Chapter 8 Systems of Equations and Inequalities

8.1 Systems of Linear Equations in Two Variables

8.1 Practice Problems

1. a. Check (2, 2).

Equation (1): $x + y = 4$, $2 + 2 \overset{?}{=} 4$, $4 = 4$ ✓

Equation (2): $3x - y = 0$, $3(2) - 2 \overset{?}{=} 0$, $4 = 0$ ×

Because (2, 2) does not satisfy both equations, it is not a solution of the system.

b. Check (1, 3).

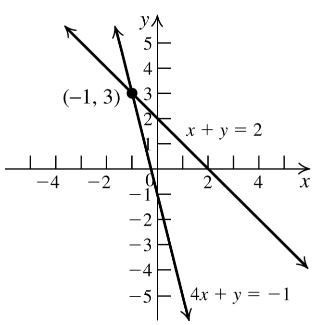
Equation (1): $x + y = 4$, $1 + 3 \overset{?}{=} 4$, $4 = 4$ ✓

Equation (2): $3x - y = 0$, $3(1) - 3 \overset{?}{=} 0$, $0 = 0$ ✓

Because (1, 3) satisfies both equations, it is a solution of the system.

2. The solution is $\{(-1, 3)\}$.

$$\begin{cases} x + y = 2 \\ 4x + y = -1 \end{cases}$$



3. $\begin{cases} x - y = 5 & (1) \\ 2x + y = 7 & (2) \end{cases}$

Solve the equation (1) for x, then substitute that expression into equation (2) and solve for y.

$x - y = 5 \Rightarrow x = y + 5$

$2x + y = 7 \Rightarrow 2(y + 5) + y = 7 \Rightarrow$

$3y + 10 = 7 \Rightarrow 3y = -3 \Rightarrow y = -1$

Now substitute the value for y into equation (1) and solve for x.

$x - y = 5 \Rightarrow x - (-1) = 5 \Rightarrow x = 4$

The solution is $\{(4, -1)\}$.

4. $\begin{cases} x - 3y = 1 & (1) \\ -2x + 6y = 3 & (2) \end{cases}$

Solve the equation (1) for x, then substitute that expression into the equation (2) and solve for y.

$x - 3y = 1 \Rightarrow x = 1 + 3y$

$-2x + 6y = 3 \Rightarrow -2(1 + 3y) + 6y = 3 \Rightarrow$

$-2 - 6y + 6y = 3 \Rightarrow -2 = 3$

Since the equation $-2 = 3$ is false, the system is inconsistent. The solution set is $\varnothing$.

5. $\begin{cases} -2x + y = -3 & (1) \\ 4x - 2y = 6 & (2) \end{cases}$

Solve the equation (1) for y, then substitute that expression into equation (2) and solve for x.

$-2x + y = -3 \Rightarrow y = 2x - 3$

$4x - 2y = 6 \Rightarrow 4x - 2(2x - 3) = 6 \Rightarrow$

$4x - 4x + 6 = 6 \Rightarrow 6 = 6$

The equation $6 = 6$ is true for every value of x. Thus, any value of x can be used in the equation $y = 2x - 3$. The solutions of the system are of the form $\{(x, 2x - 3)\}$.

6. $\begin{cases} 3x + 2y = 3 & (1) \\ 9x - 4y = 4 & (2) \end{cases}$

Multiply the equation (1) by 2, then add the two equations.

$\begin{cases} 3x + 2y = 3 \\ 9x - 4y = 4 \end{cases} \Rightarrow \begin{cases} 6x + 4y = 6 \\ 9x - 4y = 4 \end{cases} \Rightarrow 15x = 10 \Rightarrow$

$x = \frac{2}{3}$

Now substitute $x = \frac{2}{3}$ into the equation (2) and solve for y:

$9x - 4y = 4 \Rightarrow 9\left(\frac{2}{3}\right) - 4y = 4 \Rightarrow 6 - 4y = 4 \Rightarrow$

$-4y = -2 \Rightarrow y = \frac{1}{2}$

The solution is $\left\{\left(\frac{2}{3}, \frac{1}{2}\right)\right\}$.

7. $\begin{cases} p = 20 + 0.002x & (1) \\ p = 77 - 0.008x & (2) \end{cases}$

Solve by substitution.

$20 + 0.002x = 77 - 0.008x \Rightarrow 0.010x = 57 \Rightarrow$

$x = 5700$

Substitute this value into equation (1) and solve for p.

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$p = 20 + 0.002x \Rightarrow p = 20 + 0.002(5700) \Rightarrow$
$p = 31.4$

The equilibrium point is (5700, 31.4).

8. Let x = the amount invested at 12%.
Let y = the amount invested at 8%.
Then $0.12x$ = the income from the 12% investment and $0.08y$ = the income from the 8% investment. The system of equations is

$$\begin{cases} x + y = 150,000 & (1) \\ 0.12x + 0.08y = 15,400 & (2) \end{cases}$$

We will use the elimination method to solve the system.

$$\begin{array}{rl} -12x - 12y = -1,800,000 & \text{Multiply by } -12. \\ 12x + 8y = 1,540,000 & \text{Multiply by 100.} \\ \hline -4y = -260,000 & \text{Add.} \\ y = \dfrac{-260,000}{-4} = 65,000 & \\ & \text{Solve for } y. \end{array}$$

Back-substitute $y = 65,000$ into equation (1) and solve for x.

$$x + 65,000 = 150,000$$
$$x = 85,000$$

Check:
12% of 85,000 = 10,200
8% of 65,000 = 5200
85,000 + 65,000 = 150,000 and
10,200 + 5200 = 15,400, as given.
Solution: \$85,000 was invested at 12% and \$65,000 was invested at 8%.

8.1 Basic Concepts and Skills

1. The ordered pair (a, b) is a solution of a system of equations in x and y provided that when x is replaced with a and y is replaced with b, the resulting equations are true.

2. The two nongraphical methods for solving a system of equations are the substitution and elimination methods.

3. If, in the process of solving a system of equations, you get an equation of the form $0 = k$, where k is not zero, then the system is inconsistent.

4. If, in the process of solving a system of equations, you get an equation of the form $0 = 0$, then the system has dependent equations.

5. False. A system consisting of two identical equations has an infinite number of solutions.

6. False. If the system consists of dependent equations, there are an infinite number of solutions.

7. Substituting each ordered pair into the system $\begin{cases} 2x + 3y = 3 \\ 3x - 4y = 13 \end{cases}$, we find that (3, –1) is a solution.

$$\begin{cases} 2(3) + 3(-1) = 6 - 3 = 3 \\ 3(3) - 4(-1) = 9 + 4 = 13 \end{cases}$$

8. Substituting each ordered pair into the system $\begin{cases} x + 2y = 6 \\ 3x + 6y = 18 \end{cases}$, we find that (2, 2), (–2, 4), and (0, 3) are solutions.

$$\begin{cases} 2 + 2(2) = 2 + 4 = 6 \\ 3(2) + 6(2) = 6 + 12 = 18 \end{cases}$$
$$\begin{cases} -2 + 2(4) = -2 + 8 = 6 \\ 3(-2) + 6(4) = -6 + 24 = 18 \end{cases}$$
$$\begin{cases} 0 + 2(3) = 0 + 6 = 6 \\ 3(0) + 6(3) = 0 + 18 = 18 \end{cases}$$

9. Substituting each ordered pair into the system $\begin{cases} 5x - 2y = 7 \\ -10x + 4y = 11 \end{cases}$, we find that none of the ordered pairs are solutions.

10. Substituting each ordered pair into the system $\begin{cases} x - 2y = -5 \\ 3x - y = 5 \end{cases}$, we find that (3, 4) is a solution. Check: $\begin{cases} 3 - 2(4) = 3 - 8 = -5 \\ 3(3) - 4 = 9 - 4 = 5 \end{cases}$

11. Substituting each ordered pair into the system $\begin{cases} x + y = 1 \\ \frac{1}{2}x + \frac{1}{3}y = 2 \end{cases}$, we find that (10, –9) is a solution.

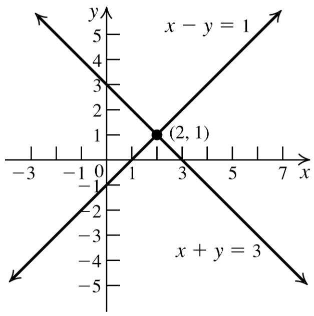
Check: $\begin{cases} 10 - 9 = 1 \\ \frac{1}{2}(10) + \frac{1}{3}(-9) = 5 - 3 = 2 \end{cases}$

12. Substituting each ordered pair into the system $\begin{cases} \frac{2}{x} + \frac{3}{y} = 2 \\ \frac{6}{x} + \frac{18}{y} = 9 \end{cases}$, we find that (2,3) is a solution.

Check: $\begin{cases} \frac{2}{2} + \frac{3}{3} = 1 + 1 = 2 \\ \frac{6}{2} + \frac{18}{3} = 3 + 6 = 9 \end{cases}$

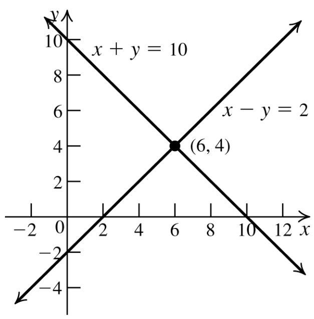
13. The solution is $\{(2, 1)\}$.

$$\begin{cases} 2+1=3 \\ 2-1=1 \end{cases}$$



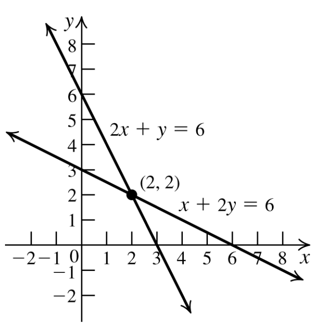
14. The solution is $\{(6, 4)\}$.

$$\begin{cases} 6+4=10 \\ 6-4=2 \end{cases}$$



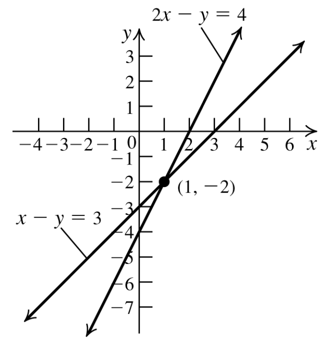
15. The solution is $\{(2, 2)\}$.

$$\begin{cases} 2+2(2)=2+4=6 \\ 2(2)+2=4+2=6 \end{cases}$$

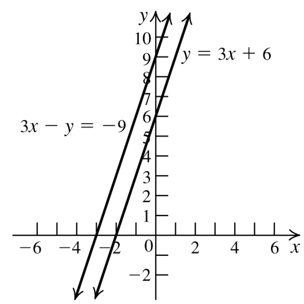


16. The solution is $\{(1, -2)\}$.

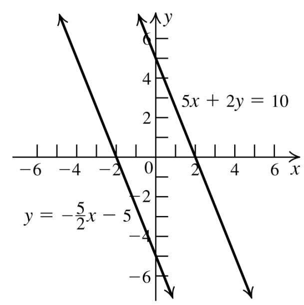
$$\begin{cases} 2(1)-(-2)=2+2=4 \\ 1-(-2)=1+2=3 \end{cases}$$



17. The system is inconsistent.

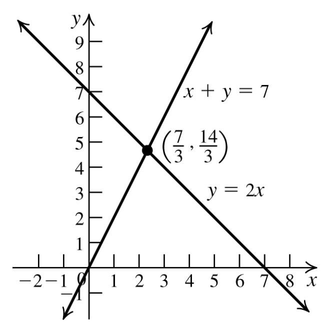


18. The system is inconsistent.



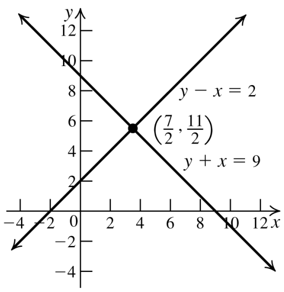
19. The solution is $\left\{\left(\frac{7}{3}, \frac{14}{3}\right)\right\}$.

$$\begin{cases} \frac{7}{3}+\frac{14}{3}=\frac{21}{3}=7 \\ \frac{14}{3}=2\left(\frac{7}{3}\right) \end{cases}$$

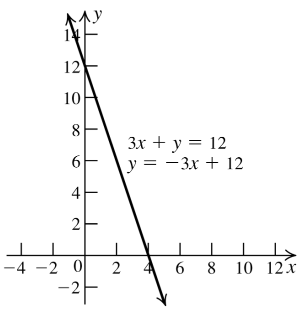


20. The solution is $\left\{\left(\frac{7}{2}, \frac{11}{2}\right)\right\}$.

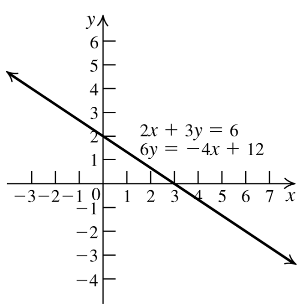
$$\begin{cases} \frac{11}{2} - \frac{7}{2} = \frac{4}{2} = 2 \\ \frac{11}{2} + \frac{7}{2} = \frac{18}{2} = 9 \end{cases}$$



21. The system is dependent. The general solution is $\{(x, 12 - 3x)\}$.



22. The system is dependent. The general solution is $\left\{\left(x, 2 - \frac{2}{3}x\right)\right\}$.



23. The slopes of the two lines are different, so the system is independent. There is a solution, so the system is consistent.

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26. The slopes of the two lines are different, so the system is independent. There is a solution, so the system is consistent.

27. The slopes and y-intercepts of the two lines are the same, so they coincide, and the system is dependent. There is a solution, so the system is consistent.

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29. The slopes of the two lines are different, so the system is independent. There is a solution, so the system is consistent.

30. The slopes of the two lines are different, so the system is independent. There is a solution, so the system is consistent.

31. The slopes of the two lines are the same while the y-intercepts are different, so the system is inconsistent.

32. The slopes of the two lines are the same while the y-intercepts are different, so the system is inconsistent.

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35. The slopes of the two lines are different, so the system is independent. There is a solution, so the system is consistent.

36. The slopes and y-intercepts of the two lines are the same, so they coincide, and the system is dependent. There is a solution, so the system is consistent.

In exercises 37–46, check your answers by substituting the values of x and y into both of the original equations.

37. Substitute $y = 2x + 1$ into

$$5x + 2y = 9 \Rightarrow 5x + 2(2x + 1) = 9 \Rightarrow$$
$$5x + 4x + 2 = 9 \Rightarrow 9x = 7 \Rightarrow x = 7/9$$

Use this value of x to find y: $y = 2\left(\frac{7}{9}\right) + 1 \Rightarrow$

$y = \frac{23}{9}$. The solution is $\left\{\left(\frac{7}{9}, \frac{23}{9}\right)\right\}$.

38. Substitute $x = 3y - 1$ into
$2x - 3y = 7 \Rightarrow 2(3y - 1) - 3y = 7 \Rightarrow$
$6y - 2 - 3y = 7 \Rightarrow 3y = 9 \Rightarrow y = 3$
Use this value of y to find x: $x = 3(3) - 1 = 8$.
The solution is $\{(8, 3)\}$.

39. Solve the second equation for y, and then substitute $y = 7 - x$ into the first equation:
$3x - (7 - x) = 5 \Rightarrow 3x - 7 + x = 5 \Rightarrow 4x = 12 \Rightarrow$
$x = 3$. Substitute this value into the second equation to find $y: 3 + y = 7 \Rightarrow y = 4$
The solution is $\{(3, 4)\}$.

40. Solve the second equation for y, and then substitute $y = 3x + 7$ into the first equation:
$2x + (3x + 7) = 2 \Rightarrow 5x = -5 \Rightarrow x = -1$.
Substitute this value into the second equation to find y: $3(-1) - y = -7 \Rightarrow -3 - y = -7 \Rightarrow y = 4$.
The solution is $\{(-1, 4)\}$.

41. Solve the first equation for y, and then substitute $y = 2x - 5$ into the second equation:
$-4x + 2(2x - 5) = 7 \Rightarrow -4x + 4x - 10 = 7 \Rightarrow$
$-10 \neq 7 \Rightarrow$ there is no solution.
Solution set: $\varnothing$

42. Solve the first equation for y, and then substitute $y = -\frac{3}{2}x + \frac{5}{2}$ into the second equation:
$-9x - 6\left(-\frac{3}{2}x + \frac{5}{2}\right) = 15 \Rightarrow -9x + 9x - 15 = 15 \Rightarrow$
$-15 \neq 15 \Rightarrow$ there is no solution.
Solution set: $\varnothing$

43. Solve the first equation for y, and then substitute $y = 3 - \frac{2}{3}x$ into the second equation:
$3x + 2\left(3 - \frac{2}{3}x\right) = 1 \Rightarrow 3x + 6 - \frac{4}{3}x = 1 \Rightarrow$
$\frac{5}{3}x = -5 \Rightarrow x = -3$. Substitute this value into the first equation to find y:
$\frac{2}{3}(-3) + y = 3 \Rightarrow y = 5$.
The solution is $\{(-3, 5)\}$.

44. Solve the first equation for x, and then substitute $x = 2y + 3$ into the second equation:
$4(2y + 3) + 6y = 3 \Rightarrow 8y + 12 + 6y = 3 \Rightarrow$
$14y = -9 \Rightarrow y = -\frac{9}{14}$. Substitute this value into the first equation to find x:
$x - 2\left(-\frac{9}{14}\right) = 3 \Rightarrow x + \frac{9}{7} = 3 \Rightarrow x = \frac{12}{7}$.
The solution is $\left\{\left(\frac{12}{7}, -\frac{9}{14}\right)\right\}$.

45. Solve the first equation for y, and then substitute $y = \frac{1}{2}(x - 5)$ into the second equation:
$-3x + 6\left(\frac{1}{2}(x - 5)\right) = -15 \Rightarrow -3x + 3x - 15 = -15 \Rightarrow$
$-15 = -15 \Rightarrow$ the system is dependent.
The general form of the solution is
$\left\{\left(x, \frac{1}{2}(x - 5)\right)\right\}$.

46. Solve the first equation for y and then substitute $y = 3 - x$ into the second equation:
$2x + 2(3 - x) = 6 \Rightarrow 2x + 6 - 2x = 6 \Rightarrow 6 = 6 \Rightarrow$
the system is dependent. The general form of the solution is $\{(x, 3 - x)\}$.

47. Add the two equations and solve for x:
$\begin{cases} x - y = 1 \\ x + y = 5 \end{cases} \Rightarrow 2x = 6 \Rightarrow x = 3$. Now substitute
$x = 3$ into the second equation and solve for y:
$3 + y = 5 \Rightarrow y = 2$. The solution is $\{(3, 2)\}$.

48. Multiply the first equation by 2 and the second equation by 3:
$\begin{cases} 2x - 3y = 5 \\ 3x + 2y = 14 \end{cases} \Rightarrow \begin{cases} 4x - 6y = 10 \\ 9x + 6y = 42 \end{cases}$
Add the equations and solve for x:
$\begin{cases} 4x - 6y = 10 \\ 9x + 6y = 42 \end{cases} \Rightarrow 13x = 52 \Rightarrow x = 4$.
Substitute $x = 4$ into the second equation and solve for y: $3(4) + 2y = 14 \Rightarrow 2y = 2 \Rightarrow y = 1$.
The solution is $\{(4, 1)\}$.

49. Multiply the first equation by –2:

$$\begin{cases} x + \ \ y = 0 \\ 2x + 3y = 3 \end{cases} \Rightarrow \begin{cases} -2x - 2y = 0 \\ \ \ 2x + 3y = 3 \end{cases}$$

Add the equations and solve for y:

$$\begin{cases} -2x - 2y = 0 \\ \ \ 2x + 3y = 3 \end{cases} \Rightarrow \ y = 3.$$

Substitute this value into the first equation and solve for x: $x + 3 = 0 \Rightarrow x = -3$.
The solution is {(–3, 3)}.

50. Multiply the first equation by –1:

$$\begin{cases} x + y = 3 \\ 3x + y = 1 \end{cases} \Rightarrow \begin{cases} -x - y = -3 \\ 3x + y = 1 \end{cases}$$

Add the equations and solve for x:

$$\begin{cases} -x - y = -3 \\ 3x + y = 1 \end{cases} \Rightarrow 2x = -2 \Rightarrow \ x = -1.$$ Substitute this value into the first equation and solve for y: $-1 + y = 3 \Rightarrow y = 4$.
The solution is {(–1, 4)}.

51. Multiply the first equation by 2:

$$\begin{cases} 5x - \ \ y = 5 \\ 3x + 2y = -10 \end{cases} \Rightarrow \begin{cases} 10x - 2y = 10 \\ \ \ 3x + 2y = -10 \end{cases}$$

Add the equations and solve for x:

$$\begin{cases} 10x - 2y = 10 \\ \ \ 3x + 2y = -10 \end{cases} \Rightarrow 13x = 0 \Rightarrow x = 0.$$

Substitute this value into the first equation and solve for y: $5(0) - y = 5 \Rightarrow y = -5$. The solution is {(0, –5)}.

52. Multiply the first equation by 7 and the second equation by 3:

$$\begin{cases} \ \ 3x - 2y = 1 \\ -7x + 3y = 1 \end{cases} \Rightarrow \begin{cases} \ \ 21x - 14y = 7 \\ -21x + 9y = 3 \end{cases}$$

Add the equations and solve for y:

$$\begin{cases} \ \ 21x - 14y = 7 \\ -21x + 9y = 3 \end{cases} \Rightarrow -5y = 10 \Rightarrow y = -2.$$

Substitute this value into the first equation and solve for x: $3x - 2(-2) = 1 \Rightarrow 3x = -3 \Rightarrow$ $x = -1$. The solution is {(–1, –2)}.

53. Multiply the first equation by 2:

$$\begin{cases} \ \ x - \ \ y = 2 \\ -2x + 2y = 5 \end{cases} \Rightarrow \begin{cases} \ \ 2x - 2y = 4 \\ -2x + 2y = 5 \end{cases}$$. Adding the equations gives $0 = 9 \Rightarrow$ the equations are inconsistent, and there is no solution.
Solution set: ∅

54. Multiply the first equation by –2:

$$\begin{cases} x + \ \ y = 5 \\ 2x + 2y = -10 \end{cases} \Rightarrow \begin{cases} -2x - 2y = -10 \\ \ \ 2x + 2y = -10 \end{cases}.$$

Adding the equations gives $0 = -20 \Rightarrow$ the equations are inconsistent, and there is no solution. Solution set: ∅

55. Multiply the second equation by –2:

$$\begin{cases} 4x + 6y = 12 \\ 2x + 3y = 6 \end{cases} \Rightarrow \begin{cases} \ \ 4x + 6y = 12 \\ -4x - 6y = -12 \end{cases}$$. Adding the equations gives $0 = 0 \Rightarrow$ the equations are dependent. Solving the second equation for y, we have $2x + 3y = 6 \Rightarrow y = -\dfrac{2x}{3} + 2$.
The general form of the solution is

$$\left\{\left(x, -\frac{2x}{3} + 2\right)\right\}.$$

56. Multiply the first equation by 2:

$$\begin{cases} \ \ 4x + \ \ 7y = -3 \\ -8x - 14y = 6 \end{cases} \Rightarrow \begin{cases} \ \ 8x + 14y = -6 \\ -8x - 14y = 6 \end{cases}$$. Adding the equations gives $0 = 0 \Rightarrow$ the equations are dependent. Solving the first equation for y, we have $4x + 7y = -3 \Rightarrow y = \dfrac{-4x - 3}{7}$.
The general form of the solution is

$$\left\{\left(x, \frac{-4x - 3}{7}\right)\right\}.$$

57. Solve the first equation for y, and then substitute this value into the second equation to solve for x: $2x + y = 9 \Rightarrow y = -2x + 9$.
$2x - 3(-2x + 9) = 5 \Rightarrow 8x - 27 = 5 \Rightarrow x = 4$.
Substitute this value into the first equation and solve for y: $2(4) + y = 9 \Rightarrow y = 1$.
The solution is {(4, 1)}.

58. Add the equations and solve for x:

$$\begin{cases} x + 2y = 10 \\ x - 2y = -6 \end{cases} \Rightarrow 2x = 4 \Rightarrow x = 2.$$ Substitute this value into the first equation and solve for y: $2 + 2y = 10 \Rightarrow y = 4$.
The solution is {(2, 4)}.

59. Multiply the second equation by –2 and then add the equations:

$$\begin{cases} 2x + 5y = 2 \\ \ \ x + 3y = 2 \end{cases} \Rightarrow \begin{cases} \ \ 2x + 5y = 2 \\ -2x - 6y = -4 \end{cases} \Rightarrow -y = -2 \Rightarrow$$

$y = 2$. Substitute this value into the second equation and solve for x: $x + 3(2) = 2 \Rightarrow x = -4$.
The solution is {(–4, 2)}.

60. Solve the first equation for *y*, and then substitute this value into the second equation to solve for *x*: $4x - y = 6 \Rightarrow y = 4x - 6$.
$3x - 4(4x - 6) = 11 \Rightarrow -13x + 24 = 11 \Rightarrow x = 1.$
Substitute this value into the first equation and solve for *y*: $4(1) - y = 6 \Rightarrow y = -2$.
The solution is {(1, –2)}.

61. Multiply the second equation by –3 and then add the equations:
$$\begin{cases} 2x + 3y = 7 \\ 3x + \ \ y = 7 \end{cases} \Rightarrow \begin{cases} 2x + 3y = 7 \\ -9x - 3y = -21 \end{cases} \Rightarrow$$
$-7x = -14 \Rightarrow x = 2.$ Substitute this value into the second equation and solve for *y*:
$3(2) + y = 7 \Rightarrow y = 1.$ The solution is {(2, 1)}.

62. Substitute the expression for *x* from the first equation into the second equation, and solve for *y*: $3y + 4 = 5y + 10 \Rightarrow y = -3$. Substitute this value into the first equation and solve for *x*:
$x = 3(-3) + 4 = -5.$
The solution is {(–5, –3)}.

63. Multiply the first equation by –2 and the second equation by 3, then add the equations to solve for *x*:
$$\begin{cases} 2x + 3y = 9 \\ 3x + 2y = 11 \end{cases} \Rightarrow \begin{cases} -4x - 6y = -18 \\ 9x + 6y = 33 \end{cases} \Rightarrow$$
$5x = 15 \Rightarrow x = 3.$ Substitute this value into the first equation to solve for *y*:
$2(3) + 3y = 9 \Rightarrow 3y = 3 \Rightarrow y = 1.$
The solution is {(3, 1)}.

64. Substitute the expression for *y* from the second equation into the first equation, and solve for *x*:
$$3x - 4\left(\frac{2x+1}{3}\right) = 0 \Rightarrow 9x - 4(2x+1) = 0 \Rightarrow$$
$9x - 8x - 4 = 0 \Rightarrow x = 4.$ Substitute this value into the second equation to solve for *y*:
$y = \dfrac{2(4)+1}{3} = 3.$ The solution is {(4, 3)}.

65. First, simplify both equations:
$$\begin{cases} \dfrac{x}{4} + \dfrac{y}{6} = 1 \\ x + 2(x - y) = 7 \end{cases} \Rightarrow \begin{cases} 6x + 4y = 24 \\ 3x - 2y = 7 \end{cases}$$
. Now multiply the second equation by 2, and then add the equations:
$$\begin{cases} 6x + 4y = 24 \\ 3x - 2y = 7 \end{cases} \Rightarrow \begin{cases} 6x + 4y = 24 \\ 6x - 4y = 14 \end{cases} \Rightarrow$$
$12x = 38 \Rightarrow x = \dfrac{19}{6}.$

Substitute this value into the second equation and solve for *y*:
$$\frac{19}{6} + 2\left(\frac{19}{6} - y\right) = 7 \Rightarrow \frac{57}{6} - 2y = 7 \Rightarrow$$
$-2y = -\dfrac{5}{2} \Rightarrow y = \dfrac{5}{4}.$
The solution is $\left\{\left(\dfrac{19}{6}, \dfrac{5}{4}\right)\right\}$.

66. First, simplify the first equation:
$$\begin{cases} \dfrac{x}{3} + \dfrac{y}{5} = 12 \\ x - y = 4 \end{cases} \Rightarrow \begin{cases} 5x + 3y = 180 \\ x - \ \ y = 4 \end{cases}$$
. Multiply the second equation by 3 and add the equations to solve for *x*:
$$\begin{cases} 5x + 3y = 180 \\ x - \ \ y = 4 \end{cases} \Rightarrow \begin{cases} 5x + 3y = 180 \\ 3x - 3y = 12 \end{cases} \Rightarrow 8x = 192 \Rightarrow$$
$x = 24.$ Substitute this value into the second equation to solve for *y*: $24 - y = 4 \Rightarrow y = 20.$
The solution is {(24, 20)}.

67. Simplify the first equation:
$3x = 2(x + y) \Rightarrow 3x = 2x + 2y \Rightarrow x = 2y.$
Substitute this expression for *x* into the second equation: $3(2y) - 5y = 2 \Rightarrow y = 2.$ Substitute this value into the first equation to solve for *x*:
$3x = 2(x + 2) \Rightarrow 3x = 2x + 4 \Rightarrow x = 4.$
The solution is {(4, 2)}.

68. Solve the first equation for *y*, and then substitute this expression into the second equation to solve for *x*:
$y + x + 2 = 0 \Rightarrow y = -x - 2.$
$-x - 2 + 2x + 1 = 0 \Rightarrow x = 1.$ Substitute this value into the first equation to solve for *y*:
$y + 1 + 2 = 0 \Rightarrow y = -3.$
The solution is {(1, –3)}.

