the asymptotes of a rational function.

107. $\dfrac{2x^2-3x+1}{x-2}$
$x-2=0 \Rightarrow x=2 \Rightarrow$ the vertical asymptote is $x = 2$. Since the largest exponent in the numerator is greater than the largest exponent in the denominator, there is no horizontal asymptote.

$$\begin{array}{r|l} & 2x+1 \\ x-2 & 2x^2-3x+1 \\ & \underline{2x^2-4x} \\ & x+1 \\ & \underline{x-2} \\ & -3 \end{array}$$

The oblique asymptote is $y=2x+1$.

108. $\dfrac{x^2-4x-5}{x-3}$
$x-3=0 \Rightarrow x=3 \Rightarrow$ the vertical asymptote is $x = 3$. Since the largest exponent in the numerator is greater than the largest exponent in the denominator, there is no horizontal asymptote.

$$\begin{array}{r|l} & x-1 \\ x-3 & x^2-4x-5 \\ & \underline{x^2-3x} \\ & -x-5 \\ & \underline{-x-3} \\ & -2 \end{array}$$

The oblique asymptote is $y=x-1$.

10.4 The Hyperbola

10.4 Practice Problems

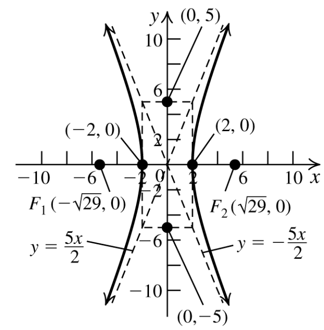
1. The transverse axis is on the y-axis.

2. Rewrite the equation in standard form to determine a and b. $x^2 - 4y^2 = 8 \Rightarrow$
$\frac{x^2}{8} - \frac{y^2}{2} = 1 \Rightarrow a^2 = 8 \Rightarrow a = 2\sqrt{2}$ and
$b^2 = 2 \Rightarrow b = \sqrt{2}.$
The transverse axis of the hyperbola is along the x-axis, so the vertices are $\left(-2\sqrt{2}, 0\right)$ and $\left(2\sqrt{2}, 0\right).$
To find the foci, we need c:
$c^2 = a^2 + b^2 \Rightarrow c^2 = 8 + 2 \Rightarrow c = \sqrt{10}.$ The foci are $\left(-\sqrt{10}, 0\right)$ and $\left(\sqrt{10}, 0\right).$

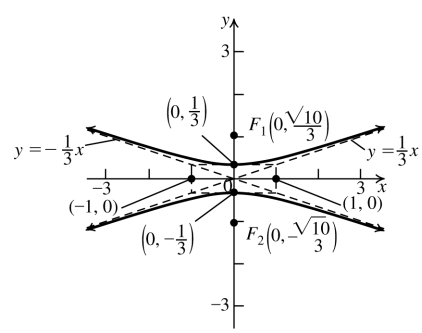
3. Since the foci of the hyperbola, (0, −6) and (0, 6) lie on the y-axis, the transverse axis also lies on the y-axis, and $c = 6$. The center of the hyperbola is (0, 0), so the standard form of this hyperbola is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1.$
The vertices are (0, −4) and (0, 4), so $a = 4$.
$c^2 = a^2 + b^2 \Rightarrow 6^2 = 4^2 + b^2 \Rightarrow b^2 = 20.$
The equation is $\frac{y^2}{16} - \frac{x^2}{20} = 1.$

4. The hyperbola $\frac{y^2}{4} - \frac{x^2}{9} = 1$ is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1,$ so $b = 3$ and $a = 2$. The asymptotes are of the form $y = \frac{a}{b}x$ and $y = -\frac{a}{b}x.$ They are $y = \frac{2}{3}x$ and $y = -\frac{2}{3}x.$

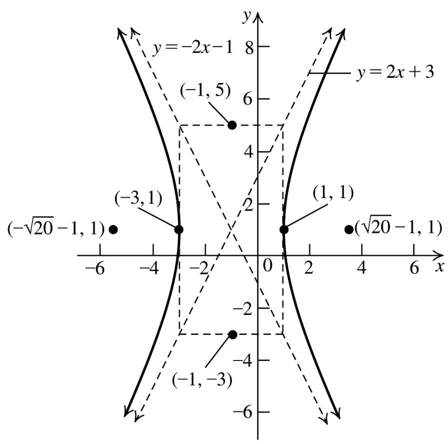
5. **a.** $25x^2 - 4y^2 = 100 \Rightarrow \frac{x^2}{4} - \frac{y^2}{25} = 1 \Rightarrow a = 2,$ $b = 5.$
$c^2 = a^2 + b^2 \Rightarrow c^2 = 4 + 25 \Rightarrow c = \sqrt{29}.$
The vertices are (2, 0) and (−2, 0). The endpoints of the conjugate axis are (0, 5) and (0, −5), and the foci are $\left(-\sqrt{29}, 0\right)$ and $\left(\sqrt{29}, 0\right).$
The asymptotes are $y = \frac{b}{a}x = \frac{5}{2}x$ and $y = -\frac{b}{a}x = -\frac{5}{2}x.$



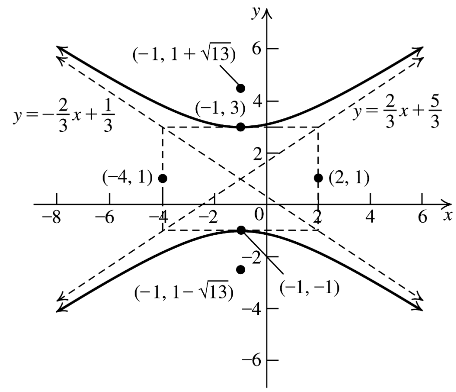
b. $9y^2 - x^2 = 1 \Rightarrow \frac{y^2}{1/9} - \frac{x^2}{1} = 1 \Rightarrow a^2 = \frac{1}{9} \Rightarrow$
$a = \frac{1}{3}$ and $b = 1.$
$c^2 = a^2 + b^2 \Rightarrow c^2 = \frac{1}{9} + 1 \Rightarrow c = \frac{\sqrt{10}}{3}.$
The vertices are $\left(0, \frac{1}{3}\right)$ and $\left(0, -\frac{1}{3}\right).$
The endpoints of the conjugate axis are (1, 0) and (−1, 0), and the foci are $\left(0, \frac{\sqrt{10}}{3}\right)$ and $\left(0, -\frac{\sqrt{10}}{3}\right).$
The asymptotes are $y = \frac{a}{b}x = \frac{1/3}{1}x = \frac{1}{3}x$ and $y = -\frac{a}{b}x = -\frac{1/3}{1}x = -\frac{1}{3}x.$



6. a. $\frac{(x+1)^2}{4} - \frac{(y-1)^2}{16} = 1 \Rightarrow a = 2,\ b = 4,$ and center (−1, 1). The vertices are $(-1-2, 1) = (-3, 1)$ and $(-1+2, 1) = (1, 1)$. The endpoints of the conjugate axes are $(-1, 1-4) = (-1, -3)$ and $(-1, 1+4) = (-1, 5)$. The asymptotes are $y - k = \pm\frac{b}{a}(x-h) \Rightarrow y - 1 = \pm 2(x+1)$.
$c^2 = a^2 + b^2 \Rightarrow c^2 = 4 + 16 \Rightarrow c = \pm\sqrt{20}$
The foci are $\left(-\sqrt{20}-1, 1\right)$ and $\left(\sqrt{20}-1, 1\right)$.



b. $\frac{(y-1)^2}{4} - \frac{(x+1)^2}{9} = 1 \Rightarrow a = 2,\ b = 3,$ and center (−1, 1). The vertices are $(-1, 1-2) = (-1, -1)$ and $(-1, 1+2) = (-1, 3)$. The endpoints of the conjugate axes are $(-1-3, 1) = (-4, 1)$ and $(-1+3, 1) = (2, 1)$ The asymptotes are $y - k = \pm\frac{a}{b}(x-h) \Rightarrow y - 1 = \pm\frac{2}{3}(x+1)$.
$c^2 = a^2 + b^2 \Rightarrow c^2 = 4 + 9 \Rightarrow c = \pm\sqrt{13}$
The foci are $\left(-1, 1+\sqrt{13}\right)$ and $\left(-1, 1-\sqrt{13}\right)$.



7. Complete the square to put the equation in standard form:

$$\left(x^2 - 2x\right) - 4\left(y^2 - 4y\right) = 20$$
$$\left(x^2 - 2x + 1\right) - 4\left(y^2 - 4y + 4\right) = 20 + 1 - 16$$
$$(x-1)^2 - 4(y-2)^2 = 5$$
$$\frac{(x-1)^2}{5} - \frac{4(y-2)^2}{5} = 1$$
$$\frac{(x-1)^2}{5} - \frac{(y-2)^2}{5/4} = 1$$

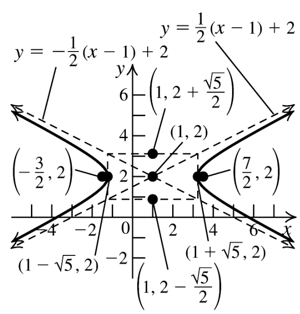
The center is (1, 2), $a = \sqrt{5}$ and $b = \frac{\sqrt{5}}{2}$.
The vertices are $\left(1+\sqrt{5}, 2\right)$ and $\left(1-\sqrt{5}, 2\right)$.
The endpoints of the conjugate axis are $\left(1, 2+\frac{\sqrt{5}}{2}\right)$ and $\left(1, 2-\frac{\sqrt{5}}{2}\right)$.

The asymptotes are $y - k = \pm\frac{b}{a}(x-h) \Rightarrow$

$$y - 2 = \pm\frac{\sqrt{5}/2}{\sqrt{5}}(x-1) \Rightarrow y - 2 = \pm\frac{1}{2}(x-1).$$

$c^2 = a^2 + b^2 \Rightarrow c^2 = 5 + \frac{5}{4} = \frac{25}{4} \Rightarrow c = \frac{5}{2}.$

The foci are $\left(1+\frac{5}{2}, 2\right) = \left(\frac{7}{2}, 2\right)$ and $\left(1-\frac{5}{2}, 2\right) = \left(-\frac{3}{2}, 2\right)$.



8. The hyperbola has foci (−150, 0) and (150, 0), so $c = 150$. The difference of the distances from the boat to A and B is 260 miles, so $a = 130$.
$c^2 = a^2 + b^2 \Rightarrow$
$150^2 = 130^2 + b^2 \Rightarrow b^2 = 5600.$
The equation of the hyperbola is
$$\frac{x^2}{130^2} - \frac{y^2}{5600} = 1 \Rightarrow \frac{x^2}{16,900} - \frac{y^2}{5600} = 1.$$

10.4 Basic Concepts and Skills

1. A hyperbola is a set of all points in the plane, the absolute value of the difference of whose distances form two fixed points is constant.

2. The line segment joining the two vertices of a hyperbola is called the transverse axis.

3. The standard equation of the hyperbola with center $(0, 0)$, vertices $(\pm a, 0)$, and foci $(\pm c, 0)$ is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $b^2 = \underline{c^2 - a^2}$.

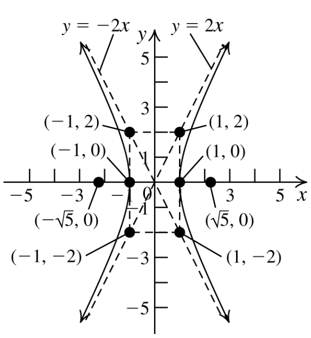
4. For the hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, the vertices ae $\underline{(0, a)}$ and $\underline{(0, -a)}$, and the foci are $(0, \pm c)$, where $c^2 = \underline{a^2 + b^2}$.

5. True **6.** True

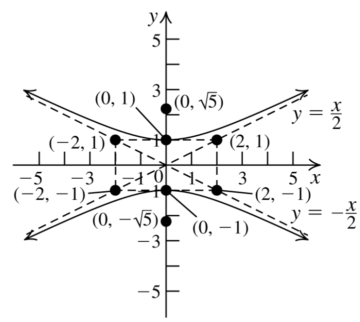
7. g **8.** c **9.** h **10.** e

11. d **12.** a **13.** b **14.** f

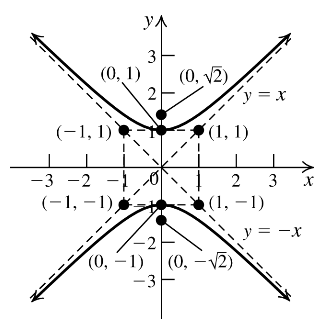
15. $a^2 = 1 \Rightarrow$ the vertices are (1, 0) and (−1, 0).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 1 + 4 \Rightarrow c = \sqrt{5} \Rightarrow$
the foci are $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$.
Transverse axis: x-axis. Hyperbola opens left and right. Vertices of the fundamental rectangle: (1, 2), (−1, 2), (−1, −2), (1, −2).
Asymptotes: $y = \pm 2x$.



16. $a^2 = 1 \Rightarrow$ the vertices are (0, 1) and (0, −1).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 1 + 4 \Rightarrow c = \sqrt{5} \Rightarrow$
the foci are $(0, \sqrt{5})$ and $(0, -\sqrt{5})$.
Transverse axis: y-axis. Hyperbola opens up and down. Vertices of the fundamental rectangle: (2, 1), (−2, 1), (−2, −1), (2, −1).
Asymptotes: $y = \pm \frac{1}{2}x$.



17. $x^2 - y^2 = -1 \Rightarrow y^2 - x^2 = 1 \Rightarrow a^2 = 1 \Rightarrow$ the vertices are (0, 1) and (0, −1).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 1 + 1 \Rightarrow c = \sqrt{2} \Rightarrow$
the foci are $(0, \sqrt{2})$ and $(0, -\sqrt{2})$.
Transverse axis: y-axis. Hyperbola opens up and down. Vertices of the fundamental rectangle: (1, 1), (−1, 1), (−1, −1), (1, −1).
Asymptotes: $y = \pm x$.



18. $y^2 - x^2 = -1 \Rightarrow x^2 - y^2 = 1 \Rightarrow a^2 = 1 \Rightarrow$ the vertices are (1, 0) and (−1, 0).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 1 + 1 \Rightarrow c = \sqrt{2} \Rightarrow$
the foci are $(\sqrt{2}, 0)$ and $(-\sqrt{2}, 0)$.
Transverse axis: x-axis. Hyperbola opens left and right. Vertices of the fundamental rectangle: (1, 1), (−1, 1), (−1, −1), (1, −1).
Asymptotes: $y = \pm x$.

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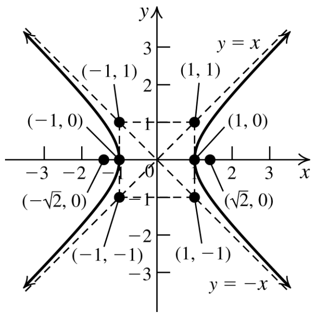
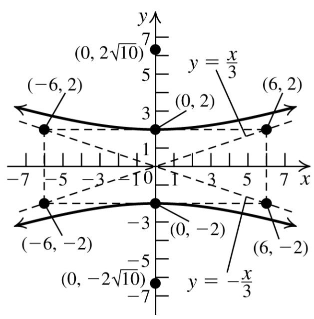


19. $9y^2 - x^2 = 36 \Rightarrow \frac{y^2}{4} - \frac{x^2}{36} = 1 \Rightarrow a^2 = 4 \Rightarrow$ the vertices are (0, 2) and (0, −2).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 4 + 36 \Rightarrow c = 2\sqrt{10} \Rightarrow$ the foci are $\left(0, 2\sqrt{10}\right)$ and $\left(0, -2\sqrt{10}\right)$.

Transverse axis: *y*-axis. Hyperbola opens up and down. Vertices of the fundamental rectangle: (6, 2), (−6, 2), (−6, −2), (6, −2).

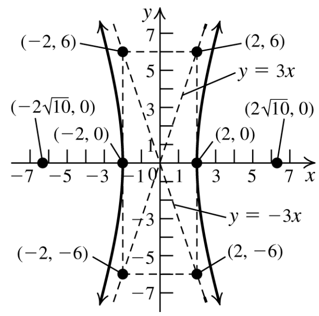
Asymptotes: $y = \pm\frac{a}{b}x = \pm\frac{1}{3}x$



20. $9x^2 - y^2 = 36 \Rightarrow \frac{x^2}{4} - \frac{y^2}{36} = 1 \Rightarrow a^2 = 4 \Rightarrow$ the vertices are (2, 0) and (−2, 0).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 4 + 36 \Rightarrow c = 2\sqrt{10} \Rightarrow$ the foci are $\left(2\sqrt{10}, 0\right)$ and $\left(-2\sqrt{10}, 0\right)$.

Transverse axis: *x*-axis. Hyperbola opens left and right. Vertices of the fundamental rectangle: (2, 6), (−2, 6), (−2, −6), (2, −6).

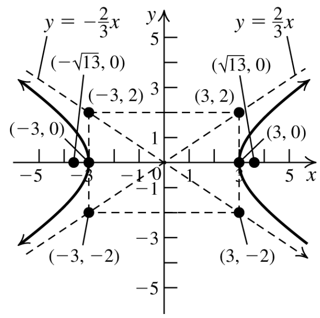
Asymptotes: $y = \pm\frac{b}{a}x = \pm 3x$.



21. $4x^2 - 9y^2 - 36 = 0 \Rightarrow \frac{x^2}{9} - \frac{y^2}{4} = 1 \Rightarrow$
$a^2 = 9 \Rightarrow$ the vertices are (3, 0) and (−3, 0).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 9 + 4 \Rightarrow c = \sqrt{13} \Rightarrow$ the foci are $\left(\sqrt{13}, 0\right)$ and $\left(-\sqrt{13}, 0\right)$.

Transverse axis: *x*-axis. Hyperbola opens left and right. Vertices of the fundamental rectangle: (3, 2), (−3, 2), (−3, −2), (3, −2).

Asymptotes: $y = \pm\frac{b}{a}x = \pm\frac{2}{3}x$



22. $4x^2 - 9y^2 + 36 = 0 \Rightarrow \frac{y^2}{4} - \frac{x^2}{9} = 1 \Rightarrow$
$a^2 = 4 \Rightarrow$ the vertices are (0, 2) and (0, −2).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 4 + 9 \Rightarrow c = \sqrt{13} \Rightarrow$ the foci are $\left(0, \sqrt{13}\right)$ and $\left(0, -\sqrt{13}\right)$.

Transverse axis: *y*-axis. Hyperbola opens up and down. Vertices of the fundamental rectangle: (3, 2), (−3, 2), (−3, −2), (3, −2).

Asymptotes: $y = \pm\frac{a}{b}x = \pm\frac{2}{3}x$

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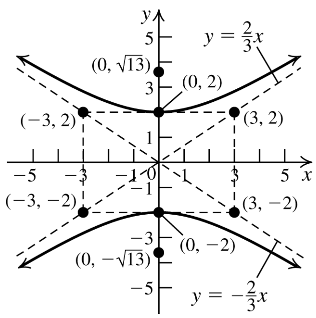
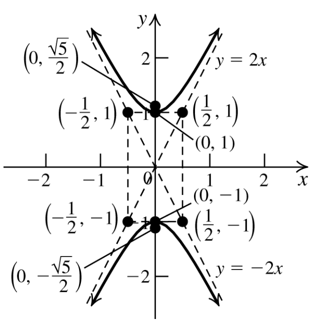
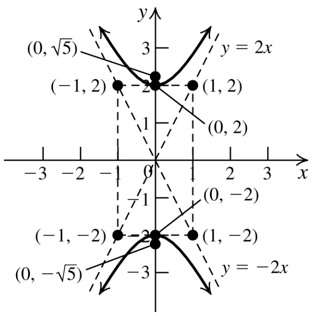


23. $y = \pm\sqrt{4x^2+1} \Rightarrow y^2 - \dfrac{x^2}{1/4} = 1 \Rightarrow a^2 = 1 \Rightarrow$ the vertices are (0, 1) and (0, −1).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 1 + \dfrac{1}{4} \Rightarrow c = \dfrac{\sqrt{5}}{2} \Rightarrow$ the foci are $\left(0, \dfrac{\sqrt{5}}{2}\right)$ and $\left(0, -\dfrac{\sqrt{5}}{2}\right)$. Transverse axis: y-axis. Hyperbola opens up and down. Vertices of the fundamental rectangle: $\left(\dfrac{1}{2}, 1\right), \left(-\dfrac{1}{2}, 1\right), \left(-\dfrac{1}{2}, -1\right)$, and $\left(\dfrac{1}{2}, -1\right)$.