69. Multiply the first equation by –2, then add the equations:
$$\begin{cases} 0.2x + 0.7y = 1.5 \\ 0.4x - 0.3y = 1.3 \end{cases} \Rightarrow \begin{cases} -0.4x - 1.4y = -3 \\ 0.4x - 0.3y = 1.3 \end{cases} \Rightarrow$$
$-1.7y = -1.7 \Rightarrow y = 1$. Substitute this value into the first equation to solve for *x*:
$0.2x + 0.7(1) = 1.5 \Rightarrow 0.2x = 0.8 \Rightarrow x = 4.$
The solution is {(4, 1)}.

70. Solve the first equation for y, then substitute this expression into the second equation to solve for x: $0.6x + y = -1 \Rightarrow y = -0.6x - 1.$

$x - 0.5(-0.6x - 1) = 7 \Rightarrow x + 0.3x + 0.5 = 7 \Rightarrow$

$1.3x = 6.5 \Rightarrow x = 5.$ Substitute this value into the first equation to solve for y:

$0.6(5) + y = -1 \Rightarrow 3 + y = -1 \Rightarrow y = -4.$

The solution is $\{(5, -4)\}$.

71. $\begin{cases} \dfrac{x}{2} + \dfrac{y}{3} = 1 & (1) \\ \dfrac{3x}{4} + \dfrac{y}{2} = 1 & (2) \end{cases}$

Clear the fractions by multiplying equation (1) by 6 and equation (2) by 8.

$\begin{cases} 3x + 2y = 6 & (1) \\ 6x + 4y = 8 & (2) \end{cases}$

Now multiply equation (1) by −2, then add the two equations.

$$\begin{array}{r} -6x - 4y = -24 \\ \underline{6x + 4y = \quad 8} \\ 0 = -16 \end{array} \quad \text{False}$$

The system is inconsistent.
The solution set is $\varnothing$.

72. $\begin{cases} \dfrac{x}{3} - \dfrac{y}{5} = 2 & (1) \\ 6y - 10x = 25 & (2) \end{cases}$

Clear the fractions by multiplying equation (1) by 15.

$\begin{cases} 5x - 3y = 30 & (1) \\ -10x + 6y = 25 & (2) \end{cases}$

Now multiply equation (1) by 2, then add the two equations.

$$\begin{array}{r} 10x - 6y = 60 \\ \underline{-10x + 6y = 25} \\ 0 = 85 \end{array} \quad \text{False}$$

The system is inconsistent.
The solution set is $\varnothing$.

73. $\begin{cases} \dfrac{x}{3} - \dfrac{y}{2} = 1 & (1) \\ \dfrac{3y}{8} - \dfrac{x}{4} = -\dfrac{3}{4} & (2) \end{cases}$

Clear the fractions by multiplying equation (1) by 6 and equation (2) by 8.

$\begin{cases} 2x - 3y = 6 & (1) \\ 3y - 2x = -6 & (2) \end{cases}$

Now add the two equations.

$$\begin{array}{r} 2x - 3y = 6 \\ \underline{-2x + 3y = -6} \\ 0 = 0 \end{array} \quad \checkmark$$

The system is dependent.
Solve equation (1) for y in terms of x.

$\dfrac{x}{3} - \dfrac{y}{2} = 1 \Rightarrow 2x - 3y = 6 \Rightarrow -3y = -2x + 6 \Rightarrow$

$3y = 2(x - 3) \Rightarrow y = \dfrac{2}{3}(x - 3)$

The solution set is $\left\{\left(x, \dfrac{2}{3}(x - 3)\right)\right\}$.

74. $\begin{cases} \dfrac{x}{5} - \dfrac{y}{2} = 1 & (1) \\ 15y - 6x = -30 & (2) \end{cases}$

Clear the fractions by multiplying equation (1) by 10.

$\begin{cases} 2x - 5y = 10 & (1) \\ -6x + 15y = -30 & (2) \end{cases}$

Multiply equation (1) by 3, then add the two equations.

$$\begin{array}{r} 6x - 15y = 30 \\ \underline{-6x + 15y = -30} \\ 0 = 0 \end{array}$$

Solve equation (1) for y in terms of x.

$\dfrac{1}{5}x - \dfrac{1}{2}y = 1 \Rightarrow 2x - 5y = 10 \Rightarrow$

$-5y = -2x + 10 \Rightarrow 5y = 2(x - 5) \Rightarrow$

$y = \dfrac{2}{5}(x - 5)$

The solution set is $\left\{\left(x, \dfrac{2}{5}(x - 5)\right)\right\}$.

75. $\begin{cases} \dfrac{2}{x} + \dfrac{5}{y} = -5 & (1) \\ \dfrac{3}{x} - \dfrac{2}{y} = -17 & (2) \end{cases}$

Let $u = \dfrac{1}{x}$ and $v = \dfrac{1}{y}$. Then the system becomes $\begin{cases} 2u + 5v = -5 \\ 3u - 2v = -17 \end{cases}$. Multiply the first equation by 2 and the second equation by 5, then add the equations: $\begin{cases} 2u + 5v = -5 \\ 3u - 2v = -17 \end{cases} \Rightarrow$

$\begin{cases} 4u + 10v = -10 \\ 15u - 10v = -85 \end{cases} \Rightarrow 19u = -95 \Rightarrow u = -5.$

Substitute this value into the first equation to solve for v:

$2(-5) + 5v = -5 \Rightarrow 5v = 5 \Rightarrow v = 1.$

$u = \dfrac{1}{x} \Rightarrow -5 = \dfrac{1}{x} \Rightarrow x = -\dfrac{1}{5}.$

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$v = \frac{1}{y} \Rightarrow 1 = \frac{1}{y} \Rightarrow y = 1.$

The solution is $\left\{\left(-\frac{1}{5}, 1\right)\right\}.$

76. $\begin{cases} \frac{2}{x} + \frac{1}{y} = 3 & (1) \\ \frac{4}{x} - \frac{2}{y} = 0 & (2) \end{cases}$

Let $u = \frac{1}{x}$ and $v = \frac{1}{y}$. Then the system becomes $\begin{cases} 2u + v = 3 \\ 4u - 2v = 0 \end{cases}$. Multiply the first equation by 2 and then add the equations:

$\begin{cases} 2u + v = 3 \\ 4u - 2v = 0 \end{cases} \Rightarrow \begin{cases} 4u + 2v = 6 \\ 4u - 2v = 0 \end{cases} \Rightarrow 8u = 6 \Rightarrow$

$u = \frac{3}{4}$. Substitute this value into the first equation to solve for v: $2\left(\frac{3}{4}\right) + v = 3 \Rightarrow v = \frac{3}{2}.$

$u = \frac{1}{x} \Rightarrow \frac{3}{4} = \frac{1}{x} \Rightarrow x = \frac{4}{3}.$

$v = \frac{1}{y} \Rightarrow \frac{3}{2} = \frac{1}{y} \Rightarrow y = \frac{2}{3}.$

The solution is $\left\{\left(\frac{4}{3}, \frac{2}{3}\right)\right\}.$

77. $\begin{cases} \frac{3}{x} + \frac{1}{y} = 4 & (1) \\ \frac{6}{x} - \frac{1}{y} = 2 & (2) \end{cases}$

Let $u = \frac{1}{x}$ and $v = \frac{1}{y}$. Then the system becomes $\begin{cases} 3u + v = 4 \\ 6u - v = 2 \end{cases}$. Add the equations and solve for u: $9u = 6 \Rightarrow u = \frac{2}{3}$. Substitute this value into the first equation and solve for v:

$3\left(\frac{2}{3}\right) + v = 4 \Rightarrow v = 2.$

$u = \frac{1}{x} \Rightarrow \frac{2}{3} = \frac{1}{x} \Rightarrow x = \frac{3}{2}.$

$v = \frac{1}{y} \Rightarrow 2 = \frac{1}{y} \Rightarrow y = \frac{1}{2}.$

The solution is $\left\{\left(\frac{3}{2}, \frac{1}{2}\right)\right\}.$

78. $\begin{cases} \frac{6}{x} + \frac{3}{y} = 0 & (1) \\ \frac{4}{x} + \frac{9}{y} = -1 & (2) \end{cases}$

Let $u = \frac{1}{x}$ and $v = \frac{1}{y}$. Then the system becomes $\begin{cases} 6u + 3v = 0 \\ 4u + 9v = -1 \end{cases}$. Multiply the first equation by –3 and add the equations to solve for u: $\begin{cases} 6u + 3v = 0 \\ 4u + 9v = -1 \end{cases} \Rightarrow \begin{cases} -18u - 9v = 0 \\ 4u + 9v = -1 \end{cases} \Rightarrow$

$-14u = -1 \Rightarrow u = \frac{1}{14}$. Substitute this value into the first equation to solve for v:

$6\left(\frac{1}{14}\right) + 3v = 0 \Rightarrow 3v = -\frac{3}{7} \Rightarrow v = -\frac{1}{7}.$

$u = \frac{1}{x} \Rightarrow \frac{1}{14} = \frac{1}{x} \Rightarrow x = 14. \quad v = \frac{1}{y} \Rightarrow$

$-\frac{1}{7} = \frac{1}{y} \Rightarrow y = -7.$

The solution is {(14, –7)}.

79. $\begin{cases} \frac{5}{x} + \frac{10}{y} = 3 & (1) \\ \frac{2}{x} - \frac{12}{y} = -2 & (2) \end{cases}$

Let $u = \frac{1}{x}$ and $v = \frac{1}{y}$. Then the system becomes $\begin{cases} 5u + 10v = 3 \\ 2u - 12v = -2 \end{cases}$. Multiply the first equation by 2 and the second equation by –5, then add the equation to solve for v:

$\begin{cases} 5u + 10v = 3 \\ 2u - 12v = -2 \end{cases} \Rightarrow \begin{cases} 10u + 20v = 6 \\ -10u + 60v = 10 \end{cases} \Rightarrow$

$80v = 16 \Rightarrow v = \frac{1}{5}$. Substitute this value into the first equation to solve for u:

$5u + 10\left(\frac{1}{5}\right) = 3 \Rightarrow 5u = 1 \Rightarrow u = \frac{1}{5}.$

$u = \frac{1}{x} \Rightarrow \frac{1}{5} = \frac{1}{x} \Rightarrow x = 5.$

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$v = \frac{1}{y} \Rightarrow \frac{1}{5} = \frac{1}{y} \Rightarrow y = 5.$

The solution is $\{(5, 5)\}$.

80. $\begin{cases} \frac{3}{x} + \frac{4}{y} = 1 & (1) \\ \frac{6}{x} + \frac{4}{y} = 3 & (2) \end{cases}$

Let $u = \frac{1}{x}$ and $v = \frac{1}{y}$. Then the system becomes $\begin{cases} 3u + 4v = 1 \\ 6u + 4v = 3 \end{cases}$. Multiply the second equation by –1 and then add the equations to solve for u: $\begin{cases} 3u + 4v = 1 \\ 6u + 4v = 3 \end{cases} \Rightarrow \begin{cases} 3u + 4v = 1 \\ -6u - 4v = -3 \end{cases} \Rightarrow$

$-3u = -2 \Rightarrow u = \frac{2}{3}$. Substitute this value into the first equation to solve for v:

$3\left(\frac{2}{3}\right) + 4v = 1 \Rightarrow 4v = -1 \Rightarrow v = -\frac{1}{4}.$

$u = \frac{1}{x} \Rightarrow \frac{2}{3} = \frac{1}{x} \Rightarrow x = \frac{3}{2}.$

$v = \frac{1}{y} \Rightarrow -\frac{1}{4} = \frac{1}{y} \Rightarrow y = -4.$

The solution is $\left\{\left(\frac{3}{2}, -4\right)\right\}$.

81. $\begin{cases} \frac{2}{x} + \frac{1}{y} = 4 & (1) \\ x + 2y = 6xy & (2) \end{cases}$

Divide equation (2) by $6xy$.

$$\begin{cases} \frac{2}{x} + \frac{1}{y} = 4 \\ \frac{1}{6y} + \frac{1}{3x} = 1 \end{cases} \Rightarrow \begin{cases} \frac{2}{x} + \frac{1}{y} = 4 & (1) \\ \frac{1}{3x} + \frac{1}{6y} = 1 & (2) \end{cases}$$

Let $u = \frac{1}{x}$ and $v = \frac{1}{y}$. Then the system becomes $\begin{cases} 2u + v = 4 \\ \frac{1}{3}u + \frac{1}{6}v = 1 \end{cases}$.

Clear the fractions in equation (2) by multiplying by 6.

$$\begin{cases} 2u + v = 4 \\ \frac{1}{3}u + \frac{1}{6}v = 1 \end{cases} \Rightarrow \begin{cases} 2u + v = 4 \\ 2u + v = 6 \end{cases}$$

Multiply equation (2) by –1, then add the equations.

$$\begin{array}{r} 2u + v = 4 \\ \underline{-2u - v = -6} \\ 0 = -2 \end{array} \quad \text{False}$$

The system is inconsistent.
The solution set is $\varnothing$.

82. $\begin{cases} \frac{1}{x} - \frac{2}{y} = 3 & (1) \\ 2x - y = 5xy & (2) \end{cases}$

Divide equation (2) by $5xy$.

$$\begin{cases} \frac{1}{x} - \frac{2}{y} = 3 \\ \frac{2}{5y} - \frac{1}{5x} = 1 \end{cases} \Rightarrow \begin{cases} \frac{1}{x} - \frac{2}{y} = 3 & (1) \\ -\frac{1}{5x} + \frac{2}{5y} = 1 & (2) \end{cases}$$

Let $u = \frac{1}{x}$ and $v = \frac{1}{y}$. Then the system becomes $\begin{cases} u - 2v = 3 \\ -\frac{1}{5}u + \frac{2}{5}v = 1 \end{cases}$.

Clear the fractions in equation (2) by multiplying by 5.

$$\begin{cases} u - 2v = 3 \\ -\frac{1}{5}u + \frac{2}{5}v = 1 \end{cases} \Rightarrow \begin{cases} u - 2v = 3 \\ -u + 2v = 10 \end{cases}$$

Add the equations.

$$\begin{array}{r} u - 2v = 3 \\ \underline{-u + 2v = 10} \\ 0 = 13 \end{array} \quad \text{False}$$

The system is inconsistent.
The solution set is $\varnothing$.

8.1 Applying the Concepts

In exercises 83–86, the equilibrium point is the point that satisfies both the demand equation and the supply equation.

83. Add the equations to solve for p:

$\begin{cases} 2p + x = 140 \\ 12p - x = 280 \end{cases} \Rightarrow 14p = 420 \Rightarrow p = 30$.

Substitute this value into the first equation to solve for x: $2(30) + x = 140 \Rightarrow x = 80$.

The equilibrium point is (80, 30).

84. Add the equations to solve for p:

$\begin{cases} 7p + x = 150 \\ 10p - x = 20 \end{cases} \Rightarrow 17p = 170 \Rightarrow p = 10$.

Substitute this value into the first equation to solve for x: $7(10) + x = 150 \Rightarrow x = 80$.

The equilibrium point is (80, 10).

85. Multiply the second equation by –1, then add the equations to solve for x:
$\begin{cases} 2p + x = 25 \\ x - p = 13 \end{cases} \Rightarrow \begin{cases} 2p + x = 25 \\ p - x = -13 \end{cases} \Rightarrow 3p = 12 \Rightarrow$
$p = 4.$ Substitute this value into the second equation to solve for x: $x - 4 = 13 \Rightarrow x = 17.$
The equilibrium point is (17, 4).

86. Multiply the second equation by –1, then add the equations to solve for p:
$\begin{cases} p + 2x = 96 \\ p - \ \ x = 39 \end{cases} \Rightarrow \begin{cases} p + 2x = 96 \\ -p + \ \ x = -39 \end{cases} \Rightarrow 3x = 57 \Rightarrow$
$x = 19.$ Substitute this value into the second equation to solve for p: $p - 19 = 39 \Rightarrow p = 58.$
The equilibrium point is (19, 58).

87. Let x = the diameter of the largest pizza, and let y = the diameter of the smallest pizza. Then
$\begin{cases} x + y = 29 \\ x - y = 13 \end{cases} \Rightarrow 2x = 42 \Rightarrow x = 21.$
$21 + y = 29 \Rightarrow y = 8.$
The largest pizza has diameter 21 inches, and the smallest pizza has diameter 8 inches.

88. Let x = the number of calories in the hamburger from Boston Burger, and let y = the number of calories in the hamburger from Carmen's Broiler. Then
$\begin{cases} x + y = 1130 \\ x - y = 40 \end{cases} \Rightarrow 2x = 1170 \Rightarrow x = 585.$
Substitute this value into the first equation to solve for y: $585 + y = 1130 \Rightarrow y = 545.$
A Boston burger has 585 calories while the Carmen's Broiler burger has 545 calories.

89. Let x = the percentage of paper trash, and let y = the percentage of plastic trash. If the total amount of trash is t, then
$\begin{cases} x + y = 48 \\ x = 5y \end{cases}.$
Substitute the expression for x from the second equation into the first equation to solve for y: $5y + y = 48 \Rightarrow y = 8$. Substitute this value into the first equation to solve for x:
$x + 8 = 48 \Rightarrow x = 40.$ So, 8% of the trash is plastic and 40% is paper.

90. Let x = the amount spent for food. Let y = the amount spent for clothes. Then $\begin{cases} x + y = 1000 \\ x = 4y \end{cases}.$
Substitute the expression for x from the second equation into the first equation to solve for y:
$4y + y = 1000 \Rightarrow y = 200.$ Substitute this value into the second equation to solve for x:
$x = 4(200) = 800.$ They spent \$800 for food and \$200 for clothes.

91. Let x = the number of beads, and let y = the number of doubloons . Then
$\begin{cases} 0.4x + 0.3y = 265 \\ x + \ \ y = 770 \end{cases}.$ Multiply the second equation by –0.3 and then add the equations to solve for x: $\begin{cases} 0.4x + 0.3y = 265 \\ x + \ \ y = 770 \end{cases} \Rightarrow$
$\begin{cases} 0.4x + 0.3y = 265 \\ -0.3x - 0.3y = -231 \end{cases} \Rightarrow 0.1x = 34 \Rightarrow x = 340.$
Substitute this value into the second equation to solve for y: $340 + y = 770 \Rightarrow y = 430.$ Levon bought 340 beads and 430 doubloons.

92. Let x = the amount of 24¢ candy, and let y = the amount of 18¢ candy. Then
$\begin{cases} x + \ \ y = 135 \\ 0.24x + 0.18y = 26.70 \end{cases}.$
Multiply the first equation by –0.18 and the add the equations to solve for x:
$\begin{cases} x + \ \ y = 135 \\ 0.24x + 0.18y = 26.70 \end{cases} \Rightarrow$
$\begin{cases} -0.18x - 0.18y = -24.30 \\ 0.24x + 0.18y = 26.70 \end{cases} \Rightarrow 0.06x = 2.40 \Rightarrow$
$x = 40.$ Substitute this value into the first equation to solve for y:
$40 + y = 135 \Rightarrow y = 95.$ Janet bought 40 pieces of 24¢ candy and 95 pieces of 18¢ candy.

93. Let x = the number of Egg McMuffins, and let y = the number of Breakfast Burritos. Then
$\begin{cases} 27x + 21y = 123 \\ 17x + 13y = 77 \end{cases}$. Multiply the first equation by 13 and the second equation by –21, then add the equations to solve for x:
$\begin{cases} 27x + 21y = 123 \\ 17x + 13y = 77 \end{cases} \Rightarrow \begin{cases} 351x + 273y = 1599 \\ -357x - 273y = -1617 \end{cases} \Rightarrow$
$-6x = -18 \Rightarrow x = 3.$ Substitute this value into the first equation to solve for y:
$27(3) + 21y = 123 \Rightarrow 21y = 42 \Rightarrow y = 2.$ You will need to eat three Egg McMuffins and two Breakfast Burritos.

94. Let x = the number of Egg McMuffins, and let y = the number of Breakfast Burritos. Then $\begin{cases} 12x + 20y = 68 \\ 27x + 21y = 129 \end{cases}$.

Multiply the first equation by 21 and the second equation by –20, then add the equations to solve for x.

$\begin{cases} 12x + 20y = 68 \\ 27x + 21y = 129 \end{cases} \Rightarrow \begin{cases} 252x + 420y = 1428 \\ -540x - 420y = -2580 \end{cases} \Rightarrow$

$-288x = -1152 \Rightarrow x = 4.$ Substitute this value into the first equation to solve for y:

$12(4) + 20y = 68 \Rightarrow 20y = 20 \Rightarrow y = 1.$ You will need to eat four Egg McMuffins and one Breakfast Burrito.

95. Let x = the amount invested at 7.5%, and let y = the amount invested at 12%. Then $\begin{cases} x + y = 50,000 \\ 0.075x + 0.12y = 5190 \end{cases}$. Solve the first equation for x and substitute this value into the second equation to solve for y:

$x + y = 50,000 \Rightarrow x = 50,000 - y.$

$0.075(50,000 - y) + 0.12y = 5190 \Rightarrow$
$3750 - 0.075y + 0.12y = 5190 \Rightarrow$
$0.045y = 1440 \Rightarrow y = \$32,000$.

Substitute this value into the first equation to solve for x:

$x + 32,000 = 50,000 \Rightarrow x = 18,000.$

Mrs. García invested $18,000 at 7.5% and $32,000 at 12%.

96. Let x = the amount invested at 8%, and let y = the amount invested at 10.5%. Then $\begin{cases} x + y = 30,000 \\ 0.08x + 0.105y = 2550 \end{cases}$. Solve the first equation for x and substitute this value into the second equation to solve for y:

$x + y = 30,000 \Rightarrow x = 30,000 - y$.

$0.08(30,000 - y) + 0.105y = 2550 \Rightarrow$
$2400 - 0.08y + 0.105y = 2550 \Rightarrow$
$0.025y = 150 \Rightarrow y = 6000.$ Substitute this value into the first equation to solve for x:

$x + 6000 = 30,000 \Rightarrow x = 24,000$. Mr. Sharma invested $24,000 at 8% and $6000 at 10.5%.

97. Let x = the amount earned tutoring, and let y = the amount earned working at McDougal's.

Then $\begin{cases} x = 2y \\ \dfrac{x + y}{2} = 11.25 \end{cases}$.

Substitute the expression for x from the first equation into the second equation and solve for y.

$\dfrac{2y + y}{2} = 11.25 \Rightarrow 3y = 22.50 \Rightarrow y = 7.50.$

Substitute this value into the first equation to solve for x: $x = 2(7.50) = 15.$ She earned $15 tutoring and $7.50 working at McDougal's.

98. Let x = the hourly amount earned by the plumber, and let y = the hourly amount earned by the apprentice. Then $\begin{cases} x = 15 + y \\ 40x + 40y = 2200 \end{cases}$.

Substitute the expression for x from the first equation into the second equation and solve for y: $40(15 + y) + 40y = 2200 \Rightarrow$

$600 + 80y = 2200 \Rightarrow y = 20.$ Substitute this value into the first equation to solve for x:

$x = 15 + 20 = 35.$ The plumber earns $35 per hour and the apprentice earns $20 per hour.

99. Let x = the number of pounds of the herb, and let y = the number of pounds of tea. Then $\begin{cases} x + y = 100 \\ 5.50x + 3.20y = 3.66(100) \end{cases}$. Solve the first equation for x, and substitute this expression into the second equation to solve for y:

$x + y = 100 \Rightarrow x = 100 - y.$

$5.5(100 - y) + 3.20y = 3.66(100) \Rightarrow$
$550 - 5.5y + 3.2y = 366 \Rightarrow -2.3y = -184 \Rightarrow$
$y = 80.$ Substitute this value into the first equation to solve for x: $x + 80 = 100 \Rightarrow x = 20.$

There are 20 pounds of the herb and 80 pounds of tea in the mixture.

100. Let x = the amount of 60% solution, and let y = the amount of water added. Then,

Amount of solution	% acid	Amount of acid
x	0.6	$0.6x$
$x + y$	0.4	$0.4(x + y)$
$x + y + 1$	0.3	$0.3(x + y + 1)$

The amount of acid is the same in all three solutions, so we have

$\begin{cases} 0.6x = 0.4(x + y) \\ 0.4(x + y) = 0.3(x + y + 1) \end{cases} \Rightarrow \begin{cases} 0.2x - 0.4y = 0 \\ 0.1x + 0.1y = 0.3 \end{cases}$

Multiply the second equation by –2 and add the equations to solve for y:

$\begin{cases} 0.2x - 0.4y = 0 \\ -0.2x - 0.2y = -0.6 \end{cases} \Rightarrow -0.6y = -0.6 \Rightarrow y = 1$

Substitute this value into the first equation to solve for x: $0.2x - 0.4(1) = 0 \Rightarrow x = 2.$

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So, there were 2 liters of the 60% solution and 1 liter of water added to make the 40% solution. Adding one more liter to make the 30% solution means there are 4 liters of the 30% solution.

101. Let x = the speed of the plane in still air, and let y = the wind speed. So, the speed of the plane with the wind is $x + y$, and the speed of the plane against the wind is $x - y$. Then, using the fact that rate × time = distance, we have $\begin{cases} 5(x+y)=3000 \\ 6(x-y)=3000 \end{cases} \Rightarrow \begin{cases} x+y=600 \\ x-y=500 \end{cases}$. Add the equations to solve for x: $2x = 1100 \Rightarrow x = 550.$ Substitute this value into the first equation to solve for y: $5(550 + y) = 3000 \Rightarrow$ $550 + y = 600 \Rightarrow y = 50.$ The speed of the plane is 550 kph, and the wind speed is 50 kph.

102. Let x = the speed of the boat in still water, and let y = the speed of the current. The speed of the boat against the current (upstream) is $x - y$, and the speed of the boat with the current (downstream) is $x + y$. Then, using the fact that rate × time = distance, we have $\begin{cases} 2(x-y)=12 \\ 3(x-2y)=12 \end{cases} \Rightarrow \begin{cases} x-y=6 \\ x-2y=4 \end{cases}$. Multiply the second equation by –1 and add the equations to solve for y:

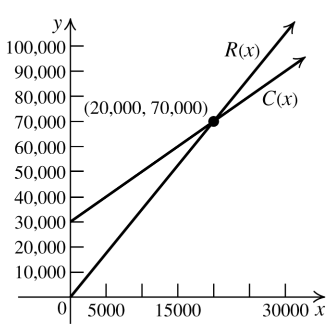
$\begin{cases} x-\ \ y=6 \\ x-2y=4 \end{cases} \Rightarrow \begin{cases} x-\ \ y=6 \\ -x+2y=-4 \end{cases} \Rightarrow y=2.$

Substitute this value into the first equation to solve for x: $2(x-2)=12 \Rightarrow 2x=16 \Rightarrow x=8.$

So, the speed of the boat in still water is 8 miles per hour and the speed of the current is 2 miles per hour. The speed of the boat going downstream is 10 miles per hour and the 12 mile trip will take $12/10 =$ 1.2 hours.

103. a. $C(x) = y = 2x + 30,000;\ R(x) = y = 3.50x$

b.



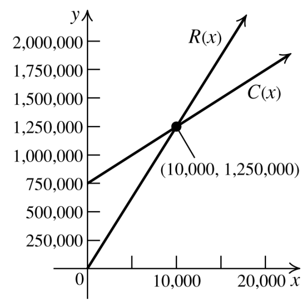
c. $\begin{cases} y = 2x + 30,000 \\ y = 3.5x \end{cases}$.

Substitute the expression for y from the second equation into the first equation to solve for x:

$2x + 30,000 = 3.5x \Rightarrow 1.5x = 30,000 \Rightarrow$
$x = 20,000$ magazines

104. a. $C(x) = y = 50x + 750,000$;
$R(x) = y = 125x$

b.



c. $\begin{cases} y = 50x + 750,000 \\ y = 125x \end{cases}$

Substitute the expression for y from the second equation into the first equation to solve for x:

$125x = 50x + 750,000 \Rightarrow$
$75x = 750,000 \Rightarrow x = 10,000$ MP4's

105. Let x = Shanaysha's weekly sales. Then her salary at store B is $150 + 0.04x$.
$150 + 0.04x > 400 \Rightarrow 0.04x > 250 \Rightarrow x > 6250.$
Her weekly sales should be more than \$6250.

106. Let x = the yearly sales premiums. Then Sheena's salary at company A is $25,000 + 0.02x$, and her salary at company B is $30,000 + 0.01x$.
$30,000 + 0.01x > 25,000 + 0.02x \Rightarrow$
$5000 > 0.01x \Rightarrow 500,000 > x$. If Sheena's yearly sales premium is less than \$500,000, then the offer from company B is the better offer.

8.1 Beyond the Basics

107. $\begin{cases} 2\log_3 x + 3\log_3 y = 8 & (1) \\ 3\log_3 x - \ \log_3 y = 1 & (2) \end{cases}$

Multiply equation (2) by 3. Then add the two equations.

$$\begin{array}{r} 2\log_3 x + 3\log_3 y = 8 \\ \underline{9\log_3 x - 3\log_3 y = 3} \\ 11\log_3 x = 11 \end{array}$$

Solve for x.

$11\log_3 x = 11 \Rightarrow \log_3 x = 1 \Rightarrow x = 3$

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(*continued*)

Substitute $x = 3$ into equation (2) and solve for y:

$3\log_3 3 - \log_3 y = 1 \Rightarrow 3(1) - \log_3 y = 1 \Rightarrow$

$-\log_3 y = -2 \Rightarrow \log_3 y = 2 \Rightarrow y = 3^2 = 9$

Be sure to check your answer in the original system of equations. The solution is $\{(3, 9)\}$.

108. $\begin{cases} 3\log_2(x+y) - 5\log_2(x-y) = 2 & (1) \\ 2\log_2(x+y) + 3\log_2(x-y) = 14 & (2) \end{cases}$

Let $u = x + y$, and let $v = x - y$. Then the system becomes

$\begin{cases} 3\log_2 u - 5\log_2 v = 2 & (1) \\ 2\log_2 u + 3\log_2 v = 14 & (2) \end{cases}$

Multiply equation (1) by 3 and equation (2) by 5. Then add the two equations.

$$\begin{array}{r} 9\log_2 u - 15\log_2 v = 6 \\ \underline{10\log_2 u + 15\log_2 v = 70} \\ 19\log_2 u = 76 \end{array}$$

Solve for u.

$19\log_2 u = 76 \Rightarrow \log_2 u = 4 \Rightarrow u = 2^4 = 16$

Substitute $u = 16$ into equation (2) and solve for v:

$2\log_2 16 + 3\log_2 v = 14 \Rightarrow$

$2(4) + 3\log_2 v = 14 \Rightarrow 3\log_2 v = 6 \Rightarrow$

$\log_2 v = 2 \Rightarrow v = 2^2 = 4$

Now solve for x and y using $u = 16$, $v = 4$.

$$\begin{array}{l} x + y = 16 \\ \underline{x - y = 4} \\ \quad 2x = 20 \Rightarrow x = 10 \end{array}$$

$10 + y = 16 \Rightarrow y = 6$

Be sure to check your answer in the original system of equations. The solution is $\{(10, 6)\}$.

109. $\begin{cases} 3e^x - 4e^y = 4 & (1) \\ 2e^x + 5e^y = 18 & (2) \end{cases}$

Multiply equation (1) by 5 and equation (2) by 4. Then add the equations.

$$\begin{array}{r} 15e^x - 20e^y = 20 \\ \underline{8e^x + 20e^y = 72} \\ 23e^x = 92 \end{array}$$

Now solve for x.

$23e^x = 92 \Rightarrow e^x = \dfrac{92}{23} \Rightarrow x = \ln 4.$

Substitute this value in equation (1) and solve for y.

$3e^{\ln 4} - 4e^y = 4 \Rightarrow 3(4) - 4e^y = 4 \Rightarrow$

$-4e^y = -8 \Rightarrow e^y = 2 \Rightarrow y = \ln 2$

110. $\begin{cases} e^{x+2y} - 3e^{x-y} = 16 & (1) \\ 3e^{x+2y} - 12e^{x-y} = 46 & (2) \end{cases}$

Let $u = x + 2y$, and let $v = x - y$. Then the system becomes

$\begin{cases} e^u - 3e^v = 16 & (1) \\ 3e^u - 12e^v = 46 & (2) \end{cases}$

Multiply equation (1) by −4. Then add the equations.

$$\begin{array}{r} -4e^u + 12e^v = -64 \\ \underline{3e^u - 12e^v = 46} \\ -e^u = -18 \end{array}$$

Now solve for u.

$-e^u = -18 \Rightarrow e^u = 18 \Rightarrow x = \ln 18.$

Substitute this value in equation (1) and solve for v.

$e^{\ln 18} - 3e^v = 16 \Rightarrow 18 - 3e^v = 16 \Rightarrow$

$-3e^v = -2 \Rightarrow e^v = \dfrac{2}{3} \Rightarrow v = \ln\left(\dfrac{2}{3}\right)$

Now solve for x and y using $u = \ln 18$, $v = \ln\left(\dfrac{2}{3}\right)$.

$\begin{cases} x + 2y = \ln 18 = \ln(9 \cdot 2) = \ln 9 + \ln 2 & (3) \\ x - y = \ln\left(\dfrac{2}{3}\right) = \ln 2 - \ln 3 & (4) \end{cases}$

Multiply equation (4) by 2, then add and solve for x.

$$\begin{array}{r} x + 2y = \ln 9 + \ln 2 \\ \underline{2x - 2y = 2\ln 2 - 2\ln 3} \\ 3x = \ln 9 + 3\ln 2 - 2\ln 3 \\ = \ln(3^2) + 3\ln 2 - 2\ln 3 \\ = 2\ln 3 + 3\ln 2 - 2\ln 3 \\ 3x = 3\ln 2 \Rightarrow x = \ln 2 \end{array}$$

Substitute this value in equation (4) and solve for y.

$\ln 2 - y = \ln 2 - \ln 3 \Rightarrow -y = -\ln 3 \Rightarrow y = \ln 3$

Be sure to check your answer in the original system of equations. The solution is $\{(\ln 2, \ln 3)\}$.

111. First solve the system $\begin{cases} x+2y=7 \\ 3x+5y=11 \end{cases}$.

Multiply the first equation by –3, then add the equations to solve for *y*:

$$\begin{cases} x+2y=7 \\ 3x+5y=11 \end{cases} \Rightarrow \begin{cases} -3x-6y=-21 \\ 3x+5y=11 \end{cases} \Rightarrow -y=-10 \Rightarrow$$

$y=10$. Substitute this value into the first equation to solve for *x*: $x+2(10)=7 \Rightarrow$ $x=-13$. Now substitute (–13, 10) into the third equation to solve for *c*:

$-13c+3(10)=4 \Rightarrow -13c=-26 \Rightarrow c=2.$

112. $\begin{cases} 2x+cy=11 \\ 5x-7y=5 \end{cases}$

The system is inconsistent if the lines are parallel, i.e., if the coefficients of one line are multiples of the coefficients of the other line, but the constant in the first line is not the same multiple of the constant in the second line.

$2x=\frac{2}{5}(5x)$, so $c=\frac{2}{5}(-7)=-\frac{14}{5}$

113. If $(x-2)$ is a factor of both $f(x)$ and $g(x)$, then the remainder when each function is divided by $(x-2)$ is zero. Use synthetic division to find the remaining factors.

$$\begin{array}{r|rrrr} 2 & 1 & -4 & a & b \\ & & 2 & -4 & 2a-8 \\ \hline & 1 & -2 & a-4 & 2a+b-8 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -a & b & 8 \\ & & 2 & -2a+4 & -4a+2b+8 \\ \hline & 1 & 2-a & -2a+b+4 & -4a+2b+16 \end{array}$$

This gives us the system

$$\begin{cases} 2a+b-8=0 \\ -4a+2b+16=0 \end{cases} \Rightarrow \begin{cases} 4a+2b-16=0 \\ -4a+2b+16=0 \end{cases} \Rightarrow$$

$4b=0 \Rightarrow b=0$

Substituting $b=0$ into the first equation gives

$2a+0-8=0 \Rightarrow a=4$

The solution is $a=4$ and $b=0$.

114. $\ell_1=a_1x+b_1y+c_1=0$

$\ell_2=a_2x+b_2y+c_2=0 \Rightarrow \ell_1+k\ell_2=0 \quad (1)$

If $P(h, k)$ is the point of intersection of ℓ_1 and ℓ_2, then $a_1h+b_1k+c_1=0$ and $a_2h+b_2k+c_2=0$. Since $\ell_1+k\ell_2=0$, *P* lies on this line also. Therefore,

$$\ell_1+k\ell_2=a_1x+b_1y+c_1+k\left(a_2x+b_2y+c_2\right)=0$$

is an equation of the line passing through *P*.

115. To find the intersection of the two lines, solve the system $\begin{cases} 2x+y=3 \\ x-3y=12 \end{cases}$. Multiply the first equation by 3, then add the equations to solve for *x*: $\begin{cases} 2x+y=3 \\ x-3y=12 \end{cases} \Rightarrow \begin{cases} 6x+3y=9 \\ x-3y=12 \end{cases} \Rightarrow$

$7x=21 \Rightarrow x=3$. Substitute this value into the first equation to solve for *y*:

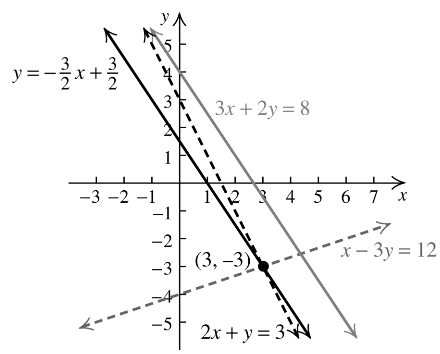
$2(3)+y=3 \Rightarrow y=-3$. So the point of intersection is (3, –3).

Solve the equation $3x+2y=8$ for *y* to find the slope: $y=-\frac{3}{2}x+4 \Rightarrow$ the slope is $-\frac{3}{2}$.

The equation of the line is

$y+3=-\frac{3}{2}(x-3) \Rightarrow y=-\frac{3}{2}x+\frac{3}{2}$.

Verify graphically:



116. To find the intersection of the two lines, solve the system $\begin{cases} 5x+2y=7 \\ 6x-5y=38 \end{cases}$. Multiply the first equation by 5 and the second equation by 2, then add the equations to solve for *x*:

$$\begin{cases} 5x+2y=7 \\ 6x-5y=38 \end{cases} \Rightarrow \begin{cases} 25x+10y=35 \\ 12x-10y=76 \end{cases} \Rightarrow 37x=111 \Rightarrow$$

$x=3$. Substitute this value into the first equation to solve for *y*: $5(3)+2y=7 \Rightarrow$ $2y=-8 \Rightarrow y=-4$. So the line passes through the point (3, –4). Because the line also passes through the point (1, 3), the slope of the line is $\frac{3-(-4)}{1-3}=-\frac{7}{2}$. The equation of the line is $y-3=-\frac{7}{2}(x-1) \Rightarrow y=-\frac{7}{2}x+\frac{13}{2}$. Verify graphically:

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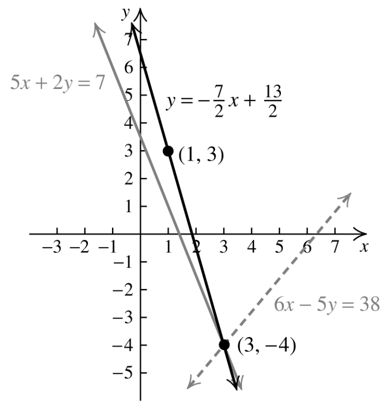


117. To find the intersection of the two lines, solve the system $\begin{cases} 2x+5y+7=0 \\ 13x-10y+3=0 \end{cases}$. Multiply the first equation by 2, then add the equations to solve for x:

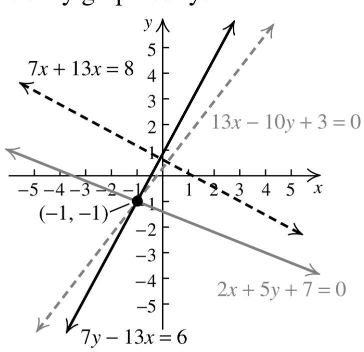
$$\begin{cases} 2x+5y+7=0 \\ 13x-10y+3=0 \end{cases} \Rightarrow \begin{cases} 4x+10y+14=0 \\ 13x-10y+3=0 \end{cases} \Rightarrow$$

$17x+17=0 \Rightarrow x=-1$. Substitute this value into the first equation to solve for y:
$2(-1)+5y+7=0 \Rightarrow 5+5y=0 \Rightarrow y=-1$.

The slope of the line $7x+13y=8$ is $-\dfrac{7}{13}$, so the slope of the perpendicular line is $\dfrac{13}{7}$. The equation of the line is $y+1=\dfrac{13}{7}(x+1) \Rightarrow$

$y=\dfrac{13}{7}x+\dfrac{6}{7}$.

Verify graphically:



8.1 Critical Thinking/Discussion/Writing

118. a. There is only one solution if the equations are independent. Solve for x by multiplying the first equation by b_2 and multiplying the second equation by $-b_1$, then adding the equations:

$$\begin{cases} a_1x+b_1y=c_1 \\ a_2x+b_2y=c_2 \end{cases} \Rightarrow$$
$$\begin{cases} b_2a_1x+b_2b_1y=b_2c_1 \\ -b_1a_2x-b_1b_2y=-b_1c_2 \end{cases} \Rightarrow$$
$$b_2a_1x-b_1a_2x=b_2c_1-b_1c_2 \Rightarrow$$
$$x(b_2a_1-b_1a_2)=b_2c_1-b_1c_2 \Rightarrow$$
$$x=\frac{b_2c_1-b_1c_2}{a_1b_2-a_2b_1}$$

Solve for y by multiplying the first equation by a_2 and multiplying the second equation by $-a_1$, then adding the equations:

$$\begin{cases} a_1x+b_1y=c_1 \\ a_2x+b_2y=c_2 \end{cases} \Rightarrow$$
$$\begin{cases} a_1a_2x+a_2b_1y=a_2c_1 \\ -a_1a_2x-a_1b_2y=-a_1c_2 \end{cases} \Rightarrow$$
$$a_2b_1y-a_1b_2y=a_2c_1-a_1c_2 \Rightarrow$$
$$y(a_2b_1-a_1b_2)=a_2c_1-a_1c_2 \Rightarrow$$
$$y=\frac{a_2c_1-a_1c_2}{a_2b_1-a_1b_2}=\frac{a_1c_2-a_2c_1}{a_1b_2-a_2b_1}.$$

Note that the denominator of x and y, $a_1b_2-a_2b_1$, cannot equal zero.

b. There is no solution if the equations are inconsistent. They are inconsistent if they are parallel. The slopes of the lines are equal and $\dfrac{b_1}{a_1}=\dfrac{b_2}{a_2} \Rightarrow \dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}$. The intercepts are different. If $c_1=c_2$, then $\dfrac{c_1}{a_1} \neq \dfrac{c_2}{a_2}$.

c. There are infinitely many solutions if the equations are dependent. From part (b), we have $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}$. The intercepts of the lines must be the same, so $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=\dfrac{c_1}{c_2}$.

119. a. The slope of ℓ_1 is $-\frac{3}{4}$, so the slope of the perpendicular is $\frac{4}{3}$. The equation of ℓ_2 is

$$y-8=\frac{4}{3}(x-5)\Rightarrow 3y-24=4x-20\Rightarrow$$
$$3y-4x=4.$$

b. To find Q, solve the system $\begin{cases}3x+4y=12\\-4x+3y=4\end{cases}$.

Multiply the first equation by 4 and the second equation by 3, then add the equations to solve for x: $\begin{cases}3x+4y=12\\-4x+3y=4\end{cases}\Rightarrow$

$$\begin{cases}12x+16y=48\\-12x+\ 9y=12\end{cases}\Rightarrow 25y=60\Rightarrow y=\frac{12}{5}.$$

Substitute this value into the first equation to solve for x: $3x+4\left(\frac{12}{5}\right)=12\Rightarrow 3x=\frac{12}{5}\Rightarrow$

$x=\frac{4}{5}$. The coordinates of Q are $\left(\frac{4}{5},\frac{12}{5}\right)$.

c. $d(P,Q)=\sqrt{\left(5-\frac{4}{5}\right)^2+\left(8-\frac{12}{5}\right)^2}=\sqrt{49}=7$

120. a. The slope of ℓ_1 is $-\frac{a}{b}$, so the slope of the perpendicular is $\frac{b}{a}$. The equation of ℓ_2 is

$$y-y_1=\frac{b}{a}(x-x_1)\Rightarrow$$
$$ay-ay_1=bx-bx_1\Rightarrow ay-bx=ay_1-bx_1 \text{ or}$$
$$b(x-x_1)-a(y-y_1)=0.$$

b. The intersection of ℓ_1 and ℓ_2 is the solution of the system

$$\begin{cases}ax+by=-c\\-bx+ay=ay_1-bx_1\end{cases}\Rightarrow$$
$$\begin{cases}abx+b^2y=-bc\\-abx+a^2y=a^2y_1-abx_1\end{cases}\Rightarrow$$
$$y(a^2+b^2)=-bc+a^2y_1-abx_1\Rightarrow$$
$$y=\frac{-bc+a^2y_1-abx_1}{a^2+b^2}$$

$$\begin{cases}ax+by=-c\\-bx+ay=ay_1-bx_1\end{cases}\Rightarrow$$
$$\begin{cases}-a^2x-aby=ac\\-b^2x+aby=aby_1-b^2x_1\end{cases}\Rightarrow$$
$$-x(a^2+b^2)=ac+aby_1-b^2x_1\Rightarrow$$
$$x=\frac{-ac-aby_1+b^2x_1}{a^2+b^2}=-\frac{ac+aby_1-b^2x_1}{a^2+b^2}$$

The intersection of the two lines is

$$\left(-\frac{ac+aby_1-b^2x_1}{a^2+b^2},\frac{-abx_1+a^2y_1-bc}{a^2+b^2}\right).$$

c. Now find the distance between (x_1,y_1) and $\left(-\frac{ac+aby_1-b^2x_1}{a^2+b^2},\frac{-bc+a^2y_1-abx_1}{a^2+b^2}\right)$:

$$\sqrt{\left(x_1+\frac{ac+aby_1-b^2x_1}{a^2+b^2}\right)^2+\left(y_1-\frac{-bc+a^2y_1-abx_1}{a^2+b^2}\right)^2}$$
$$=\sqrt{\left(\frac{a(ax_1+by_1+c)}{a^2+b^2}\right)^2+\left(\frac{b(ax_1+by_1+c}{a^2+b^2}\right)^2}$$
$$=\sqrt{\frac{a^2(ax_1+by_1+c)^2}{\left(a^2+b^2\right)^2}+\frac{b^2(ax_1+by_1+c)^2}{\left(a^2+b^2\right)^2}}$$
$$=\sqrt{\frac{(a^2+b^2)(ax_1+by_1+c)^2}{\left(a^2+b^2\right)^2}}$$
$$=\sqrt{\frac{(ax_1+by_1+c)^2}{a^2+b^2}}=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}.$$

121. a. $a=1,b=1,c=-7,x_1=2,y_1=3$

$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}=\frac{|1(2)+1(3)-7|}{\sqrt{1^2+1^2}}=\frac{2}{\sqrt{2}}=\sqrt{2}$$

b. $a=2,b=-1,c=3,x_1=-2,y_1=5$

$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}=\frac{|2(-2)-1(5)+3|}{\sqrt{2^2+(-1)^2}}=\frac{6}{\sqrt{5}}=\frac{6\sqrt{5}}{5}$$

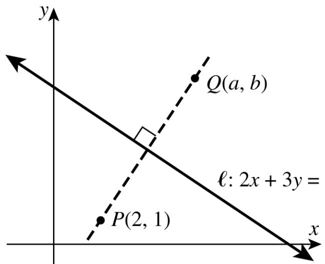
c. $a=5,b=-2,c=-7,x_1=3,y_1=4$

$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}=\frac{|5(3)-2(4)-7|}{\sqrt{3^2+4^2}}=0$$

d. $a = a, b = b, c = c, x_1 = 0, y_1 = 0$

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|a(0) + b(0) + c|}{\sqrt{a^2 + b^2}} = \frac{|c|}{\sqrt{a^2 + b^2}}$$

122.



Let $Q(a, b)$ be the reflection point. Using the hint, we have two equations, one developed from the fact that the midpoint of $\overline{PQ}$ lies on line ℓ, and the other developed from the fact that ℓ is perpendicular to the line containing the segment $\overline{PQ}$.

Equation (1): The midpoint of $\overline{PQ}$ is $M\left(\frac{a+2}{2}, \frac{b+1}{2}\right)$. Since M lies on line ℓ, we have $2\left(\frac{a+2}{2}\right) + 3\left(\frac{b+1}{2}\right) = 20$. We can simplify this to

$(a+2) + \frac{3}{2}b + \frac{3}{2} = 20 \Rightarrow a + \frac{3}{2}b = \frac{33}{2} \Rightarrow$
$2a + 3b = 33$

Equation (2): The slope of ℓ is $-\frac{2}{3}$, so the slope of the line perpendicular to line ℓ is $\frac{3}{2}$.

Using the formula for the slope of $\overline{PQ}$, we have

$\frac{3}{2} = \frac{b-1}{a-2} \Rightarrow 3a - 6 = 2b - 2 \Rightarrow 3a - 2b = 4.$

So, the system is

$$\begin{cases} 2a + 3b = 33 & (1) \\ 3a - 2b = 4 & (2) \end{cases}$$

Multiply the first equation by 2 and the second equation by 3. Then add the two equations, then solve for a.

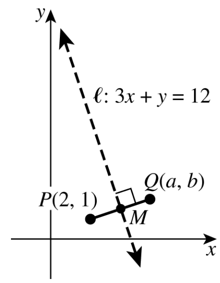
$$\begin{array}{r} 4a + 6b = 66 \\ \underline{9a - 6b = 12} \\ 13a = 78 \Rightarrow a = 6 \end{array}$$

Substitute $a = 6$ into equation (2) and solve for b.

$3(6) - 2b = 4 \Rightarrow -2b = -14 \Rightarrow b = 7$

Be sure to check your answer.
The reflection point is $Q(6, 7)$.

123.



Let $Q(a, b)$ be the reflection point. Using the hint from exercise 122, we have two equations, one developed from the fact that the midpoint of $\overline{PQ}$ lies on line ℓ, and the other developed from the fact that ℓ is perpendicular to the line containing the segment $\overline{PQ}$.

Equation (1): The midpoint of $\overline{PQ}$ is $M\left(\frac{a+2}{2}, \frac{b+1}{2}\right)$. Since M lies on line ℓ, we have $3\left(\frac{a+2}{2}\right) + \left(\frac{b+1}{2}\right) = 12$. We can simplify this to

$3(a+2) + (b+1) = 24 \Rightarrow$
$3a + 6 + b + 1 = 24 \Rightarrow 3a + b = 17$

Equation (2): The slope of ℓ is -3, so the slope of the line perpendicular to line ℓ is $\frac{1}{3}$.

Using the formula for the slope of $\overline{PQ}$, we have

$\frac{1}{3} = \frac{b-1}{a-2} \Rightarrow 3b - 3 = a - 2 \Rightarrow a - 3b = -1.$

So, the system is

$$\begin{cases} 3a + b = 17 & (1) \\ a - 3b = -1 & (2) \end{cases}$$

Multiply the first equation by 3. Then add the two equations, then solve for a.

$$\begin{array}{r} 9a + 3b = 51 \\ \underline{a - 3b = -1} \\ 10a = 50 \Rightarrow a = 5 \end{array}$$

Substitute $a = 5$ into equation (1) and solve for b.

$3(5) + b = 17 \Rightarrow b = 2$

Be sure to check your answer.
The reflection point is $Q(5, 2)$.

8.1 Maintaining Skills

124. $3x-(2x-1)-(4x+3)=-5$
$3x-2x+1-4x-3=-5$
$-3x-2=-5$
$-3x=-3 \Rightarrow x=1$

125. $y+2z=7;\ z=2$
$y+2(2)=7 \Rightarrow y+4=7 \Rightarrow y=3$

126. $x-3y=5;\ y=-3$
$x-3(-3)=5 \Rightarrow x+9=5 \Rightarrow x=-4$

127. $2x+3y+5z=21;\ y=1;\ z=2$
$2x+3(1)+5(2)=21 \Rightarrow 2x+13=21 \Rightarrow$
$2x=8 \Rightarrow x=4$

128. $3x-5y+z=2;\ x=-2,\ z=3$
$3(-2)-5y+3=2 \Rightarrow -5y-3=2 \Rightarrow$
$-5y=5 \Rightarrow y=-1$

129. $2x+3y-2z=5;\ y-z=4,\ z=-3$
$y-(-3)=4 \Rightarrow y+3=4 \Rightarrow y=1$
$2x+3(1)-2(-3)=5 \Rightarrow 2x+9=5 \Rightarrow$
$2x=-4 \Rightarrow x=-2$

130. $3x+2y+z=2;\ y+2z=1,\ z=2$
$y+2(2)=1 \Rightarrow y+4=1 \Rightarrow y=-3$
$3x+2(-3)+2=2 \Rightarrow 3x-4=2 \Rightarrow$
$3x=6 \Rightarrow x=2$

8.2 Systems of Linear Equations in Three Variables

8.2 Practice Problems

1. Substituting the ordered triple into the system

$$\begin{cases} x+y+z=1 \\ 3x+4y+z=-4, \\ 2x+y+2z=5 \end{cases}$$

we find that (2, −3, 2) is a solution.

$$\begin{cases} 2+(-3)+2=1 \\ 3(2)+4(-3)+2=-4 \\ 2(2)+(-3)+2(2)=5 \end{cases}$$

2.
$$\begin{cases} 2x+5y=1 & (1) \\ x-3y+2z=1 & (2) \\ -x+2y+z=7 & (3) \end{cases}$$

Interchange equations (1) and (2).

$$\begin{cases} x-3y+2z=1 & (2) \\ 2x+5y=1 & (1) \\ -x+2y+z=7 & (3) \end{cases}$$

Multiply equation (2) by −2, then add the resulting equation and equation (1) to eliminate x, then replace equation (1) with the resulting equation.

$$\begin{array}{rl} -2x+6y-4z=-2 & (2) \\ 2x+5y=1 & (1) \\ \hline 11y-4z=-1 & (4) \end{array}$$

$$\begin{cases} x-3y+2z=1 & (2) \\ 11y-4z=-1 & (4) \\ -x+2y+z=7 & (3) \end{cases}$$

Now add equations (2) and (3), then replace equation (3) with the resulting equation.

$$\begin{array}{rl} x-3y+2z=1 & (2) \\ -x+2y+z=7 & (3) \\ \hline -y+3z=8 & (5) \end{array}$$

$$\begin{cases} x-3y+2z=1 & (2) \\ 11y-4z=-1 & (4) \\ -y+3z=8 & (5) \end{cases}$$

Now multiply equation (5) by 11, add the resulting equation to equation (4), then solve for z.

$$\begin{array}{rl} 11y-4z=-1 & (4) \\ -11y+33z=88 & (5) \\ \hline 29z=87 & \\ z=3 & (6) \end{array}$$

The system becomes

$$\begin{cases} x-3y+2z=1 & (2) \\ 11y-4z=-1 & (4). \\ z=3 & (6) \end{cases}$$

Now substitute $z = 3$ into equation (4) and solve for y.
$11y-4(3)=-1 \Rightarrow 11y=11 \Rightarrow y=1$
Substitute the values of y and z into equation (2) and solve for x.
$x-3(1)+2(3)=1 \Rightarrow x+3=1 \Rightarrow x=-2$
So, we have $x = -2$, $y = 1$, and $z = 3$. Now check in the original system.

$$\begin{cases} 2(-2)+5(1)=1 \ \checkmark \\ -2-3(1)+2(3)=1 \ \checkmark \\ -(-2)+2(1)+3=7 \ \checkmark \end{cases}$$

The solution is {(−2, 1, 3)}.

3.
$$\begin{cases} 2x+2y+2z=12 & (1) \\ -3x+y-11z=-6 & (2) \\ 2x+y+4z=-8 & (3) \end{cases}$$

Divide equation (1) by 2 and replace equation (1) with the resulting equation.

$$\begin{cases} x+y+z=6 & (4) \\ -3x+y-11z=-6 & (2) \\ 2x+y+4z=-8 & (3) \end{cases}$$

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Multiply equation (4) by 3, add the resulting equation to equation (2), and replace equation (2) with the result. Similarly, multiply equation (4) by −2, add the resulting equation to equation (3), and replace equation (3) with the result.

$$\begin{cases} x + y + z = 6 & (4) \\ 4y - 8z = 12 & (5) \\ -y + 2z = -20 & (6) \end{cases}$$

Multiply equation (6) by 4, add the resulting equation to equation (5), and replace equation (6) with the result.

$$\begin{aligned} 4y - 8z &= 12 \\ -4y + 8z &= -80 \\ \hline 0 &= -68 \end{aligned}$$

$$\begin{cases} x + y + z = 6 & (4) \\ 4y - 8z = 12 & (5) \\ 0 = -68 & (7) \end{cases}$$

Since equation (7) is false, the solution set of the system is $\varnothing$.

4. $$\begin{cases} x + y + z = 5 & (1) \\ -4x - y - 8z = -29 & (2) \\ 2x + 5y - 2z = 1 & (3) \end{cases}$$

Multiply equation (1) by 4, add the resulting equation to equation (2), and replace equation (2) with the result. Similarly, multiply equation (1) by −2, add the resulting equation to equation (3), and replace equation (3) with the resulting equation.

$$\begin{cases} x + y + z = 5 & (1) \\ 3y - 4z = -9 & (4) \\ 3y - 4z = -9 & (5) \end{cases}$$

Now multiply equation (4) by −1, add the result to equation (5), and replace equation (5) with the resulting equation.

$$\begin{cases} x + y + z = 5 & (1) \\ 3y - 4z = -9 & (4) \\ 0 = 0 & (6) \end{cases}$$

Solve equation (4) for y.

$$3y - 4z = -9 \Rightarrow y = \frac{-9 + 4z}{3} = -3 + \frac{4}{3}z$$

Substitute this expression into equation (1) and solve for x.

$$x + y + z = 5 \Rightarrow x + \left(-3 + \frac{4}{3}z\right) + z = 5 \Rightarrow$$

$$x + \frac{7}{3}z - 3 = 5 \Rightarrow x = -\frac{7}{3}z + 8$$

The solution set is $\left\{\left(8 - \frac{7}{3}z, -3 + \frac{4}{3}z, z\right)\right\}$.

5. $$\begin{cases} x + 3y + 2z = 4 & (1) \\ 2x + 7y - z = 5 & (2) \end{cases}$$

Eliminate x.

$$\begin{aligned} -2x - 6y - 4z &= -8 \\ 2x + 7y - z &= 5 \\ \hline y - 5z &= -3 \end{aligned}$$

$$\begin{cases} x + 3y + 2z = 4 & (1) \\ y - 5z = -3 & (3) \end{cases}$$

Solve equation (3) for y in terms of z.

$y - 5z = -3 \Rightarrow y = 5z - 3$

Substitute this expression into equation (1) and then solve for x in terms of z.

$x + 3(5z - 3) + 2z = 4 \Rightarrow$

$x + 15z - 9 + 2z = 4 \Rightarrow x = -17z + 13$

The solution set is $\{(-17z + 13, 5z - 3, z)\}$.