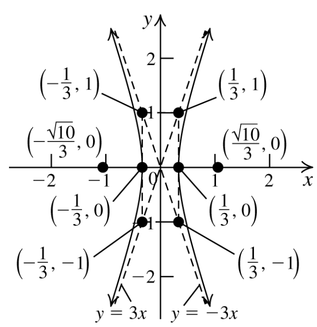
Asymptotes: $y = \pm\dfrac{a}{b}x = \pm 2x$



24. $y = \pm 2\sqrt{x^2+1} \Rightarrow y^2 - 4x^2 = 4 \Rightarrow \dfrac{y^2}{4} - x^2 = 1 \Rightarrow a^2 = 4 \Rightarrow$ the vertices are (0, 2) and (0, −2).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 4 + 1 \Rightarrow c = \sqrt{5} \Rightarrow$ the foci are $(0, \sqrt{5})$ and $(0, -\sqrt{5})$. Transverse axis: y-axis. Hyperbola opens up and down.
Vertices of the fundamental rectangle: (1, 2), (−1, 2), (−1, −2), (1, −2). Asymptotes:
$y = \pm\dfrac{a}{b}x = \pm 2x$.



25. $y = \pm\sqrt{9x^2-1} \Rightarrow 9x^2 - y^2 = 1 \Rightarrow \dfrac{x^2}{1/9} - y^2 = 1 \Rightarrow a^2 = \dfrac{1}{9} \Rightarrow$ the vertices are $\left(\dfrac{1}{3}, 0\right)$ and $\left(-\dfrac{1}{3}, 0\right)$. $c^2 = a^2 + b^2 \Rightarrow$ $c^2 = \dfrac{1}{9} + 1 \Rightarrow c = \dfrac{\sqrt{10}}{3} \Rightarrow$ the foci are $\left(\dfrac{\sqrt{10}}{3}, 0\right)$ and $\left(-\dfrac{\sqrt{10}}{3}, 0\right)$.
Transverse axis: x-axis. Hyperbola opens left and right. Vertices of the fundamental rectangle: $\left(\dfrac{1}{3}, 1\right), \left(-\dfrac{1}{3}, 1\right), \left(-\dfrac{1}{3}, -1\right)$, and $\left(\dfrac{1}{3}, -1\right)$. Asymptotes: $y = \pm\dfrac{b}{a}x \Rightarrow y = \pm 3x$



26. $y = \pm 3\sqrt{x^2-4} \Rightarrow y^2 - 9x^2 = -36 \Rightarrow \dfrac{x^2}{4} - \dfrac{y^2}{36} = 1 \Rightarrow a^2 = 4 \Rightarrow$ the vertices are (2, 0) and (−2, 0). $c^2 = a^2 + b^2 \Rightarrow$ $c^2 = 4 + 36 \Rightarrow c = 2\sqrt{10} \Rightarrow$ the foci are $(2\sqrt{10}, 0)$ and $(-2\sqrt{10}, 0)$. Transverse axis: x-axis. Hyperbola opens left and right. Vertices of the fundamental rectangle: (2, 6), (−2, 6), (−2, −6), (2, −6).

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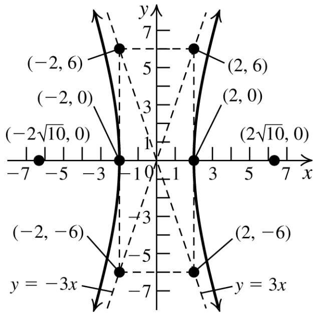
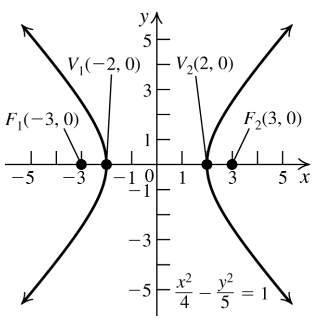
Asymptotes: $y = \pm\frac{b}{a}x \Rightarrow y = \pm 3x$.

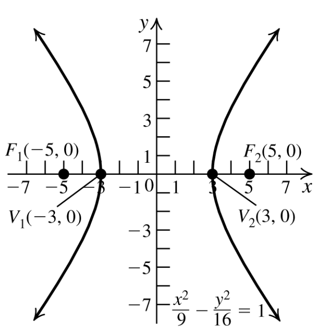


27. The transverse axis is the *x*-axis and the center is (0, 0), so the equation is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

$a = 2, c = 3 \Rightarrow 9 = 4 + b^2 \Rightarrow b^2 = 5$. The equation is $\frac{x^2}{4} - \frac{y^2}{5} = 1$.

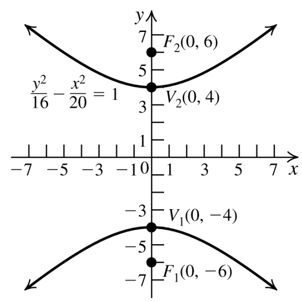


28. The transverse axis is the *x*-axis and the center is (0, 0), so the equation is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. $a = 3, c = 5 \Rightarrow 25 = 9 + b^2 \Rightarrow$ $b^2 = 16$. The equation is $\frac{x^2}{9} - \frac{y^2}{16} = 1$.



29. The transverse axis is the *y*-axis and the center is (0, 0), so the equation is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

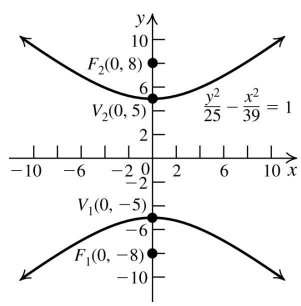
$a = 4, c = 6 \Rightarrow 36 = 16 + b^2 \Rightarrow b^2 = 20$. The equation is $\frac{y^2}{16} - \frac{x^2}{20} = 1$.



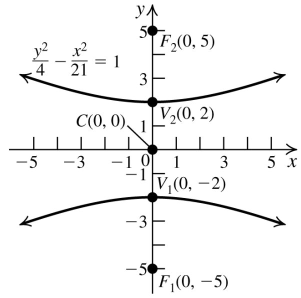
30. The transverse axis is the *y*-axis and the center is (0, 0), so the equation is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

$a = 5, c = 8 \Rightarrow 64 = 25 + b^2 \Rightarrow b^2 = 39$.

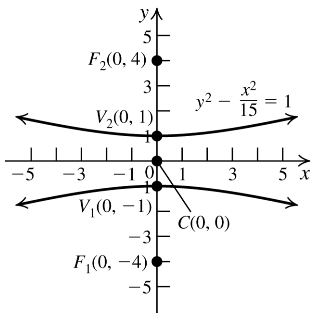
The equation is $\frac{y^2}{25} - \frac{x^2}{39} = 1$.



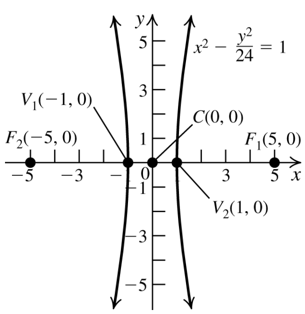
31. The transverse axis is the *y*-axis and the center is (0, 0), so the equation is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$. $a = 2, c = 5 \Rightarrow 25 = 4 + b^2 \Rightarrow$ $b^2 = 21$. The equation is $\frac{y^2}{4} - \frac{x^2}{21} = 1$.



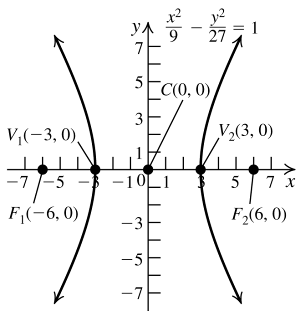
32. The transverse axis is the y-axis and the center is (0, 0), so the equation is of the form $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$. $a=1$, $c=4 \Rightarrow 16=1+b^2 \Rightarrow$ $b^2=15$. The equation is $y^2-\frac{x^2}{15}=1$.



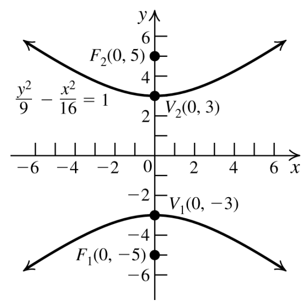
33. The transverse axis is the x-axis and the center is (0, 0), so the equation is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. $a=1$, $c=5 \Rightarrow 25=1+b^2 \Rightarrow$ $b^2=24$. The equation is $x^2-\frac{y^2}{24}=1$.



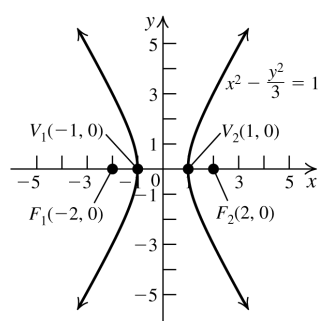
34. The transverse axis is the x-axis and the center is (0, 0), so the equation is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. $a=3$, $c=6 \Rightarrow 36=9+b^2 \Rightarrow$ $b^2=27$. The equation is $\frac{x^2}{9}-\frac{y^2}{27}=1$.



35. The transverse axis is the y-axis and the center is (0, 0), so the equation is of the form $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$.
Length of the transverse axis $= 6 \Rightarrow a=3$.
$c=5 \Rightarrow 25=9+b^2 \Rightarrow b^2=16$.
The equation is $\frac{y^2}{9}-\frac{x^2}{16}=1$.



36. The transverse axis is the x-axis and the center is (0, 0), so the equation is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.
Length of the transverse axis $= 2 \Rightarrow a=1$.
$c=2 \Rightarrow 4=1+b^2 \Rightarrow b^2=3$.
The equation is $x^2-\frac{y^2}{3}=1$.

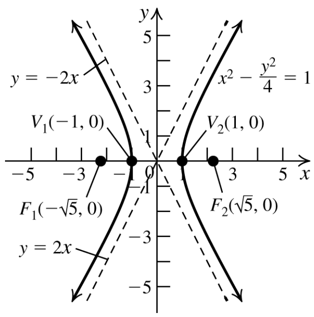


37. The transverse axis is the x-axis and the center is (0, 0), so the equation is of the form

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.\quad y=2x=\frac{b}{a}x\Rightarrow b=2a.$$

$$c^2=a^2+b^2\Rightarrow 5=a^2+(2a)^2\Rightarrow a=1, b=2.$$

The equation is $x^2-\frac{y^2}{4}=1$.



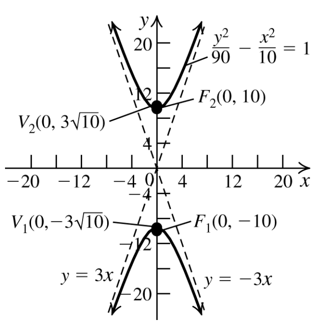
38. The transverse axis is the y-axis and the center is (0, 0), so the equation is of the form

$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1.\quad y=3x=\frac{a}{b}x\Rightarrow a=3b.$$

$$c^2=a^2+b^2\Rightarrow 100=(3b)^2+b^2\Rightarrow$$

$$b=\sqrt{10}, a=3\sqrt{10}.$$

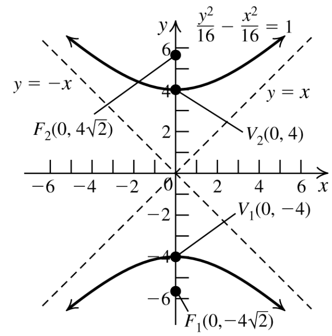
The equation is $\frac{y^2}{90}-\frac{x^2}{10}=1$.



39. The transverse axis is the y-axis and the center is (0, 0), so the equation is of the form

$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1.\quad y=x=\frac{a}{b}x\Rightarrow a=b.$$

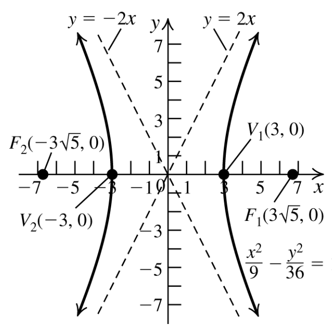
$a=4\Rightarrow b=4$. The equation is $\frac{y^2}{16}-\frac{x^2}{16}=1$.



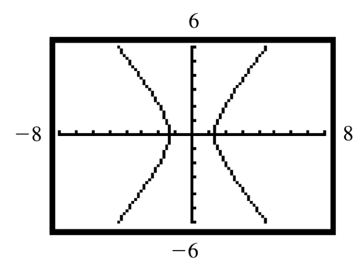
40. The transverse axis is the x-axis and the center is (0, 0), so the equation is of the form

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.\quad y=2x=\frac{b}{a}x\Rightarrow b=2a.$$

$a=3\Rightarrow b=6$. The equation is $\frac{x^2}{9}-\frac{y^2}{36}=1$.



41.



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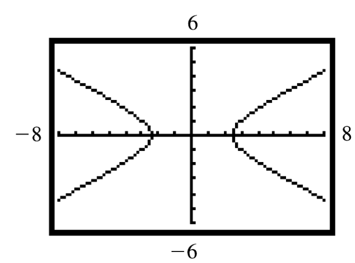

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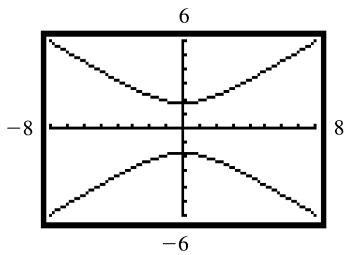

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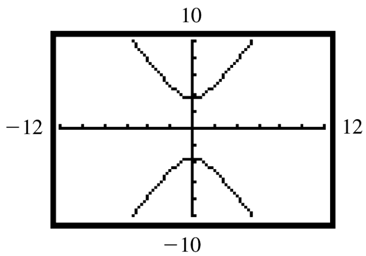

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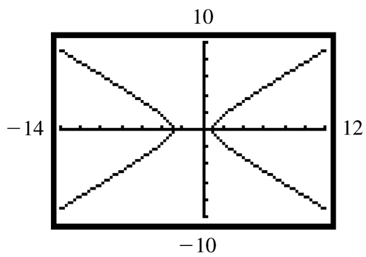

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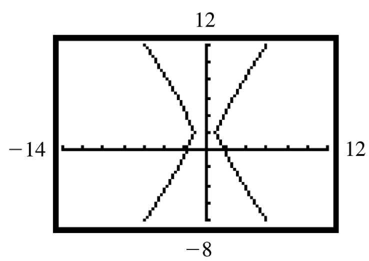

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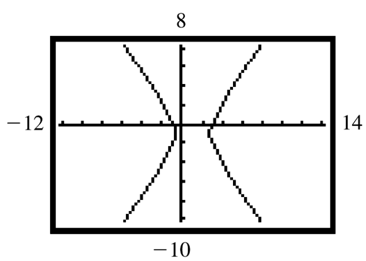
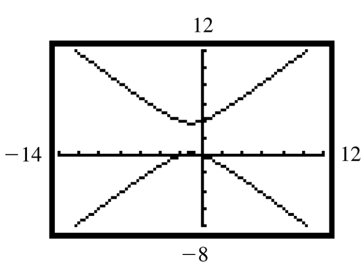
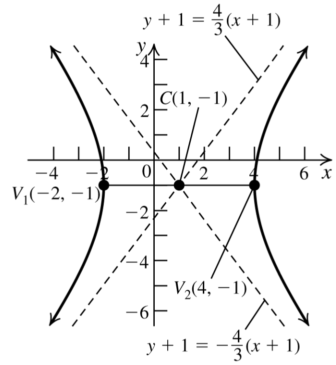


48.



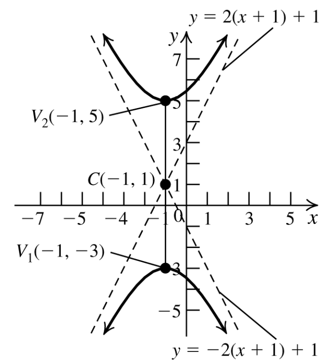
49. Center: (1, −1); vertices: (4, −1), (−2, −1); transverse axis: $y = -1$; asymptotes:

$$y - k = \pm\frac{b}{a}(x - h) \Rightarrow y + 1 = \pm\frac{4}{3}(x - 1)$$



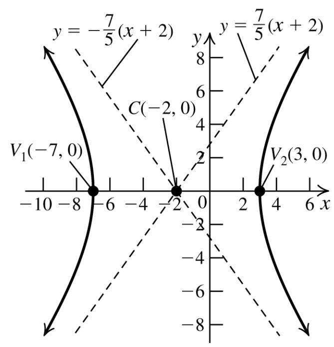
50. Center: (−1, 1); vertices: (−1, 5), (−1, −3); transverse axis: $x = -1$; asymptotes:

$$y - k = \pm\frac{a}{b}(x - h) \Rightarrow y - 1 = \pm 2(x + 1)$$



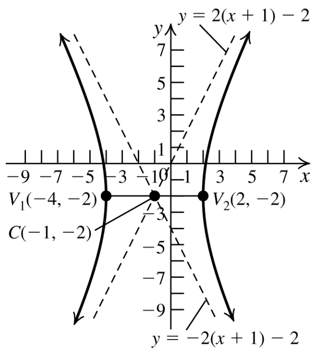
51. Center: (−2, 0); vertices: (3, 0), (−7, 0); transverse axis: x-axis ($y = 0$); asymptotes:

$$y - k = \pm\frac{b}{a}(x - h) \Rightarrow y = \pm\frac{7}{5}(x + 2)$$



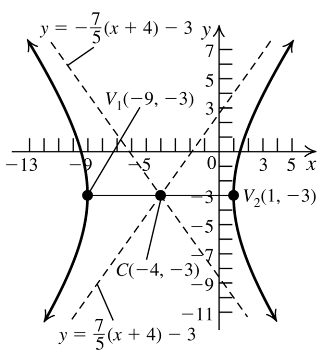
52. Center: (−1, −2); vertices: (2, −2), (−4, −2); transverse axis: $y = -2$; asymptotes:

$$y - k = \pm\frac{b}{a}(x-h) \Rightarrow y + 2 = \pm 2(x+1).$$



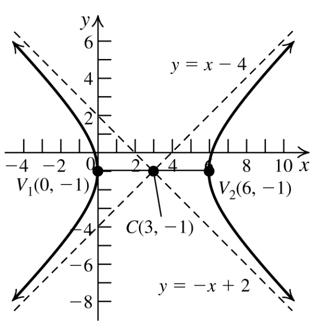
53. Center: (−4, −3); vertices: (1, −3), (−9, −3); transverse axis: $y = -3$; asymptotes:

$$y - k = \pm\frac{b}{a}(x-h) \Rightarrow y + 3 = \pm\frac{7}{5}(x+4).$$



54. Center: (3, −1); vertices: (6, −1), (0, −1); transverse axis: $y = -1$; asymptotes:

$$y - k = \pm\frac{b}{a}(x-h) \Rightarrow y + 1 = \pm(x-3) \Rightarrow$$
$$y = x - 4 \text{ or } y = -x + 2$$

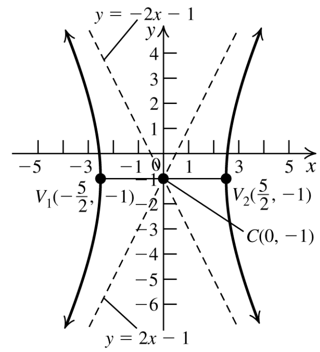


55. $4x^2 - (y+1)^2 = 25 \Rightarrow \dfrac{x^2}{25/4} - \dfrac{(y+1)^2}{25} = 1 \Rightarrow$

center: (0, −1); vertices: $\left(\frac{5}{2}, -1\right), \left(-\frac{5}{2}, -1\right)$;

transverse axis: $y = -1$; asymptotes:

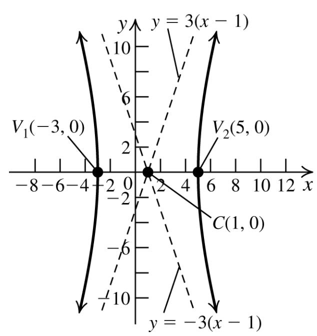
$$y - k = \pm\frac{b}{a}(x-h) \Rightarrow y + 1 = \pm 2x.$$



56. $9(x-1)^2 - y^2 = 144 \Rightarrow \dfrac{(x-1)^2}{144/9} - \dfrac{y^2}{144} = 1 \Rightarrow$

$\dfrac{(x-1)^2}{16} - \dfrac{y^2}{144} = 1 \Rightarrow$ center: (1, 0);
vertices: (5, 0), (−3, 0) ; transverse axis: *x*-axis ($y = 0$); asymptotes:

$$y - k = \pm\frac{b}{a}(x-h) \Rightarrow y = \pm 3(x-1).$$



57. $(y+1)^2 - 9(x-2)^2 = 25 \Rightarrow$

$\dfrac{(y+1)^2}{25} - \dfrac{(x-2)^2}{25/9} = 1 \Rightarrow$ center: (2, −1);
vertices: (2, 4), (2, −6);
transverse axis: $x = 2$; asymptotes:

$$y - k = \pm\frac{a}{b}(x-h) \Rightarrow y + 1 = \pm 3(x-2).$$

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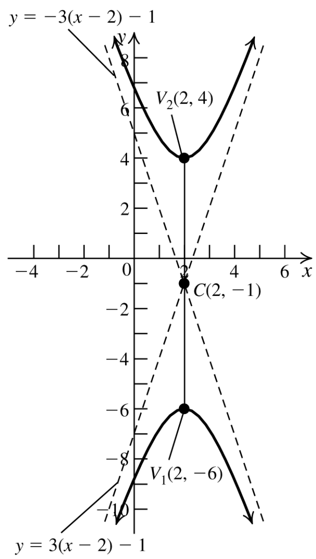


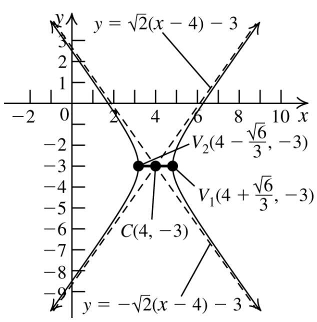
58. $6(x-4)^2 - 3(y+3)^2 = 4 \Rightarrow$

$\dfrac{(x-4)^2}{2/3} - \dfrac{(y+3)^2}{4/3} = 1 \Rightarrow$ center: (4, −3);

vertices: $\left(4+\dfrac{\sqrt{6}}{3}, -3\right), \left(4-\dfrac{\sqrt{6}}{3}, -3\right)$;

transverse axis: $y = -3$; asymptotes:

$y - k = \pm\dfrac{b}{a}(x-h) \Rightarrow y + 3 = \pm\sqrt{2}(x-4)$.



59. Complete the square to put the equation in standard form: $x^2 - y^2 + 6x = 36 \Rightarrow$

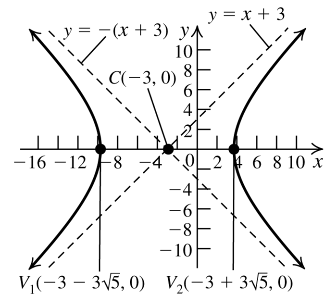
$x^2 + 6x + 9 - y^2 = 36 + 9 \Rightarrow$

$\dfrac{(x+3)^2}{45} - \dfrac{y^2}{45} = 1 \Rightarrow$ center: (−3, 0); vertices:

$\left(-3+3\sqrt{5}, 0\right), \left(-3-3\sqrt{5}, 0\right)$; transverse axis:

x-axis ($y = 0$); asymptotes:

$y - k = \pm\dfrac{b}{a}(x-h) \Rightarrow y = \pm(x+3)$.



60. Complete the square to put the equation in standard form: $x^2 + 4x - 4y^2 = 12 \Rightarrow$

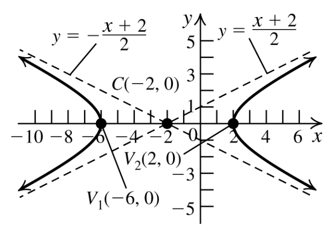
$x^2 + 4x + 4 - 4y^2 = 12 + 4 \Rightarrow$

$\dfrac{(x+2)^2}{16} - \dfrac{y^2}{4} = 1 \Rightarrow$ center: (−2, 0);

vertices: $(2, 0), (-6, 0)$; transverse axis:

x-axis ($y = 0$); asymptotes:

$y - k = \pm\dfrac{b}{a}(x-h) \Rightarrow y = \pm\dfrac{1}{2}(x+2)$



61. Complete the square to put the equation in standard form: $x^2 - 4y^2 - 4x = 0 \Rightarrow$

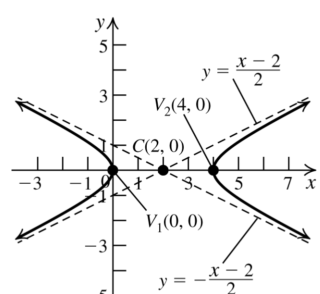
$x^2 - 4x + 4 - 4y^2 = 0 + 4 \Rightarrow$

$\dfrac{(x-2)^2}{4} - y^2 = 1 \Rightarrow$ center: (2, 0); vertices:

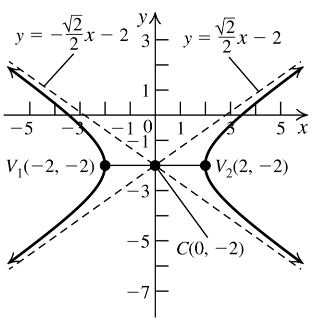
$(4, 0), (0, 0)$; transverse axis: x-axis ($y = 0$);

asymptotes: $y - k = \pm\dfrac{b}{a}(x-h) \Rightarrow$

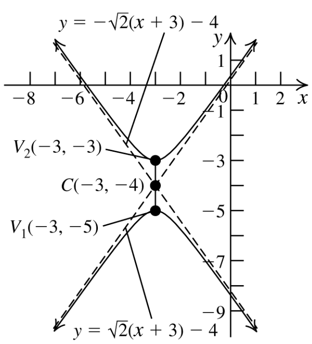
$y = \pm\dfrac{1}{2}(x-2)$



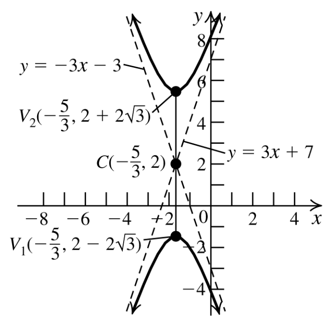
62. Complete the square to put the equation in standard form: $x^2 - 2y^2 - 8y = 12 \Rightarrow$
$x^2 - 2(y^2 + 4y + 4) = 12 - 8 \Rightarrow$
$\frac{x^2}{4} - \frac{(y+2)^2}{2} = 1 \Rightarrow$ center: (0, −2); vertices: $(2,-2), (-2,-2)$; transverse axis: $y = -2$;
asymptotes: $y - k = \pm\frac{b}{a}(x - h) \Rightarrow$
$y + 2 = \pm\frac{\sqrt{2}}{2}x$



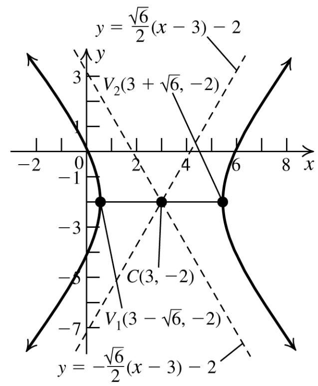
63. Complete the square to put the equation in standard form: $2x^2 - y^2 + 12x - 8y + 3 = 0 \Rightarrow$
$2(x^2 + 6x + 9) - (y^2 + 8y + 16) = -3 + 18 - 16 \Rightarrow$
$(y+4)^2 - \frac{(x+3)^2}{1/2} = 1 \Rightarrow$ center: (−3, −4);
vertices: $(-3,-3), (-3,-5)$; transverse axis: $x = -3$; asymptotes: $y - k = \pm\frac{a}{b}(x - h) \Rightarrow$
$y + 4 = \pm\sqrt{2}(x + 3)$



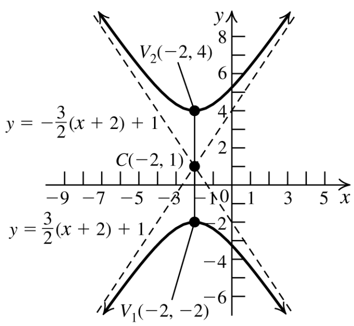
64. Complete the square to put the equation in standard form: $y^2 - 9x^2 - 4y - 30x = 33 \Rightarrow$
$y^2 - 4y + 4 - 9\left(x^2 + \frac{10}{3}x + \frac{25}{9}\right) = 33 + 4 - 25 \Rightarrow$
$\frac{(y-2)^2}{12} - \frac{\left(x + \frac{5}{3}\right)^2}{4/3} = 1 \Rightarrow$ center: $\left(-\frac{5}{3}, 2\right)$;
vertices: $\left(-\frac{5}{3}, 2 + 2\sqrt{3}\right), \left(-\frac{5}{3}, 2 - 2\sqrt{3}\right)$;
transverse axis: $x = -\frac{5}{3}$; asymptotes:
$y - k = \pm\frac{a}{b}(x - h) \Rightarrow y - 2 = \pm\frac{2\sqrt{3}}{\frac{2\sqrt{3}}{3}}\left(x + \frac{5}{3}\right) \Rightarrow$
$y - 2 = \pm 3\left(x + \frac{5}{3}\right) \Rightarrow y = 3x + 7$ or $y = -3x - 3$



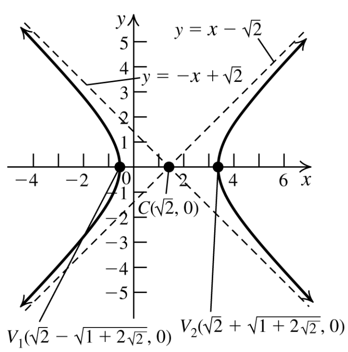
65. Complete the square to put the equation in standard form:
$3x^2 - 18x - 2y^2 - 8y + 1 = 0 \Rightarrow$
$3(x^2 - 6x + 9) - 2(y^2 + 4y + 4) = -1 + 27 - 8 \Rightarrow$
$\frac{(x-3)^2}{6} - \frac{(y+2)^2}{9} = 1 \Rightarrow$ center: (3, −2);
vertices: $\left(3 + \sqrt{6}, -2\right), \left(3 - \sqrt{6}, -2\right)$;
transverse axis: $y = -2$; asymptotes:
$y - k = \pm\frac{b}{a}(x - h) \Rightarrow y + 2 = \pm\frac{\sqrt{6}}{2}(x - 3)$



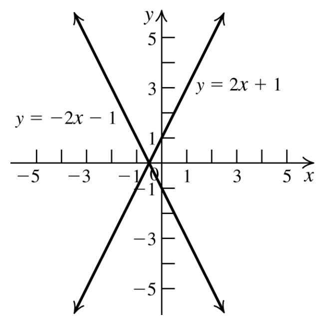
66. Complete the square to put the equation in standard form: $4y^2 - 9x^2 - 8y - 36x = 68 \Rightarrow$
$4(y^2 - 2y + 1) - 9(x^2 + 4x + 4) = 68 + 4 - 36 \Rightarrow$
$\frac{(y-1)^2}{9} - \frac{(x+2)^2}{4} = 1 \Rightarrow$ center: $(-2, 1)$;
vertices: $(-2, 4)$, $(-2, -2)$;
transverse axis: $x = -2$; asymptotes:
$y - k = \pm\frac{a}{b}(x - h) \Rightarrow\ y - 1 = \pm\frac{3}{2}(x + 2)$



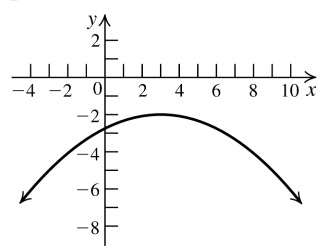
67. Complete the square to put the equation in standard form: $y^2 + 2\sqrt{2} - x^2 + 2\sqrt{2}x = 1 \Rightarrow$
$y^2 - \left(x^2 - 2\sqrt{2}x + 2\right) = 1 - 2\sqrt{2} - 2 \Rightarrow$
$\frac{(x-\sqrt{2})^2}{1+2\sqrt{2}} - \frac{y^2}{1+2\sqrt{2}} = 1 \Rightarrow$ center: $\left(\sqrt{2}, 0\right)$;
vertices: $\left(\sqrt{2} + \sqrt{1+2\sqrt{2}}, 0\right)$,
$\left(\sqrt{2} - \sqrt{1+2\sqrt{2}}, 0\right)$; transverse axis: x-axis
($y = 0$); asymptotes: $y - k = \pm\frac{b}{a}(x - h) \Rightarrow$
$y = \pm\left(x - \sqrt{2}\right)$



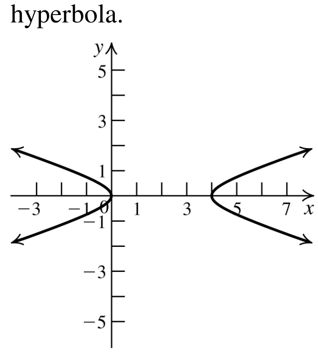
68. Complete the square to put the equation in standard form: $x^2 + x = \frac{y^2 - 1}{4} \Rightarrow$
$4\left(x^2 + x + \frac{1}{4}\right) - y^2 = -1 + 1 \Rightarrow$
$4\left(x + \frac{1}{2}\right)^2 - y^2 = 0 \Rightarrow 4\left(x + \frac{1}{2}\right)^2 = y^2 \Rightarrow$
$y = \pm 2\left(x + \frac{1}{2}\right) \Rightarrow y = 2x + 1$ or $y = -2x - 1$.
This is not a hyperbola, but a pair of lines.



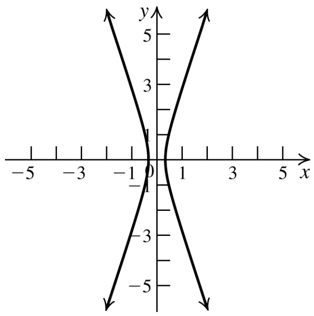
69. $x^2 - 6x + 12y + 33 = 0 \Rightarrow$
$x^2 - 6x + 9 = -12y - 33 + 9 \Rightarrow$
$(x - 3)^2 = -12(y + 2) \Rightarrow$ the conic is a parabola.



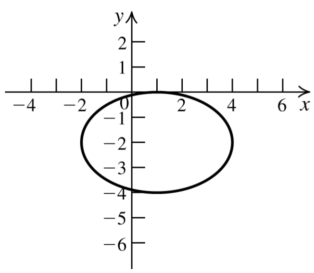
70. $9y^2 = x^2 - 4x \Rightarrow 9y^2 + 4 = x^2 - 4x + 4 \Rightarrow$
$\frac{(x-2)^2}{4} - \frac{y^2}{4/9} = 1 \Rightarrow$ the conic is a hyperbola.



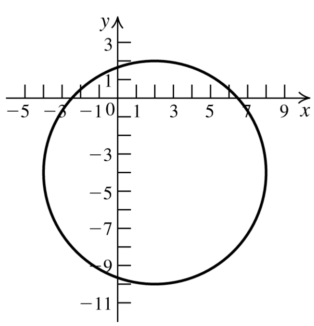
71. $y^2 - 9x^2 = -1 \Rightarrow \dfrac{x^2}{1/9} - y^2 = 1 \Rightarrow$ the conic is a hyperbola.



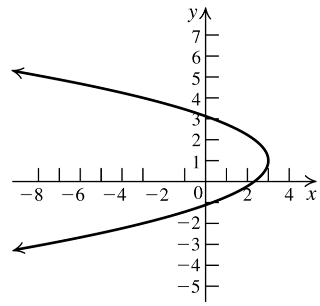
72. $4x^2 + 9y^2 - 8x + 36y + 4 = 0 \Rightarrow$
$4(x^2 - 2x + 1) + 9(y^2 + 4y + 4) = -4 + 4 + 36 \Rightarrow$
$\dfrac{(x-1)^2}{9} + \dfrac{(y+2)^2}{4} = 1 \Rightarrow$ the conic is an ellipse.



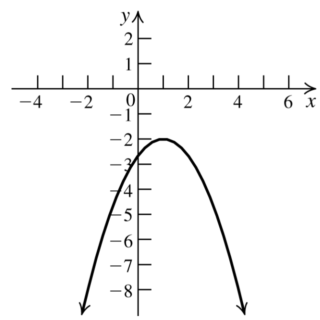
73. $x^2 + y^2 - 4x + 8y = 16 \Rightarrow$
$x^2 - 4x + 4 + y^2 + 8y + 16 = 16 + 4 + 16 \Rightarrow$
$(x-2)^2 + (y+4)^2 = 36 \Rightarrow$ the conic is a circle.



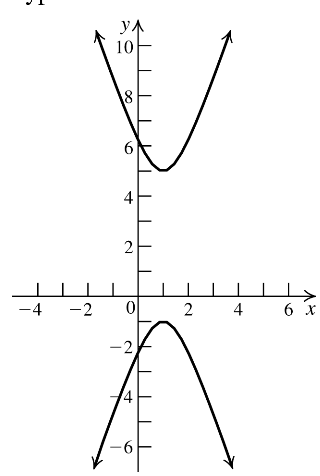
74. $2y^2 + 3x - 4y - 7 = 0 \Rightarrow$
$2(y^2 - 2y + 1) = -3x + 7 + 2 \Rightarrow$
$2(y-1)^2 = -3(x-3) \Rightarrow$ the conic is a parabola.



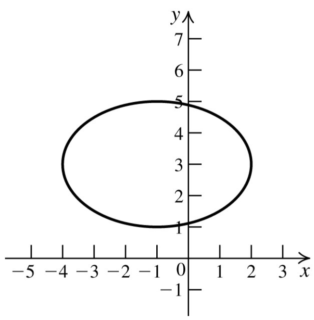
75. $2x^2 - 4x + 3y + 8 = 0 \Rightarrow$
$2(x^2 - 2x + 1) = -3y - 8 + 2 \Rightarrow$
$2(x-1)^2 = -3(y+2) \Rightarrow$ the conic is a parabola.



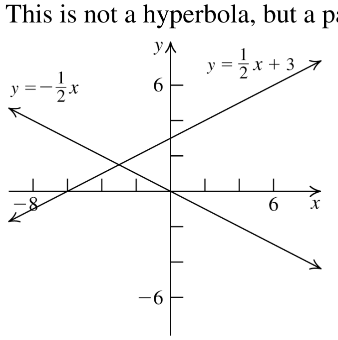
76. $y^2 - 9x^2 + 18x - 4y = 14 \Rightarrow$
$y^2 - 4y + 4 - 9(x^2 - 2x + 1) = 14 + 4 - 9 \Rightarrow$
$(y-2)^2 - 9(x-1)^2 = 9 \Rightarrow$
$\dfrac{(y-2)^2}{9} - (x-1)^2 = 1 \Rightarrow$ the conic is a hyperbola.



77. $4x^2 + 9y^2 + 8x - 54y + 49 = 0 \Rightarrow$
$4(x^2 + 2x + 1) + 9(y^2 - 6y + 9) = -49 + 4 + 81 \Rightarrow$
$\frac{(x+1)^2}{9} + \frac{(y-3)^2}{4} = 1 \Rightarrow$ the conic is an ellipse.

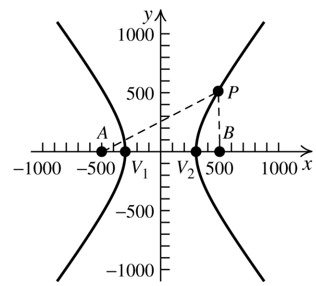


78. Complete the square to put the equation in standard form:
$x^2 - 4y^2 + 6x + 12y = 0 \Rightarrow$
$\left(x^2 + 6x + 9\right) - 4\left(y^2 - 3y + \frac{9}{4}\right) = 9 - 9 \Rightarrow$
$\left(x+3\right)^2 - 4\left(y - \frac{3}{2}\right)^2 = 0 \Rightarrow$
$\left(x+3\right)^2 = 4\left(y - \frac{3}{2}\right)^2 \Rightarrow \frac{\left(x+3\right)^2}{4} = \left(y - \frac{3}{2}\right)^2 \Rightarrow$
$\pm\frac{1}{2}\left(x+3\right) = y - \frac{3}{2} \Rightarrow \frac{1}{2}x + \frac{3}{2} = y - \frac{3}{2}$ or
$-\frac{1}{2}x - \frac{3}{2} = y - \frac{3}{2} \Rightarrow y = \frac{1}{2}x + 3$ or $y = -\frac{1}{2}x$
This is not a hyperbola, but a pair of lines.

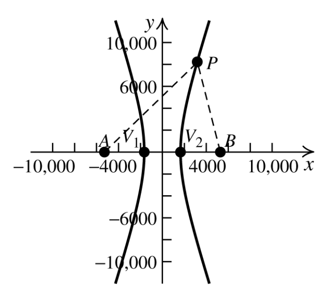


10.4 Applying the Concepts

79. A hyperbola is the set of all points in the plane whose distances from two fixed points have a constant difference. The difference in the distances of the location of the explosion from point *A* to point *B* is 600 meters, so *A* and *B* are the foci of the hyperbola. Let the coordinates of *A* be (−500, 0) and those of *B* be (500, 0), so *c* = 500. The distance between V_1 and V_2 is 600, so *a* = 300.
$c^2 = a^2 + b^2 \Rightarrow 500^2 = 300^2 + b^2 \Rightarrow$
$b^2 = 160{,}000$.
The equation is $\frac{x^2}{90{,}000} - \frac{y^2}{160{,}000} = 1$.



80. A hyperbola is the set of all points in the plane whose distances from two fixed points have a constant difference. The difference in the distances of the location of the explosion from point *A* to point *B* is 3 sec × 1100 ft/sec = 3300 ft, so *A* and *B* are the foci of the hyperbola. Let the coordinates of *A* be (−5280, 0) and those of *B* be (5280, 0), so *c* = 5280. The distance between V_1 and V_2 is 3300, so *a* = 1650.
$c^2 = a^2 + b^2 \Rightarrow 5280^2 = 1650^2 + b^2 \Rightarrow$
$b^2 = 25{,}155{,}900$.
The equation is
$\frac{x^2}{2{,}722{,}500} - \frac{y^2}{25{,}155{,}900} = 1$.



81. Using the same reasoning as in exercises 79 and 80, Nicole and Juan are located at the foci of the hyperbola, (−4000, 0) and (4000, 0). The distance between the vertices is 3300, so *a* = 1650. $c^2 = a^2 + b^2 \Rightarrow$
$4000^2 = 1650^2 + b^2 \Rightarrow b^2 = 13{,}277{,}500$.

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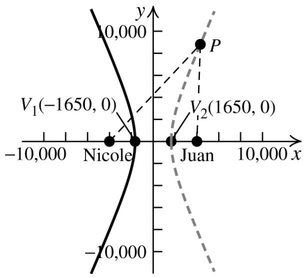
The equation is $\dfrac{x^2}{2{,}722{,}500} - \dfrac{y^2}{13{,}277{,}500} = 1.$

Because Nicole hears the thunder first, the graph consists only of the part of the hyperbola closest to Nicole:



82. This is impossible. Because Valerie is midway between Nicole and Juan, she is located at the origin. Therefore, she must hear the thunder before Juan does.

83. Let the coordinates of *A* and *B* be (−150, 0) and (150, 0), respectively. At 300,000 km per sec, the difference in the distances the signals travel is 150 km, so *a* = 75. $c^2 = a^2 + b^2 \Rightarrow$ $150^2 = 75^2 + b^2 \Rightarrow b^2 = 16{,}875$. The equation is $\dfrac{x^2}{5625} - \dfrac{y^2}{16{,}875} = 1.$

84. Because the ship receives the signal from point *A* first, it must be located closer to *A*. So, when it reaches the coastline, the ship is at (−75, 0).

85. The bullet reaches the target in $t = \dfrac{d}{r} = \dfrac{1600 \text{ ft}}{2000 \text{ ft/sec}} = \dfrac{4}{5}$ second. The person hears the crack of the gun and the thud of the bullet at the same time. So,

$$\frac{d_1}{1100} = \frac{4}{5} + \frac{d_2}{1100} \Rightarrow \frac{d_1}{1100} - \frac{d_2}{1100} = \frac{4}{5} \Rightarrow$$

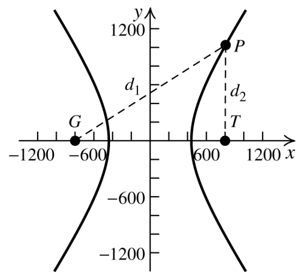
$d_1 - d_2 = \dfrac{4}{5}(1100) = 880,$ a constant. So *P* lies on a hyperbola with *a* = 440. The foci are at (−800, 0) and (800, 0), so

$b^2 = c^2 - a^2 \Rightarrow$

$b^2 = 800^2 - 440^2 = 446{,}400$

The equation of the hyperbola is

$$\frac{x^2}{193{,}600} - \frac{y^2}{446{,}400} = 1.$$



86. Let the coordinates of *A* be (−125, 0) and those of *B* be (125, 0). The difference in the distances of the location from the plane to point *A* and to point *B* is 500 microseconds × 980 feet per microsecond = 490,000 feet = 92.8 miles. So, *a* = 245,000 feet = 46.4 miles and *c* = 125 miles.

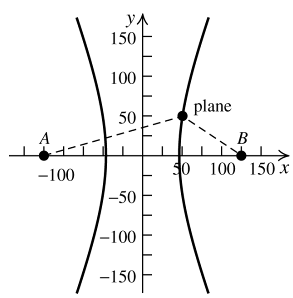
$c^2 = a^2 + b^2 \Rightarrow 125^2 = 46.4^2 + b^2 \Rightarrow$

$b^2 = 13{,}472.04$. The equation is

$\dfrac{x^2}{2152.96} - \dfrac{y^2}{13{,}472.04} = 1$. The plane is flying along the line *y* = 50, so

$\dfrac{x^2}{2152.96} - \dfrac{50^2}{13{,}472.04} = 1 \Rightarrow x = \pm 50.52$

miles. Since the signal from point *B* arrives sooner than the signal from point *A*, the plane is located closer to *B*. It is at approximately (50.52, 50).



87. *A* and *B* are the foci of hyperbola 1, and the difference of the distances is 120, so *c* = 100 and *a* = 60. $c^2 = a^2 + b^2 \Rightarrow$

$100^2 = 60^2 + b^2 \Rightarrow b^2 = 80^2.$ The equation of hyperbola 1 is $\dfrac{x^2}{3600} - \dfrac{y^2}{6400} = 1$. *C* and *D* are the foci of hyperbola 2, and the difference of the distances is 80, so *c* = 150 and *a* = 40.

$c^2 = a^2 + b^2 \Rightarrow 150^2 = 40^2 + b^2 \Rightarrow$

$b^2 = 20{,}900.$

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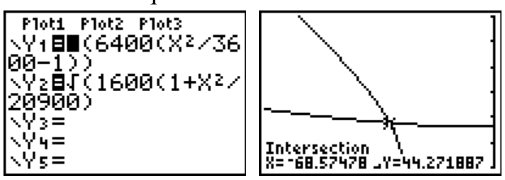
The equation of hyperbola 2 is

$$\frac{y^2}{1600} - \frac{x^2}{20,900} = 1.$$

The ship is located at the intersection of the two hyperbolas, in the second quadrant. Solve the system

$$\begin{cases} \dfrac{x^2}{3600} - \dfrac{y^2}{6400} = 1 \\ \dfrac{y^2}{1600} - \dfrac{x^2}{20,900} = 1 \end{cases}$$

or use a graphing calculator to find the intersection. Since the ship is in the second quadrant, graph only those portions of the curve in that quadrant:



The ship is at approximately (−68.5748, 44.2719).

88. The fishing boat is located at the intersection of two hyperbolas, one with foci at *A* and *B*, and the other with foci at *A* and *C*. For the first hyperbola, *c* = 200 and *a* = 150.

$c^2 = a^2 + b^2 \Rightarrow 200^2 = 150^2 + b^2 \Rightarrow$

$b^2 = 17,500.$ The equation of the first hyperbola is $\dfrac{x^2}{22,500} - \dfrac{y^2}{17,500} = 1$. For the second hyperbola, *c* = 500 and *a* = 200.

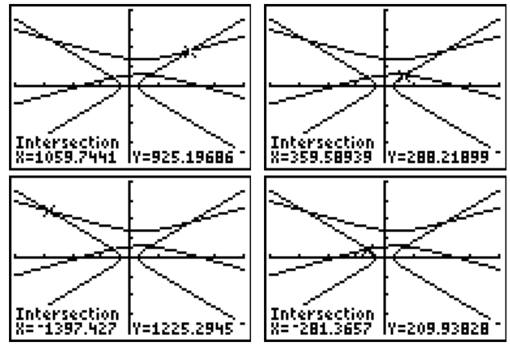
$500^2 = 200^2 + b^2 \Rightarrow b^2 = 210,000$. The center of the second hyperbola is (200, 500), so the equation of the second hyperbola is

$$\frac{(y-500)^2}{40,000} - \frac{(x-200)^2}{210,000} = 1.$$

Solve the system

$$\begin{cases} \dfrac{x^2}{22,500} - \dfrac{y^2}{17,500} = 1 \\ \dfrac{(y-500)^2}{40,000} - \dfrac{(x-200)^2}{210,000} = 1 \end{cases}$$

or use a graphing calculator to find the intersections (the possible locations of the boat).



10.4 Beyond the Basics

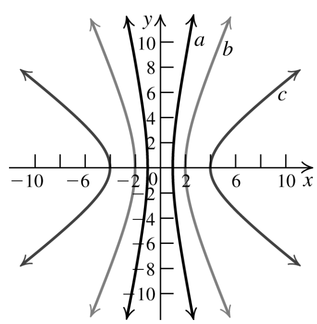
89. For all three parts, *c* = 5 and the transverse axis is the *x*-axis.

a. *a* = 1, so $b^2 = 25 - 1 = 24$. The equation is $x^2 - \dfrac{y^2}{24} = 1$.

b. *a* = 2, so $b^2 = 25 - 4 = 21$. The equation is $\dfrac{x^2}{4} - \dfrac{y^2}{21} = 1$.

c. *a* = 4, so $b^2 = 25 - 16 = 9$. The equation is $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$.

90.



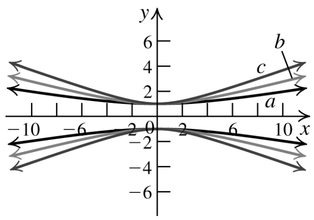
91. For all three parts, *a* = 1 and the transverse axis is the *y*-axis.

a. *c* = 6, so $b^2 = 36 - 1 = 35$. The equation is $y^2 - \dfrac{x^2}{35} = 1$.

b. *c* = 4, so $b^2 = 16 - 1 = 15$. The equation is $y^2 - \dfrac{x^2}{15} = 1$.

c. *c* = 3, so $b^2 = 9 - 1 = 8$. The equation is $y^2 - \dfrac{x^2}{8} = 1$.

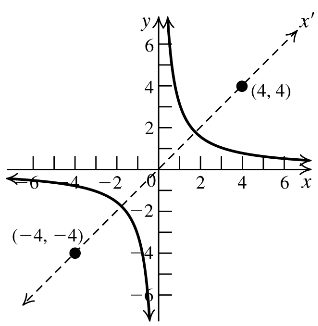
92.



93. $\sqrt{(x+c)^2+y^2}-\sqrt{(x-c)^2+y^2}=\pm 2a \Rightarrow \sqrt{(x+c)^2+y^2}=\pm 2a+\sqrt{(x-c)^2+y^2} \Rightarrow$
$(x+c)^2+y^2=4a^2 \pm 4a\sqrt{(x-c)^2+y^2}+(x-c)^2+y^2 \Rightarrow$
$x^2+2xc+c^2+y^2=4a^2 \pm 4a\sqrt{(x-c)^2+y^2}+x^2-2xc+c^2+y^2 \Rightarrow$
$xc=a^2 \pm a\sqrt{(x-c)^2+y^2} \Rightarrow xc-a^2=\pm a\sqrt{(x-c)^2+y^2} \Rightarrow a^4-2a^2xc+x^2c^2=a^2(x^2-2xc+c^2+y^2) \Rightarrow$
$a^4+x^2c^2=a^2x^2+a^2c^2+a^2y^2 \Rightarrow x^2c^2-a^2x^2-a^2y^2=-a^4+a^2c^2 \Rightarrow (c^2-a^2)x^2-a^2y^2=a^2(c^2-a^2)$

94. a. Using the distance formula, the equation of the hyperbola with foci $(p,-p)$ and $(-p,-p)$ with distance difference 2*p* is $\sqrt{(x+p)^2+(y+p)^2}-\sqrt{(x-p)^2+(y-p)^2}=2p \Rightarrow$
$(x+p)^2+(y+p)^2-2\sqrt{(x+p)^2+(y+p)^2}\sqrt{(x-p)^2+(y-p)^2}+(x-p)^2+(y-p)^2=4p^2 \Rightarrow$
$2x^2+2y^2+4p^2-2\sqrt{(x+p)^2+(y+p)^2}\sqrt{(x-p)^2+(y-p)^2}=4p^2 \Rightarrow$
$x^2+y^2=\sqrt{(x+p)^2+(y+p)^2}\sqrt{(x-p)^2+(y-p)^2} \Rightarrow$
$x^4+2x^2y^2+y^4=4p^4+x^4-8p^2xy+2x^2y^2+y^4 \Rightarrow 8p^2xy=4p^4 \Rightarrow xy=\frac{p^2}{2}$

b.



$xy=8=\frac{p^2}{2} \Rightarrow p=\pm 4$.

The foci are (4, 4) and (−4, −4).

95. Assume that the transverse axis is the *x*-axis. Then the equation of the hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. The hyperbola is equilateral, so *a* = *b*, and the equations of the asymptotes are $y=\pm x$. These lines are perpendicular to each other. Similarly, it can be shown that the asymptotes are perpendicular to each other if the transverse axis is the *y*-axis.

96. Assume that the transverse axis is the *x*-axis and the conjugate axis is the *y*-axis. Then the equation of the hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.
$c^2=a^2+b^2$, so one of the foci is $\left(\sqrt{a^2+b^2},0\right)$. The coordinates of the endpoints of the latus rectum are $\left(\sqrt{a^2+b^2},y\right)$ and $\left(\sqrt{a^2+b^2},-y\right)$. Substitute the expression for the *x*-coordinate into the equation to solve for *y*:

$$\frac{\left(\sqrt{a^2+b^2}\right)^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow \frac{a^2+b^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow$$
$$b^2\left(\frac{a^2+b^2}{a^2}-1\right)=\frac{b^4}{a^2}=y^2 \Rightarrow y=\pm\frac{b^2}{a}$$

So, the length of the latus rectum is $\frac{2b^2}{a}$.

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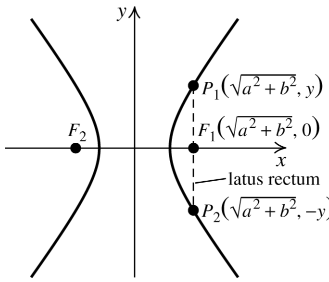


97. Since $b > 0$ and $c^2 = a^2 + b^2$, it follows that $c^2 > a^2$ and $c > a$. So, $e > 1$. When $e = 1$, the hyperbola becomes the union of two rays.

98. $a = 4, b = 3 \Rightarrow c^2 = 16 + 9 = 25 \Rightarrow c = 5$

$e = \frac{c}{a} = \frac{5}{4}$. Length of latus rectum $= \frac{2b^2}{a} = \frac{9}{2}$.

99. $36x^2 - 25y^2 = 900 \Rightarrow \frac{x^2}{25} - \frac{y^2}{36} = 1$

$a = 5, b = 6 \Rightarrow c^2 = 25 + 36 \Rightarrow c = \sqrt{61}$

$e = \frac{c}{a} = \frac{\sqrt{61}}{5}$.

Length of latus rectum $= \frac{2b^2}{a} = \frac{72}{5}$.

100. $x^2 - y^2 = 49 \Rightarrow \frac{x^2}{49} - \frac{y^2}{49} = 1$

$a = 7, b = 7 \Rightarrow c^2 = 49 + 49 \Rightarrow c = 7\sqrt{2}$

$e = \frac{c}{a} = \sqrt{2}$.

Length of latus rectum $= \frac{2b^2}{a} = 14$.

101. $8(x-1)^2 - (y+2)^2 = 2 \Rightarrow$

$\frac{(x-1)^2}{1/4} - \frac{(y+2)^2}{2} = 1$

$a = \frac{1}{2}, b = \sqrt{2} \Rightarrow c^2 = \frac{1}{4} + 2 \Rightarrow c = \frac{3}{2} \Rightarrow$

$e = \frac{c}{a} = 3$

Length of latus rectum $= \frac{2b^2}{a} = 8$.

102. Rearrange the equation $5x^2 - 4y^2 - 10x - 8y - 19 = 0$ as $5(x^2 - 2x) - 4(y^2 + 2y) = 19$, complete the square and write the equation in standard form:

$5(x^2 - 2x + 1) - 4(y^2 + 2y + 1) = 19 + 5 - 4 \Rightarrow$

$5(x-1)^2 - 4(y+1)^2 = 20 \Rightarrow$

$\frac{(x-1)^2}{4} - \frac{(y+1)^2}{5} = 1$. So, $a = 2, b = \sqrt{5}$ and

$c^2 = 4 + 5 \Rightarrow c = 3 \Rightarrow e = \frac{c}{a} = \frac{3}{2}$.

Length of latus rectum $= \frac{2b^2}{a} = 5$.

103. $e = \frac{c}{a} \Rightarrow e^2 = \frac{c^2}{a^2} \Rightarrow e^2 = \frac{a^2 + b^2}{a^2} = 1 + \frac{b^2}{a^2} \Rightarrow$

$e^2 - 1 = \frac{b^2}{a^2} \Rightarrow a^2(e^2 - 1) = b^2$

104. $\frac{2b^2}{a} = 10 \Rightarrow b^2 = 5a$. $c = 6 \Rightarrow$

$36 = a^2 + b^2 = a^2 + 5a \Rightarrow$

$a^2 + 5a - 36 = 0 \Rightarrow (a+9)(a-4) = 0 \Rightarrow$

$a = -9$ (reject this) or $a = 4$. $a = 4 \Rightarrow b = 2\sqrt{5}$

The equation is $\frac{x^2}{16} - \frac{y^2}{20} = 1$.