6. The system is

$$\begin{cases} x + y = 0.65 \text{ (beam 1)} \\ x + z = 0.70 \text{ (beam 2)} \\ y + z = 0.55 \text{ (beam 3)} \end{cases}.$$

Multiply the first equation by –1, add the result to the second equation, and replace the second equation with the new equation:

$-1(x + y = 0.65) \Rightarrow -x - y = -0.65$

$$\begin{cases} -x - y = -0.65 \\ x + z = 0.70 \end{cases} \Rightarrow -y + z = 0.05$$

$$\begin{cases} x + y = 0.65 \\ x + z = 0.70 \\ y + z = 0.55 \end{cases} \Rightarrow \begin{cases} x + y = 0.65 \\ -y + z = 0.05 \\ y + z = 0.55 \end{cases}$$

Add the second and third equations and solve for z: $2z = 0.60 \Rightarrow z = 0.30$. Substituting this value into the original second equation, we have $x + 0.30 = 0.70 \Rightarrow x = 0.40$. Substituting this value into the original first equation, we have $0.40 + y = 0.65 \Rightarrow y = 0.25$. Referring to table 8.1, we see that cell A contains bone (since $x = 0.40$), cell B contains healthy tissue (since $y = 0.25$), and cell C contains tumorous tissue (since $z = 0.30$).

8.2 Basic Concepts and Skills

1. Systems of equations that have the same solution set are called equivalent systems.
2. Adding a constant multiple of one equation to another equation in the system produces an equivalent system.
3. If any of the equations in a system has no solution, then the system is inconsistent.

4. If the number of equations is different from the number of variables in a linear system, then the system is called a nonsquare system.

5. True

6. True

7. Substituting the ordered triple into the system
$$\begin{cases} 2x-2y-3z=1 \\ \quad 3y+2z=-1 \\ \quad y+z=0 \end{cases},$$
we find that (1, –1,1) is a solution.
$$\begin{cases} 2(1)-2(-1)-3(1)=1 \\ \quad 3(-1)+2(1)=-1 \\ \quad -1+1=0 \end{cases}$$

8. Substituting the ordered triple into the system
$$\begin{cases} 3x-2y+z=2 \\ \quad y+z=5 \\ x \quad +3z=8 \end{cases},$$
we find that (2, 3, 2) is a solution.
$$\begin{cases} 3(2)-2(3)+2=2 \\ \quad 3+2=5 \\ 2 \quad +3(2)=8 \end{cases}$$

9. Substituting the ordered triple into the system
$$\begin{cases} x+3y-2z=0 \\ 2x-y+4z=5 \\ x-11y+14z=0 \end{cases},$$
we find that (–10, 8, 7) is not a solution.
$$\begin{cases} -10+3(8)-2(7)=0 \\ 2(-10)-8+4(7)=0\neq 5 \\ -10-11(8)+14(7)=0 \end{cases}$$

10. Substituting the ordered triple into the system
$$\begin{cases} x-4y+7z=14 \\ 3x+8y-2z=13 \\ 7x-8y+26z=5 \end{cases}$$
we find that {(4, 1, 2)} is not a solution.
$$\begin{cases} 4-4(1)+7(2)=14 \\ 3(4)+8(1)-2(2)=16\neq 13 \\ 7(4)-8(1)+26(2)=72\neq 5 \end{cases}$$

11. To solve the system $\begin{cases} x+y+z=4 \\ \quad y-2z=4 \\ \quad z=-1 \end{cases}$, substitute $z=-1$ into the second equation, and solve for *y*: $y-2(-1)=4 \Rightarrow y=2$. Now substitute the values for *y* and *z* into the first equation, and solve for *x*:
$x+2-1=4 \Rightarrow x=3$.
The solution is {(3, 2, –1)}.

12. To solve the system $\begin{cases} x+3y+z=3 \\ \quad y+2z=8 \\ \quad z=5 \end{cases}$, substitute $z=5$ into the second equation, and solve for *y*: $y+2(5)=8 \Rightarrow y=-2$. Now substitute the values for *y* and *z* into the first equation, and solve for *x*:
$x+3(-2)+5=3 \Rightarrow x=4$.
The solution is {(4, –2, 5)}.

13. To solve the system $\begin{cases} x-5y+3z=-1 \\ \quad y-2z=-6 \\ \quad z=4 \end{cases}$, substitute $z=4$ into the second equation, and solve for *y*: $y-2(4)=-6 \Rightarrow y=2$. Now substitute the values for *y* and *z* into the first equation, and solve for *x*:
$x-5(2)+3(4)=-1 \Rightarrow x=-3$.
The solution is {(–3, 2, 4)}.

14. To solve the system $\begin{cases} x+3y+5z=0 \\ \quad y+7z=2 \\ \quad z=\frac{1}{2} \end{cases}$, substitute $z=\frac{1}{2}$ into the second equation, and solve for *y*: $y+7\left(\frac{1}{2}\right)=2 \Rightarrow y=-\frac{3}{2}$. Now substitute the values for *y* and *z* into the first equation, and solve for *x*:
$x+3\left(-\frac{3}{2}\right)+5\left(\frac{1}{2}\right)=0 \Rightarrow x=2$.

The solution is $\left\{\left(2,-\frac{3}{2},\frac{1}{2}\right)\right\}$.

15. $\begin{cases} 2x-2y-3z=1 \\ \quad 3y+2z=-1 \\ \quad y+z=0 \end{cases} \Rightarrow \begin{cases} 2x-2y-3z=1 \\ \quad y+z=0 \\ \quad 3y+2z=-1 \end{cases}$.

Multiply the first equation by $1/2$:
$$\frac{1}{2}(2x-2y-3z=1) \Rightarrow x-y-\frac{3}{2}z=\frac{1}{2}.$$
Now multiply the second equation by –3, add the result to the third equation, and replace the third equation with the new equation:
$-3(y+z=0) \Rightarrow -3y-3z=0$.
$$\begin{cases} -3y-3z=0 \\ 3y+2z=-1 \end{cases} \Rightarrow -z=-1 \Rightarrow z=1.$$

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So the system becomes

$$\begin{cases} x - y - \frac{3}{2}z = \frac{1}{2} \\ y + z = 0 \\ z = 1 \end{cases}.$$

16. $\begin{cases} 3x - 2y + z = 2 \\ y + z = 5 \\ x + 3z = 8 \end{cases} \Rightarrow \begin{cases} x + 3z = 8 \\ y + z = 5 \\ 3x - 2y + z = 2 \end{cases}.$

Multiply the first equation by –3, add the result to the third equation, and replace the third equation with the new equation:

$-3(x + 3z = 8) \Rightarrow -3x - 9z = -24$

$\begin{cases} -3x - 9z = -24 \\ 3x - 2y + z = 2 \end{cases} \Rightarrow -2y - 8z = -22$

The system becomes

$\begin{cases} x + 3z = 8 \\ y + z = 5 \\ -2y - 8z = -22 \end{cases}.$

Now multiply the second equation by 2, add the result to the third equation, and replace the third equation with the new equation:

$2(y + z = 5) \Rightarrow 2y + 2z = 10.$

$\begin{cases} 2y + 2z = 10 \\ -2y - 8z = -22 \end{cases} \Rightarrow -6z = -12 \Rightarrow z = 2$

The system becomes

$\begin{cases} x + 3z = 8 \\ y + z = 5 \\ z = 2 \end{cases}.$

17. Multiply the first equation by –2, add the result to the second equation, and replace the second equation with the new equation:

$-2(x + 3y - 2z = 0) \Rightarrow -2x - 6y + 4z = 0$

$\begin{cases} -2x - 6y + 4z = 0 \\ 2x - y + 4z = 5 \end{cases} \Rightarrow -7y + 8z = 5.$

The system becomes:

$\begin{cases} x + 3y - 2z = 0 \\ 2x - y + 4z = 5 \\ x - 11y + 14z = 0 \end{cases} \Rightarrow \begin{cases} x + 3y - 2z = 0 \\ -7y + 8z = 5 \\ x - 11y + 14z = 0 \end{cases}$

Multiply the first equation by –1, add the result to the third equation, and replace the third equation with the new equation:

$-1(x + 3y - 2z = 0) \Rightarrow -x - 3y + 2z = 0$

$\begin{cases} -x - 3y + 2z = 0 \\ x - 11y + 14z = 5 \end{cases} \Rightarrow -14y + 16z = 5.$

The system becomes:

$\begin{cases} x + 3y - 2z = 0 \\ -7y + 8z = 5 \\ x - 11y + 14z = 0 \end{cases} \Rightarrow \begin{cases} x + 3y - 2z = 0 \\ -7y + 8z = 5 \\ -14y + 16z = 5 \end{cases}$

Multiply the second equation by –2, add the result to the third equation, and replace the third equation with the new equation:

$-2(-7y + 8z = 5) \Rightarrow 14y - 16z = -10$

$\begin{cases} 14y - 16z = -10 \\ -14y + 16z = 5 \end{cases} \Rightarrow 0 = -5.$

The system becomes:

$\begin{cases} x + 3y - 2z = 0 \\ -7y + 8z = 5 \\ x - 11y + 14z = 0 \end{cases} \Rightarrow \begin{cases} x + 3y - 2z = 0 \\ y - \frac{8}{7}z = -\frac{5}{7} \\ 0 = -5 \end{cases}$

18. Multiply the first equation by –3, add the result to the second equation, and replace the second equation with the new equation:

$-3(x - 4y + 7z = 14) \Rightarrow -3x + 12y - 21z = -42$

$\begin{cases} -3x + 12y - 21z = -42 \\ 3x + 8y - 2z = 13 \end{cases} \Rightarrow 20y - 23z = -29$

$\begin{cases} x - 4y + 7z = 14 \\ 3x + 8y - 2z = 13 \\ 7x - 8y + 26z = 5 \end{cases} \Rightarrow \begin{cases} x - 4y + 7z = 14 \\ 20y - 23z = -29 \\ 7x - 8y + 26z = 5 \end{cases}$

Multiply the first equation by –7, add the result to the third equation, and replace the third equation with the new equation:

$-7(x - 4y + 7z = 14) \Rightarrow -7x + 28y - 49z = -98$

$\begin{cases} -7x + 28y - 49z = -98 \\ 7x - 8y + 26z = 5 \end{cases} \Rightarrow 20y - 23z = -93$

$\begin{cases} x - 4y + 7z = 14 \\ 20y - 23z = -29 \\ 7x - 8y + 26z = 5 \end{cases} \Rightarrow \begin{cases} x - 4y + 7z = 14 \\ 20y - 23z = -29 \\ 20y - 23z = -93 \end{cases}$

Multiply the second equation by –1, add the result to the third equation, and replace the third equation with the new equation:

$-1(20y - 23z = -29) \Rightarrow -20y + 23z = 29$

$\begin{cases} -20y + 23z = 29 \\ 20y - 23z = -93 \end{cases} \Rightarrow 0 = -64$

$\begin{cases} x - 4y + 7z = 14 \\ 20y - 23z = -29 \\ 20y - 23z = -93 \end{cases} \Rightarrow \begin{cases} x - 4y + 7z = 14 \\ y - \frac{23}{20}z = -\frac{29}{20} \\ 0 = -64 \end{cases}$

19. $\begin{cases} x - y + z = 6 \\ 2y + 3z = 5 \\ 2z = 6 \end{cases} \Rightarrow \begin{cases} x - y + z = 6 \\ 2y + 3z = 5 \\ z = 3 \end{cases}.$

Substitute $z = 3$ into the second equation to solve for y: $2y + 3(3) = 5 \Rightarrow y = -2.$

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Now substitute the values for y and z into the first equation to solve for x:
$x-(-2)+3=6 \Rightarrow x=1.$
The solution is $\{(1, -2, 3)\}$.

20. $\begin{cases} 4x+5y+2z=-3 \\ 3y-z=14 \\ -3z=15 \end{cases} \Rightarrow \begin{cases} 4x+5y+2z=-3 \\ y-\frac{1}{3}z=\frac{14}{3}. \\ z=-5 \end{cases}$

Substitute $z=-5$ into the second equation to solve for y: $3y-(-5)=14 \Rightarrow y=3.$ Now substitute the values for y and z into the first equation to solve for x:
$4x+5(3)+2(-5)=-3 \Rightarrow x=-2.$
The solution is $\{(-2, 3, -5)\}$.

21. Multiply the first equation by $-3/4$ add the result to the second equation, then replace the second equation with the new equation:

$-\frac{3}{4}(4x+4y+4z=7) \Rightarrow$

$-3x-3y-3z=-\frac{21}{4}$

$\begin{cases} -3x-3y-3z=-\frac{21}{4} \\ 3x-8y=14 \end{cases} \Rightarrow -11y-3z=\frac{35}{4} \Rightarrow$

$y+\frac{3}{11}z=-\frac{35}{44}$

$\begin{cases} 4x+4y+4z=7 \\ 3x-8y=14 \\ 4z=-1 \end{cases} \Rightarrow \begin{cases} x+y+z=\frac{7}{4} \\ y+\frac{3}{11}z=-\frac{35}{44} \\ z=-\frac{1}{4} \end{cases}$

Substitute $z=-1/4$ into the second equation to solve for y:

$y+\frac{3}{11}\left(-\frac{1}{4}\right)=-\frac{35}{44} \Rightarrow y=-\frac{8}{11}.$

Now substitute the values for y and z into the first equation to solve for x:

$4x+4\left(-\frac{8}{11}\right)+4\left(-\frac{1}{4}\right)=7 \Rightarrow x=\frac{30}{11}.$

The solution is $\left\{\left(\frac{30}{11}, -\frac{8}{11}, -\frac{1}{4}\right)\right\}$.

22. $\begin{cases} 5x+10y+10z=0 \\ 2y+3z=-0.6 \\ 4z=1.6 \end{cases} \Rightarrow$

$\begin{cases} x+2y+2z=0 \\ y+\frac{3}{2}z=-0.3 \\ z=0.4 \end{cases}$

Substitute $z=0.4$ into the second equation to solve for y: $y+\frac{3}{2}(0.4)=-0.3 \Rightarrow y=-0.9.$

Now substitute the values for y and z into the first equation to solve for x:
$5x+10(-0.9)+10(0.4)=0 \Rightarrow x=1.$
The solution is $\{(1, -0.9, 0.4)\}$.

In exercises 23–44, be sure to check the answers by substituting the values into the original system of equations.

23. Multiply the first equation by -1, add the result to the second equation, and replace the second equation with the new equation:
$-1(x+y+z=6) \Rightarrow -x-y-z=-6$

$\begin{cases} -x-y-z=-6 \\ x-y+z=2 \end{cases} \Rightarrow -2y=-4$

$\begin{cases} x+y+z=6 \\ x-y+z=2 \\ 2x+y-z=1 \end{cases} \Rightarrow \begin{cases} x+y+z=6 \\ -2y=-4 \\ 2x+y-z=1 \end{cases}$

Simplify the new second equation, and then multiply the first equation by -2, add the result to the third equation, and replace the third equation with the new equation:
$-2(x+y+z=6) \Rightarrow -2x-2y-2z=-12$

$\begin{cases} -2x-2y-2z=-12 \\ 2x+y-z=1 \end{cases} \Rightarrow -y-3z=-11$

$\begin{cases} x+y+z=6 \\ -2y=-4 \\ 2x+y-z=1 \end{cases} \Rightarrow \begin{cases} x+y+z=6 \\ y=2 \\ -y-3z=-11 \end{cases}$

Add the second and third equations, replacing the third equation with the new equation:

$\begin{cases} x+y+z=6 \\ y=2 \\ -y-3z=-11 \end{cases} \Rightarrow \begin{cases} x+y+z=6 \\ y=2 \\ -3z=-9 \end{cases} \Rightarrow$

$\begin{cases} x+y+z=6 \\ y=2 \\ z=3 \end{cases}$

Substitute the values for y and z into the original first equation to solve for x:
$x+2+3=6 \Rightarrow x=1.$
The solution is $\{(1, 2, 3)\}$.

24. Multiply the first equation by –3, add the result to the third equation, then replace the third equation with the new equation:

$-3(x+y+z=6) \Rightarrow -3x-3y-3z=-18$

$\begin{cases} -3x-3y-3z=-18 \\ 3x-2y+3z=8 \end{cases} \Rightarrow -5y-=-10 \Rightarrow y=2$

$\begin{cases} x+\ \ y+\ z=6 \\ 2x+3y-\ z=5 \\ 3x-2y+3z=8 \end{cases} \Rightarrow \begin{cases} x+\ \ y+\ z=6 \\ 2x+3y-\ z=5 \\ y\quad =2 \end{cases}$

Multiply the first equation by –2, add the result to the second equation, then replace the second equation with the new equation:

$-2(x+y+z=6) \Rightarrow -2x-2y-2z=-12$

$\begin{cases} -2x-2y-2z=-12 \\ 2x+3y-\ z=5 \end{cases} \Rightarrow y-3z=-7$

$\begin{cases} x+\ \ y+\ z=6 \\ 2x+3y-\ z=5 \\ y\quad =2 \end{cases} \Rightarrow \begin{cases} x+y+\ z=6 \\ y-3z=-7 \\ y\quad =2 \end{cases}$

Switch the second and third equations and subtract the new third equation from the new second equation:

$\begin{cases} x+y+\ z=6 \\ y-3z=-7 \\ y\quad =2 \end{cases} \Rightarrow \begin{cases} x+y+\ z=6 \\ y\quad =2 \\ y-3z=-7 \end{cases} \Rightarrow$

$\begin{cases} x+y+\ z=6 \\ y\quad =2 \\ 3z=9 \end{cases} \Rightarrow \begin{cases} x+y+\ z=6 \\ y\quad =2 \\ z=3 \end{cases}$

Substitute the values for y and z into the first equation to solve for x: $x+2+3=6 \Rightarrow x=1$.

The solution is $\{(1,2,3)\}$.

25. Switch the first and second equations.

$\begin{cases} 2x+3y+\ z=9 \\ x+2y+3z=6 \\ 3x+\ y+2z=8 \end{cases} \Rightarrow \begin{cases} x+2y+3z=6 \\ 2x+3y+\ z=9 \\ 3x+\ y+2z=8 \end{cases}$

Multiply the first equation by –2, add the result to the second equation, then replace the second equation with the new equation:

$-2(x+2y+3z=6)=-2x-4y-6z=-12$

$\begin{cases} -2x-4y-6z=-12 \\ 2x+3y+\ z=9 \end{cases} \Rightarrow -y-5z=-3 \Rightarrow$

$y+5z=3$

$\begin{cases} x+2y+3z=6 \\ 2x+3y+\ z=9 \\ 3x+\ y+2z=8 \end{cases} \Rightarrow \begin{cases} x+2y+3z=6 \\ y+5z=3 \\ 3x+\ y+2z=8 \end{cases}$

Multiply the first equation by –3, add the result to the third equation, then replace the third equation with the new equation:

$-3(x+2y+3z=6)=-3x-6y-9z=-18$

$\begin{cases} -3x-6y-9z=-18 \\ 3x+\ y+2z=8 \end{cases} \Rightarrow -5y-7z=-10 \Rightarrow$

$5y+7z=10$

$\begin{cases} x+2y+3z=6 \\ y+5z=3 \\ 3x+\ y+2z=8 \end{cases} \Rightarrow \begin{cases} x+2y+3z=6 \\ y+5z=3 \\ 5y+7z=10 \end{cases}$

Multiply the second equation by –5, and add the result to the third equation, then replace the third equation with the new equation:

$-5(y+5z=3) \Rightarrow -5y-25z=-15$

$\begin{cases} -5y-25z=-15 \\ 5y+\ 7z=10 \end{cases} \Rightarrow -18z=-5 \Rightarrow z=\frac{5}{18}$

$\begin{cases} x+2y+3z=6 \\ y+5z=3 \\ 5y+7z=10 \end{cases} \Rightarrow \begin{cases} x+2y+3z=6 \\ y+5z=3 \\ z=\frac{5}{18} \end{cases}$

Substitute the value of z into the second equation to solve for y:

$y+5\left(\frac{5}{18}\right)=3 \Rightarrow y=\frac{29}{18}.$

Substitute the values for y and z into the first equation to solve for x:

$x+2\left(\frac{29}{18}\right)+3\left(\frac{5}{18}\right)=6 \Rightarrow x=\frac{35}{18}.$

The solution is $\left\{\left(\frac{35}{18},\frac{29}{18},\frac{5}{18}\right)\right\}$.

26. Switch the first and second equations.

$\begin{cases} 4x+2y+3z=6 \\ x+2y+2z=1 \\ 2x-\ y+\ z=-1 \end{cases} \Rightarrow \begin{cases} x+2y+2z=1 \\ 4x+2y+3z=6 \\ 2x-\ y+\ z=-1 \end{cases}$

Multiply the first equation by –4, add the result to the second equation, then replace the second equation with the new equation:

$-4(x+2y+2z=1)=-4x-8y-8z=-4$

$\begin{cases} -4x-8y-8z=-4 \\ 4x+2y+3z=6 \end{cases} \Rightarrow -6y-5z=2$

$\begin{cases} x+2y+2z=1 \\ 4x+2y+3z=6 \\ 2x-\ y+\ z=-1 \end{cases} \Rightarrow \begin{cases} x+2y+2z=1 \\ -6y-5z=2 \\ 2x-\ y+\ z=-1 \end{cases}$

Multiply the first equation by –2, add the result to the third equation, then replace the third equation with the new equation:

$-2(x+2y+2z=1)=-2x-4y-4z=-2$

$\begin{cases} -2x-4y-4z=-2 \\ 2x-\ y+\ z=-1 \end{cases} \Rightarrow -5y-3z=-3 \Rightarrow$

$5y+3z=3$

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$$\begin{cases} x+2y+2z=1 \\ 4x+2y+3z=6 \\ 2x-\ \ y+\ \ z=-1 \end{cases} \Rightarrow \begin{cases} x+2y+2z=1 \\ -6y-5z=2 \\ 5y+3z=3 \end{cases}$$

Multiply the second equation by $5/6$, then add the result to the third equation to solve for z:

$$\frac{5}{6}(-6y-5z=2) \Rightarrow -5y-\frac{25}{6}z=\frac{10}{6}$$

$$\begin{cases} -5y-\dfrac{25}{6}z=\dfrac{10}{6} \\ 5y+\ \ 3z=3 \end{cases} \Rightarrow -\frac{7}{6}z=\frac{28}{6} \Rightarrow z=-4$$

$$\begin{cases} x+2y+2z=1 \\ -6y-5z=2 \\ 5y+3z=3 \end{cases} \Rightarrow \begin{cases} x+2y+2z=1 \\ -6y-5z=2 \\ z=-4 \end{cases}$$

Substitute $z=-4$ into the second equation to solve for y: $-6y-5(-4)=2 \Rightarrow y=3$.

Substitute the values for y and z into the first equation to solve for x:

$x+2(3)+2(-4)=1 \Rightarrow x=3.$

The solution is $\{(3, 3, -4)\}$.

27. $$\begin{cases} x+\ \ y-2z=5 & (1) \\ 2x-\ \ y-\ \ z=-4 & (2) \\ x-2y+\ \ z=-2 & (3) \end{cases}$$

Add equations (1) and (2), then replace equation (2) with the new equation (4).

$$\begin{cases} x+\ \ y-2z=5 & (1) \\ 2x-\ \ y-\ \ z=-4 & (2) \end{cases} \Rightarrow 3x-3z=1 \quad (4)$$

$$\begin{cases} x+\ \ y-2z=5 \\ 2x-\ \ y-\ \ z=-4 \\ x-2y+\ \ z=-2 \end{cases} \Rightarrow \begin{cases} x+\ \ y-2z=5 & (1) \\ 3x\qquad -3z=1 & (4) \\ x-2y+\ \ z=-2 & (3) \end{cases}$$

Multiply the first equation by 2, add the result to the third equation, then replace the third equation with the new equation (5)

$$2(x+y-2z=5) \Rightarrow 2x+2y-4z=10$$

$$\begin{cases} 2x+2y-4z=10 \\ x-2y+\ \ z=-2 \end{cases} \Rightarrow 3x-3z=8 \quad (5)$$

$$\begin{cases} x+\ \ y-2z=5 \\ 3x\qquad -3z=1 \\ x-2y+\ \ z=-2 \end{cases} \Rightarrow \begin{cases} x+\ \ y-2z=5 & (1) \\ 3x\qquad -3z=1 & (4) \\ 3x\qquad -3z=8 & (5) \end{cases}$$

Multiplying the equation (4) by −1, and adding the result to equation (5) gives:

$$-1(3x-3z=1) \Rightarrow -3x+3z=-1$$

$$\begin{cases} -3x+3z=-1 \\ 3x-3z=8 \end{cases} \Rightarrow 0=-7 \text{ False}$$

Thus, the system is inconsistent and the solution set is $\varnothing$.

28. $$\begin{cases} x+\ y+4z=6 \\ x+2y-2z=8 \\ 7x+10y+10z=60 \end{cases}$$

Multiply the first equation by –1, add the result to the second equation, then replace the second equation with the new equation:

$$-1(x+y+4z=6) \Rightarrow -x-y-4z=-6$$

$$\begin{cases} -x-\ \ y-4z=-6 \\ x+2y-2z=8 \end{cases} \Rightarrow y-6z=2$$

$$\begin{cases} x+\ y+4z=6 \\ x+2y-2z=8 \\ 7x+10y+10z=60 \end{cases} \Rightarrow \begin{cases} x+\ y+4z=6 \\ y-6z=2 \\ 7x+10y+10z=60 \end{cases}$$

Multiply the first equation by –7, add the result to the third equation, then replace the third equation:

$$-7(x+y+4z=6) \Rightarrow -7x-7y-28z=-42$$

$$\begin{cases} -7x\ -7y-28z=-42 \\ 7x+10y+10z=60 \end{cases} \Rightarrow 3y-18z=18$$

$$\begin{cases} x+\ y+4z=6 \\ y-6z=2 \\ 7x+10y+10z=60 \end{cases} \Rightarrow \begin{cases} x+\ y+4z=6 \\ y-6z=2 \\ 3y-18z=18 \end{cases}$$

Multiply the second equation by −3, and add the result to the third equation:

$$-3(y-6z=2) \Rightarrow -3y+18z=-6$$

$$\begin{cases} -3y+18z=-6 \\ 3y-18z=18 \end{cases} \Rightarrow 0=12 \quad \text{False}$$

Thus, the system is inconsistent and the solution set is $\varnothing$.

29. Multiply the second equation by –2, add the result to the first equation, then replace the second equation with the new equation:

$$-2(x+3y-z=-2) \Rightarrow -2x-6y+2z=4$$

$$\begin{cases} -2x-6y+2z=4 \\ 2x+3y+2z=7 \end{cases} \Rightarrow -3y+4z=11$$

$$\begin{cases} 2x+3y+2z=7 \\ x+3y-\ \ z=-2 \\ x-\ \ y+2z=8 \end{cases} \Rightarrow \begin{cases} 2x+3y+2z=7 \\ -3y+4z=11 \\ x-\ \ y+2z=8 \end{cases}$$

Multiply the third equation by –2, add the result to the first equation, then replace the third equation with the new equation:

$$-2(x-y+2z=8) \Rightarrow -2x+2y-4z=-16$$

$$\begin{cases} -2x+2y-4z=-16 \\ 2x+3y+2z=7 \end{cases} \Rightarrow 5y-2z=-9$$

$$\begin{cases} 2x+3y+2z=7 \\ x+3y-\ \ z=-2 \\ x-\ \ y+2z=8 \end{cases} \Rightarrow \begin{cases} 2x+3y+2z=7 \\ -3y+4z=11 \\ 5y-2z=-9 \end{cases}$$

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Multiply the third equation by 2, and add the result to the second equation to solve for y:

$2(5y-2z=-9) \Rightarrow 10y-4z=-18$

$$\begin{cases} 10y-4z=-18 \\ -3y+4z=11 \end{cases} \Rightarrow 7y=-7 \Rightarrow y=-1$$

$$\begin{cases} 2x+3y+2z=7 \\ \quad -3y+4z=11 \\ \quad 5y-2z=-9 \end{cases} \Rightarrow \begin{cases} 2x+3y+2z=7 \\ \quad y=-1 \\ \quad 5y-2z=-9 \end{cases}$$

Multiply the second equation by −5, then add the result to the third equation and replace the third equation with the new equation:

$$\begin{cases} -5y=5 \\ 5y-2z=-9 \end{cases} \Rightarrow -2z=-4 \Rightarrow z=2$$

$$\begin{cases} 2x+3y+2z=7 \\ \quad y=-1 \\ \quad 5y-2z=-9 \end{cases} \Rightarrow \begin{cases} 2x+3y+2z=7 \\ \quad y=-1 \\ \quad z=2 \end{cases}$$

Substitute the values for y and z into the first equation to solve for x:

$2x+3(-1)+2(2)=7 \Rightarrow x=3.$

The solution is $\{(3, -1, 2)\}$.