105. The equation of the path is $\sqrt{(x-3)^2 + y^2} = 2|x+1|$. Square both sides, simplify, complete the square, and write the equation in standard form:

$(x-3)^2 + y^2 = 4(x+1)^2 \Rightarrow$

$x^2 - 6x + 9 + y^2 = 4x^2 + 8x + 4 \Rightarrow$

$3x^2 + 14x - y^2 = 5 \Rightarrow$

$3\left(x^2 + \frac{14}{3}x + \frac{49}{9}\right) - y^2 = 5 + \frac{49}{3} \Rightarrow$

$3\left(x + \frac{7}{3}\right)^2 - y^2 = \frac{64}{3} \Rightarrow \frac{\left(x + \frac{7}{3}\right)^2}{64/9} - \frac{y^2}{64/3} = 1$

$c = \sqrt{\frac{64}{9} + \frac{64}{3}} = \frac{16}{3}; a = \frac{8}{3}; e = \frac{c}{a} = 2$

106. The equation of the path is $\sqrt{x^2 + (y+3)^2} = 3|y-1|$. Square both sides, simplify, complete the square, and write the equation in standard form:

$x^2 + (y+3)^2 = 9(y-1)^2 \Rightarrow$

$x^2 + y^2 + 6y + 9 = 9y^2 - 18y + 9 \Rightarrow$

(*continued on next page*)

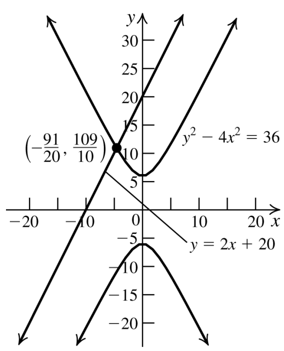
(*continued*)

$$8y^2-24y-x^2=0\Rightarrow$$
$$8\left(y^2+3y+\frac{9}{4}\right)-x^2=18\Rightarrow$$
$$8\left(y+\frac{3}{2}\right)^2-x^2=18\Rightarrow\frac{\left(y+\frac{3}{2}\right)^2}{9/4}-\frac{x^2}{18}=1$$
$$c=\sqrt{\frac{9}{4}+18}=\frac{9}{2};a=\frac{3}{2};e=\frac{c}{a}=3$$

107. Solve using substitution:

$$\begin{cases}y-2x-20=0\\y^2-4x^2=36\end{cases}\Rightarrow\begin{cases}y=2x+20\\y^2-4x^2=36\end{cases}\Rightarrow$$
$$(2x+20)^2-4x^2=36\Rightarrow 80x+400=36\Rightarrow$$
$$x=-\frac{91}{20};y=2\left(-\frac{91}{20}\right)+20=\frac{109}{10}$$

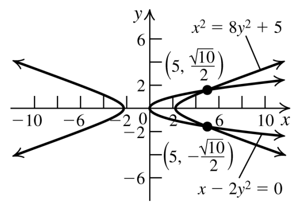
The only point of intersection is $\left(-\frac{91}{20},\frac{109}{10}\right)$.



108. Solve using substitution:

$$\begin{cases}x-2y^2=0\\x^2=8y^2+5\end{cases}\Rightarrow\begin{cases}x=2y^2\\x^2=8y^2+5\end{cases}\Rightarrow$$
$$(2y^2)^2=8y^2+5\Rightarrow 4y^4-8y^2-5=0\Rightarrow$$
$$(2y^2-5)(2y^2+1)=0\Rightarrow y^2=\frac{5}{2}\Rightarrow y=\pm\frac{\sqrt{10}}{2}$$
$$\text{or } y^2=-\frac{1}{2}\text{ (reject this); } x=2\left(\pm\frac{\sqrt{10}}{2}\right)^2=5$$

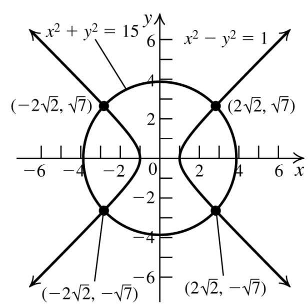
The points of intersection are $\left(5,\frac{\sqrt{10}}{2}\right)$ and $\left(5,-\frac{\sqrt{10}}{2}\right)$.



109. Solve using elimination:

$$\begin{cases}x^2+y^2=15\\x^2-y^2=1\end{cases}\Rightarrow 2x^2=16\Rightarrow x=\pm2\sqrt{2}$$
$$\left(\pm2\sqrt{2}\right)^2+y^2=15\Rightarrow y^2=7\Rightarrow y=\pm\sqrt{7}$$

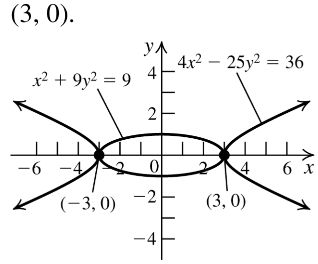
The points of intersection are $\left(2\sqrt{2},\sqrt{7}\right)$, $\left(2\sqrt{2},-\sqrt{7}\right),\left(-2\sqrt{2},-\sqrt{7}\right)$, and $\left(-2\sqrt{2},\sqrt{7}\right)$.



110. Solve using substitution:

$$\begin{cases}x^2+\ \ 9y^2=9\\4x^2-25y^2=36\end{cases}\Rightarrow\begin{cases}-4x^2-36y^2=-36\\\ \ 4x^2-25y^2=36\end{cases}\Rightarrow$$
$$-61y^2=0\Rightarrow y=0;x^2=9\Rightarrow x=\pm3$$

The points of intersection are (–3, 0) and (3, 0).



111. Solve using substitution:

$$\begin{cases}9x^2+4y^2=72\\\ \ x^2-9y^2=-77\end{cases}\Rightarrow\begin{cases}9x^2+4y^2=72\\\ \ x^2=9y^2-77\end{cases}\Rightarrow$$
$$9(9y^2-77)+4y^2=72\Rightarrow$$
$$85y^2-693=72\Rightarrow y=\pm3$$
$$x^2=9(9)-77\Rightarrow x=\pm2$$

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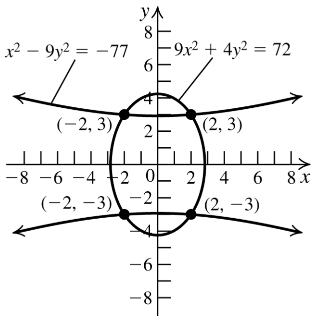
(*continued*)

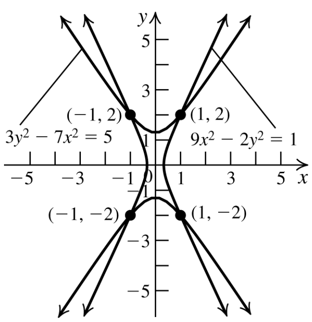
The points of intersection are (−2, −3), (−2, 3), (2, −3), and (2, 3).



112. Solve using elimination:

$$\begin{cases} 3y^2 - 7x^2 = 5 \\ 9x^2 - 2y^2 = 1 \end{cases} \Rightarrow \begin{cases} -14x^2 + 6y^2 = 10 \\ 27x^2 - 6y^2 = 3 \end{cases} \Rightarrow 13x^2 = 13 \Rightarrow x = \pm 1; 3y^2 - 7 = 5 \Rightarrow y = \pm 2$$

Thus, the points of intersection are (–1, –2), (–1, 2), (1, 2), and (1, –2).



113. The equation of the hyperbola is

$$\frac{x^2}{5/3} - \frac{y^2}{5/7} = 1 \Rightarrow \frac{3x^2}{5} - \frac{7y^2}{5} = 1 \Rightarrow 3x^2 - 7y^2 = 5 \Rightarrow 7y^2 = 3x^2 - 5$$

Step 1: Let m = the slope of the tangent line. Then the equation of the tangent line is

$$y - 1 = m(x-2) \Rightarrow y = m(x-2) + 1$$

Step 2: $7y^2 = 7\left[m(x-2)+1\right]^2 = 7\left[m^2(x-2)^2 + 2m(x-2) + 1\right] = 7m^2(x-2)^2 + 14m(x-2) + 7$

Step 3: $3x^2 - 5 = 7m^2(x-2)^2 + 14m(x-2) + 7 \Rightarrow 3x^2 - 5 = 7m^2\left(x^2 - 4x + 4\right) + 14mx - 28m + 7 \Rightarrow$

$$3x^2 - 5 = 7m^2x^2 - 28m^2x + 28m^2 + 14mx - 28m + 7 \Rightarrow$$
$$3x^2 - 7m^2x^2 + 28m^2x - 14mx - 28m^2 + 28m - 12 = 0 \Rightarrow$$
$$\left(3 - 7m^2\right)x^2 + \left(28m^2 - 14m\right)x - \left(28m^2 - 28m + 12\right) = 0$$

Now set the discriminant equal to 0 and solve for x:

$$\left(28m^2 - 14m\right)^2 - 4\left(3 - 7m^2\right)\left[-\left(28m^2 - 28m + 12\right)\right] = 0 \Rightarrow$$
$$784m^4 - 784m^3 + 196m^2 - 784m^4 + 784m^3 - 336m + 144 = 0 \Rightarrow$$
$$196m^2 - 336m + 144 = 0 \Rightarrow 49m^2 - 84m + 36 = 0 \Rightarrow (7m-6)^2 = 0 \Rightarrow m = \frac{6}{7}$$

Step 4: The equation of the tangent line is $y - 1 = \frac{6}{7}(x-2) \Rightarrow y = \frac{6}{7}x - \frac{5}{7}$.

114. Let $p = x_1$ and $q = y_1$. We need to find the equation of the tangent line to the hyperbola with equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (p, q). First notice that $y^2 = \frac{b^2x^2}{a^2} - b^2$, and the tangent line equation is $y = m(x-p)+q$. Now solve $\frac{b^2x^2}{a^2} - b^2 = [m(x-p)+q]^2$. First multiply both sides by a^2, then expand the right side:

$b^2x^2 - a^2b^2 = a^2[m(x-p)+q]^2 = a^2m^2x^2 - 2a^2m^2xp + 2a^2mxq + a^2m^2p^2 - 2a^2mpq + a^2q^2$ Now combine like terms, factoring where possible, and set the equation equal to 0:

$$\left(b^2 - a^2m^2\right)x^2 + \left(2a^2m^2p - 2a^2mq\right)x - a^2m^2p^2 + 2a^2mpq - a^2q^2 + a^2b^2 = 0$$

Since the roots of the equation must be equal for the line to be tangent to the ellipse, set the discriminant equal to 0, then solve for m.

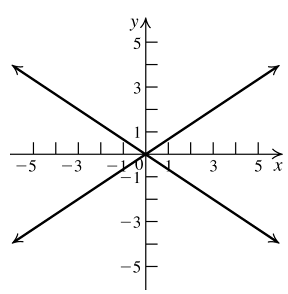
$$\left(2a^2m^2p - 2a^2mq\right)^2 - 4\left(b^2 - a^2m^2\right)\left(-a^2m^2p^2 + 2a^2mpq - a^2q^2 + a^2b^2\right) = 0 \Rightarrow$$
$$4a^4 - 8a^4m^3pq + 4a^4m^2q^2 - 4\left(b^2 - a^2m^2\right)\left(-a^2m^2p^2 + 2a^2mpq - a^2q^2 + a^2b^2\right) = 0 \Rightarrow$$
$$a^4m^4p^2 - 2a^4m^3pq + a^4m^2q^2 - \left(b^2 - a^2m^2\right)\left(-a^2m^2p^2 + 2a^2mpq - a^2q^2 + a^2b^2\right) = 0 \Rightarrow$$
$$a^2m^4p^2 - 2a^2m^3pq + a^2m^2q^2 + \left(b^2 + a^2m^2\right)\left(-m^2p^2 + 2mpq - q^2 + b^2\right) = 0 \Rightarrow$$
$$a^2m^4p^2 - 2a^2m^3pq + a^2m^2q^2 - b^2m^2p^2 + 2b^2mpq - b^2q^2 + b^4$$
$$- a^2m^4p^2 + 2a^2m^3pq - a^2m^2q^2 + a^2m^2b^2 = 0 \Rightarrow$$
$$-b^2m^2p^2 + 2b^2mpq - b^2q^2 + b^4 + a^2m^2b^2 = 0 \Rightarrow -m^2p^2 + 2mpq - q^2 + b^2 + a^2m^2 = 0$$
$$-m^2p^2 + 2mpq - b^2 + \frac{b^2p^2}{a^2} + b^2 + a^2m^2 = 0 \Rightarrow -m^2p^2 + 2mpq + \frac{b^2p^2}{a^2} + a^2m^2 = 0 \Rightarrow$$
$$-a^2m^2p^2 + 2a^2mpq + b^2p^2 + a^4m^2 = 0 \Rightarrow a^2\left(a^2 - p^2\right)m^2 + 2a^2mpq + b^2p^2 = 0 \Rightarrow$$
$$a^2\left(a^2 - \left[a^2 - \frac{a^2q^2}{b^2}\right]\right)m^2 + 2a^2mpq + b^2p^2 = 0 \Rightarrow a^2\left(\frac{a^2q^2}{b^2}\right)m^2 + 2a^2mpq + b^2p^2 = 0 \Rightarrow$$
$$a^4q^2m^2 + 2a^2b^2mpq + b^4p^2 = 0 \Rightarrow \left(a^2qm + b^2p\right)^2 = 0$$

Thus, $m = -\frac{b^2p}{a^2q}$, and the equation for the tangent line is $y = -\frac{b^2p}{a^2q}(x-p)+q$ or $y = -\frac{b^2p}{a^2q}x + \frac{b^2p^2}{a^2q} + q$.

10.4 Critical Thinking/Writing/Discussion

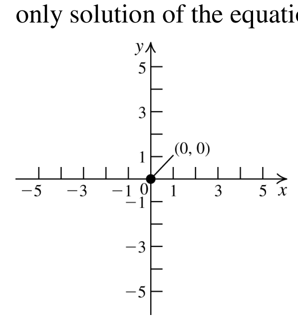
115. a. $4x^2 - 9y^2 = 0 \Rightarrow (2x+3y)(2x-3y) = 0$.

The graph consists of two lines, $y = \pm\frac{2}{3}x$.

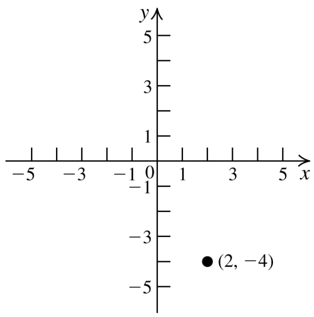


b. All the variable terms on the left side of the equation are always nonnegative, so there are no x or y values that would satisfy the equation. There is no graph.

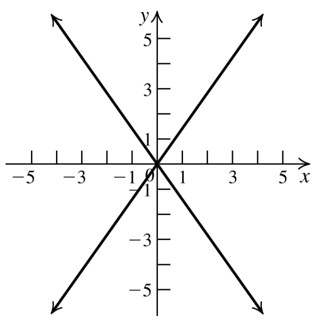
c. Since both x^2 and y^2 are nonnegative, the only solution of the equation is (0, 0).



d. Complete the square:
$x^2 + y^2 - 4x + 8y = -20 \Rightarrow$
$(x^2 - 4x + 4) + (y^2 + 8y + 16) = -20 + 4 + 16 \Rightarrow$
$(x-2)^2 + (y+4)^2 = 0 \Rightarrow x = 2$ and $y = -4$.



e. $y^2 - 2x^2 = 0 \Rightarrow y^2 = 2x^2 \Rightarrow y = \pm x\sqrt{2}$.
The graph consists of two lines.



116. $Ax^2 + Cy^2 + Dx + Ey + F = 0,\ AC \neq 0$
Rewriting the equation, we have

$$A\left(x^2 + \frac{D}{A}x + \frac{D^2}{4A^2}\right) + C\left(y^2 + \frac{E}{C}y + \frac{E^2}{4C^2}\right) = \frac{D^2}{4A} + \frac{E^2}{4C} - F \Rightarrow$$

$$A\left(x + \frac{D}{2A}\right)^2 + C\left(y + \frac{E}{2C}\right)^2 = \frac{D^2}{4A} + \frac{E^2}{4C} - F.$$

If the term on the right side is not zero, then the equation can be rewritten as

$$\frac{A\left(x + \frac{D}{2A}\right)^2}{\frac{D^2}{4A} + \frac{E^2}{4C} - F} + \frac{C\left(y + \frac{E}{2C}\right)^2}{\frac{D^2}{4A} + \frac{E^2}{4C} - F} = 1 \Rightarrow$$

$$\frac{\left(x + \frac{D}{2A}\right)^2}{\frac{D^2}{4A^2} + \frac{E^2}{4AC} - \frac{F}{A}} + \frac{\left(y + \frac{E}{2C}\right)^2}{\frac{D^2}{4AC} + \frac{E^2}{4C^2} - \frac{F}{C}} = 1$$

(i) If $A > 0$ and $C > 0$, and if $\frac{D^2}{4A} + \frac{E^2}{4C} - F = 0$, the graph consists of one point, $\left(-\frac{D}{2A}, -\frac{E}{2C}\right)$. If $\frac{D^2}{4A} + \frac{E^2}{4C} - F > 0$, the graph is an ellipse. If $\frac{D^2}{4A} + \frac{E^2}{4C} - F < 0$, then there is no graph.

(ii) If $A > 0$ and $C < 0$, and if $\frac{D^2}{4A} + \frac{E^2}{4C} - F = 0$, the graph consists of two lines, $\sqrt{A}\left(x + \frac{D}{2A}\right) = \pm\sqrt{-C}\left(y + \frac{E}{2C}\right)$. If $\frac{D^2}{4A} + \frac{E^2}{4C} - F > 0$, the graph is a hyperbola with a horizontal transverse axis. If $\frac{D^2}{4A} + \frac{E^2}{4C} - F < 0$, then the graph is a hyperbola with a vertical transverse axis.

(iii) If $A < 0$ and $C > 0$, and if $\frac{D^2}{4A} + \frac{E^2}{4C} - F = 0$, the graph consists of two lines, $\sqrt{-A}\left(x + \frac{D}{2A}\right) = \pm\sqrt{C}\left(y + \frac{E}{2C}\right)$. If $\frac{D^2}{4A} + \frac{E^2}{4C} - F > 0$, the graph is a hyperbola with a vertical transverse axis. If $\frac{D^2}{4A} + \frac{E^2}{4C} - F < 0$, then the graph is a hyperbola with a horizontal transverse axis.

(iv) If $A < 0$ and $C < 0$, and if $\frac{D^2}{4A} + \frac{E^2}{4C} - F = 0$, the graph consists of one point, $\left(-\frac{D}{2A}, -\frac{E}{2C}\right)$. If $\frac{D^2}{4A} + \frac{E^2}{4C} - F > 0$, there is no graph. If $\frac{D^2}{4A} + \frac{E^2}{4C} - F < 0$, then the graph is an ellipse.

117. The hyperbolas $x^2 - y^2 = -1$ and $x^2 - y^2 = 1$ have the same asymptotes because $a = b = 1$ for both graphs. The asymptotes are $y = x$ and $y = -x$.

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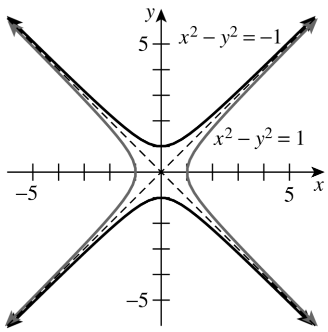
Note that the graphs are the reflections of each other across $y = x$ and across $y = -x$. Also note that the graph of one can be obtained by rotating the graph of the other about the origin 90°.



10.4 Maintaining Skills

118. $f(3) = \frac{1}{1+3} = \frac{1}{4}$

119. $g(10) = 2(10) + 1 = 21$

120. $h(5) = (-1)^3 3^{5-1} = -3^4 = -81$

121. $f(6) = \frac{16(6)}{6^2 + 12} = \frac{96}{48} = 2$

122. $(-1)^{17} = -1$

123. $(-1)^8 \left(\frac{1}{8+1}\right) = \frac{1}{9}$

124. $(-1)^5 2^5 = 32$

125. $(-1)^{7-2}(2(7)+1) + (-1)^7 = -1(15) + (-1)$
$= -16$

126. $\frac{3 \cdot \pi \cdot e^2 \cdot 7 \cdot 11 \cdot 19 \cdot a}{a \cdot 3 \cdot \pi \cdot e^2 \cdot 19} = 7 \cdot 11 = 77$

127. $\frac{a \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10}{3 \cdot 5 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdot a} = 2 \cdot 4 \cdot 6 = 48$

128. $c(1 - 5a + 3b) = c - 5ac + 3bc$
True

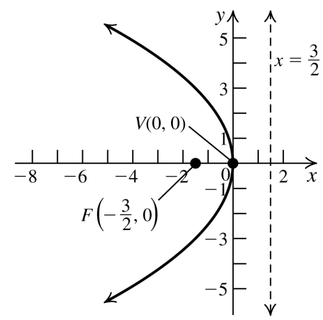
129. $3a - 2b + 6c - 5d = 3a + 6c - 2b - 5d$
$= (3a + 6c) - (2b + 5d)$
True

Chapter 10 Review Exercises

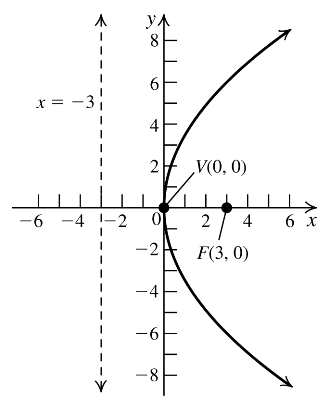
1. $y^2 = -6x \Rightarrow -6 = 4a \Rightarrow a = -\frac{3}{2}$

Vertex: (0, 0), focus: $\left(-\frac{3}{2}, 0\right)$, axis: x-axis,

directrix: $x = \frac{3}{2}$.