30. Switch the first and second equations:

$$\begin{cases} x-y+2z=3 \\ 2x+2y+z=3 \\ x+y+3z=4 \end{cases} \Rightarrow \begin{cases} 2x+2y+z=3 \\ x-y+2z=3 \\ x+y+3z=4 \end{cases}$$

Multiply the third equation by –2, add the result to the first equation, and replace the third equation with the new equation:

$-2(x+y+3z=4) \Rightarrow -2x-2y-6z=-8$

$$\begin{cases} 2x+2y+z=3 \\ -2x-2y-6z=-8 \end{cases} \Rightarrow -5z=-5 \Rightarrow z=1$$

$$\begin{cases} 2x+2y+z=3 \\ x-y+2z=3 \\ x+y+3z=4 \end{cases} \Rightarrow \begin{cases} x-y+2z=3 \\ 2x+2y+z=3 \\ z=1 \end{cases}$$

Multiply the first equation by –2, add the result to the second equation, and replace the second equation with the new equation, then switch the first and second equations:

$-2(x-y+2z=3) \Rightarrow -2x+2y-4z=-6$

$$\begin{cases} 2x+2y+z=3 \\ -2x+2y-4z=-6 \end{cases} \Rightarrow 4y-3z=-3$$

$$\begin{cases} x-y+2z=3 \\ 2x+2y+z=3 \\ z=1 \end{cases} \Rightarrow \begin{cases} 2x+2y+z=3 \\ x-y+2z=3 \\ z=1 \end{cases}$$

$$\begin{cases} 2x+2y+z=3 \\ x-y+2z=3 \\ z=1 \end{cases} \Rightarrow \begin{cases} 2x+2y+z=3 \\ 4y-3z=-3 \\ z=1 \end{cases}$$

Substitute $z = 1$ into the second equation to solve for y: $4y-3(1)=-3 \Rightarrow y=0.$ Substitute the values for y and z into the first equation to solve for x: $2x+2(0)+1=3 \Rightarrow x=1.$

The solution is $\{(1, 0, 1)\}$.

31. Switch the first and third equations:

$$\begin{cases} 4x-2y+z=5 \\ 2x+y-2z=4 \\ x+3y-2z=6 \end{cases} \Rightarrow \begin{cases} x+3y-2z=6 \\ 2x+y-2z=4 \\ 4x-2y+z=5 \end{cases}$$

Multiply the first equation by –2, add the result to the second equation, and replace the second equation with the new equation:

$-2(x+3y-2z=6) \Rightarrow -2x-6y+4z=-12$

$$\begin{cases} -2x-6y+4z=-12 \\ 2x+y-2z=4 \end{cases} \Rightarrow -5y+2z=-8$$

$$\begin{cases} x+3y-2z=6 \\ 2x+y-2z=4 \\ 4x-2y+z=5 \end{cases} \Rightarrow \begin{cases} x+3y-2z=6 \\ -5y+2z=-8 \\ 4x-2y+z=5 \end{cases}$$

Multiply the first equation by –4, add the result to the third equation, and replace the third equation with the new equation:

$-4(x+3y-2z=6) \Rightarrow -4x-12y+8z=-24$

$$\begin{cases} -4x-12y+8z=-24 \\ 4x-2y+z=5 \end{cases} \Rightarrow -14y+9z=-19$$

$$\begin{cases} x+3y-2z=6 \\ -5y+2z=-8 \\ 4x-2y+z=5 \end{cases} \Rightarrow \begin{cases} x+3y-2z=6 \\ -5y+2z=-8 \\ -14y+9z=-19 \end{cases}$$

Multiply the second equation by $-\frac{14}{5}$ and add the result to the third equation and replace the third equation with the new equation:

$-\frac{14}{5}(-5y+2z=-8) \Rightarrow 14y-\frac{28}{5}z=\frac{112}{5}$

$$\begin{cases} 14y-\frac{28}{5}z=\frac{112}{5} \\ -14y+9z=-19 \end{cases} \Rightarrow \frac{17}{5}z=\frac{17}{5} \Rightarrow z=1$$

$$\begin{cases} x+3y-2z=6 \\ -5y+2z=-8 \\ -14y+9z=-19 \end{cases} \Rightarrow \begin{cases} x+3y-2z=6 \\ -5y+2z=-8 \\ z=1 \end{cases}$$

Substituting $z = 1$ into the second equation, we have $-5y+2(1)=-8 \Rightarrow y=2.$ Substitute the values for y and z into the first equation to solve for x: $x+3(2)-2(1)=6 \Rightarrow x=2.$

The solution is $\{(2, 2, 1)\}$.

32. Multiply the first equation by –2, add the result to the second equation, and replace the second equation with the new equation:

$-2(x-3y+2z=9) \Rightarrow -2x+6y-4z=-18$

$\begin{cases} -2x+6y-4z=-18 \\ 2x+4y-3z=-9 \end{cases} \Rightarrow 10y-7z=-27$

$\begin{cases} x-3y+2z=9 \\ 2x+4y-3z=-9 \\ 3x-2y+5z=12 \end{cases} \Rightarrow \begin{cases} x-3y+2z=9 \\ 10y-7z=-27 \\ 3x-2y+5z=12 \end{cases}$

Multiply the first equation by –3, add the result to the third equation, and replace the third equation with the new equation:

$-3(x-3y+2z=9) \Rightarrow -3x+9y-6z=-27$

$\begin{cases} -3x+9y-6z=-27 \\ 3x-2y+5z=12 \end{cases} \Rightarrow 7y-z=-15$

$\begin{cases} x-3y+2z=9 \\ 10y-7z=-27 \\ 3x-2y+5z=12 \end{cases} \Rightarrow \begin{cases} x-3y+2z=9 \\ 10y-7z=-27 \\ 7y-z=-15 \end{cases}$

Multiply the second equation by $-7/10$ and add the result to the third equation to solve for z:

$-\frac{7}{10}(10y-7z=-27) \Rightarrow -7y+\frac{49}{10}z=\frac{189}{10}$

$\begin{cases} -7y+\frac{49}{10}z=\frac{189}{10} \\ 7y-\quad z=-15 \end{cases} \Rightarrow \frac{39}{10}z=\frac{39}{10} \Rightarrow z=1$

$\begin{cases} x-3y+2z=9 \\ 10y-7z=-27 \\ 7y-z=-15 \end{cases} \Rightarrow \begin{cases} x-3y+2z=9 \\ 10y-7z=-27 \\ z=1 \end{cases}$

Substitute $z = 1$ into the second equation to solve for y: $10y-7(1)=-27 \Rightarrow y=-2$.

Substitute the values for y and z into the first equation to solve for x:

$x-3(-2)+2(1)=9 \Rightarrow x=1$.

The solution is {(1, –2, 1)}.

33. Multiply the second equation by –1 and then add the result to the third equation:

$-(x+y-z=1) \Rightarrow -x-y+z=-1$

$\begin{cases} -x-y+z=-1 \\ x+y+2z=4 \end{cases} \Rightarrow 3z=3 \Rightarrow z=1$

Multiply the second equation by –2, add the result to the first equation, and replace the second equation with the new equation:

$-2(x+y-z=1) \Rightarrow -2x-2y+2z=-2$

$\begin{cases} 2x+y+z=6 \\ -2x-2y+2z=-2 \end{cases} \Rightarrow -y+3z=4$

$\begin{cases} 2x+y+z=6 \\ x+y-z=1 \\ x+y+2z=4 \end{cases} \Rightarrow \begin{cases} 2x+y+z=6 \\ -y+3z=4 \\ z=1 \end{cases}$

Substitute $z = 1$ into the second equation to solve for y: $-y+3(1)=4 \Rightarrow y=-1$.

Substitute the values for y and z into the first equation to solve for x: $2x-1+1=6 \Rightarrow x=3$.

The solution is (3, –1, 1).

34. Switch the first and second equations:

$\begin{cases} 2x+y-3z=7 \\ x-y-2z=4 \\ 3x+3y+2z=4 \end{cases} \Rightarrow \begin{cases} x-y-2z=4 \\ 2x+y-3z=7 \\ 3x+3y+2z=4 \end{cases}$

Multiply the first equation by –2, add the result to the second equation, and replace the second equation with the new equation:

$-2(x-y-2z=4) \Rightarrow -2x+2y+4z=-8$

$\begin{cases} 2x+y-3z=7 \\ -2x+2y+4z=-8 \end{cases} \Rightarrow 3y+z=-1$

$\begin{cases} x-y-2z=4 \\ 2x+y-3z=7 \\ 3x+3y+2z=4 \end{cases} \Rightarrow \begin{cases} x-y-2z=4 \\ 3y+z=-1 \\ 3x+3y+2z=4 \end{cases}$

Multiply the first equation by –3, add the result to the third equation, and replace the third equation with the new equation:

$-3(x-y-2z=4) \Rightarrow -3x+3y+6z=-12$

$\begin{cases} -3x+3y+6z=-12 \\ 3x+3y+2z=4 \end{cases} \Rightarrow 6y+8z=-8$

$\begin{cases} x-y-2z=4 \\ 3y+z=-1 \\ 3x+3y+2z=4 \end{cases} \Rightarrow \begin{cases} x-y-2z=4 \\ 3y+z=-1 \\ 6y+8z=-8 \end{cases}$

Multiply the second equation by –2 and add the result to the third equation to solve for z:

$-2(3y+z=-1) \Rightarrow -6y-2z=2$

$\begin{cases} -6y-2z=2 \\ 6y+8z=-8 \end{cases} \Rightarrow 6z=-6 \Rightarrow z=-1$

$\begin{cases} x-y-2z=4 \\ 3y+z=-1 \\ 6y+8z=-8 \end{cases} \Rightarrow \begin{cases} x-y-2z=4 \\ 3y+z=-1 \\ z=-1 \end{cases}$

Substitute $z = –1$ into the second equation to solve for y: $3y-1=-1 \Rightarrow y=0$. Substitute the values for y and z into the first equation to solve for x: $x-0-2(-1)=4 \Rightarrow x=2$.

The solution is {(2, 0, –1)}.

35. $\begin{cases} x-y-z=3 \\ x+9y+z=3 \\ 2x+3y-z=6 \end{cases}$

Multiply the first equation by −1, add the result to the second equation, and replace the second equation with the new equation:

$-1(x-y-z=3) \Rightarrow -x+y+z=-3$

$\begin{cases} -x+y+z=-3 \\ x+9y+z=3 \end{cases} \Rightarrow 10y+2z=0$

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$$\begin{cases} x - y - z = 3 \\ x + 9y + z = 3 \\ 2x + 3y - z = 6 \end{cases} \Rightarrow \begin{cases} x - y - z = 3 \\ 10y + 2z = 0 \\ 2x + 3y - z = 6 \end{cases}$$

Multiply the first equation by −2, add the result to the third equation, and replace the third equation with the new equation:

$-2(x - y - z = 3) \Rightarrow -2x + 2y + 2z = -6$

$$\begin{cases} -2x + 2y + 2z = -6 \\ 2x + 3y - z = 6 \end{cases} \Rightarrow 5y + z = 0$$

$$\begin{cases} x - y - z = 3 \\ 10y + 2z = 0 \\ 2x + 3y - z = 6 \end{cases} \Rightarrow \begin{cases} x - y - z = 3 \\ 10y + 2z = 0 \\ 5y + z = 0 \end{cases}$$

Multiply the third equation by −2, then add the result to the second equation:

$-2(5y + z = 0) \Rightarrow -10y - 2z = 0$

$$\begin{cases} 10y + 2z = 0 \\ -10y - 2z = 0 \end{cases} \Rightarrow 0 = 0$$

The equation 0 = 0 is equivalent to $0z = 0$, which is true for every value of z.
Solving the second equation for y, we have

$10y + 2z = 0 \Rightarrow y = -\frac{1}{5}z$. Substituting this into the first equation, we have

$x - y - z = 3 \Rightarrow$

$x - \left(-\frac{1}{5}z\right) - z = 3 \Rightarrow x = 3 + \frac{4}{5}z$

Thus, the solution is $\left\{\left(3 + \frac{4}{5}z, -\frac{1}{5}z, z\right)\right\}$.

36. $$\begin{cases} x + y - z = 2 \\ 3x - y - z = 10 \\ 3x + y - 2z = 8 \end{cases}$$

Multiply the first equation by −3, add the result to the second equation, and replace the second equation with the new equation:

$-3(x + y - z = 2) \Rightarrow -3x - 3y + 3z = -6$

$$\begin{cases} -3x - 3y + 3z = -6 \\ 3x - y - z = 10 \end{cases} \Rightarrow -4y + 2z = 4$$

$$\begin{cases} x + y - z = 2 \\ 3x - y - z = 10 \\ 3x + y - 2z = 8 \end{cases} \Rightarrow \begin{cases} x + y - z = 2 \\ -4y + 2z = 4 \\ 3x + y - 2z = 8 \end{cases}$$

Multiply the first equation by −3, add the result to the third equation, and replace the third equation with the new equation:

$-3(x + y - z = 2) \Rightarrow -3x - 3y + 3z = -6$

$$\begin{cases} -3x - 3y + 3z = -6 \\ 3x + y - 2z = 8 \end{cases} \Rightarrow -2y + z = 2$$

$$\begin{cases} x + y - z = 2 \\ -4y + z = 4 \\ 3x + y - 2z = 8 \end{cases} \Rightarrow \begin{cases} x + y - z = 2 \\ -4y + 2z = 4 \\ -2y + z = 2 \end{cases}$$

Multiply the third equation by −2, then add the result to the second equation:

$-2(-2y + z = 2) \Rightarrow 4y - 2z = -4$

$$\begin{cases} -4y + 2z = 4 \\ -2y + z = 2 \end{cases} \Rightarrow \begin{cases} -4y + 2z = 4 \\ 4y - 2z = -4 \end{cases} \Rightarrow 0 = 0$$

The equation 0 = 0 is equivalent to $0z = 0$, which is true for every value of z. Solving the second equation for y, we have

$-4y + 2z = 4 \Rightarrow y = -1 + \frac{1}{2}z$. Substituting this into the first equation, we have

$x + y - z = 2 \Rightarrow$

$x + \left(-1 + \frac{1}{2}z\right) - z = 2 \Rightarrow x = 3 + \frac{1}{2}z$

Thus, the solution is $\left\{\left(3 + \frac{1}{2}z, -1 + \frac{1}{2}z, z\right)\right\}$.

37. Rearrange the equations as shown:

$$\begin{cases} x + y = 0 \\ y + 2z = -4 \\ y + z = 4 - x \end{cases} \Rightarrow \begin{cases} x + y = 0 \\ y + 2z = -4 \\ x + y + z = 4 \end{cases} \Rightarrow$$

$$\begin{cases} x + y + z = 4 \\ y + 2z = -4 \\ x + y = 0 \end{cases}$$

Subtract the third equation from the first equation, and replace the third equation:

$$\begin{cases} x + y + z = 4 \\ y + 2z = -4 \\ x + y = 0 \end{cases} \Rightarrow \begin{cases} x + y + z = 4 \\ y + 2z = -4 \\ z = 4 \end{cases}$$

Substitute z = 4 into the second equation to solve for y: $y + 2(4) = -4 \Rightarrow y = -12$.

Substitute the values for x and y into the first equation to solve for x:

$x - 12 + 4 = 4 \Rightarrow x = 12$.

The solution is {(12, −12, 4)}.

38. $$\begin{cases} 2x + 4y + 3z = 6 \\ x + 2z = -1 \\ x - 2y + z = -5 \end{cases}$$

Subtract the third equation from the second equation and replace the second equation with the new equation:

$$\begin{cases} 2x + 4y + 3z = 6 \\ x + 2z = -1 \\ x - 2y + z = -5 \end{cases} \Rightarrow \begin{cases} 2x + 4y + 3z = 6 \\ 2y + z = 4 \\ x - 2y + z = -5 \end{cases}$$

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Multiply the third equation by −2, add the result to the first equation and replace the third equation with the new equation:

$-2(x-2y+z=-5)=-2x+4y-2z=10$

$$\begin{cases} 2x+4y+3z=6 \\ -2x+4y-2z=10 \end{cases} \Rightarrow 8y+z=16$$

$$\begin{cases} 2x+4y+3z=6 \\ \quad\;\; 2y+\;z=4 \\ x-2y+\;z=-5 \end{cases} \Rightarrow \begin{cases} 2x+4y+3z=6 \\ \quad\;\; 2y+\;z=4 \\ \quad\;\; 8y+\;z=16 \end{cases}$$

Multiply the second equation by −4, add to the third equation, and replace the third equation with the new equation:

$-4(2y+z)=-4(4)\Rightarrow -8y-4z=-16$

$$\begin{cases} -8y-4z=-16 \\ \;\; 8y+\;\;z=16 \end{cases} \Rightarrow -3z=0\Rightarrow z=0$$

$$\begin{cases} 2x+4y+3z=6 \\ \quad\;\; 2y+\;z=4 \\ \qquad\quad\;\; z=0 \end{cases}$$

Substitute $z=0$ into the second equation to solve for y: $2y+0=4\Rightarrow y=2.$ Substitute the values for y and z into the first equation to solve for x: $2x+4(2)+3(0)=6\Rightarrow x=-1.$ The solution is {(–1, 2, 0)}.

39.
$$\begin{cases} x+\;\;y+\;\;z=6 & (1) \\ x+2y+3z=14 & (2) \\ x+4y+7z=30 & (3) \end{cases}$$

Multiply equation (1) by −1, add the result to equation (2), and replace equation (2) with the new equation (4).

$-1(x+y+z=6)\Rightarrow -x-y-z=-6$

$$\begin{cases} -x-\;\;y-\;\;z=-6 \\ \;\;\, x+2y+3z=14 \end{cases} \Rightarrow y+2z=8 \quad (4)$$

$$\begin{cases} x+y+z=6 \\ x+2y+3z=3 \\ x+4y+7z=30 \end{cases} \Rightarrow \begin{cases} x+y+z=6 & \\ \quad\;\; y+2z=8 & (4) \\ x+4y+7z=30 & \end{cases}$$

Multiply equation (1) by −1, add the result to equation (3), and replace equation (3) with the new equation (5).

$-1(x+y+z=6)\Rightarrow -x-y-z=-6$

$$\begin{cases} -x-\;\;y-\;\;z=-6 \\ \;\;\, x+4y+7z=30 \end{cases} \Rightarrow 3y+6z=24 \quad (5)$$

$$\begin{cases} x+y+z=6 & \\ \;\; y+2z=8 & (4) \\ 3y+6z=24 & (5) \end{cases}$$

Multiply equation (4) by −3, add the result to equation (5), then replace equation (5) with the result (6).

$-3(y+2z=8)\Rightarrow -3y-6z=-24$

$$\begin{cases} -3y-6z=-24 & \\ \;\; 3y+6z=24 & (6) \end{cases} \Rightarrow 0=0$$

$$\begin{cases} x+y+z=6 & (1) \\ \;\; y+2z=8 & (4) \\ \qquad\;\; 0=0 & (6) \end{cases}$$

The system is now in triangular form. Solve equation (4) for y: $y+2z=8\Rightarrow y=-2z+8$

Substitute the expression for y into equation (1) and solve for x.

$x+(-2z+8)+z=6\Rightarrow x-z+8=6\Rightarrow$
$x=z-2$

The solution set for the system is $\{(z-2,\,-2z+8,\,z)\}.$

40.
$$\begin{cases} x-\;\;y+\;\;z=3 & (1) \\ 2x+\;\;y-\;\;z=2 & (2) \\ x+2y-2z=-1 & (3) \end{cases}$$

Multiply equation (1) by –2, add it to equation (2), and replace equation (2) with the result (4).

$-2(x-y+z=3)\Rightarrow -2x+2y-2z=-6$

$$\begin{cases} -2x+2y-2z=-6 \\ \;\; 2x+\;\;y-\;\;z=2 \end{cases} \Rightarrow 3y-3z=-4 \quad (4)$$

$$\begin{cases} x-\;\;y+\;\;z=3 \\ 2x+\;\;y-\;\;z=2 \\ x+2y-2z=-1 \end{cases} \Rightarrow \begin{cases} x-\;\;y+\;\;z=3 & \\ \quad\; 3y-3z=-4 & (4) \\ x+2y-2z=-1 & \end{cases}$$

Multiply the equation (1) by −1, add the result to equation (3), and replace equation (3) with the new equation (5).

$-1(x-y+z=3)\Rightarrow -x+y-z=-3$

$$\begin{cases} -x+\;\;y-\;\;z=-3 \\ \;\;\, x+2y-2z=-1 \end{cases} \Rightarrow 3y-3z=-4 \quad (5)$$

$$\begin{cases} x-\;\;y+\;\;z=3 \\ \quad\; 3y-3z=-4 \\ x+2y-2z=-1 \end{cases} \Rightarrow \begin{cases} x-\;\;y+\;\;z=3 & \\ \quad\; 3y-3z=-4 & (4) \\ \quad\; 3y-3z=-4 & (5) \end{cases}$$

Multiply equation (4) by −1, add the result to equation (5), then replace equation (5) with the result (6).

$-1(3y-3z=-4)=-3y+3z=4$

$$\begin{cases} 3y-3z=-4 \\ -3y+3z=4 \end{cases} \Rightarrow 0=0 \quad (6)$$

$$\begin{cases} x-\;\;y+\;\;z=3 \\ \quad\; 3y-3z=-4 \\ \quad\; 3y-3z=-4 \end{cases} \Rightarrow \begin{cases} x-\;\;y+\;\;z=3 & (1) \\ \quad\; 3y-3z=-4 & (4) \\ \qquad\qquad 0=0 & (6) \end{cases}$$

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The system is now in triangular form. Solve equation (4) for y:

$3y-3z=-4 \Rightarrow y-z=-\frac{4}{3} \Rightarrow y=z-\frac{4}{3}$

Substitute the expression for y into equation (1) and solve for x.

$x-\left(z-\frac{4}{3}\right)+z=3 \Rightarrow x+\frac{4}{3}=3 \Rightarrow x=\frac{5}{3}$

The solution set for the system is $\left\{\left(\frac{5}{3}, z-\frac{4}{3}, z\right)\right\}$.

41. Multiply the second equation by −2, add the result to the third equation, and replace the second equation with the result:

$-2(2x+y=8) \Rightarrow -4x-2y=-16$

$\begin{cases} 2y+3z=1 \\ -4x-2y=-16 \end{cases} \Rightarrow -4x+3z=-15$

$\begin{cases} 3x-2z=11 \\ 2x+y=8 \\ 2y+3z=1 \end{cases} \Rightarrow \begin{cases} 3x-2z=11 \\ -4x+3z=-15 \\ 2y+3z=1 \end{cases}$

Multiply the first equation by 4 and the second equation by 3, add the two result and replace the second equation:

$4(3x-2z=11) \Rightarrow 12x-8z=44$

$3(-4x+3z=-15) \Rightarrow -12x+9z=-45$

$\begin{cases} 12x-8z=44 \\ -12x+9z=-45 \end{cases} \Rightarrow z=-1$

$\begin{cases} 3x-2z=11 \\ -4x+3z=-15 \\ 2y+3z=1 \end{cases} \Rightarrow$

$\begin{cases} 3x-2z=11 \\ z=-1 \\ 2y+3z=1 \end{cases} \Rightarrow \begin{cases} 3x-2z=11 \\ 2y+3z=1 \\ z=-1 \end{cases}$

Substitute $z=-1$ into the first and second equations to solve for x and y:

$2y+3(-1)=1 \Rightarrow y=2.$

$3x-2(-1)=11 \Rightarrow x=3.$

The solution is {(3, 2, –1)}.

42. Multiply the second equation by −2, add the result to the first equation, and replace the second equation with the new equation:

$-2(x+2z=3) \Rightarrow -2x-4z=-6$

$\begin{cases} 2x+y=4 \\ -2x-4z=-6 \end{cases} \Rightarrow y-4z=-2$

$\begin{cases} 2x+y=4 \\ x+2z=3 \\ 3y-z=5 \end{cases} \Rightarrow \begin{cases} 2x+y=4 \\ y-4z=-2 \\ 3y-z=5 \end{cases}$

Multiply the second equation by −3, add the result to the third equation, and solve for z:

$-3(y-4z=-2) \Rightarrow -3y+12z=6$

$\begin{cases} -3y+12z=6 \\ 3y-z=5 \end{cases} \Rightarrow 11z=11 \Rightarrow z=1$

$\begin{cases} 2x+y=4 \\ y-4z=-2 \\ 3y-z=5 \end{cases} \Rightarrow \begin{cases} 2x+y=4 \\ y-4z=-2 \\ z=1 \end{cases}$

Substitute $z = 1$ into the second equation to solve for y: $y-4(1)=-2 \Rightarrow y=2$. Substitute $y = 2$ into the first equation to solve for x:

$2x+2=4 \Rightarrow x=1.$

The solution is {(1, 2, 1)}.

43. Multiplying the second equation by –1, and then adding the two equations, we have:

$\begin{cases} 2x+6y+11=0 \\ -6y+18z-1=0 \end{cases} \Rightarrow 2x+18z+10=0 \Rightarrow$

$2x+18z=-10 \Rightarrow x=-5-9z$

$6y-18z+1=0 \Rightarrow y=-\frac{1}{6}+3z.$

The solution is $\left\{\left(-5-9z, -\frac{1}{6}+3z, z\right)\right\}$.

The system is dependent.

44. Multiplying the first equation by –2, and then adding the two equations, we have

$-2(3x+5y-15)=0 \Rightarrow -6x-10y+30=0$

$\begin{cases} -6x-10y=-30 \\ 6x+20y-6z=11 \end{cases} \Rightarrow 10y-6z=-19 \Rightarrow$

$y=\frac{3}{5}z-\frac{19}{10}$. Substituting this expression into the first equation, we have

$3x+5\left(\frac{3}{5}z-\frac{19}{10}\right)=15 \Rightarrow 3x+3z=\frac{49}{2} \Rightarrow$

$x=\frac{49}{6}-z.$

The solution is $\left\{\left(\frac{49}{6}-z, \frac{3}{5}z-\frac{19}{10}, z\right)\right\}$.

The system is dependent.

8.2 Applying the Concepts

45. Let x = the amount invested at 4%, y = the amount invested at 5%, and z = the amount invested at 6%. Then, we have the system:

$\begin{cases} x+y+z=20{,}000 \\ 0.04x+0.05y+0.06z=1060 \Rightarrow \\ 0.06z=2(0.05y) \end{cases}$

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$$\begin{cases} x + y + z = 20{,}000 \\ 4x + 5y + 6z = 106{,}000 \Rightarrow \\ 6z = 10y \end{cases}$$

$$\begin{cases} x + y + z = 20{,}000 \\ 4x + 5y + 6z = 106{,}000 \\ -10y + 6z = 0 \end{cases}$$

Multiply the first equation by −4, add the result to the second equation, and replace the second equation with the new equation:

$-4(x + y + z = 20{,}000) \Rightarrow$
$-4x - 4y - 4z = -80{,}000$

$$\begin{cases} -4x - 4y - 4z = -80{,}000 \\ 4x + 5y + 6z = 106{,}000 \end{cases} \Rightarrow y + 2z = 26{,}000$$

$$\begin{cases} x + y + z = 20{,}000 \\ y + 2z = 26{,}000 \\ -10y + 6z = 0 \end{cases}$$

Multiply the second equation by 10, add the result to the third equation, and solve for z:

$10(y + 2z = 26{,}000) \Rightarrow 10y + 20z = 260{,}000$

$$\begin{cases} 10y + 20z = 260{,}000 \\ -10y + 6z = 0 \end{cases} \Rightarrow 26z = 260{,}000 \Rightarrow$$

$z = 10{,}000$

$$\begin{cases} x + y + z = 20{,}000 \\ y + 2z = 26{,}000 \Rightarrow \\ -10y + 6z = 0 \end{cases}$$

$$\begin{cases} x + y + z = 20{,}000 \\ y + 2z = 26{,}000 \\ z = 10{,}000 \end{cases}$$

Substitute z = 10,000 into the second equation to solve for y:

$y + 2(10{,}000) = 26{,}000 \Rightarrow y = 6000.$ Substitut e the values for y and z into the first equation to solve for x:

$x + 6000 + 10{,}000 = 20{,}000 \Rightarrow x = 4000.$

Miguel invested \$4000 at 4%, \$6000 at 5%, and \$10,000 at 6%.

46. Let u = the units digit, t = the tens digit, and h = the hundreds digit. The value of the number is $100h + 10t + u.$ Then, we have the system

$$\begin{cases} u + t + h = 14 \\ u + h = t \\ 100u + 10t + h = 100h + 10t + u + 297 \end{cases} \Rightarrow$$

$$\begin{cases} u + t + h = 14 \\ u - t + h = 0 \\ 99u - 99h = 297 \end{cases} \Rightarrow \begin{cases} u + t + h = 14 \\ u - t + h = 0 \\ u - h = 3 \end{cases}$$

Add the first and second equations, and replace the second equation with the new equation.

$$\begin{cases} u + t + h = 14 \\ u - t + h = 0 \\ u - h = 3 \end{cases} \Rightarrow \begin{cases} u + t + h = 14 \\ 2u + 2h = 14 \\ u - h = 3 \end{cases}$$

Multiply the third equation by −2, add the result to the second equation, and solve for h:

$-2(u - h = 3) \Rightarrow -2u + 2h = -6$

$$\begin{cases} 2u + 2h = 14 \\ -2u + 2h = -6 \end{cases} \Rightarrow 4h = 8 \Rightarrow h = 2$$

$$\begin{cases} u + t + h = 14 \\ 2u + 2h = 14 \Rightarrow \\ u - h = 3 \end{cases} \begin{cases} u + t + h = 14 \\ 2u + 2h = 14 \\ h = 2 \end{cases}$$

Substitute h = 2 into the second equation to solve for u: $2u + 2(2) = 14 \Rightarrow u = 5.$

Substitute the values for u and h into the first equation to solve for t: $5 + t + 2 = 14 \Rightarrow t = 7.$ The original number was 275.

47. Let a = the number of hours Alex worked, b = the number of hours Becky worked, and c = the number of hours Courtney worked. Then, we have the system:

$$\begin{cases} a + b + c = 6 \quad \Rightarrow \\ 124a + 118b + 132c = 741 \\ b + c = 2a \end{cases}$$

$$\begin{cases} a + b + c = 6 \\ 124a + 118b + 132c = 741 \\ -2a + b + c = 0 \end{cases}$$

Subtract the third equation from the first, and replace the first equation with the result:

$$\begin{cases} a + b + c = 6 \\ 124a + 118b + 132c = 741 \Rightarrow \\ -2a + b + c = 0 \end{cases}$$

$$\begin{cases} 3a = 6 \\ 124a + 118b + 132c = 741 \Rightarrow \\ -2a + b + c = 0 \end{cases}$$

$$\begin{cases} a = 2 \\ 124a + 118b + 132c = 741 \Rightarrow \\ -2a + b + c = 0 \end{cases}$$

Substitute a = 2 into the second and third equations and simplify:

$$\begin{cases} a = 2 \\ 124(2) + 118b + 132c = 741 \Rightarrow \\ -2(2) + b + c = 0 \end{cases}$$

$$\begin{cases} a = 2 \\ 118b + 132c = 493 \\ b + c = 4 \end{cases}$$

Multiply the third equation by −118, and add the result to the second equation to solve for c.