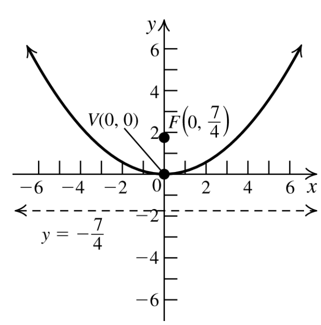
2. $y^2 = 12x \Rightarrow 12 = 4a \Rightarrow a = 3$.
Vertex: (0, 0), focus: (3, 0), axis: x-axis, directrix: $x = -3$.



3. $x^2 = 7y \Rightarrow 7 = 4a \Rightarrow \frac{7}{4} = a$.

Vertex: (0, 0), focus: $\left(0, \frac{7}{4}\right)$, axis: y-axis,

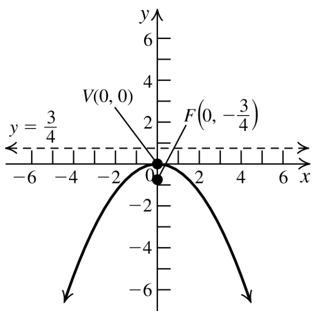
directrix: $y = -\frac{7}{4}$.



4. $x^2 = -3y \Rightarrow -3 = 4a \Rightarrow -\frac{3}{4} = a$

Vertex: (0, 0), focus: $\left(0, -\frac{3}{4}\right)$, axis: y-axis,

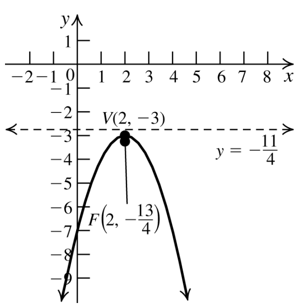
directrix: $y = \frac{3}{4}$.



5. $(x-2)^2 = -(y+3) \Rightarrow -1 = 4a \Rightarrow -\frac{1}{4} = a$.

Vertex: (2, −3), focus: $\left(2, -\frac{13}{4}\right)$, axis: $x = 2$,

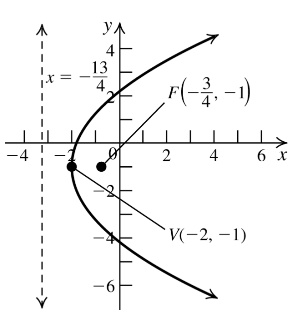
directrix: $y = -\frac{11}{4}$.



6. $(y+1)^2 = 5(x+2) \Rightarrow 5 = 4a \Rightarrow \frac{5}{4} = a$.

Vertex: (−2, −1), focus: $\left(-\frac{3}{4}, -1\right)$,

axis: $y = -1$, directrix: $x = -\frac{13}{4}$.



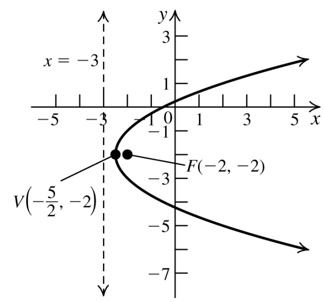
7. Rearrange the equation and complete the square to put the equation in standard form:

$y^2 = -4y + 2x + 1 \Rightarrow y^2 + 4y + 4 = 2x + 1 + 4 \Rightarrow$

$(y+2)^2 = 2\left(x + \frac{5}{2}\right) \Rightarrow 2 = 4a \Rightarrow a = \frac{1}{2}$.

Vertex: $\left(-\frac{5}{2}, -2\right)$, focus: (−2, −2),

axis: $y = -2$, directrix: $x = -3$.



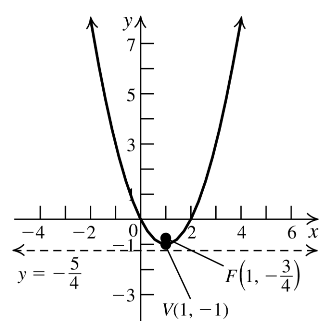
8. Rearrange the equation and complete the square to put the equation in standard form:

$-x^2 + 2x + y = 0 \Rightarrow -(x^2 - 2x + 1) = -y - 1 \Rightarrow$

$(x-1)^2 = (y+1) \Rightarrow 1 = 4a \Rightarrow a = \frac{1}{4}$.

Vertex: (1, −1), focus: $\left(1, -\frac{3}{4}\right)$, axis: $x = 1$,

directrix: $y = -\frac{5}{4}$.



9. The vertex is (0, 0) and the focus is (−3, 0), so the graph opens to the left. The general form is $(y-k)^2 = -4a(x-h)$.

$a = -3 \Rightarrow y^2 = -12x$.

10. The vertex is (0, 0) and the focus is (0, 4), so the graph opens up. The general form is $(x-h)^2 = 4a(y-k)$.

$a = 4 \Rightarrow x^2 = 16y$.

11. The focus is (0, 4) and the directrix is $y = -4$, so the vertex is (0, 0), and the graph opens up. The general form is $(x-h)^2 = 4a(y-k)$.

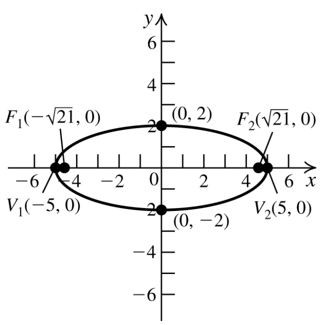
$a = 4 \Rightarrow x^2 = 16y$.

12. The focus is (−3, 0) and the directrix is $x = 3$, so the vertex is (0, 0), and the graph opens to the left. The general form is

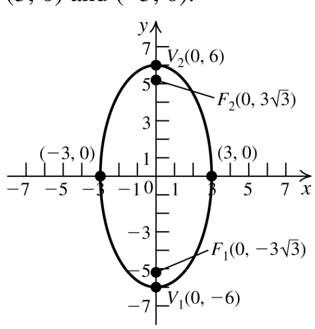
$(y-k)^2 = -4a(x-h)$.

$a = 3 \Rightarrow y^2 = -12x$.

13. $a^2 = 25 \Rightarrow$ the vertices are (5, 0) and (−5, 0).
$b^2 = a^2 - c^2 \Rightarrow 4 = 25 - c^2 \Rightarrow c = \pm\sqrt{21} \Rightarrow$
the foci are $\left(\sqrt{21}, 0\right)$ and $\left(-\sqrt{21}, 0\right)$.
Endpoints of the minor axis: (0, −2) and (0, 2).

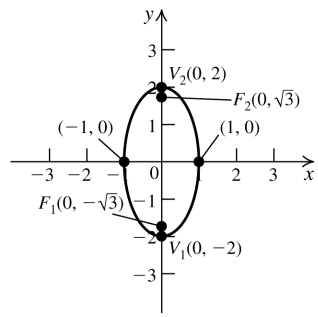


14. $a^2 = 36 \Rightarrow$ the vertices are (0, 6) and (0, −6). $b^2 = a^2 - c^2 \Rightarrow 9 = 36 - c^2 \Rightarrow$
$c = \pm 3\sqrt{3} \Rightarrow$ the foci are $\left(0, 3\sqrt{3}\right)$ and
$\left(0, -3\sqrt{3}\right)$. Endpoints of the minor axis: (3, 0) and (−3, 0).



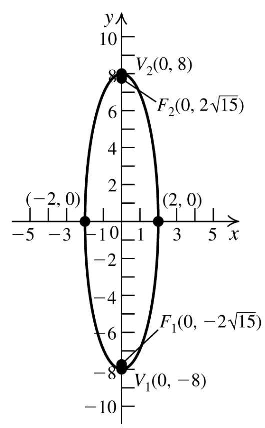
15. $4x^2 + y^2 = 4 \Rightarrow x^2 + \frac{y^2}{4} = 1$.

$a^2 = 4 \Rightarrow$ the vertices are (0, 2) and (0, −2).
$b^2 = a^2 - c^2 \Rightarrow 1 = 4 - c^2 \Rightarrow c = \pm\sqrt{3} \Rightarrow$
the foci are $\left(0, \sqrt{3}\right)$ and $\left(0, -\sqrt{3}\right)$.
Endpoints of the minor axis: (1, 0) and (−1, 0).



16. $16x^2 + y^2 = 64 \Rightarrow \frac{x^2}{4} + \frac{y^2}{64} = 1$.

$a^2 = 64 \Rightarrow$ the vertices are (0, 8) and (0, −8).
$b^2 = a^2 - c^2 \Rightarrow 4 = 64 - c^2 \Rightarrow c = \pm 2\sqrt{15} \Rightarrow$
the foci are $\left(0, 2\sqrt{15}\right)$ and $\left(0, -2\sqrt{15}\right)$.
Endpoints of the minor axis: (2, 0) and (−2, 0).



17. $16(x+1)^2 + 9(y+4)^2 = 144 \Rightarrow$

$$\frac{(x+1)^2}{9} + \frac{(y+4)^2}{16} = 1.$$

The center is (−1, −4) and $a^2 = 16 \Rightarrow$ the vertices are (−1, −8) and (−1, 0).
$b^2 = a^2 - c^2 \Rightarrow 9 = 16 - c^2 \Rightarrow c = \pm\sqrt{7}$.
The foci are $\left(-1, -4 - \sqrt{7}\right)$ and $\left(-1, -4 + \sqrt{7}\right)$.
Endpoints of the minor axis: (−4, −4) and (2, −4).

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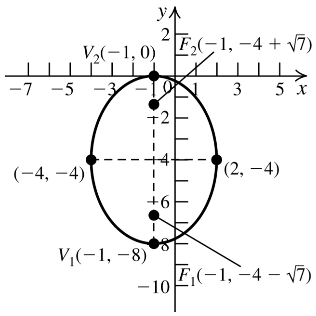


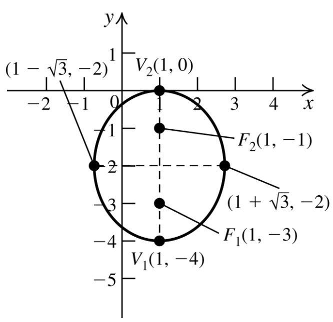
18. $4(x-1)^2+3(y+2)^2=12 \Rightarrow$

$$\frac{(x-1)^2}{3}+\frac{(y+2)^2}{4}=1.$$

The center is (1, −2) and $a^2=4 \Rightarrow$ the vertices are (1, 0) and (1, −4).

$b^2=a^2-c^2 \Rightarrow 3=4-c^2 \Rightarrow c=\pm 1$

The foci are (1, −1) and (1, −3). Endpoints of the minor axis: $\left(1+\sqrt{3},-2\right)$ and $\left(1-\sqrt{3},-2\right)$.



19. Rearrange the equation and complete the square to put the equation in standard form:

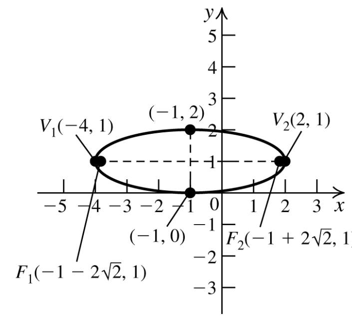
$$x^2+9y^2+2x-18y+1=0 \Rightarrow$$
$$x^2+2x+1+9(y^2-2y+1)=-1+1+9 \Rightarrow$$
$$(x+1)^2+9(y-1)^2=9 \Rightarrow \frac{(x+1)^2}{9}+(y-1)^2=1$$

The center is (−1, 1) and $a^2=9 \Rightarrow$ the vertices are (−4, 1) and (2, 1).

$b^2=a^2-c^2 \Rightarrow 1=9-c^2 \Rightarrow c=\pm 2\sqrt{2}$

The foci are $\left(-1-2\sqrt{2},1\right)$ and $\left(-1+2\sqrt{2},1\right)$.

Endpoints of the minor axis: (−1, 2) and (−1, 0).



20. Rearrange the equation and complete the square to put the equation in standard form:

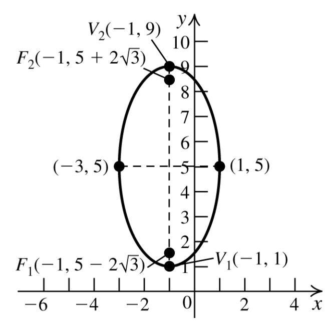
$$4x^2+y^2+8x-10y+13=0 \Rightarrow$$
$$4(x^2+2x+1)+(y^2-10y+25)=-13+4+25 \Rightarrow$$
$$4(x+1)^2+(y-5)^2=16 \Rightarrow$$
$$\frac{(x+1)^2}{4}+\frac{(y-5)^2}{16}=1.$$

The center is (−1, 5) and $a^2=16 \Rightarrow$ the vertices are (−1, 1) and (−1, 9).

$b^2=a^2-c^2 \Rightarrow 4=16-c^2 \Rightarrow c=\pm 2\sqrt{3}$.

The foci are $\left(-1,5+2\sqrt{3}\right)$ and $\left(-1,5-2\sqrt{3}\right)$.

Endpoints of the minor axis: (−3, 5) and (1, 5).



21. $a=4$, $b=2$, major axis: x-axis. $\dfrac{x^2}{16}+\dfrac{y^2}{4}=1$

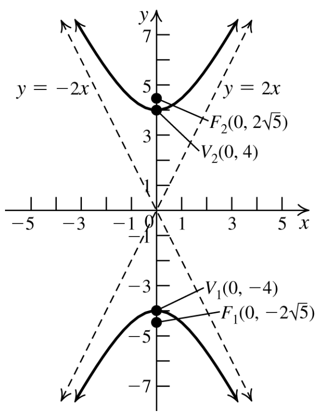
22. $a=6$, $b=2$, major axis: y-axis: $\dfrac{x^2}{4}+\dfrac{y^2}{36}=1$

23. $a=10$, center (0, 0), major axis: x-axis, $c=5$, so $b^2=100-25=75$. $\dfrac{x^2}{100}+\dfrac{y^2}{75}=1$

24. $b=8$, center (0, 0), major axis: y-axis, $c=6$, so $8^2=a^2-6^2 \Rightarrow a^2=100$. $\dfrac{x^2}{64}+\dfrac{y^2}{100}=1$

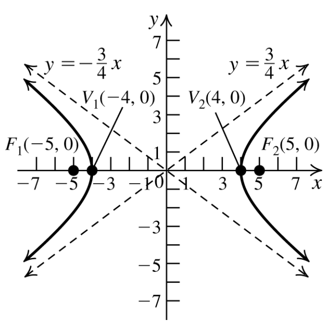
25. $a^2 = 16 \Rightarrow$ the vertices are (0, 4) and (0, −4).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 16 + 4 \Rightarrow c = \pm 2\sqrt{5} \Rightarrow$
the foci are $\left(0, 2\sqrt{5}\right)$ and $\left(0, -2\sqrt{5}\right)$.

Asymptotes: $y = \pm \dfrac{a}{b}x = \pm 2x$.

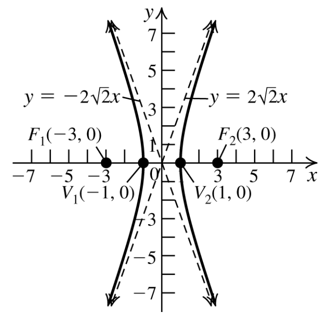


26. $a^2 = 16 \Rightarrow$ the vertices are (4, 0) and (−4, 0).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 16 + 9 \Rightarrow c = \pm 5 \Rightarrow$
the foci are $(5, 0)$ and $(-5, 0)$.

Asymptotes: $y = \pm \dfrac{b}{a}x = \pm \dfrac{3}{4}x$.

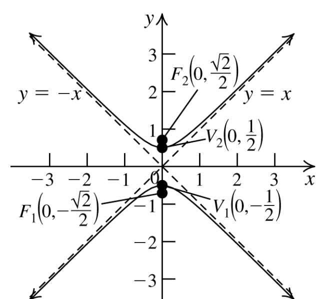


27. $8x^2 - y^2 = 8 \Rightarrow x^2 - \dfrac{y^2}{8} = 1 \Rightarrow a^2 = 1 \Rightarrow$ the vertices are (−1, 0) and (1, 0).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 1 + 8 \Rightarrow c = \pm 3 \Rightarrow$ the foci are (3, 0) and (−3, 0). Asymptotes:
$y = \pm \dfrac{b}{a}x = \pm 2\sqrt{2}x$.

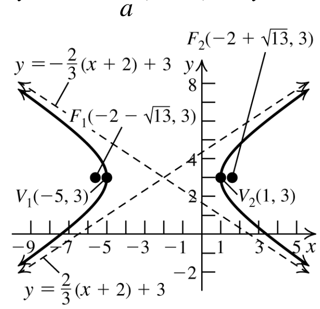


28. $4y^2 - 4x^2 = 1 \Rightarrow \dfrac{y^2}{1/4} - \dfrac{x^2}{1/4} = 1 \Rightarrow a^2 = \dfrac{1}{4} \Rightarrow$
the vertices are $\left(0, \dfrac{1}{2}\right)$ and $\left(0, -\dfrac{1}{2}\right)$.
$c^2 = a^2 + b^2 \Rightarrow c^2 = \dfrac{1}{4} + \dfrac{1}{4} \Rightarrow c = \dfrac{\sqrt{2}}{2} \Rightarrow$ the foci are $\left(0, \dfrac{\sqrt{2}}{2}\right)$ and $\left(0, -\dfrac{\sqrt{2}}{2}\right)$.

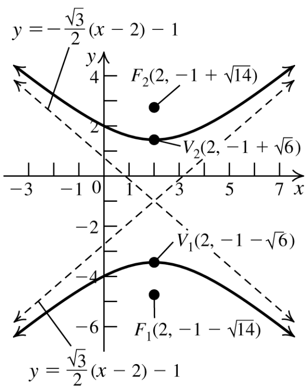
Asymptotes: $y = \pm \dfrac{a}{b}x = \pm x$.



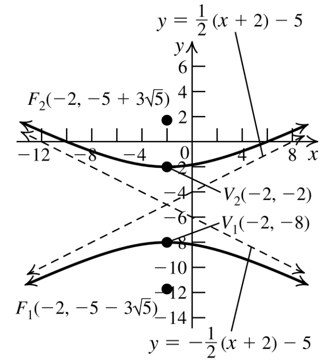
29. The center is (−2, 3). $a^2 = 9 \Rightarrow$ the vertices are (1, 3) and (−5, 3).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 9 + 4 \Rightarrow c = \pm\sqrt{13} \Rightarrow$
the foci are $\left(-2 + \sqrt{13}, 3\right)$ and $\left(-2 - \sqrt{13}, 3\right)$.
Asymptotes:
$y - k = \pm \dfrac{b}{a}(x - h) \Rightarrow y - 3 = \pm \dfrac{2}{3}(x + 2)$.