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$-118(b+c=4) \Rightarrow -118b-118c=-472$

$$\begin{cases} 118b+132c=493 \\ -118b-118c=-472 \end{cases} \Rightarrow 14c=21 \Rightarrow c=1.5$$

Substitute $c = 1.5$ into the third equation to solve for b: $b+1.5=4 \Rightarrow b=2.5$.

Alex worked 2 hours, Becky worked 2.5 hours, and Courtney worked 1.5 hours.

48. Let x = the number of 18-year olds, y = the number of 19-year olds, and z = the number of 20-year olds. Then, we have the system

$$\begin{cases} x+y+z=38 \\ \dfrac{18x+19y+20z}{38}=18.5 \Rightarrow \\ x=8+y+z \end{cases}$$

$$\begin{cases} x+y+z=38 \\ 18x+19y+20z=703 \\ x-y-z=8 \end{cases}$$

Add the first and third equations, and replace the first equation with the result:

$$\begin{cases} x+y+z=38 \\ 18x+19y+20z=703 \Rightarrow \\ x-y-z=8 \end{cases}$$

$$\begin{cases} 2x=46 \\ 18x+19y+20z=703 \Rightarrow \\ x-y-z=8 \end{cases}$$

$$\begin{cases} x=23 \\ 18x+19y+20z=703 \\ x-y-z=8 \end{cases}$$

Substitute $x = 23$ into the second and third equations, then solve for y and z:

$$\begin{cases} x=23 \\ 18(23)+19y+20z=703 \Rightarrow \\ 23-y-z=8 \end{cases}$$

$$\begin{cases} x=23 \\ 19y+20z=289 \Rightarrow \\ -y-z=-15 \end{cases}$$

$$\begin{cases} x=23 \\ 19y+20z=289 \\ -19y-19z=-285 \end{cases} \Rightarrow \begin{cases} x=23 \\ 19y+20z=289 \\ z=4 \end{cases}$$

$19y+20(4)=289 \Rightarrow y=11$

There are twenty-three 18-year olds, eleven 19-year olds, and four 20-year olds.

49. Let n = the number of nickels, d = the number of dimes, and q = the number of quarters. Then, we have the system

$$\begin{cases} n+d+q=300 \\ d=3(n+q) \Rightarrow \\ 0.05n+0.1d+0.25q=30.05 \end{cases}$$

$$\begin{cases} n+d+q=300 \\ -3n+d-3q=0 \\ 5n+10d+25q=3005 \end{cases}$$

Multiply the first equation by 3, add the result to the second equation, and replace the second equation with the new equation:

$3(n+d+q=300) \Rightarrow 3n+3d+3q=900$

$$\begin{cases} 3n+3d+3q=900 \\ -3n+d-3q=0 \end{cases} \Rightarrow 4d=900 \Rightarrow d=225$$

$$\begin{cases} n+d+q=300 \\ d=225 \\ 5n+10d+25q=3005 \end{cases}$$

Multiply the first equation by −5, add the result to the third equation, and replace the third equation with the new equation:

$-5(n+d+q=300) \Rightarrow -5n-5d-5q=-1500$

$$\begin{cases} -5n-5d-5q=-1500 \\ 5n+10d+25q=3005 \end{cases} \Rightarrow 5d+20q=1505$$

$$\begin{cases} n+d+q=300 \\ d=225 \Rightarrow \\ 5n+10d+25q=3005 \end{cases}$$

$$\begin{cases} n+d+q=300 \\ d=225 \\ 5d+20q=1505 \end{cases}$$

Substitute $d = 225$ into the third equation to solve for q, then substitute the values for d and q into the first equation to solve for n:

$5(225)+20q=1505 \Rightarrow q=19.$

$n+225+19=300 \Rightarrow n=56.$

There are 56 nickels, 225 dimes, and 19 quarters.

50. Let t = the number of touchdowns, f = the number of field goals, and p = the number of extra points. Then, we have the system

$$\begin{cases} 6t+3f+p=46 \\ 2+6t=2(3f+p) \Rightarrow \\ 2(6t)=5(3f) \end{cases} \begin{cases} 6t+3f+p=46 \\ 6t-6f-2p=-2 \Rightarrow \\ 12t=15f \end{cases}$$

$$\begin{cases} 6t+3f+p=46 \\ 6t-6f-2p=-2 \\ 12t-15f=0 \end{cases}$$

Subtract the second equation from the first equation, and replace the second equation with the new equation:

$$\begin{cases} 6t+3f+p=46 \\ 6t-6f-2p=-2 \Rightarrow \\ 12t-15f=0 \end{cases} \begin{cases} 6t+3f+p=46 \\ 9f+3p=48 \\ 12t-15f=0 \end{cases}$$

Multiply the first equation by −2, add the result to the third equation, and replace the third equation with the new equation.

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$-2(6t+3f+p=46) \Rightarrow -12t-6f-2p=-92$

$\begin{cases} -12t-6f-2p=-92 \\ 12t-15f \qquad =0 \end{cases} \Rightarrow -21f-2p=-92$

$\begin{cases} 6t+3f+p=46 \\ 9f+3p=48 \\ 12t-15f=0 \end{cases} \Rightarrow \begin{cases} 6t+3f+p=46 \\ 9f+3p=48 \\ -21f-2p=-92 \end{cases}$

Multiply the second equation by $\frac{21}{9}$ and add the result to the third equation to solve for p:

$\frac{21}{9}(9f+3p=48) \Rightarrow 21f+7p=112$

$\begin{cases} 21f+7p=112 \\ -21f-2p=-92 \end{cases} \Rightarrow 5p=20 \Rightarrow p=4.$

Substitute $p = 4$ into the second equation to solve for f, and then substitute the values for p and f into the first equation to solve for t:

$9f+3(4)=48 \Rightarrow f=4.$

$6t+3(4)+4=46 \Rightarrow t=5.$

There were 5 touchdowns, 4 field goals, and 4 extra points.

51. Let x = the number of daytime hours Amy worked, y = the number of night hours Amy worked, and z = the number of holiday hours Amy worked. Then, we have the system

$\begin{cases} x+y+z=53 \\ 7.40x+9.20y+11.75z=452.20 \\ x=y+z+9 \end{cases} \Rightarrow$

$\begin{cases} x+y+z=53 \\ 7.40x+9.20y+11.75z=452.20 \\ x-y-z=9 \end{cases}$

Add the first and third equations, and replace the first equation with the result:

$\begin{cases} x+y+z=53 \\ 7.40x+9.20y+11.75z=452.20 \\ x-y-z=9 \end{cases} \Rightarrow$

$\begin{cases} 2x=62 \\ 7.40x+9.20y+11.75z=452.20 \\ x-y-z=9 \end{cases} \Rightarrow$

$\begin{cases} x=31 \\ 7.40x+9.20y+11.75z=452.20 \\ x-y-z=9 \end{cases}$

Substitute $x = 31$ into the second and third equations and simplify:

$\begin{cases} x=31 \\ 7.40x+9.20y+11.75z=452.20 \\ x-y-z=9 \end{cases} \Rightarrow$

$\begin{cases} x=31 \\ 7.40(31)+9.20y+11.75z=452.20 \\ 31-y-z=9 \end{cases} \Rightarrow$

$\begin{cases} x=31 \\ 9.20y+11.75z=222.80 \\ -y-z=-22 \end{cases}$

Multiply the third equation by 9.2, add the result to the second equation, and solve for z:

$9.2(-y-z=-22) \Rightarrow -9.2y-9.2z=202.4$

$\begin{cases} 9.20y+11.75z=222.80 \\ -9.20y-9.20z=-202.40 \end{cases} \Rightarrow$

$2.55z=20.40 \Rightarrow z=8$

$\begin{cases} x=31 \\ 9.20y+11.75z=222.80 \\ -y-z=-22 \end{cases} \Rightarrow$

$\begin{cases} x=31 \\ 9.20y+11.75z=222.80 \\ z=8 \end{cases}$

$9.20y+11.75(8)=222.80 \Rightarrow y=14$

Amy worked 31 daytime hours, 14 night hours, and 8 holiday hours.

52. $\begin{cases} A=B+C \\ A+B+C=100 \\ 4A+5B+6C=480 \end{cases} \Rightarrow \begin{cases} A-B-C=0 \\ A+B+C=100 \\ 4A+5B+6C=480 \end{cases}$

Add the first and second equations, and replace the first equation with the new equation:

$\begin{cases} A-B-C=0 \\ A+B+C=100 \\ 4A+5B+6C=480 \end{cases} \Rightarrow$

$\begin{cases} 2A=100 \\ A+B+C=100 \\ 4A+5B+6C=480 \end{cases} \Rightarrow$

$\begin{cases} A=50 \\ A+B+C=100 \\ 4A+5B+6C=480 \end{cases}$

Substitute $A = 50$ into the second and third equations and simplify:

$\begin{cases} A=50 \\ A+B+C=100 \\ 4A+5B+6C=480 \end{cases} \Rightarrow$

$\begin{cases} A=50 \\ 50+B+C=100 \\ 4(50)+5B+6C=480 \end{cases} \Rightarrow \begin{cases} A=50 \\ B+C=50 \\ 5B+6C=280 \end{cases}$

Multiply the second equation by −5, and add the result to the third equation to solve for C.

$-5(B+C=50) \Rightarrow -5B-5C=-250$

$\begin{cases} -5B-5C=-250 \\ 5B+6C=280 \end{cases} \Rightarrow C=30$

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$$\begin{cases} A = 50 \\ B + C = 50 \Rightarrow B = 20 \\ C = 30 \end{cases}$$

She purchased 50 units at \$4, 20 units at \$5, and 30 units at \$6.

53. The system is

$$\begin{cases} x + y = 0.54 \text{ (beam 1)} \\ x + z = 0.40 \text{ (beam 2)} \\ y + z = 0.52 \text{ (beam 3)} \end{cases}$$

Multiply the first equation by –1, add the result to the second equation, and replace the second equation with the new equation:

$$-1(x + y = 0.54) \Rightarrow -x - y = -0.54$$

$$\begin{cases} -x - y = -0.54 \\ x + z = 0.40 \end{cases} \Rightarrow -y + z = -0.14$$

$$\begin{cases} x + y = 0.54 \\ x + z = 0.40 \\ y + z = 0.52 \end{cases} \Rightarrow \begin{cases} x + y = 0.54 \\ -y + z = -0.14 \\ y + z = 0.52 \end{cases}$$

Add the second and third equations and solve for z: $2z = 0.38 \Rightarrow z = 0.19$. Substituting this value into the original second equation, we have $x + 0.19 = 0.40 \Rightarrow x = 0.21$. Substituting this value into the original first equation, we have $0.21 + y = 0.54 \Rightarrow y = 0.33$.

Referring to table 8.1 in the text, we see that cell A contains healthy tissue (since $x = 0.21$), cell B contains tumorous tissue (since $y = 0.33$), and cell C contains healthy tissue (since $z = 0.19$).

54. The system is

$$\begin{cases} x + y = 0.65 \text{ (beam 1)} \\ x + z = 0.80 \text{ (beam 2)} \\ y + z = 0.75 \text{ (beam 3)} \end{cases}$$

Multiply the first equation by –1, add the result to the second equation, and replace the second equation with the new equation:

$$-1(x + y = 0.65) \Rightarrow -x - y = -0.65$$

$$\begin{cases} -x - y = -0.65 \\ x + z = 0.80 \end{cases} \Rightarrow -y + z = 0.15$$

$$\begin{cases} x + y = 0.65 \\ x + z = 0.80 \\ y + z = 0.75 \end{cases} \Rightarrow \begin{cases} x + y = 0.65 \\ -y + z = 0.15 \\ y + z = 0.75 \end{cases}$$

Add the second and third equations and solve for z: $2z = 0.90 \Rightarrow z = 0.45$.
Substituting this value into the original second equation, we have
$x + 0.45 = 0.80 \Rightarrow x = 0.35$. Substituting this value into the original first equation, we have $0.35 + y = 0.65 \Rightarrow y = 0.30$.

Referring to table 8.1 in the text, we see that cell A contains tumorous tissue (since $x = 0.35$), cell B contains tumorous tissue (since $y = 0.30$), and cell C contains bone (since $z = 0.45$).

55. The system is

$$\begin{cases} x + y = 0.51 \text{ (beam 1)} \\ x + z = 0.49 \text{ (beam 2)} \\ y + z = 0.44 \text{ (beam 3)} \end{cases}$$

Multiply the first equation by –1, add the result to the second equation, and replace the second equation with the new equation:

$$-1(x + y = 0.51) \Rightarrow -x - y = -0.51$$

$$\begin{cases} -x - y = -0.51 \\ x + z = 0.49 \end{cases} \Rightarrow -y + z = -0.02$$

$$\begin{cases} x + y = 0.51 \\ x + z = 0.49 \\ y + z = 0.44 \end{cases} \Rightarrow \begin{cases} x + y = 0.51 \\ -y + z = -0.02 \\ y + z = 0.44 \end{cases}$$

Add the second and third equations and solve for z: $2z = 0.42 \Rightarrow z = 0.21$. Substituting this value into the original second equation, we have $x + 0.21 = 0.49 \Rightarrow x = 0.28$. Substituting this value into the original first equation, we have $0.28 + y = 0.51 \Rightarrow y = 0.23$. Referring to table 8.1 in the text, we see that cell A contains healthy tissue (since $x = 0.28$), cell B contains healthy tissue (since $y = 0.23$), and cell C contains healthy tissue (since $z = 0.21$).

56. The system is

$$\begin{cases} x + y = 0.44 \text{ (beam 1)} \\ x + z = 2.21 \text{ (beam 2)} \\ y + z = 2.23 \text{ (beam 3)} \end{cases}$$

Multiply the first equation by –1, add the result to the second equation, and replace the second equation with the new equation:

$$-1(x + y = 0.44) \Rightarrow -x - y = -0.44$$

$$\begin{cases} -x - y = -0.44 \\ x + z = 2.21 \end{cases} \Rightarrow -y + z = 1.77$$

$$\begin{cases} x + y = 0.44 \\ x + z = 2.21 \\ y + z = 2.23 \end{cases} \Rightarrow \begin{cases} x + y = 0.44 \\ -y + z = 1.77 \\ y + z = 2.23 \end{cases}$$

Add the second and third equations and solve for z: $2z = 4 \Rightarrow z = 2$. Substituting this value into the original second equation, we have $x + 2 = 2.21 \Rightarrow x = 0.21$. Substituting this value into the original first equation, we have $0.21 + y = 0.44 \Rightarrow y = 0.23$. Referring to table 8.1 in the text, we see that cell A contains healthy tissue (since $x = 0.21$), cell B contains healthy tissue (since $y = 0.23$), and cell C contains metal (since $z = 2$).

8.2 Beyond the Basics

57. Because each of the ordered triples satisfies the given equation, we can find the values of the coefficients by solving the system

$$\begin{cases}1+0b+0c=d\\0+1b+0c=d\\0+0b+1c=d\end{cases}\Rightarrow\begin{cases}1=d\\b=d\\c=d\end{cases}\Rightarrow b=c=d=1.$$

The equation is $x+y+z=1$.

58. Because each of the ordered triples satisfies the given equation, we can find the values of the coefficients by solving the system

$$\begin{cases}\frac{1}{3}+0b+0c=d\\0+4b+3c=d\\1+2b+2c=d\end{cases}\Rightarrow\begin{cases}\frac{1}{3}=d\\4b+3c=d\\2b+2c=d-1\end{cases}\Rightarrow$$

$$\begin{cases}4b+3c=\frac{1}{3}\\2b+2c=\frac{1}{3}-1\end{cases}\Rightarrow\begin{cases}12b+9c=1\\b+c=-\frac{1}{3}\end{cases}\Rightarrow$$

$$\begin{cases}12b+\ \ 9c=1\\-12b-12c=4\end{cases}\Rightarrow -3c=5\Rightarrow c=-\frac{5}{3}$$

$$1+2b+2\left(-\frac{5}{3}\right)=\frac{1}{3}\Rightarrow 2b=\frac{8}{3}\Rightarrow b=\frac{4}{3}$$

The equation is $x+\frac{4}{3}y-\frac{5}{3}z=\frac{1}{3}$.

59. Because each of the ordered triples satisfies the given equation, we can find the values of the coefficients by solving the system

$$\begin{cases}3-4b+0c=d\\0+\frac{1}{4}b+\frac{1}{2}c=d\\1+1b-4c=d\end{cases}\Rightarrow\begin{cases}-4b-\qquad d=-3\\b+2c-4d=0\\b-4c-\ d=-1\end{cases}$$

Multiplying the second equation by 2, adding it to the third equation, and replacing the second equation with the new equation, we have

$$2(b+2c-4d=0)\Rightarrow 2b+4c-8d=0$$

$$\begin{cases}2b+4c-8d=0\\b-4c-\ d=-1\end{cases}\Rightarrow 3b-9d=-1$$

$$\begin{cases}-4b-\qquad d=-3\\b+2c-4d=0\\b-4c-\ d=-1\end{cases}\Rightarrow\begin{cases}-4b-\qquad d=-3\\3b-\qquad 9d=-1\\b-4c-\ d=1\end{cases}$$

From the first equation, we have $d=3-4b$. Substituting this into the second equation, we have

$$3b-9(3-4b)=-1\Rightarrow 39b=26\Rightarrow b=\frac{2}{3}.$$

Then $d=3-4\left(\frac{2}{3}\right)-3=\frac{1}{3}$.

$$\frac{1}{4}\left(\frac{2}{3}\right)+\frac{1}{2}c=\frac{1}{3}\Rightarrow\frac{1}{2}c=\frac{1}{6}\Rightarrow c=\frac{1}{3}.$$

The equation is $x+\frac{2}{3}y+\frac{1}{3}z=\frac{1}{3}$.

60. Because each of the ordered triples satisfies the given equation, we can find the values of the coefficients by solving the system

$$\begin{cases}0+1b-10c=d\\\frac{1}{8}+0b+\frac{1}{4}c=d\\1+\frac{1}{3}b-2c=d\end{cases}\Rightarrow\begin{cases}b-10c-\ d=0\\2c-8d=-1\\b-\ 6c-3d=-3\end{cases}$$

Multiplying the third equation by –1, adding the result to the first equation, and replacing the third equation with the new equation, we have

$$-1(b-6c-3d=-3)\Rightarrow -b+6c+3d=3$$

$$\begin{cases}b-10c-\ d=0\\-b+\ 6c+3d=3\end{cases}\Rightarrow -4c+2d=3$$

$$\begin{cases}b-10c-\ d=0\\2c-8d=-1\\b-\ 6c-3d=-3\end{cases}\Rightarrow\begin{cases}b-10c-\ d=0\\2c-8d=-1\\-4c+2d=3\end{cases}$$

Multiplying the second equation by 2 and adding it to the third equation, we have

$$2(2c-8d=-1)\Rightarrow 4c-16d=-2$$

$$\begin{cases}4c-16d=-2\\-4c+\ 2d=3\end{cases}\Rightarrow -14d=1\Rightarrow d=-\frac{1}{14}.$$

Substituting this value into the second equation, we have

$$2c-8\left(-\frac{1}{14}\right)=-1\Rightarrow 2c=-\frac{11}{7}\Rightarrow$$

$c=-\frac{11}{14}$. Substituting the values for c and d into the first equation, we have

$$b-10\left(-\frac{11}{14}\right)=-\frac{1}{14}\Rightarrow b=-\frac{111}{14}.$$

The equation is $x-\frac{111}{14}y-\frac{11}{14}z=-\frac{1}{14}$.

61. Because each of the ordered triples satisfies the given equation, we can find the values of the coefficients by solving the system

$$\begin{cases} a(0)^2 + b(0) + c = 1 \\ a(-1)^2 + b(-1) + c = 0 \\ a(1)^2 + b(1) + c = 4 \end{cases} \Rightarrow \begin{cases} c = 1 \\ a - b + c = 0 \\ a + b + c = 4 \end{cases}$$

Substituting $c = 1$ into the second and third equations and then solving for b and c, we have

$$\begin{cases} a - b + 1 = 0 \\ a + b + 1 = 4 \end{cases} \Rightarrow 2a = 2 \Rightarrow a = 1$$

$$1 - b + 1 = 0 \Rightarrow b = 2$$

The equation is $y = x^2 + 2x + 1$.

62. Because each of the ordered triples satisfies the given equation, we can find the values of the coefficients by solving the system

$$\begin{cases} a(0)^2 + b(0) + c = 2 \\ a(-1)^2 + b(-1) + c = 30 \\ a(2)^2 + b(2) + c = 6 \end{cases} \Rightarrow \begin{cases} c = 2 \\ a - b + c = 30 \\ 4a + 2b + c = 6 \end{cases}$$

Substituting $c = 2$ into the second and third equations and then solving for b and c, we have

$$\begin{cases} a - b + 2 = 30 \\ 4a + 2b + 2 = 6 \end{cases} \Rightarrow \begin{cases} 2a - 2b + 4 = 60 \\ 4a + 2b + 2 = 6 \end{cases} \Rightarrow$$

$$6a = 60 \Rightarrow a = 10$$

$$10 - b + 2 = 30 \Rightarrow b = -18$$

The equation is $y = 10x^2 - 18x + 2$.

63. Because each of the ordered triples satisfies the given equation, we can find the values of the coefficients by solving the system

$$\begin{cases} a(1)^2 + b(1) + c = 2 \\ a(-1)^2 + b(-1) + c = 4 \\ a(2)^2 + b(2) + c = 4 \end{cases} \Rightarrow \begin{cases} c = 2 \\ a - b + c = 4 \\ 4a + 2b + c = 4 \end{cases}$$

Substituting $c = 2$ into the second and third equations and then solving for b and c, we have

$$\begin{cases} a - b + 2 = 4 \\ 4a + 2b + 2 = 4 \end{cases} \Rightarrow \begin{cases} 2a - 2b + 4 = 8 \\ 4a + 2b + 2 = 4 \end{cases} \Rightarrow$$

$$6a = 6 \Rightarrow a = 1$$

$$1 - b + 2 = 4 \Rightarrow b = -1$$

The equation is $y = x^2 - x + 2$.

64. Because each of the ordered triples satisfies the given equation, we can find the values of the coefficients by solving the system

$$\begin{cases} a(0)^2 + b(0) + c = 3 \\ a(-1)^2 + b(-1) + c = 4 \\ a(1)^2 + b(1) + c = 6 \end{cases} \Rightarrow \begin{cases} c = 3 \\ a - b + c = 4 \\ a + b + c = 6 \end{cases}$$

Substituting $c = 3$ into the second and third equations and then solving for b and c, we have

$$\begin{cases} a - b + 3 = 4 \\ a + b + 3 = 6 \end{cases} \Rightarrow 2a = 4 \Rightarrow a = 2$$

$$2 - b + 3 = 4 \Rightarrow b = 1$$

The equation is $y = 2x^2 + x + 3$.

65. Because each of the ordered triples satisfies the given equation, we can find the values of the coefficients by solving the system

$$\begin{cases} 0^2 + 4^2 + a(0) + b(4) + c = 0 \\ \left(2\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2 + a\left(2\sqrt{2}\right) + b\left(2\sqrt{2}\right) + c = 0 \\ (-4)^2 + 0^2 + a(-4) + b(0) + c = 0 \end{cases} \Rightarrow$$

$$\begin{cases} 4b + c = -16 \\ \left(2\sqrt{2}\right)a + \left(2\sqrt{2}\right)b + c = -16 \\ -4a + c = -16 \end{cases}$$

From the third equation, we have $c = 4a - 16$. Substituting this expression into the first and second equations, we have

$$\begin{cases} 4b + 4a - 16 = -16 \\ \left(2\sqrt{2}\right)a + \left(2\sqrt{2}\right)b + 4a - 16 = -16 \end{cases} \Rightarrow$$

$$\begin{cases} 4a + 4b = 0 \\ \left(2\sqrt{2} + 4\right)a + \left(2\sqrt{2}\right)b = 0 \end{cases} \Rightarrow$$

$$\begin{cases} a + b = 0 \\ \left(2\sqrt{2} + 4\right)a + \left(2\sqrt{2}\right)b = 0 \end{cases} \Rightarrow$$

$$\begin{cases} a = -b \\ \left(2\sqrt{2} + 4\right)a + \left(2\sqrt{2}\right)b = 0 \end{cases} \Rightarrow$$

$$\left(2\sqrt{2} + 4\right)(-b) + \left(2\sqrt{2}\right)b = 0 \Rightarrow$$

$-4b = 0 \Rightarrow b = 0$. Substituting this value into the first equation, we have $4(0) + c = -16 \Rightarrow c = -16$. Substituting into the third equation, we have $-4a - 16 = -16 \Rightarrow a = 0$. The equation is $x^2 + y^2 - 16 = 0$.

66. Because each of the ordered triples satisfies the given equation, we can find the values of the coefficients by solving the system

$$\begin{cases} 0^2 + 3^2 + a(0) + b(3) + c = 0 \\ 0^2 + (-1)^2 + a(0) + b(-1) + c = 0 \Rightarrow \\ \left(\sqrt{3}\right)^2 + 2^2 + a\left(\sqrt{3}\right) + b(2) + c = 0 \end{cases}$$

$$\begin{cases} 3b + c = -9 \\ -b + c = -1 \\ \left(\sqrt{3}\right)a + 2b + c = -7 \end{cases}$$

Solve the first two equations for a and b:

$$\begin{cases} 3b + c = -9 \\ -b + c = -1 \end{cases} \Rightarrow \begin{cases} 3b + c = -9 \\ -3b + 3c = -3 \end{cases} \Rightarrow$$

$4c = -12 \Rightarrow c = -3$

$3b - 3 = -9 \Rightarrow b = -2.$ Substituting these values into the third equation, we have

$a\sqrt{3} + 2(-2) - 3 = -7 \Rightarrow a\sqrt{3} = 0 \Rightarrow a = 0.$

The equation is $x^2 + y^2 - 2y - 3 = 0.$

67. Because each of the ordered triples satisfies the given equation, we can find the values of the coefficients by solving the system

$$\begin{cases} 1^2 + 2^2 + a(1) + b(2) + c = 0 \\ 6^2 + (-3)^2 + a(6) + b(-3) + c = 0 \Rightarrow \\ (4)^2 + 1^2 + a(4) + b(1) + c = 0 \end{cases}$$

$$\begin{cases} a + 2b + c = -5 \\ 6a - 3b + c = -45 \\ 4a + b + c = -17 \end{cases}$$

Multiply the first equation by –6, add the result to the second equation, then replace the second equation with the new equation. Similarly, multiply the first equation by –4, add that result to the third equation, then replace the third equation with the new equation:

$-6(a + 2b + c = -5) \Rightarrow -6a - 12b - 6c = 30$

$$\begin{cases} -6a - 12b - 6c = 30 \\ 6a - 3b + c = -45 \end{cases} \Rightarrow -15b - 5c = -15 \Rightarrow$$

$3b + c = 3$

$-4(a + 2b + c = -5) \Rightarrow -4a - 8b - 4c = 20$

$$\begin{cases} -4a - 8b - 4c = 20 \\ 4a + b + c = -17 \end{cases} \Rightarrow -7b - 3c = 3$$

$$\begin{cases} a + 2b + c = -5 \\ 6a - 3b + c = -45 \Rightarrow \\ 4a + b + c = -17 \end{cases} \begin{cases} a + 2b + c = -5 \\ 3b + c = 3 \\ -7b - 3c = 3 \end{cases}$$

From the second equation, we have $c = -3b + 3.$ Substitute this expression into the third equation, and solve for b.

$-7b - 3(-3b + 3) = 3 \Rightarrow 2b = 12 \Rightarrow b = 6.$

So, $c = -3(6) + 3 \Rightarrow c = -15.$ Substituting into the original first equation, we have $a + 2(6) - 15 = -5 \Rightarrow a = -2.$

The equation is $x^2 + y^2 - 2x + 6y - 15 = 0.$

68. Because each of the ordered triples satisfies the given equation, we can find the values of the coefficients by solving the system

$$\begin{cases} 5^2 + 6^2 + a(5) + b(6) + c = 0 \\ (-1)^2 + 6^2 + a(-1) + b(6) + c = 0 \Rightarrow \\ 3^2 + 2^2 + a(3) + b(2) + c = 0 \end{cases}$$

$$\begin{cases} 5a + 6b + c = -61 \\ -a + 6b + c = -37 \\ 3a + 2b + c = -13 \end{cases}$$

Switch the first and second equations. Multiply the first equation by 5, add the result to the second equation, then replace the second equation with the new equation. Similarly, multiply the first equation by 3, add that result to the third equation, then replace the third equation with the new equation:

$$\begin{cases} 5a + 6b + c = -61 \\ -a + 6b + c = -37 \Rightarrow \\ 3a + 2b + c = -13 \end{cases} \begin{cases} -a + 6b + c = -37 \\ 5a + 6b + c = -61 \\ 3a + 2b + c = -13 \end{cases}$$

$5(-a + 6b + c = -37) \Rightarrow -5a + 30b + 5c = -185$

$$\begin{cases} -5a + 30b + 5c = -185 \\ 5a + 6b + c = -61 \end{cases} \Rightarrow 36b + 6c = -246 \Rightarrow$$

$6b + c = -41$

$3(-a + 6b + c = -37) \Rightarrow -3a + 18b + 3c = -111$

$$\begin{cases} -3a + 18b + 3c = -111 \\ 3a + 2b + c = -13 \end{cases} \Rightarrow 20b + 4c = -124 \Rightarrow$$

$5b + c = -31$

$$\begin{cases} -a + 6b + c = -37 \\ 5a + 6b + c = -61 \Rightarrow \\ 3a + 2b + c = -13 \end{cases} \begin{cases} -a + 6b + c = -37 \\ 6b + c = -41 \\ 5b + c = -31 \end{cases}$$

Multiplying the third equation by –1, then adding the result to the second equation, we have

$$\begin{cases} 6b + c = -41 \\ -5b - c = 31 \end{cases} \Rightarrow b = -10.$$

So, $5(-10) + c = -31 \Rightarrow c = 19$

Substituting into the original second equation, we have $-a + 6(-10) + 19 = -37 \Rightarrow a = -4.$

The equation is $x^2 + y^2 - 4x - 10y + 19 = 0.$

69. Letting $u = 1/x, v = 1/y, w = 1/z$, we have

$$\begin{cases} \frac{1}{x} + \frac{3}{y} - \frac{1}{z} = 5 \\ \frac{2}{x} + \frac{4}{y} + \frac{6}{z} = 4 \\ \frac{2}{x} + \frac{3}{y} + \frac{1}{z} = 3 \end{cases} \Rightarrow \begin{cases} u + 3v - w = 5 \\ 2u + 4v + 6w = 4 \\ 2u + 3v + w = 3 \end{cases}$$

Multiplying the first equation by –2, adding the result to the second and third equations, and replacing those equations with the new equations, we have

$$-2(u + 3v - w = 5) \Rightarrow -2u - 6v + 2w = -10$$

$$\begin{cases} -2u - 6v + 2w = -10 \\ 2u + 4v + 6w = 4 \end{cases} \Rightarrow -2v + 8w = -6 \Rightarrow v - 4w = 3$$

$$\begin{cases} -2u - 6v + 2w = -10 \\ 2u + 3v + \ \ w = 3 \end{cases} \Rightarrow -3v + 3w = -7$$

$$\begin{cases} u + 3v - w = 5 \\ 2u + 4v + 6w = 4 \\ 2u + 3v + w = 3 \end{cases} \Rightarrow \begin{cases} u + 3v - \ \ w = 5 \\ v - 4w = 3 \\ -3v + 3w = -7 \end{cases}$$

Multiplying the second equation by 3 and adding the result to the third equation, we have

$$3(v - 4w = 3) \Rightarrow 3v - 12w = 9$$

$$\begin{cases} 3v - 12w = 9 \\ -3v + \ 3w = -7 \end{cases} \Rightarrow -9w = 2 \Rightarrow w = -\frac{2}{9}.$$

Substituting, we have

$$v - 4\left(-\frac{2}{9}\right) = 3 \Rightarrow v = \frac{19}{9} \text{ and}$$

$$u + 3\left(\frac{19}{9}\right) - \left(-\frac{2}{9}\right) = 5 \Rightarrow u = -\frac{14}{9}.$$

So, $x = -\frac{9}{14}, y = \frac{9}{19}, z = -\frac{9}{2}.$

The solution set is $\left\{\left(-\frac{9}{14}, \frac{9}{19}, -\frac{9}{2}\right)\right\}.$

70. Letting $u = 1/x, v = 1/y, w = 1/z$, we have

$$\begin{cases} \frac{1}{x} + \frac{2}{y} + \frac{3}{z} = 8 \\ \frac{2}{x} + \frac{5}{y} + \frac{9}{z} = 16 \\ \frac{3}{x} - \frac{4}{y} - \frac{5}{z} = 32 \end{cases} \Rightarrow \begin{cases} u + 2v + 3w = 8 \\ 2u + 5v + 9w = 16 \\ 3u - 4v - 5w = 32 \end{cases}$$

Multiply the first equation by –2, add the result to the second equation, and replace the second equation with the new equation. Similarly, multiply the first equation by –3, add that result to the third equation, and replace the third equation with that new equation:

$$-2(u + 2v + 3w = 8) \Rightarrow -2u - 4v - 6w = -16$$

$$\begin{cases} -2u - 4v - 6w = -16 \\ 2u + 5v + 9w = 16 \end{cases} \Rightarrow v + 3w = 0$$

$$-3(u + 2v + 3w = 8) \Rightarrow -3u - 6v - 9w = -24$$

$$\begin{cases} -3u - 6v - 9w = -24 \\ 3u - 4v - 5w = 32 \end{cases} \Rightarrow -10v - 14w = 8 \Rightarrow -5v - 7w = 4$$

$$\begin{cases} u + 2v + 3w = 8 \\ 2u + 5v + 9w = 16 \\ 3u - 4v - 5w = 32 \end{cases} \Rightarrow \begin{cases} u + 2v + 3w = 8 \\ v + 3w = 0 \\ -5v - 7w = 4 \end{cases}$$

Multiplying the second equation by 5 and adding the result to the third equation, we have

$$5(v + 3w = 0) \Rightarrow 5v + 15w = 0$$

$$\begin{cases} 5v + 15w = 0 \\ -5v - 7w = 4 \end{cases} \Rightarrow 8w = 4 \Rightarrow w = \frac{1}{2}.$$

Substituting, we have

$$v + 3\left(\frac{1}{2}\right) = 0 \Rightarrow v = -\frac{3}{2} \text{ and}$$

$$u + 2\left(-\frac{3}{2}\right) + 3\left(\frac{1}{2}\right) = 8 \Rightarrow u = \frac{19}{2}.$$

So, $x = \frac{2}{19}, y = -\frac{2}{3}, z = 2.$

The solution set is $\left\{\left(\frac{2}{19}, -\frac{2}{3}, 2\right)\right\}.$

71. Solve the first three equations in the system for x, y, and z:

$$-3(x + 2y - 5z = 9) \Rightarrow -3x - 6y + 15z = -27$$

$$\begin{cases} -3x - 6y + 15z = -27 \\ 3x - \ y + \ 2z = 14 \end{cases} \Rightarrow -7y + 17z = -13$$

$$-2(x + 2y - 5z = 9) \Rightarrow -2x - 4y + 10z = -18$$

$$\begin{cases} -2x - 4y + 10z = -18 \\ 2x + 3y - \ \ z = 3 \end{cases} \Rightarrow -y + 9z = -15$$

$$\begin{cases} x + 2y - 5z = 9 \\ 3x - \ y + 2z = 14 \\ 2x + 3y - \ z = 3 \end{cases} \Rightarrow \begin{cases} x + 2y - \ 5z = 9 \\ -7y + 17z = -13 \\ -y + \ 9z = -15 \end{cases}$$

$$-7(-y + 9z = -15) \Rightarrow 7y - 63z = 105$$

$$\begin{cases} -7y + 17z = -13 \\ 7y - 63z = 105 \end{cases} \Rightarrow -46z = 92 \Rightarrow z = -2$$

$$-7y + 17(-2) = -13 \Rightarrow -7y = 21 \Rightarrow y = -3$$

$$x + 2(-3) - 5(-2) = 9 \Rightarrow x = 5.$$

Substituting $x = 5$, $y = -3$, and $z = -2$ into the fourth equation of the original system, we have

$$5c - 5(-3) + (-2) + 3 = 0 \Rightarrow$$

$$5c = -16 \Rightarrow c = -\frac{16}{5}.$$

72. Solve the first, third, and fourth equations in the system for x, y, and z:

$-1(x+5y+z=42) \Rightarrow -x-5y-z=-42$

$\begin{cases} -x-5y-\ z=-42 \\ \ x-\ y+3z=4 \end{cases} \Rightarrow -6y+2z=-38 \Rightarrow$

$-3y+z=-19$

$\begin{cases} -x-5y-z=-42 \\ \ x+\ y-z=0 \end{cases} \Rightarrow -4y-2z=-42 \Rightarrow$

$-2y-z=-21$

$\begin{cases} x+5y+z=42 \\ x-y+3z=4 \\ x+y-z=0 \end{cases} \Rightarrow \begin{cases} x+5y+z=42 \\ -3y+z=-19 \\ -2y-z=-21 \end{cases}$

$\begin{cases} -3y+z=-19 \\ -2y-z=-21 \end{cases} \Rightarrow -5y=-40 \Rightarrow y=8$

$-3(8)+z=-19 \Rightarrow z=5$

$x+5(8)+5=42 \Rightarrow x=-3.$

Substituting $x = -3$, $y = 8$ and $z = 5$ into the second equation of the original system, we have $3(-3)+8-3(5)=c \Rightarrow c=-16.$

73. Each of the ordered triples satisfies the given equation, so find the values of the coefficients by solving the system

$\begin{cases} a(-1)^2+\ b(-1)+\ c=-1 \\ a(0)^2+b(0)+c=5 \\ a(2)^2+\ b(2)+\ c=5 \end{cases} \Rightarrow \begin{cases} a-\ b+c=-1 \\ c=5 \\ 4a+2b+c=5 \end{cases}$

Substitute $c = 5$ into the first and third equations and then solve those equations for a and b

$\begin{cases} a-\ b+5=-1 \\ 4a+2b+5=5 \end{cases} \Rightarrow \begin{cases} a-b=-6 \\ 2a+b=0 \end{cases} \Rightarrow$

$3a=-6 \Rightarrow a=-2$

$-2-b+5=-1 \Rightarrow b=-4.$

The equation is $y=-2x^2+4x+5.$

74. a. Let x = the amount grid cell A weakens the beam, y = the amount grid cell B weakens the beam, z = the amount grid cell C weakens the beam, and w = the amount grid cell D weakens the beam. Then we have

$\begin{cases} x+y & =0.60 \text{ (beam 1)} \\ z+w & =0.75 \text{ (beam 2)} \\ x+w & =0.65 \text{ (beam 3)} \\ y+z & =0.70 \text{ (beam 4)} \end{cases}$

Subtract the third equation from the first, and replace the third equation with the result:

$\begin{cases} x+y=0.60 \\ z+w=0.75 \\ x+w=0.65 \\ y+z=0.70 \end{cases} \Rightarrow \begin{cases} x+y=0.60 \\ z+w=0.75 \\ y-w=-0.05 \\ y+z=0.70 \end{cases}$

Add the second and third equations, and replace the third equation with the result:

$\begin{cases} x+y=0.60 \\ z+w=0.75 \\ y-w=-0.05 \\ y+z=0.70 \end{cases} \Rightarrow$

$\begin{cases} x+y=0.60 \\ z+w=0.75 \\ y+z=0.70 \\ y+z=0.70 \end{cases}$

Note that the third and fourth equations are the same and $z=0.70-y.$ So the second equation becomes $0.70-y+w=0.75 \Rightarrow$ $y=-0.05+w$ and the first equation becomes

$x+(-0.05+w)=0.60 \Rightarrow x=0.65-w.$

$z=0.70-(-0.05+w) \Rightarrow z=0.075-w.$

There are infinitely many solutions for the system.

b. The system is

$\begin{cases} x+y & =0.60 \text{ (beam 1)} \\ z+w & =0.75 \text{ (beam 2)} \\ x+w & =0.65 \text{ (beam 3)} \\ y+z & =0.70 \text{ (beam 4)} \\ y+w & =0.85 \text{ (beam 5)} \\ x+z & =0.50 \text{ (beam 6)} \end{cases}$

Subtract equation 3 from equation 1,and replace equation 3:

$\begin{cases} x+y=0.60 \\ z+w=0.75 \\ x+w=0.65 \\ y+z=0.70 \\ y+w=0.85 \\ x+z=0.50 \end{cases} \Rightarrow \begin{cases} x+y=0.60 \\ z+w=0.75 \\ y-w=-0.05 \\ y+z=0.70 \\ y+w=0.85 \\ x+z=0.50 \end{cases}$

Add equations 3 and 5 to solve for y:

$\begin{cases} x+y=0.60 \\ z+w=0.75 \\ y-w=-0.05 \\ y+z=0.70 \\ y+w=0.85 \\ x+z=0.50 \end{cases} \Rightarrow$

$\begin{cases} x+y=0.60 \\ z+w=0.75 \\ 2y=0.80 \\ y+z=0.70 \\ y+w=0.85 \\ x+z=0.50 \end{cases} \Rightarrow$

(*continued on next page*)

(*continued*)

$$\begin{cases} x+y = 0.60 \\ z+w = 0.75 \\ y = 0.40 \\ y+z = 0.70 \\ y+w = 0.85 \\ x+z = 0.50 \end{cases} \Rightarrow \begin{matrix} x=0.2, z=0.3, \\ w=0.45 \end{matrix}$$

Using table 8.1, we find that cell C contains tumorous tissue. Notice that it is only necessary to use beams 2, 4, and 5 to find that the cell C weakens the beam by 0.3 unit.

8.2 Critical Thinking/Discussion/Writing

Answers may vary for exercises 75 and 76.

75. $\begin{cases} 1+(-1)-2=-2 \\ 2(1)-(-1)+3(2)=9 \\ 1+(-1)+2=2 \end{cases} \Rightarrow \begin{cases} x+y-z=-2 \\ 2x-y+3z=9 \\ x+y+z=2 \end{cases}$

76. a. $\begin{cases} x+3y+3z=15 \\ x+2y+z=1 \\ 2y+4z=11 \end{cases}$ **b.** $\begin{cases} x+2y-z=4 \\ x+3y+2z=5 \\ y+3z=1 \end{cases}$

8.2 Maintaining Skills

77. $\frac{1}{2\cdot 3}=\frac{1}{2}+\frac{B}{3} \Rightarrow 1=3+2B \Rightarrow -2=2B \Rightarrow$
$B=-1$

78. $\frac{2}{5\cdot 7}=\frac{A}{5}-\frac{1}{7} \Rightarrow 2=7A-5 \Rightarrow 7=7A \Rightarrow$
$A=1$

79. $\frac{1}{n\cdot(n+1)}=\frac{A}{n}-\frac{1}{n+1} \Rightarrow 1=A(n+1)-n \Rightarrow$
$1+n=A(n+1) \Rightarrow 1=A$

80. $\frac{4}{n\cdot(n+2)}=\frac{2}{n}+\frac{B}{n+2} \Rightarrow 4=2(n+2)+Bn \Rightarrow$
$4=2n+4+Bn \Rightarrow 0=2n+Bn \Rightarrow$
$-2n=Bn \Rightarrow -2=B$

81. $2x+3=A(x+3)+B(x-1)$
$2x+3=Ax+3A+Bx-B=(A+B)x+3A-B$
Equating the coefficients gives the system
$\begin{cases} A+B=2 \\ 3A-B=3 \end{cases} \Rightarrow 4A=5 \Rightarrow A=\frac{5}{4}$
$\frac{5}{4}+B=2 \Rightarrow B=\frac{3}{4}$

82. $3x+2=A(x-2)+B(x+2)$
$3x+2=Ax-2A+Bx+2B$
$=(A+B)x-2A+2B$
Equating the coefficients gives the system
$\begin{cases} A+B=3 \\ -2A+2B=2 \end{cases} \Rightarrow \begin{cases} A+B=3 \\ A-B=-1 \end{cases} \Rightarrow 2A=2 \Rightarrow$
$A=1$
$1+B=3 \Rightarrow B=2$

83. $x^2+5x+6=(x+2)(x+3)$

84. $x^2-3x-10=(x+2)(x-5)$

85. $2x^2+5x-3=(2x-1)(x+3)$

86. $3x^2+x-2=(3x-2)(x+1)$

87. $4x^2-9=(2x-3)(2x+3)$

88. $x^3-8=(x-2)(x^2+2x+4)$

8.3 Partial-Fraction Decomposition

8.3 Practice Problems

1. $\frac{2x-7}{(x+1)(x-2)}=\frac{A}{x+1}+\frac{B}{x-2} \Rightarrow$
$2x-7=A(x-2)+B(x+1) \Rightarrow$
$2x-7=(A+B)x+(-2A+B) \Rightarrow$
$\begin{cases} A+B=2 \\ -2A+B=-7 \end{cases} \Rightarrow \begin{cases} -A-B=-2 \\ -2A+B=-7 \end{cases} \Rightarrow$
$-3A=-9 \Rightarrow A=3$
$3+B=2 \Rightarrow B=-1$
$\frac{2x-7}{(x+1)(x-2)}=\frac{3}{x+1}-\frac{1}{x-2}$

2. $\frac{3x^2+4x+3}{x^3-x}=\frac{3x^2+4x+3}{x(x-1)(x+1)}$
$=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+1} \Rightarrow$

$3x^2+4x+3$
$=A(x+1)(x-1)+Bx(x+1)+Cx(x-1)$
$=Ax^2-A+Bx^2+Bx+Cx^2-Cx$
$=(A+B+C)x^2+(B-C)x-A$

$\begin{cases} A+B+C=3 \\ B-C=4 \\ -A=3 \end{cases} \Rightarrow A=-3$

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$$\begin{cases} -3+B+C=3 \\ B-C=4 \end{cases} \Rightarrow -3+2B=7 \Rightarrow B=5$$
$$5-C=4 \Rightarrow C=1$$
$$\frac{3x^2+4x+3}{x^3-x} = -\frac{3}{x}+\frac{5}{x-1}+\frac{1}{x+1}$$

3. $$\frac{x+5}{x(x-1)^2} = \frac{A}{x}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2} \Rightarrow$$
$$x+5 = A(x-1)^2+Bx(x-1)+Cx \Rightarrow$$
$$x+5 = (A+B)x^2+(-2A-B+C)x+A$$
$$\begin{cases} A+B = 0 \\ -2A-B+C=1 \\ A = 5 \end{cases} \Rightarrow A=5,\ B=-5$$
$$-2A-B+C=1 \Rightarrow -2(5)+5+C=1 \Rightarrow C=6$$
$$\frac{x+5}{x(x-1)^2} = \frac{5}{x}-\frac{5}{(x-1)}+\frac{6}{(x-1)^2}$$

4. $$\frac{3x^2+5x-2}{x(x^2+2)} = \frac{A}{x}+\frac{Bx+C}{x^2+2} \Rightarrow$$
$$3x^2+5x-2 = A(x^2+2)+(Bx+C)x \Rightarrow$$
$$3x^2+5x-2 = (A+B)x^2+Cx+2A \Rightarrow$$
$$\begin{cases} A+B = 3 \\ C=5 \\ 2A = -2 \end{cases} \Rightarrow A=-1,\ C=5$$
$$A+B=3 \Rightarrow -1+B=3 \Rightarrow B=4$$
$$\frac{3x^2+5x-2}{x(x^2+2)} = -\frac{1}{x}+\frac{4x+5}{x^2+2}$$

5. $$\frac{x^2+3x+1}{(x^2+1)^2} = \frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2} \Rightarrow$$
$$x^2+3x+1 = (Ax+B)(x^2+1)+Cx+D \Rightarrow$$
$$x^2+3x+1 = Ax^3+Bx^2+(A+C)x+(B+D) \Rightarrow$$
$$\begin{cases} A = 0 \\ B = 1 \\ A+C = 3 \\ B+D=1 \end{cases} \Rightarrow A=0, B=1, C=3,\ D=0$$
$$\frac{x^2+3x+1}{(x^2+1)^2} = \frac{1}{x^2+1}+\frac{3x}{(x^2+1)^2}$$

6. $$\frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}+\frac{1}{6\cdot 7}+\cdots+\frac{1}{3111\cdot 3112}$$
$$=\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)+\left(\frac{1}{6}-\frac{1}{7}\right)+\cdots+\left(\frac{1}{3111}-\frac{1}{3112}\right)$$
$$=\frac{1}{4}-\frac{1}{3112}=\frac{778-1}{3112}=\frac{777}{3112}$$

7. $$R=\frac{(x+1)(x+2)}{4x+7} \Rightarrow \frac{1}{R}=\frac{4x+7}{(x+1)(x+2)}$$
$$\frac{4x+7}{(x+1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2} \Rightarrow$$
$$4x+7 = A(x+2)+B(x+1) \Rightarrow$$
$$4x+7 = (A+B)x+(2A+B) \Rightarrow$$
$$\begin{cases} A+B=4 \\ 2A+B=7 \end{cases} \Rightarrow A=3, B=1$$
$$\frac{4x+7}{(x+1)(x+2)}=\frac{3}{x+1}+\frac{1}{x+2} = \frac{1}{\frac{x+1}{3}}+\frac{1}{x+2}=\frac{1}{R}$$

This means that if two resistances $R_1 = \frac{x+1}{3}$ and $R_2 = x+2$ are connected in parallel, they will produce a total resistance given by $\frac{(x+1)(x+2)}{4x+7}$.

8.3 Basic Concepts and Skills

1. In a rational expression, if the degree of the numerator, $P(x)$, is less than the degree of the denominator, $Q(x)$, then the expression is proper.
2. The numerators in the partial-fraction decomposition of a rational expression are constants if the denominator, $Q(x)$, can be factored into distinct linear factors.
3. In a rational expression, if $(x-8)$ is a linear factor that is repeated three times in the denominator, then the portion of the expression's partial-fraction decomposition that corresponds to $(x-8)^3$ has three terms.
4. True
5. False. The factors are repeated linear factors.
6. True
7. $$\frac{1}{(x-1)(x+2)}=\frac{A}{x-1}+\frac{B}{x+2}$$
8. $$\frac{x}{(x+1)(x-3)}=\frac{A}{x+1}+\frac{B}{x-3}$$
9. $$\frac{1}{x^2+7x+6}=\frac{1}{(x+6)(x+1)}=\frac{A}{x+6}+\frac{B}{x+1}$$
10. $$\frac{3}{x^2-6x+8}=\frac{3}{(x-2)(x-4)}=\frac{A}{x-2}+\frac{B}{x-4}$$

11. $\frac{2}{x^3-x^2}=\frac{2}{x^2(x-1)}=\frac{A}{x^2}+\frac{B}{x}+\frac{C}{x-1}$

12. $\frac{x-1}{(x+2)^2(x-3)}=\frac{A}{(x+2)^2}+\frac{B}{x-3}+\frac{C}{x+2}$

13. $\frac{x^2-3x+3}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}$

14. $\frac{2x+3}{(x-1)^2(x^2+x+1)}$
$=\frac{A}{(x-1)^2}+\frac{Bx+C}{(x^2+x+1)}+\frac{D}{x-1}$

15. $\frac{3x-4}{(x^2+1)^2}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}$

16. $\frac{x-2}{(2x+3)^2(x^2+3)^2}$
$=\frac{A}{(2x+3)^2}+\frac{Bx+C}{(x^2+3)^2}+\frac{D}{2x+3}+\frac{Ex+F}{x^2+3}$

17. $\frac{2x+1}{(x+1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2}\Rightarrow$
$2x+1=A(x+2)+B(x+1)\Rightarrow$
$2x+1=(A+B)x+(2A+B)\Rightarrow$
$\begin{cases}A+B=2\\2A+B=1\end{cases}\Rightarrow\begin{cases}-A-B=-2\\2A+B=1\end{cases}\Rightarrow A=-1, B=3$
$\frac{2x+1}{(x+1)(x+2)}=-\frac{1}{x+1}+\frac{3}{x+2}=\frac{3}{x+2}-\frac{1}{x+1}$

18. $\frac{7}{(x-2)(x+5)}=\frac{A}{x-2}+\frac{B}{x+5}\Rightarrow$
$7=A(x+5)+B(x-2)\Rightarrow$
$7=(A+B)x+(5A-2B)\Rightarrow$
$\begin{cases}A+\ \ B=0\\5A-2B=7\end{cases}\Rightarrow\begin{cases}2A+2B=0\\5A-2B=7\end{cases}\Rightarrow 7A=7\Rightarrow$
$A=1, B=-1$
$\frac{7}{(x-2)(x+5)}=\frac{1}{x-2}-\frac{1}{x+5}$

19. $\frac{1}{x^2+4x+3}=\frac{1}{(x+3)(x+1)}=\frac{A}{x+3}+\frac{B}{x+1}\Rightarrow$
$1=A(x+1)+B(x+3)=(A+B)x+(A+3B)\Rightarrow$
$\begin{cases}A+\ \ B=0\\A+3B=1\end{cases}\Rightarrow\begin{cases}-A-\ \ B=0\\A+3B=1\end{cases}\Rightarrow 2B=1\Rightarrow$
$B=\frac{1}{2}, A=-\frac{1}{2}$
$\frac{1}{x^2+4x+3}=-\frac{1}{2(x+3)}+\frac{1}{2(x+1)}$

20. $\frac{x}{x^2+5x+6}=\frac{x}{(x+2)(x+3)}=\frac{A}{x+2}+\frac{B}{x+3}\Rightarrow$
$x=A(x+3)+B(x+2)\Rightarrow$
$x=(A+B)x+(3A+2B)\Rightarrow$
$\begin{cases}A+\ \ B=1\\3A+2B=0\end{cases}\Rightarrow\begin{cases}-2A-2B=-2\\3A+2B=0\end{cases}\Rightarrow A=-2,$
$B=3$
$\frac{x}{x^2+5x+6}=-\frac{2}{x+2}+\frac{3}{x+3}=\frac{3}{x+3}-\frac{2}{x+2}$

21. $\frac{2}{x^2+2x}=\frac{2}{x(x+2)}=\frac{A}{x}+\frac{B}{x+2}\Rightarrow$
$2=A(x+2)+Bx=(A+B)x+2A\Rightarrow$
$\begin{cases}A+B=0\\2A=2\end{cases}\Rightarrow A=1, B=-1\Rightarrow$
$\frac{2}{x^2+2x}=\frac{1}{x}-\frac{1}{x+2}$

22. $\frac{x+9}{x^2-9}=\frac{x+9}{(x-3)(x+3)}=\frac{A}{x-3}+\frac{B}{x+3}\Rightarrow$
$x+9=A(x+3)+B(x-3)\Rightarrow$
$x+9=(A+B)x+(3A-3B)\Rightarrow$
$\begin{cases}A+\ \ B=1\\3A-3B=9\end{cases}\Rightarrow\begin{cases}3A+3B=3\\3A-3B=9\end{cases}\Rightarrow 6A=12\Rightarrow$
$A=2, B=-1$
$\frac{x+9}{x^2-9}=\frac{2}{x-3}-\frac{1}{x+3}$

23. $\frac{x}{(x+1)(x+2)(x+3)}=\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x+3}\Rightarrow$
$x=A(x+2)(x+3)+B(x+1)(x+3)$
$+C(x+1)(x+2)\Rightarrow$
$x=A(x^2+5x+6)+B(x^2+4x+3)$
$+C(x^2+3x+2)\Rightarrow$
$x=(A+B+C)x^2+(5A+4B+3C)x$
$+(6A+3B+2C)\Rightarrow$
$\begin{cases}A+\ \ B+\ \ C=0\\5A+4B+3C=1\\6A+3B+2C=0\end{cases}$
$-5(A+\ \ B+\ \ C=0)=-5A-5B-5C=0$
$\begin{cases}-5A-5B-5C=0\\5A+4B+3C=1\end{cases}\Rightarrow -B-2C=1$
$-6(A+B+C=0)=-6A-6B-6C=0$
$\begin{cases}-6A-6B-6C=0\\6A+3B+2C=0\end{cases}\Rightarrow -3B-4C=0$
$\begin{cases}A+\ \ B+\ \ C=0\\5A+3B+3C=1\\6A+3B+2C=0\end{cases}\Rightarrow\begin{cases}A+\ \ B+\ \ C=0\\-B-2C=1\\-3B-4C=0\end{cases}$

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$$-3(-B-2C=1)=3B+6C=-3$$

$$\begin{cases}3B+6C=-3\\-3B-4C=0\end{cases}\Rightarrow 2C=-3\Rightarrow C=-\frac{3}{2}$$

$$-B-2\left(-\frac{3}{2}\right)=1\Rightarrow -B=-4\Rightarrow B=2$$

$$A+2-\frac{3}{2}=0\Rightarrow A=-\frac{1}{2}$$

$$\frac{x}{(x+1)(x+2)(x+3)} = -\frac{1}{2(x+1)}+\frac{2}{x+2}-\frac{3}{2(x+3)}$$

24. $$\frac{x^2}{(x-1)(x^2+5x+4)}=\frac{x^2}{(x-1)(x+1)(x+4)} = \frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x+4}\Rightarrow$$

$$x^2=A(x+1)(x+4)+B(x-1)(x+4) + C(x-1)(x+1)\Rightarrow$$

$$x^2=A(x^2+5x+4)+B(x^2+3x-4) + C(x^2-1)\Rightarrow$$

$$x^2=(A+B+C)x^2+(5A+3B)x + (4A-4B-C)\Rightarrow$$

$$\begin{cases}A+\ \ B+C=1\\5A+3B\qquad =0\\4A-4B-C=0\end{cases}$$

$$\begin{cases}A+\ \ B+C=1\\4A-4B-C=0\end{cases}\Rightarrow 5A-3B=1$$

$$\begin{cases}A+\ \ B+C=1\\5A+3B\qquad =0\\4A-4B-C=0\end{cases}\Rightarrow\begin{cases}A+\ \ B+C=1\\5A+3B\qquad =0\\5A-3B\qquad =1\end{cases}\Rightarrow$$

$$10A=1\Rightarrow A=\frac{1}{10}$$

$$5\left(\frac{1}{10}\right)+3B=0\Rightarrow 3B=-\frac{1}{2}\Rightarrow B=-\frac{1}{6}$$

$$\frac{1}{10}-\frac{1}{6}+C=1\Rightarrow C=\frac{16}{15}$$

$$\frac{x^2}{(x-1)(x^2+5x+4)} = \frac{1}{10(x-1)}-\frac{1}{6(x+1)}+\frac{16}{15(x+4)}$$