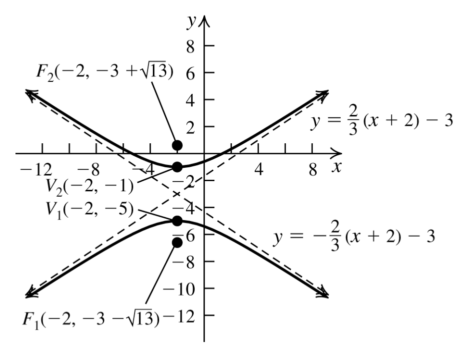
30. The center is (2, −1). $a^2 = 6 \Rightarrow$ the vertices are $\left(2, -1+\sqrt{6}\right)$ and $\left(2, -1-\sqrt{6}\right)$.
$c^2 = a^2 + b^2 \Rightarrow c^2 = 6 + 8 \Rightarrow c = \pm\sqrt{14} \Rightarrow$ the foci are $\left(2, -1+\sqrt{14}\right)$ and $\left(2, -1-\sqrt{14}\right)$.
Asymptotes:
$y - k = \pm\frac{a}{b}(x-h) \Rightarrow y + 1 = \pm\frac{\sqrt{3}}{2}(x-2)$.



31. Rearrange the equation and complete the square to put the equation in standard form:
$4y^2 - x^2 + 40y - 4x + 60 = 0 \Rightarrow$
$4(y^2 + 10y + 25) - (x^2 + 4x + 4) = -60 + 100 - 4$
$4(y+5)^2 - (x+2)^2 = 36 \Rightarrow$
$\frac{(y+5)^2}{9} - \frac{(x+2)^2}{36} = 1$.
The center is (–2, –5). $a^2 = 9 \Rightarrow$ the vertices are (–2, –5 – 3) = (–2, –8) and (–2, –5 + 3) = (–2, –2).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 9 + 36 \Rightarrow c = \pm 3\sqrt{5} \Rightarrow$ the foci are $\left(-2, -5+3\sqrt{5}\right)$ and $\left(-2, -5-3\sqrt{5}\right)$. Asymptotes:
$y - k = \pm\frac{a}{b}(x-h) \Rightarrow y + 5 = \pm\frac{1}{2}(x+2)$.



32. Rearrange the equation and complete the square to put the equation in standard form:
$4x^2 - 9y^2 + 16x - 54y - 29 = 0 \Rightarrow$
$4(x^2 + 4x + 4) - 9(y^2 + 6y + 9) = 29 + 16 - 81$
$4(x+2)^2 - 9(y+3)^2 = -36 \Rightarrow$
$\frac{(y+3)^2}{4} - \frac{(x+2)^2}{9} = 1$.
The center is (–2, –3). $a^2 = 4 \Rightarrow$ the vertices are (–2, –1) and (–2, –5).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 4 + 9 \Rightarrow c = \pm\sqrt{13} \Rightarrow$ the foci are $\left(-2, -3+\sqrt{13}\right)$ and $\left(-2, -3-\sqrt{13}\right)$. Asymptotes:
$y - k = \pm\frac{a}{b}(x-h) \Rightarrow y + 3 = \pm\frac{2}{3}(x+2)$.



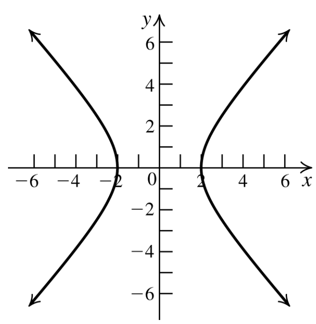
33. The vertices are $(\pm 1, 0)$ and the foci are $(\pm 2, 0)$, so the center is (0, 0), $a = 1$, $c = 2$, and the transverse axis is the x-axis.
$c^2 = a^2 + b^2 \Rightarrow 4 = 1 + b^2 \Rightarrow b^2 = 3$. The equation is $x^2 - \frac{y^2}{3} = 1$.

34. The vertices are $(0, \pm 2)$ and the foci are $(0, \pm 4)$, so the center is (0, 0), $a = 2$, $c = 4$, and the transverse axis is the y-axis. $c^2 = a^2 + b^2 \Rightarrow 16 = 4 + b^2 \Rightarrow b^2 = 12$. The equation is $\frac{y^2}{4} - \frac{x^2}{12} = 1.$

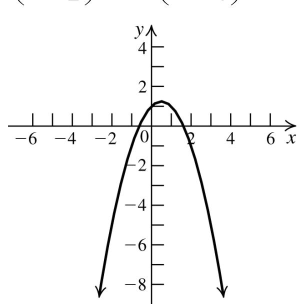
35. The vertices are $(\pm 2, 0)$ so the center is (0, 0), $a = 2$, and the transverse axis is the x-axis. The asymptotes are $y = \pm 3x \Rightarrow \pm \frac{b}{a} = \pm 3 \Rightarrow \frac{b}{2} = 3 \Rightarrow b = 6$. The equation is $\frac{x^2}{4} - \frac{y^2}{36} = 1.$

36. The vertices are $(0, \pm 3)$, so the center is (0, 0), $a = 3$, and the transverse axis is the y-axis. The asymptotes are $y = \pm x \Rightarrow \pm \frac{a}{b} = \pm 1 \Rightarrow \frac{3}{b} = 1 \Rightarrow b = 3$. The equation is $\frac{y^2}{9} - \frac{x^2}{9} = 1.$

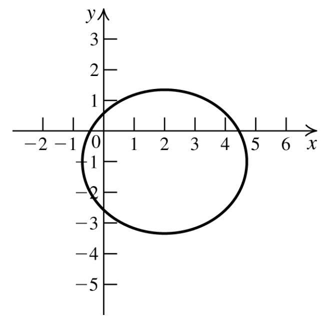
37. Hyperbola. In standard form, the equation is $\frac{x^2}{4} - \frac{y^2}{5} = 1$.



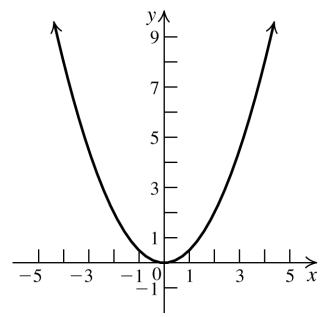
38. Parabola. In standard form, the equation is $\left(x - \frac{1}{2}\right)^2 = -\left(y - \frac{5}{4}\right)$.



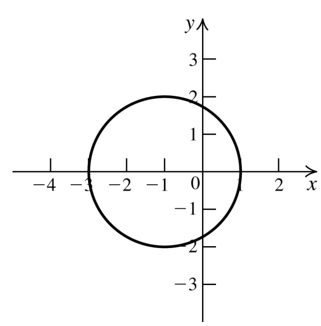
39. Ellipse. In standard form, the equation is $\frac{(x-2)^2}{22/3} + \frac{(y+1)^2}{11/2} = 1$.



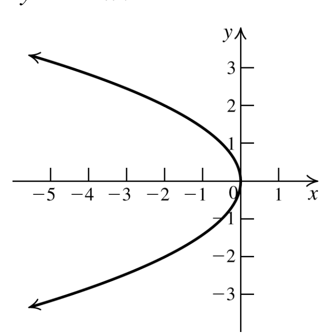
40. Parabola. In standard form, the equation is $x^2 = 2y$.



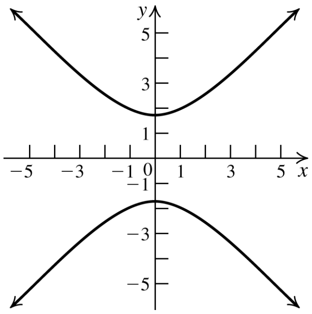
41. Circle. In standard form, the equation is $(x+1)^2 + y^2 = 4$.



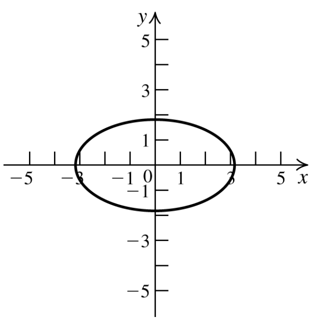
42. Parabola. In standard form, the equation is $y^2 = -2x$.



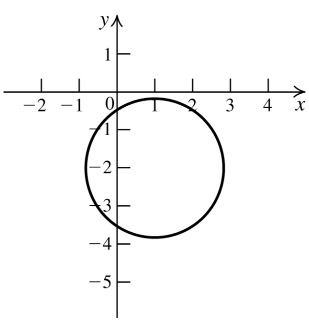
43. Hyperbola. In standard form, the equation is $y^2 - x^2 = 3$.



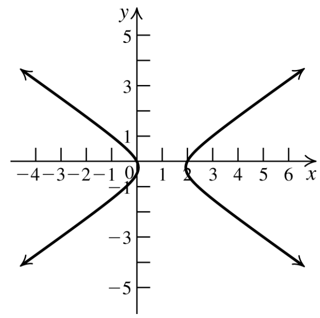
44. Ellipse. In standard form, the equation is $\frac{x^2}{10} + \frac{y^2}{10/3} = 1$.



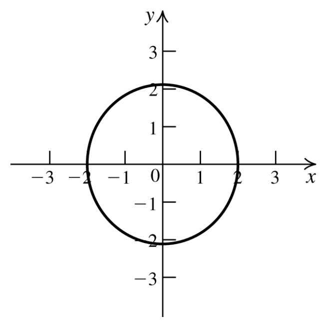
45. Circle. In standard form, the equation is $(x-1)^2 + (y+2)^2 = \frac{10}{3}$



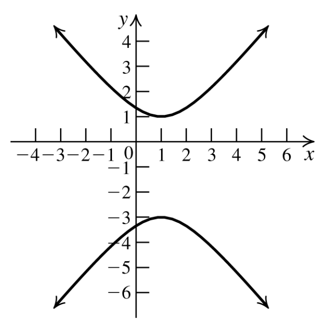
46. Hyperbola. In standard form, the equation is $\frac{(x-1)^2}{7/8} - \frac{\left(y+\frac{1}{4}\right)^2}{7/16} = 1$.



47. Ellipse. In standard form, the equation is $\frac{x^2}{4} + \frac{y^2}{9/2} = 1$.



48. Hyperbola. In standard form, the equation is $\frac{(y+1)^2}{4} - \frac{(x-1)^2}{8/3} = 1$.



49. $4x^2 + 9y^2 = 36 \Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = 1 \Rightarrow$ the vertices of the ellipse are (3, 0) and (–3, 0). $b^2 = a^2 - c^2 \Rightarrow 4 = 9 - c^2 \Rightarrow c = \pm\sqrt{5} \Rightarrow$ the foci are $\left(\sqrt{5}, 0\right)$ and $\left(-\sqrt{5}, 0\right)$. For the hyperbola, the transverse axis is the x-axis, $a = \sqrt{5}$, and $c = 3$, so $c^2 = a^2 + b^2 \Rightarrow$ $9 = 5 + b^2 \Rightarrow b^2 = 4$. The equation of the hyperbola is $\frac{x^2}{5} - \frac{y^2}{4} = 1$.

50. $9x^2 - 16y^2 = 144 \Rightarrow \dfrac{x^2}{16} - \dfrac{y^2}{9} = 1 \Rightarrow$ the vertices of the hyperbola are (4, 0) and (−4, 0). $c^2 = a^2 + b^2 \Rightarrow c^2 = 16 + 9 \Rightarrow c = \pm 5 \Rightarrow$ the foci are (−5, 0) and (5, 0). For the ellipse, a =5 and c = 4, so $b^2 = a^2 - c^2 \Rightarrow b^2 = 25 - 16 \Rightarrow b^2 = 9$. The equation of the ellipse is $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$.

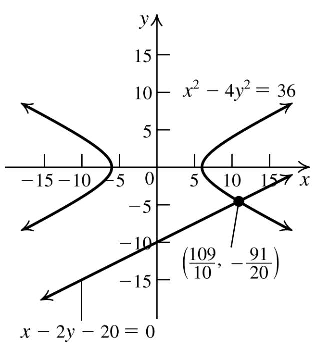
51. Solve using substitution:

$$\begin{cases} x^2 - 4y^2 = 36 \\ x - 2y - 20 = 0 \end{cases} \Rightarrow \begin{cases} x^2 - 4y^2 = 36 \\ x = 2y + 20 \end{cases} \Rightarrow$$

$$(2y + 20)^2 - 4y^2 = 36 \Rightarrow 80y + 400 = 36 \Rightarrow$$

$$y = -\frac{91}{20};\ x = 2\left(-\frac{91}{20}\right) + 20 = \frac{109}{10}$$

The only point of intersection is $\left(\dfrac{109}{10}, -\dfrac{91}{20}\right)$.



52. Solve using elimination:

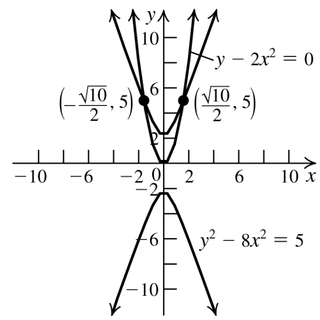
$$\begin{cases} y^2 - 8x^2 = 5 \\ y - 2x^2 = 0 \end{cases} \Rightarrow \begin{cases} y^2 - 8x^2 = 5 \\ 4y - 8x^2 = 0 \end{cases} \Rightarrow$$

$$y^2 - 4y = 5 \Rightarrow y^2 - 4y - 5 = 0 \Rightarrow$$

$$(y - 5)(y + 1) = 0 \Rightarrow y = 5 \text{ or } y = -1$$

$$5 - 2x^2 = 0 \Rightarrow x^2 = \frac{5}{2} \Rightarrow x = \pm\frac{\sqrt{10}}{2}$$

$-1 - 2x^2 = 0 \Rightarrow -1 = 2x^2 \Rightarrow$ there is no solution if $y = -1$. The points of intersection are $\left(\dfrac{\sqrt{10}}{2}, 5\right)$ and $\left(-\dfrac{\sqrt{10}}{2}, 5\right)$.



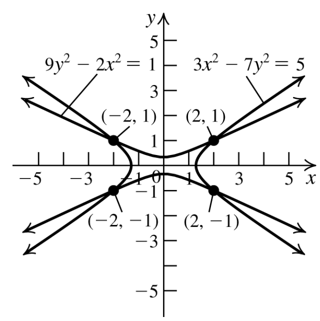
53. Solve using elimination:

$$\begin{cases} 3x^2 - 7y^2 = 5 \\ 9y^2 - 2x^2 = 1 \end{cases} \Rightarrow \begin{cases} 6x^2 - 14y^2 = 10 \\ -6x^2 + 27y^2 = 3 \end{cases} \Rightarrow$$

$$13y^2 = 13 \Rightarrow y = \pm 1$$

$$3x^2 - 7 = 5 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$$

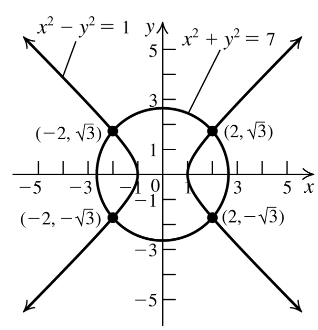
The points of intersection are (−2, −1), (−2, 1), (2, −1), and (2, 1).



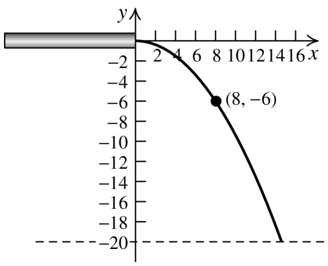
54. Solve using elimination:

$$\begin{cases} x^2 - y^2 = 1 \\ x^2 + y^2 = 7 \end{cases} \Rightarrow 2x^2 = 8 \Rightarrow x = \pm 2$$

$4 + y^2 = 7 \Rightarrow y = \pm\sqrt{3}$. The points of intersection are $\left(-2, -\sqrt{3}\right), \left(-2, \sqrt{3}\right), \left(2, -\sqrt{3}\right)$, and $\left(2, \sqrt{3}\right)$.



55. Let the end of the pipe, and therefore the vertex of the parabola, be at (0, 0). Then the ground is at $y = -20$. The water goes through the point (8, −6).



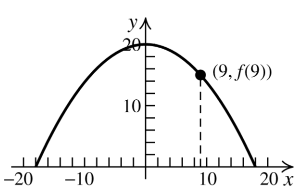
The equation is of the form $y = -4ax^2$. Substitute (8, −6) into the equation and solve for a: $-6 = -4a(64) \Rightarrow a = \frac{3}{128}$ and the equation of the parabola is $y = -\frac{3}{32}x^2$. Let $y = -20$ and solve for x:

$$-20 = -\frac{3}{32}x^2 \Rightarrow x^2 = \frac{640}{3} \Rightarrow x = \pm 14.61.$$

We are looking for the positive solution. The water hits the ground 14.6 feet from the end of the pipe.

56. Let the vertex of the parabola be at (0, 20). Then the ends of the arch are at (−18, 0) and (18, 0). The equation is of the form $y - 20 = -4ax^2$. Substitute (18, 0) into the equation and solve for a:

$0 - 20 = -4a(18)^2 \Rightarrow a = \frac{5}{324}$ and the equation of the parabola is $y - 20 = -\frac{5}{81}x^2$.



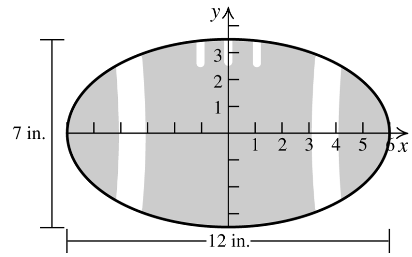
Let $x = 9$ and solve for y:

$$y - 20 = -\frac{5}{81}(81) \Rightarrow y = 15 \text{ m}.$$

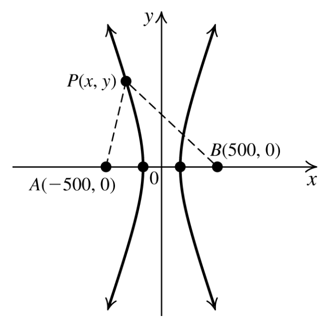
57. The equation of the ellipse is $\frac{x^2}{36} + \frac{y^2}{49/4} = 1$.

At $x = 4$, $y = \frac{7\sqrt{5}}{6} \approx 2.61$. (Reject the negative solution.)

This is the radius of the cross section. The circumference $= 2\pi(2.61) \approx 16.4$ inches.



58.



A and B are 1000 miles apart, so let the coordinates of A be (−500, 0) and let the coordinates of B be (500, 0). Let c = the cost of production at location B. Then $c - 20$ = the cost of production at location A. The delivery cost at any point P from location B is $\frac{1}{4}d(P, B)$, while the delivery cost at any point P from location A is $\frac{1}{4}d(P, A)$. So, the total cost at location A is $(c - 20) + \frac{1}{4}d(P, A)$, while the total cost at location B is $c + \frac{1}{4}d(P, B)$. P is located on a hyperbola because the two total costs are equal; therefore,

$$(c - 20) + \frac{1}{4}d(P, A) = c + \frac{1}{4}d(P, B) \Rightarrow$$

$$d(P, A) - d(P, B) = 80 = 2a \Rightarrow a = 40.$$

We have $c = 500$, so $b^2 = c^2 - a^2 = 500^2 - 40^2 = 248,400$. The equation of the hyperbola is

$$\frac{x^2}{40^2} - \frac{y^2}{248,400} = 1 \Rightarrow \frac{x}{40} = \sqrt{1 + \frac{y^2}{248,400}}.$$

Note that this gives only the portion of the hyperbola for $y \geq 0$, which is the portion that is defined for this problem.

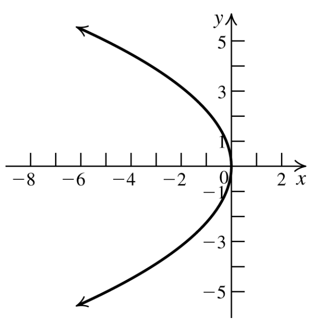
Chapter 10 Practice Test A

1. The focus is at (0, 12) and the directrix is $x = -12$, so the vertex is at (−6, 12) and the parabola opens to the right. The general form is $(y-k)^2 = 4a(x-h)$. $a = 0-(-6) = 6 \Rightarrow$
$(y-12)^2 = 4(6)(x+6) = 24(x+6)$.

2. $y^2 - 2y + 8x + 25 = 0 \Rightarrow$
$y^2 - 2y + 1 = -8x - 25 + 1 \Rightarrow$
$(y-1)^2 = -8(x+3)$

3. $x^2 = -9y = 4ay \Rightarrow a = -\frac{9}{4}$
Focus: $\left(0, -\frac{9}{4}\right)$, directrix: $y = \frac{9}{4}$.

4.



5. $(x+2)^2 = -8(y-1) = 4a(y-1) \Rightarrow a = -2$.
Vertex: (−2, 1), focus: (−2, −1), directrix: $y = 3$.

6. Let the vertex be at (0, 0). Then the parabola goes through the point (3, 2). The general form of the equation is $(x-h)^2 = 4a(y-k)$. Substitute the values for x, y, h, and k, then solve for a: $3^2 = 4a(2) \Rightarrow a = \frac{9}{8}$. The equation of the parabola is $x^2 = \frac{9}{2}y$. Now find the value of y for $x = 1$: $1 = \frac{9}{2}y \Rightarrow y = \frac{2}{9} \approx 0.22$ feet.

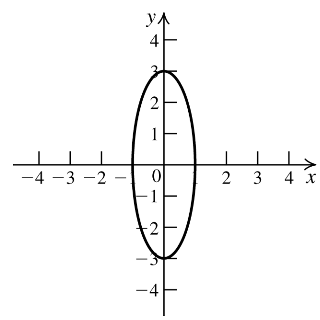
7. Vertex (3, −1) and directrix $x = -3 \Rightarrow a = 6$. The parabola opens to the right, so the general form of the equation is $(y-k)^2 = 4a(x-h)$. The equation is $(y+1)^2 = 24(x-3)$.

8. Foci (0, −2) and (0, 2) $\Rightarrow c = 2$.
Vertices (0, −4) and (0, 4) $\Rightarrow a = 4$.
$b^2 = a^2 - c^2 \Rightarrow b^2 = 4^2 - 2^2 = 12$.
The equation is $\frac{x^2}{12} + \frac{y^2}{16} = 1$.

9. Foci (−2, 0) and (2, 0) $\Rightarrow c = 2$.
y-intercepts −5 and 5 $\Rightarrow b = 5$.
$b^2 = a^2 - c^2 \Rightarrow 25 = a^2 - 2^2 \Rightarrow a^2 = 29$.
The equation is $\frac{x^2}{29} + \frac{y^2}{25} = 1$.

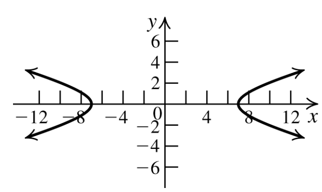
10. $a = 9$, $b = 2$. The equation is $\frac{x^2}{81} + \frac{y^2}{4} = 1$.

11.



12. $49y^2 - x^2 = 49 \Rightarrow y^2 - \frac{x^2}{49} = 1 \Rightarrow$
$a^2 = 1, b^2 = 49$. The vertices are at (0, 1) and (0, −1).
$c^2 = a^2 + b^2 \Rightarrow c^2 = 1 + 49 \Rightarrow c = \pm 5\sqrt{2}$.
The foci are at $\left(0, 5\sqrt{2}\right)$ and $\left(0, -5\sqrt{2}\right)$.

13.

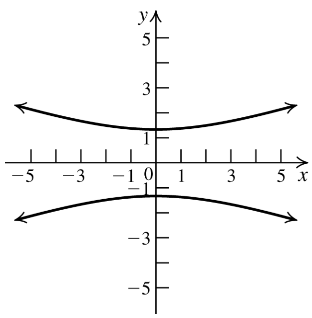


14. The center is (0, 0) and the transverse axis is the y-axis, so the equation is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.
$a = 6, c = \sqrt{45} \Rightarrow 45 = 36 + b^2 \Rightarrow b^2 = 9$.
The equation is $\frac{y^2}{36} - \frac{x^2}{9} = 1$.

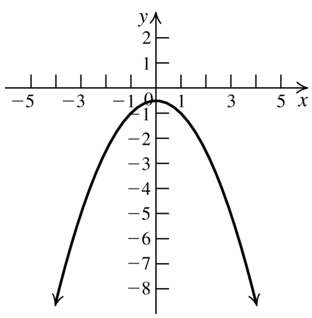
15. $y^2 - x^2 + 2x = 2 \Rightarrow$
$y^2 - (x^2 - 2x + 1) = 2 - 1 \Rightarrow y^2 - (x-1)^2 = 1$

16. $x^2 - 2x - 4y^2 - 16y = 19 \Rightarrow$
$(x^2 - 2x + 1) - 4(y^2 + 4y + 4) = 19 + 1 - 16 \Rightarrow$
$(x-1)^2 - 4(y+2)^2 = 4 \Rightarrow$
$\dfrac{(x-1)^2}{4} - (y+2)^2 = 1$

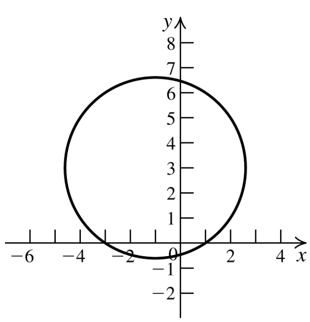
17. Hyperbola



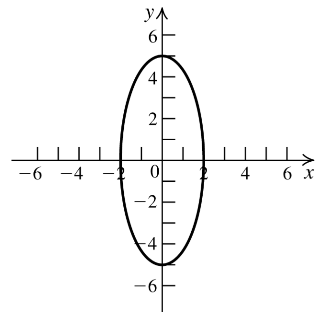
18. Parabola



19. Circle



20. Ellipse



Chapter 10 Practice Test B

1. The focus is at (−10, 0) and the directrix is $x = 10$, so the vertex is at (0, 0) and the parabola opens to the left. The general form is $(y-k)^2 = -4a(x-h)$.
$a = 0 - (-10) = 10 \Rightarrow y^2 = -4(10)x = -40x$.
The answer is B.

2. $y^2 - 4y - 5x + 24 = 0 \Rightarrow$
$y^2 - 4y + 4 = 5x - 24 + 4 \Rightarrow$
$(y-2)^2 = 5(x-4)$. The answer is B.

3. $x = 7y^2 \Rightarrow \dfrac{x}{7} = y^2 \Rightarrow \dfrac{1}{7} = 4a \Rightarrow \dfrac{1}{28} = a \Rightarrow$
the focus at $\left(\dfrac{1}{28}, 0\right)$ and the directrix is
$x = -\dfrac{1}{28}$. The answer is A.

4. The answer is A.

5. $(x-3)^2 = 12(y-1) \Rightarrow$ the vertex is (3, 1).
$12 = 4a \Rightarrow a = 3 \Rightarrow$ the focus is at (3, 4), and the directrix is $y = -2$. The answer is A.

6. Let the vertex be located at (0, 25) and one end of the base is at (90, 0). The general form of the equation is
$(x-h)^2 = -4a(y-k)$. Substitute the values for x, y, h, and k, then solve for a:
$(90-0)^2 = -4a(0-25) \Rightarrow a = 81$
The equation of the parabola is
$x^2 = -4(81)(y-25) \Rightarrow x^2 = -324(y-25)$.
Find the value of y for
$x = 45$: $45^2 = -324(y-25) \Rightarrow y = 18.75$.
The answer is D.

7. Vertex (−2, 1) and directrix $x = 2 \Rightarrow a = 4$. The parabola opens to the left, so the general form of the equation is $(y-k)^2 = -4a(x-h)$.

The equation is $(y-1)^2 = -16(x+2)$.

The answer is A.

8. Foci (−3, 0) and (3, 0) ⇒ $c = 3$.

Vertices (−5, 0) and (5, 0) ⇒ $a = 5$.

$b^2 = a^2 - c^2 \Rightarrow b^2 = 5^2 - 3^2 = 16$

The equation is $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

The answer is B.

9. Foci (0, −3) and (0, 3) ⇒ $c = 3$. y-intercepts −7 and 7 ⇒ $a = 7$. $b^2 = a^2 - c^2 \Rightarrow$ $49 = 3^2 - b^2 \Rightarrow b^2 = 40$. The equation is $\frac{x^2}{40} + \frac{y^2}{49} = 1$. The answer is D.

10. $a = 8$, $b = 4$. The equation is $\frac{x^2}{16} + \frac{y^2}{64} = 1$.

The answer is A.

11. $9(x-1)^2 + 4(y-2)^2 = 36 \Rightarrow$

$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{9} = 1$. The answer is A.

12. $a = 11$, $b = 2$. The vertices are at (11, 0) and (−11, 0). $c^2 = a^2 + b^2 \Rightarrow c^2 = 121 + 4 \Rightarrow$ $c = 5\sqrt{5}$. The foci are at $\left(-5\sqrt{5}, 0\right)$ and $\left(5\sqrt{5}, 0\right)$. The answer is C.

13. The answer is B.

14. The center is (0, 0) and the transverse axis is the y-axis, so the equation is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

$a = 5, c = 10 \Rightarrow 100 = 25 + b^2 \Rightarrow b^2 = 75$.

The equation is $\frac{y^2}{25} - \frac{x^2}{75} = 1$.

The answer is B.

15. $y^2 - 4x^2 - 2y - 16x - 19 = 0 \Rightarrow$
$y^2 - 2y + 1 - 4(x^2 + 4x + 4) = 19 + 1 - 16 \Rightarrow$
$(y-1)^2 - 4(x+2)^2 = 4 \Rightarrow$
$\frac{(y-1)^2}{4} - (x+2)^2 = 1$. The answer is C.

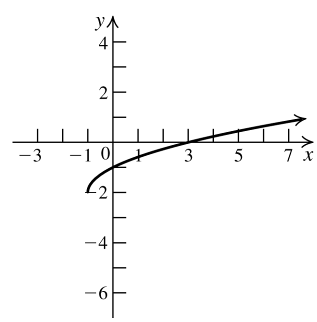
16. $4y^2 - 9x^2 - 16y - 36x - 56 = 0 \Rightarrow$
$4(y^2 - 4y + 4) - 9(x^2 + 4x + 4) = 56 + 16 - 36 \Rightarrow$
$4(y-2)^2 - 9(x+2)^2 = 36 \Rightarrow$
$\frac{(y-2)^2}{9} - \frac{(x+2)^2}{4} = 1$. The answer is C.

17. C **18.** A **19.** D **20.** D

Cumulative Review Exercises (Chapters P–10)

1. $\frac{\left((x+h)^2 - 3(x+h) + 2\right) - \left(x^2 - 3x + 2\right)}{h}$
$= \frac{x^2 + 2xh + h^2 - 3x - 3h + 2 - x^2 + 3x - 2}{h}$
$= \frac{h^2 + 2xh - 3h}{h} = h + 2x - 3$

2.

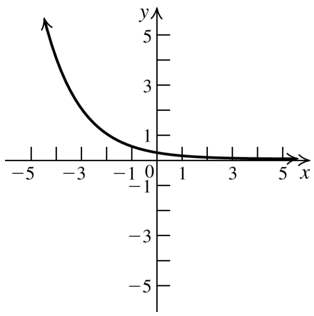


3. Switch the variables, then solve for y:

$y = 2x - 3 \Rightarrow x = 2y - 3 \Rightarrow \frac{x+3}{2} = y = f^{-1}(x)$

$f(f^{-1}(x)) = 2\left(\frac{x+3}{2}\right) - 3 = x$

4.



5. $\log_5(x-1)+\log_5(x-2)=3\log_5\sqrt[3]{6} \Rightarrow$
$\log_5\left((x-1)(x-2)\right)=\log_5\left(\sqrt[3]{6}\right)^3 \Rightarrow$
$x^2-3x+2=6 \Rightarrow x^2-3x-4=0 \Rightarrow$
$(x-4)(x+1)=0 \Rightarrow x=4$ or $x=-1$. (Reject the negative solution.) The solution is $\{4\}$.

6.
$$\log_a\sqrt[3]{x\sqrt{yz}}=\log_a\left(x\sqrt{yz}\right)^{1/3}$$
$$=\frac{1}{3}\log_a x+\frac{1}{3}\log_a\sqrt{yz}$$
$$=\frac{1}{3}\log_a x+\frac{1}{3}\log_a(yz)^{1/2}$$
$$=\frac{1}{3}\log_a x+\frac{1}{3}\cdot\frac{1}{2}\log_a(yz)$$
$$=\frac{1}{3}\log_a x+\frac{1}{6}\log_a y+\frac{1}{6}\log_a z$$

7. $\dfrac{x}{x-2}\ge 1 \Rightarrow \dfrac{x}{x-2}-1\ge 0 \Rightarrow \dfrac{2}{x-2}\ge 0$.
$x-2>0 \Rightarrow x>2$. The solution is $(2,\infty)$.

8. $I=\dfrac{V}{R}(1-e^{-0.3t}) \Rightarrow 1-\dfrac{IR}{V}=e^{-0.3t} \Rightarrow$
$$\ln\left(1-\frac{IR}{V}\right)=-0.3t \Rightarrow t=-\frac{\ln\left(1-\frac{IR}{V}\right)}{0.3}$$

9. Solve using elimination:
$$\begin{cases}1.4x-0.5y=1.3\\0.4x+1.1y=4.1\end{cases} \Rightarrow \begin{cases}0.56x-0.2y=0.52\\-0.56x-1.54y=-5.74\end{cases} \Rightarrow$$
$-1.74y=-5.22 \Rightarrow y=3$
$0.4x+1.1(3)=4.1 \Rightarrow x=2$
The solution is $\{(2, 3)\}$.

10. Switch the first and second equations:
$$\begin{cases}2x+y-4z=3\\x-2y+3z=4\\-3x+4y-z=-2\end{cases} \Rightarrow \begin{cases}x-2y+3z=4\\2x+y-4z=3\\-3x+4y-z=-2\end{cases}.$$
Multiply the first equation by −2, add the result to the second equation, and replace the second equation with the new equation:
$$\begin{cases}x-2y+3z=4\\2x+y-4z=3\\-3x+4y-z=-2\end{cases} \Rightarrow \begin{cases}x-2y+3z=4\\5y-10z=-5\\-3x+4y-z=-2\end{cases}$$
Divide the second equation by 5 and replace the equation with the result. Then multiply the first equation by 3, add the result to the third equation, and replace the third equation with the new equation.
$$\begin{cases}x-2y+3z=4\\5y-10z=-5\\-3x+4y-z=-2\end{cases} \Rightarrow \begin{cases}x-2y+3z=4\\y-2z=-1\\-2y+8z=10\end{cases}.$$
Divide the third equation by −2 and replace the equation with the result:
$$\begin{cases}x-2y+3z=4\\y-2z=-1\\-2y+8z=10\end{cases} \Rightarrow \begin{cases}x-2y+3z=4\\y-2z=-1\\y-4z=-5\end{cases}.$$
Subtract the third equation from the second equation and solve for z:
$$\begin{cases}x-2y+3z=4\\y-2z=-1\\y-4z=-5\end{cases} \Rightarrow \begin{cases}x-2y+3z=4\\y-2z=-1\\z=2\end{cases}$$
Substitute $z=2$ into the second equation and solve for y: $y-2(2)=-1 \Rightarrow y=3$. Substitute the values for y and z into the first equation and solve for x: $x-2(3)+3(2)=4 \Rightarrow x=4$. The solution is $\{(4, 3, 2)\}$.

11. Solve using substitution:
$$\begin{cases}y=2-\log x\\y-\log(x+3)=1\end{cases} \Rightarrow$$
$2-\log x-\log(x+3)=1 \Rightarrow$
$\log x+\log(x+3)=1 \Rightarrow \log\left(x(x+3)\right)=1 \Rightarrow$
$x^2+3x=10 \Rightarrow x^2+3x-10=0 \Rightarrow$
$(x+5)(x-2)=0 \Rightarrow x=-5$ or $x=2$
(Reject the negative solution.)
The solution is $\{(2, 2-\log 2)\}$.

12. Solve using substitution:
$$\begin{cases}y=x^2-1\\3x^2+8y^2=8\end{cases} \Rightarrow \begin{cases}y+1=x^2\\3x^2+8y^2=8\end{cases} \Rightarrow$$
$3(y+1)+8y^2=8 \Rightarrow 8y^2+3y-5=0 \Rightarrow$
$(y+1)(8y-5)=0 \Rightarrow y=-1$ or $y=\dfrac{5}{8}$

(*continued on next page*)

(*continued*)

$-1 = x^2 - 1 \Rightarrow x^2 = 0 \Rightarrow x = 0$

$\frac{5}{8} = x^2 - 1 \Rightarrow \frac{13}{8} = x^2 \Rightarrow x = \pm\frac{\sqrt{26}}{4}$

The solutions are $\left\{(0,-1), \left(\frac{\sqrt{26}}{4}, \frac{5}{8}\right), \left(-\frac{\sqrt{26}}{4}, \frac{5}{8}\right)\right\}$.

13. Expand by the first column:

$$\begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix} = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$$

$$= 1(-1)^{1+1}\begin{vmatrix} 5 & 8 \\ 6 & 9 \end{vmatrix} + 2(-1)^{2+1}\begin{vmatrix} 4 & 7 \\ 6 & 9 \end{vmatrix} + 3(-1)^{3+1}\begin{vmatrix} 4 & 7 \\ 5 & 8 \end{vmatrix}$$

$$= -3 - 2(-6) + 3(-3) = 0$$

14. $\begin{cases} 2x - 3y = -4 \\ 5x + 7y = 1 \end{cases} \Rightarrow D = \begin{vmatrix} 2 & -3 \\ 5 & 7 \end{vmatrix} = 29$

$D_x = \begin{vmatrix} -4 & -3 \\ 1 & 7 \end{vmatrix} = -25, D_y = \begin{vmatrix} 2 & -4 \\ 5 & 1 \end{vmatrix} = 22$

$x = -\frac{25}{29}, y = \frac{22}{29}$

15. $A = \begin{vmatrix} 3 & -2 \\ -5 & 4 \end{vmatrix} \Rightarrow A^{-1} = \frac{1}{12-10}\begin{vmatrix} 4 & 2 \\ 5 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ \frac{5}{2} & \frac{3}{2} \end{vmatrix}$

16. To find the point of intersection, solve the system

$\begin{cases} x + 2y - 3 = 0 \\ 3x + 4y - 5 = 0 \end{cases} \Rightarrow \begin{cases} -2x - 4y + 6 = 0 \\ 3x + 4y - 5 = 0 \end{cases} \Rightarrow$

$x = -1; -1 + 2y - 3 = 0 \Rightarrow y = 2$

The point of intersection is (−1, 2). The line $x - 3y + 5 = 0 \Rightarrow y = \frac{1}{3}x + \frac{5}{3} \Rightarrow$ its slope is $1/3$. The slope of the perpendicular is −3. The equation of the line through (−1, 2) with slope −3 is $y - 2 = -3(x+1) \Rightarrow y = -3x - 1$.

17. Let $u = x^2$. Then the equation becomes

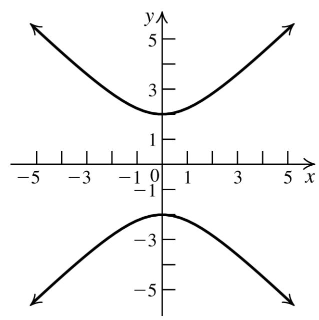
$2u^2 - 5u + 3 = 0 \Rightarrow (2u-3)(u-1) = 0 \Rightarrow$

$u = \frac{3}{2}$ or $u = 1$. Now solve for *x*:

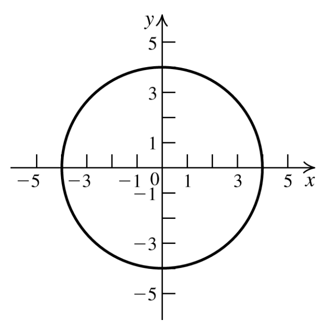
$x^2 = \frac{3}{2} \Rightarrow x = \pm\frac{\sqrt{6}}{2}; x^2 = 1 \Rightarrow x = \pm 1$.

The solutions are $\left\{\pm\frac{\sqrt{6}}{2}, \pm 1\right\}$.

18. Hyperbola



19. Circle



20.