25. $$\frac{x-1}{(x+1)^2}=\frac{A}{(x+1)^2}+\frac{B}{(x+1)}\Rightarrow$$

$$x-1=A+B(x+1)=Bx+(A+B)\Rightarrow$$

$$\begin{cases}B=1\\A+B=-1\end{cases}\Rightarrow B=1, A=-2$$

$$\frac{x-1}{(x+1)^2}=-\frac{2}{(x+1)^2}+\frac{1}{(x+1)}$$

26. $$\frac{3x+2}{x^2(x+1)}=\frac{A}{x^2}+\frac{B}{x}+\frac{C}{x+1}\Rightarrow$$

$$3x+2=A(x+1)+Bx(x+1)+Cx^2\Rightarrow$$

$$3x+2=(A+B)x+(B+C)x^2+A\Rightarrow$$

$$\begin{cases}A+B\qquad =3\\\quad B+C=0\\A\qquad\quad\ =2\end{cases}\Rightarrow A=2, B=1, C=-1$$

$$\frac{3x+2}{x^2(x+1)}=\frac{2}{x^2}+\frac{1}{x}-\frac{1}{x+1}$$

27. $$\frac{2x^2+x}{(x+1)^3}=\frac{A}{(x+1)^3}+\frac{B}{(x+1)^2}+\frac{C}{x+1}\Rightarrow$$

$$2x^2+x=A+B(x+1)+C(x+1)^2 = A+Bx+B+C(x^2+2x+1) = Cx^2+(B+2C)x+(A+B+C)\Rightarrow$$

$$\begin{cases}A+B+\ C=0\\\quad B+2C=1\\\qquad C=2\end{cases}\Rightarrow C=2, B=-3, A=1$$

$$\frac{2x^2+x}{(x+1)^3}=\frac{2}{x+1}-\frac{3}{(x+1)^2}+\frac{1}{(x+1)^3}$$

28. $$\frac{x}{(x-1)(x+1)}=\frac{A}{x-1}+\frac{B}{x+1}\Rightarrow$$

$$x=A(x+1)+B(x-1)\Rightarrow$$

$$x=Ax+A+Bx-B\Rightarrow$$

$$x=(A+B)x+(A-B)\Rightarrow$$

$$\begin{cases}A+B=1\\A-B=0\end{cases}\Rightarrow 2A=1\Rightarrow A=\frac{1}{2}, B=\frac{1}{2}$$

$$\frac{x}{x^2-1}=\frac{1}{2(x-1)}+\frac{1}{2(x+1)}$$

29. $$\frac{-x^2+3x+1}{x^3+2x^2+x}=\frac{-x^2+3x+1}{x(x+1)^2} = \frac{A}{(x+1)^2}+\frac{B}{x+1}+\frac{C}{x}\Rightarrow$$

$$-x^2+3x+1=Ax+Bx(x+1)+C(x+1)^2 = Ax+Bx^2+Bx+Cx^2+2Cx+C = (B+C)x^2+(A+B+2C)x+C$$

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$$\begin{cases} B + C = -1 \Rightarrow C = 1, B = -2 \\ A + B + 2C = 3 \\ C = 1 \end{cases}$$

$$A + 2(-2) + 3(1) = 2 \Rightarrow A = 3$$

$$\frac{-x^2 + 3x + 1}{x^3 + 2x^2 + x} = \frac{1}{x} - \frac{2}{x+1} + \frac{3}{(x+1)^2}$$

30. $$\frac{5x^2 - 8x + 2}{x^3 - 2x^2 + x} = \frac{5x^2 - 8x + 2}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \Rightarrow$$

$$5x^2 - 8x + 2 = A(x-1)^2 + Bx(x-1) + Cx = (A+B)x^2 + (-2A - B + C)x + A \Rightarrow$$

$$\begin{cases} A + B = 5 \\ -2A - B + C = -8 \Rightarrow A = 2, B = 3 \\ A = 2 \end{cases}$$

$$-2(2) - 3 + C = -8 \Rightarrow C = -1$$

$$\frac{5x^2 - 8x + 2}{x^3 - 2x^2 + x} = \frac{2}{x} + \frac{3}{x-1} - \frac{1}{(x-1)^2}$$

31. $$\frac{1}{x^2(x+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} \Rightarrow 1 = Ax(x+1)^2 + B(x+1)^2 + Cx^2(x+1) + Dx^2$$

Letting $x = -1$, we have $D = 1$. Letting $x = 0$, we have $B = 1$. Substitute the values for B and D, expand the equation, and simplify:

$$1 = Ax(x+1)^2 + 1(x+1)^2 + Cx^2(x+1) + 1x^2 = (A+C)x^3 + (2 + 2A + C)x^2 + (2 + A)x + 1 \Rightarrow$$

$$\begin{cases} A + C = 0 \\ 2A + C = -2 \Rightarrow A = -2, C = 2 \\ A = -2 \end{cases}$$

$$\frac{1}{x^2(x+1)^2} = -\frac{2}{x} + \frac{1}{x^2} + \frac{2}{x+1} + \frac{1}{(x+1)^2}$$

32. $$\frac{2}{(x-1)^2(x+3)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3} + \frac{D}{(x+3)^2} \Rightarrow$$

$$2 = A(x-1)(x+3)^2 + B(x+3)^2 + C(x-1)^2(x+3) + D(x-1)^2$$

Letting $x = 1$, we have $2 = B(4)^2 \Rightarrow B = 1/8$. Letting $x = -3$, we have $2 = D(-3-1)^2 \Rightarrow D = 1/8$. Substitute the values for B and D, expand the equation, and simplify:

$$2 = A(x-1)(x+3)^2 + \frac{1}{8}(x+3)^2 + C(x-1)^2(x+3) + \frac{1}{8}(x-1)^2 \Rightarrow$$

$$2 = (A+C)x^3 + \left(\frac{1}{4} + 5A + C\right)x^2 + \left(\frac{1}{2} + 3A - 5C\right)x + \left(\frac{5}{4} - 9A + 3C\right) \Rightarrow$$

$$\begin{cases} A + C = 0 \\ 5A + C = -\frac{1}{4} \\ 3A - 5C = -\frac{1}{2} \Rightarrow A = -\frac{1}{16}, C = \frac{1}{16} \\ -9A + 3C = \frac{3}{4} \end{cases}$$

$$\frac{2}{(x-1)^2(x+3)^2} = -\frac{1}{16(x-1)} + \frac{1}{8(x-1)^2} + \frac{1}{16(x+3)} + \frac{1}{8(x+3)^2}$$

33. $\dfrac{1}{(x^2-1)^2}=\dfrac{1}{\left((x-1)(x+1)\right)^2}=\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x+1}+\dfrac{D}{(x+1)^2}\Rightarrow$

$$1=A(x-1)(x+1)^2+B(x+1)^2+C(x-1)^2(x+1)+D(x-1)^2$$

Letting $x = 1$, we have $1=B(2)^2\Rightarrow B=1/4$. Letting $x=-1$, we have $1=D(-2)^2\Rightarrow D=1/4$.
Substitute the values for B and D, expand the equation, and simplify:

$$1=A(x-1)(x+1)^2+\frac{1}{4}(x+1)^2+C(x-1)^2(x+1)+\frac{1}{4}(x-1)^2\Rightarrow$$

$$1=(A+C)x^3+\left(\frac{1}{2}+A-C\right)x^2+(-A-C)x+\left(\frac{1}{2}-A+C\right)\Rightarrow$$

$$\begin{cases} A+C=0 \\ A-C=-\dfrac{1}{2} \\ -A-C=0 \\ -A+C=\dfrac{1}{2}\end{cases}\Rightarrow A=-\frac{1}{4},C=\frac{1}{4}$$

$$\frac{1}{(x^2-1)^2}=-\frac{1}{4(x-1)}+\frac{1}{4(x-1)^2}+\frac{1}{4(x+1)}+\frac{1}{4(x+1)^2}$$

34. $\dfrac{3}{(x^2-4)^2}=\dfrac{3}{\left((x-2)(x+2)\right)^2}=\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{C}{x+2}+\dfrac{D}{(x+2)^2}\Rightarrow$

$$3=A(x-2)(x+2)^2+B(x+2)^2+C(x-2)^2(x+2)+D(x-2)^2$$

Letting $x = 2$, we have $3=B(4)^2\Rightarrow B=3/16$. Letting $x=-2$, we have $3=D(-4)^2\Rightarrow D=3/16$.
Substitute the values for B and D, expand the equation, and simplify:

$$3=A(x-2)(x+2)^2+\frac{3}{16}(x+2)^2+C(x-2)^2(x+2)+\frac{3}{16}(x-2)^2\Rightarrow$$

$$3=(A+C)x^3+\left(\frac{3}{8}+2A-2C\right)x^2+(-4A-4C)x+\left(\frac{3}{2}-8A+8C\right)\Rightarrow$$

$$\begin{cases} A+C=0 \\ 2A-2C=-\dfrac{3}{8} \\ -4A-4C=0 \\ -8A+8C=\dfrac{3}{2}\end{cases}\Rightarrow A=-\frac{3}{32},C=\frac{3}{32}$$

$$\frac{3}{(x^2-4)^2}=-\frac{3}{32(x-2)}+\frac{3}{16(x-2)^2}+\frac{3}{32(x+2)}+\frac{3}{16(x+2)^2}$$

35. $\dfrac{x-1}{(2x-3)^2}=\dfrac{A}{2x-3}+\dfrac{B}{(2x-3)^2}\Rightarrow x-1=A(2x-3)+B\Rightarrow x-1=2Ax+(-3A+B)$

$$\begin{cases} 2A =1 \\ -3A+B=-1\end{cases}\Rightarrow A=\frac{1}{2}$$

$$-3\left(\frac{1}{2}\right)+B=-1\Rightarrow B=\frac{1}{2}$$

$$\frac{x-1}{(2x-3)^2}=\frac{1}{2(2x-3)}+\frac{1}{2(2x-3)^2}$$

36. $\frac{6x+1}{(3x+1)^2}=\frac{A}{3x+1}+\frac{B}{(3x+1)^2}\Rightarrow 6x+1=A(3x+1)+B\Rightarrow 6x+1=3Ax+(A+B)\Rightarrow$

$\begin{cases}3A &=6\\ A+B &=1\end{cases}\Rightarrow A=2, B=-1$

$\frac{6x+1}{(3x+1)^2}=\frac{2}{3x+1}-\frac{1}{(3x+1)^2}$

37. $\frac{6x+7}{4x^2+12x+9}=\frac{6x+7}{(2x+3)^2}=\frac{A}{2x+3}+\frac{B}{(2x+3)^2}\Rightarrow 6x+7=A(2x+3)+B\Rightarrow 6x+7=2Ax+(3A+B)\Rightarrow$

$\begin{cases}2A &=6\\ 3A+B &=7\end{cases}\Rightarrow A=3, B=-2$

$\frac{6x+7}{4x^2+12x+9}=\frac{3}{2x+3}-\frac{2}{(2x+3)^2}$

38. $\frac{2x+3}{9x^2+30x+25}=\frac{2x+3}{(3x+5)^2}=\frac{A}{3x+5}+\frac{B}{(3x+5)^2}\Rightarrow 2x+3=A(3x+5)+B\Rightarrow 2x+3=3Ax+(5A+B)\Rightarrow$

$\begin{cases}3A &=2\\ 5A+B &=3\end{cases}\Rightarrow A=\frac{2}{3}, B=-\frac{1}{3}$

$\frac{2x+3}{9x^2+30x+25}=\frac{2}{3(3x+5)}-\frac{1}{3(3x+5)^2}$

39. $\frac{x-3}{x^3+x^2}=\frac{x-3}{x^2(x+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}\Rightarrow x-3=Ax(x+1)+B(x+1)+Cx^2\Rightarrow$

$x-3=(A+C)x^2+(A+B)x+B\Rightarrow$

$\begin{cases}A+\quad C=0\\ A+B \quad =1\\ \quad B \quad =-3\end{cases}\Rightarrow B=-3, A=4, C=-4$

$\frac{x-3}{x^3+x^2}=\frac{4}{x}-\frac{3}{x^2}-\frac{4}{x+1}$

40. $\frac{1}{x^3+x}=\frac{1}{x(x^2+1)}=\frac{A}{x}+\frac{Bx+C}{x^2+1}\Rightarrow 1=A(x^2+1)+x(Bx+C)=(A+B)x^2+Cx+A\Rightarrow$

$\begin{cases}A+B=0\\ \quad C=0\\ A \quad =1\end{cases}\Rightarrow A=1, B=-1, C=0$

$\frac{1}{x^3+x}=\frac{1}{x}-\frac{x}{x^2+1}$

41. $\frac{x^2+2x+4}{x^3+x^2}=\frac{x^2+2x+4}{x^2(x+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}\Rightarrow x^2+2x+4=Ax(x+1)+B(x+1)+Cx^2\Rightarrow$

$x^2+2x+4=(A+C)x^2+(A+B)x+B\Rightarrow$

$\begin{cases}A+\quad C=1\\ A+B \quad =2\\ \quad B \quad =4\end{cases}\Rightarrow B=4, A=-2, C=3$

$\frac{x^2+2x+4}{x^3+x^2}=\frac{3}{x+1}-\frac{2}{x}+\frac{4}{x^2}$

42. $\dfrac{x^2+2x-1}{(x-1)(x^2+1)}=\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+1}\Rightarrow x^2+2x-1=A(x^2+1)+(Bx+C)(x-1)\Rightarrow$

$x^2+2x-1=Ax^2+A+Bx^2-Bx+Cx-C\Rightarrow x^2+2x-1=(A+B)x^2+(-B+C)x+(A-C)\Rightarrow$

$$\begin{cases}A+B\quad=1\\ -B+C=2\\ A-\quad C=-1\end{cases}\Rightarrow A=1,B=0,C=2$$

$$\frac{x^2+2x+1}{(x-1)(x^2+1)}=\frac{1}{x-1}+\frac{2}{x^2+1}$$

43. $\dfrac{x}{x^4-1}=\dfrac{x}{(x-1)(x+1)(x^2+1)}=\dfrac{A}{x-1}+\dfrac{B}{x+1}+\dfrac{Cx+D}{x^2+1}\Rightarrow$

$x=A(x+1)(x^2+1)+B(x-1)(x^2+1)+(Cx+D)(x^2-1)\Rightarrow$

$x=(A+B+C)x^3+(A-B+D)x^2+(A+B-C)x+(A-B-D)\Rightarrow$

$$\begin{cases}A+B+C\quad=0\\ A-B+\quad D=0\\ A+B-C\quad=1\\ A-B-\quad D=0\end{cases}\Rightarrow A=\frac{1}{4},B=\frac{1}{4},C=-\frac{1}{2},D=0$$

$$\frac{x}{x^4-1}=\frac{1}{4(x-1)}+\frac{1}{4(x+1)}-\frac{x}{2(x^2+1)}$$

44. $\dfrac{3x^3-5x^2+12x+4}{x^4-16}=\dfrac{3x^3-5x^2+12x+4}{(x-2)(x+2)(x^2+4)}=\dfrac{A}{x-2}+\dfrac{B}{x+2}+\dfrac{Cx+D}{x^2+4}\Rightarrow$

$3x^3-5x^2+12x+4=A(x+2)(x^2+4)+B(x-2)(x^2+4)+(Cx+D)(x^2-4)$

Letting $x=-2$, we have

$3(-2)^3-5(-2)^2+12(-2)+4=A(-2+2)((-2)^2+4)+B(-2-2)((-2)^2+4)+(C(-2)+D)((-2)^2-4)\Rightarrow$

$-64=-32B\Rightarrow B=2$

Substitute the value for B, expand the equation, and simplify:

$3x^3-5x^2+12x+4=A(x+2)(x^2+4)+2(x-2)(x^2+4)+(Cx+D)(x^2-4)\Rightarrow$

$3x^3-5x^2+12x+4=(2+A+C)x^3+(-4+2A+D)x^2+(8+4A-4C)x+(-16+8A-4D)\Rightarrow$

$$\begin{cases}A+\quad C\quad=1\\ 2A+\quad D=-1\\ 4A-4C\quad=4\\ 8A-\quad 4D=20\end{cases}\Rightarrow A=1,C=0,D=-3$$

$$\frac{3x^3-5x^2+12x+4}{x^4-16}=\frac{1}{x-2}+\frac{2}{x+2}-\frac{3}{x^2+4}$$

45. $\dfrac{1}{x(x^2+1)^2}=\dfrac{A}{x}+\dfrac{Bx+C}{x^2+1}+\dfrac{Dx+E}{(x^2+1)^2}\Rightarrow$

$1=A(x^2+1)^2+(Bx+C)x(x^2+1)+(Dx+E)x\Rightarrow 1=(A+B)x^4+Cx^3+(2A+B+D)x^2+(C+E)x+A\Rightarrow$

$$\begin{cases}A+B\quad=0\\ C\quad=0\\ 2A+B+\quad D\quad=0\\ C+\quad E=0\\ A\quad=1\end{cases}\Rightarrow A=1,B=-1,C=0,D=-1,E=0$$

$$\frac{1}{x(x^2+1)^2}=\frac{1}{x}-\frac{x}{x^2+1}-\frac{x}{(x^2+1)^2}$$

46. $\dfrac{x^2}{(x^2+2)^2}=\dfrac{Ax+B}{x^2+2}+\dfrac{Cx+D}{(x^2+2)^2}\Rightarrow x^2=(Ax+B)(x^2+2)+Cx+D\Rightarrow$

$x^2=Ax^3+Bx^2+(2A+C)x+(2B+D)\Rightarrow$

$$\begin{cases}A & =0\\ \quad B & =1\\ 2A+\quad C & =0\\ \quad 2B+\quad D & =0\end{cases}\Rightarrow A=0, B=1, C=0, D=-2$$

$$\frac{x^2}{(x^2+2)^2}=\frac{1}{x^2+2}-\frac{2}{(x^2+2)^2}$$

47. $\dfrac{2x^2+3x}{(x^2+1)(x^2+2)}=\dfrac{Ax+B}{x^2+1}+\dfrac{Cx+D}{x^2+2}\Rightarrow 2x^2+3x=(Ax+B)(x^2+2)+(Cx+D)(x^2+1)\Rightarrow$

$2x^2+3x=(A+C)x^3+(B+D)x^2+(2A+C)x+(2B+D)\Rightarrow$

$$\begin{cases}A+\quad C & =0\\ \quad B+\quad D & =2\\ 2A+\quad C & =3\\ \quad 2B+\quad D & =0\end{cases}\Rightarrow A=3, B=-2, C=-3, D=4$$

$$\frac{2x^2+3x}{(x^2+1)(x^2+2)}=\frac{3x-2}{x^2+1}+\frac{-3x+4}{x^2+2}$$

48. $\dfrac{x^2-2x}{(x^2+9)(x^2+x+7)}=\dfrac{Ax+B}{(x^2+9)}+\dfrac{Cx+D}{(x^2+x+7)}\Rightarrow x^2-2x=(Ax+B)(x^2+x+7)+(Cx+D)(x^2+9)\Rightarrow$

$x^2-2x=(A+C)x^3+(A+B+D)x^2+(7A+B+9C)x+(7B+9D)\Rightarrow$

$$\begin{cases}A+\quad C & =0\\ A+\ B+\quad D & =1\\ 7A+\ B+9C & =-2\\ \quad 7B+\quad 9D & =0\end{cases}\Rightarrow A=1, B=0, C=-1, D=0$$

$$\frac{x^2-2x}{(x^2+9)(x^2+x+7)}=\frac{x}{x^2+9}-\frac{x}{x^2+x+7}$$

8.3 Applying the Concepts

49. $\dfrac{1}{1\cdot 2}+\dfrac{1}{2\cdot 3}+\dfrac{1}{3\cdot 4}+\cdots+\dfrac{1}{n(n+1)}=\left(1-\dfrac{1}{2}\right)+\left(\dfrac{1}{2}-\dfrac{1}{3}\right)+\left(\dfrac{1}{3}-\dfrac{1}{4}\right)+\cdots+\left(\dfrac{1}{n}-\dfrac{1}{n+1}\right)=1-\dfrac{1}{n+1}=\dfrac{n}{n+1}$

50. Each term is of the form $\dfrac{2}{k(k+2)}$. Decomposing $\dfrac{2}{k(k+2)}$ we have

$\dfrac{2}{k(k+2)}=\dfrac{A}{k}+\dfrac{B}{k+2}\Rightarrow 2=Ak+2A+Bk=(A+B)k+2A\Rightarrow\begin{cases}A+B=0\\ 2A\quad =2\end{cases}\Rightarrow A=1, B=-1$

$\dfrac{2}{k(k+2)}=\dfrac{1}{k}-\dfrac{1}{k+2}.$

Thus,

$$\frac{2}{1\cdot 3}+\frac{2}{2\cdot 4}+\frac{2}{3\cdot 5}+\cdots+\frac{2}{100\cdot 102}=\left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\cdots+\left(\frac{1}{99}-\frac{1}{101}\right)+\left(\frac{1}{100}-\frac{1}{102}\right)$$

$$=1+\frac{1}{2}-\frac{1}{101}-\frac{1}{102}=\frac{7625}{5151}$$

51. Each term is of the form $\frac{2}{(2n-1)(2n+1)}$. Decomposing $\frac{2}{(2n-1)(2n+1)}$ we have

$$\frac{2}{(2n-1)(2n+1)} = \frac{A}{2n-1} + \frac{B}{2n+1} \Rightarrow 2 = A(2n+1) + B(2n-1) \Rightarrow 2 = (2A+2B)n + (A-B) \Rightarrow$$

$$\begin{cases} 2A + 2B = 0 \\ A - \quad B = 2 \end{cases} \Rightarrow A = 1, B = -1$$

$$\frac{2}{(2n-1)(2n+1)} = \frac{1}{2n-1} - \frac{1}{2n+1}.$$

Thus,

$$\frac{2}{1\cdot 3} + \frac{2}{3\cdot 5} + \frac{2}{5\cdot 7} + \cdots + \frac{2}{(2n-1)(2n+1)} = \left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{7}\right) + \cdots + \frac{1}{2n-1} - \frac{1}{2n+1}$$
$$= 1 - \frac{1}{2n+1} = \frac{2n}{2n+1}$$

52. Each term is of the form $\frac{2}{k(k+1)(k+2)}$. Decomposing $\frac{2}{k(k+1)(k+2)}$ we have

$$\frac{2}{k(k+1)(k+2)} = \frac{A}{k} + \frac{B}{k+1} + \frac{C}{k+2} \Rightarrow 2 = A(k+1)(k+2) + Bk(k+2) + Ck(k+1) \Rightarrow$$

$$2 = (A+B+C)k^2 + (3A+2B+C)k + 2A \Rightarrow \begin{cases} A + \quad B + C = 0 \\ 3A + 2B + C = 0 \\ 2A \qquad\quad = 2 \end{cases} \Rightarrow A = 1, B = -2, C = 1$$

$$\frac{2}{k(k+1)(k+2)} = \frac{1}{k} - \frac{2}{k+1} + \frac{1}{k+2}$$

$$-\frac{2}{k+1} = -\frac{1}{k+1} - \frac{1}{k+1}, \text{ so } \frac{1}{k} - \frac{2}{k+1} + \frac{1}{k+2} = \frac{1}{k} - \frac{1}{k+1} - \frac{1}{k+1} + \frac{1}{k+2}$$

$$\frac{2}{1\cdot 2\cdot 3} + \frac{2}{2\cdot 3\cdot 4} + \frac{2}{3\cdot 4\cdot 5} + \cdots + \frac{2}{100\cdot 101\cdot 102}$$
$$= \left(\frac{1}{1} - \frac{1}{2} - \frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{3} - \frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{4} - \frac{1}{4} + \frac{1}{5}\right) + \cdots + \left(\frac{1}{100} - \frac{1}{101} - \frac{1}{101} + \frac{1}{102}\right)$$
$$= 1 - \frac{1}{2} - \frac{1}{101} + \frac{1}{102} = \frac{2575}{5151}$$

53. $\frac{1}{R} = \frac{2x+4}{(x+1)(x+3)}$

$$\frac{2x+4}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3} \Rightarrow 2x+4 = A(x+3) + B(x+1) \Rightarrow 2x+4 = (A+B)x + (3A+B) \Rightarrow$$

$$\begin{cases} A + B = 2 \\ 3A + B = 4 \end{cases} \Rightarrow A = 1, B = 1$$

$$\frac{2x+4}{(x+1)(x+3)} = \frac{1}{x+1} + \frac{1}{x+3} = \frac{1}{R}.$$

This means that if two resistances $R_1 = x+1$ and $R_2 = x+3$ are connected in parallel, they will produce a total resistance given by $\frac{(x+1)(x+3)}{2x+4}$.

54. $\frac{1}{R}=\frac{7x+20}{(x+2)(x+4)}=\frac{A}{x+2}+\frac{B}{x+4}\Rightarrow 7x+20=A(x+4)+B(x+2)\Rightarrow 7x+20=(A+B)x+(4A+2B)\Rightarrow$

$\begin{cases}A+\ \ B=7\\4A+2B=20\end{cases}\Rightarrow A=3, B=4$

$\frac{7x+20}{(x+2)(x+4)}=\frac{3}{x+2}+\frac{4}{x+4}=\frac{1}{\frac{x+2}{3}}+\frac{1}{\frac{x+4}{4}}=\frac{1}{R}$

This means that if two resistances $R_1=\frac{x+2}{3}$ and $R_2=\frac{x+4}{4}$ are connected in parallel, they will produce a total resistance given by $\frac{(x+2)(x+4)}{7x+20}$.

55. $\frac{1}{R}=\frac{R_1R_2+R_2R_3+R_3R_1}{R_1R_2R_3}=\frac{A}{R_1}+\frac{B}{R_2}+\frac{C}{R_3}\Rightarrow R_1R_2+R_2R_3+R_3R_1=AR_2R_3+BR_1R_3+CR_1R_2\Rightarrow\begin{cases}A=1\\B=1\\C=1\end{cases}$

$\frac{R_1R_2+R_2R_3+R_3R_1}{R_1R_2R_3}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}=\frac{1}{R}$

This means that if three resistances $R_1, R_2,$ and R_3 are connected in parallel, they will produce a total resistance given by $\frac{R_1R_2R_3}{R_1R_2+R_2R_3+R_3R_1}$.

56. $\frac{1}{R}=\frac{3x^2+12x+8}{x(x+2)(x+4)}=\frac{A}{x}+\frac{B}{x+2}+\frac{C}{x+4}\Rightarrow 3x^2+12x+8=A(x+2)(x+4)+Bx(x+4)+Cx(x+2)\Rightarrow$

$3x^2+12x+8=(A+B+C)x^2+(6A+4B+2C)x+8A\Rightarrow\begin{cases}A+\ \ B+\ \ C=3\\6A+4B+2C=12\\8A\qquad\qquad\ =8\end{cases}\Rightarrow A=1, B=1, C=1$

$\frac{3x^2+12x+8}{x(x+2)(x+4)}=\frac{1}{x}+\frac{1}{x+2}+\frac{1}{x+4}=\frac{1}{R}$

This means that if three resistances $R_1=x,\ \ R_2=x+2,$ and $R_3=x+4$ are connected in parallel, they will produce a total resistance given by $\frac{x(x+2)(x+4)}{3x^2+12x+8}$.

8.3 Beyond the Basics

57. $\begin{cases}A+B+\ C=0\\\quad 2B-2C=10\\-4A\qquad\quad=-4\end{cases}\Rightarrow\begin{cases}A+B+C=0\\\quad B-C=5\\A\qquad\ \ =1\end{cases}$

Subtract the third equation from the first equation, and replace the first equation with the new equation:

$\begin{cases}A+B+C=0\\\quad B-C=5\\A\qquad\ \ =1\end{cases}\Rightarrow\begin{cases}B+C=-1\\B-C=5\\A\qquad=1\end{cases}$

Add the first and second equations to solve for *B*:

$\begin{cases}B+C=-1\\B-C=5\\A\qquad=1\end{cases}\Rightarrow\begin{cases}B+C=-1\\2B\qquad=4\\A\qquad=1\end{cases}\Rightarrow\begin{cases}B+C=-1\\B\qquad=2\\A\qquad=1\end{cases}$

Substitute the value for *B* into the first equation to solve for *C*:

$\begin{cases}B+C=-1\\B\qquad=2\\A\qquad=1\end{cases}\Rightarrow\begin{cases}2+C=-1\\B\qquad=2\\A\qquad=1\end{cases}\Rightarrow\begin{cases}C=-3\\B=2\\A=1\end{cases}$

58. $\frac{1}{x^3+1}=\frac{1}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}\Rightarrow 1=A(x^2-x+1)+(Bx+C)(x+1)\Rightarrow$

$1=(A+B)x^2+(-A+B+C)x+(A+C)\Rightarrow\begin{cases}A+B=0\\-A+B+C=0\\A+\quad C=1\end{cases}\Rightarrow A=\frac{1}{3},B=-\frac{1}{3},C=\frac{2}{3}$

$\frac{1}{x^3+1}=\frac{1}{3(x+1)}+\frac{-x+2}{3(x^2-x+1)}$

59. $\frac{4x}{(x^2-1)^2}=\frac{4x}{(x-1)^2(x+1)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+1}+\frac{D}{(x+1)^2}\Rightarrow$

$4x=A(x-1)(x+1)^2+B(x+1)^2+C(x-1)^2(x+1)+D(x-1)^2\Rightarrow$

$4x=(A+C)x^3+(A+B-C+D)x^2+(-A+2B-C-2D)x+(-A+B+C+D)\Rightarrow$

$\begin{cases}A+\quad C\quad=0 & (1)\\A+B-C+D=0 & (2)\\-A+2B-C-2D=4 & (3)\\-A+B+C+D=0 & (4)\end{cases}$

Add equations (1) and (3), and (2) and (4), and replace equations (3) and (4):

$\begin{cases}A+\quad C\quad=0\\A+B-C+D=0\\-A+2B-C-2D=4\\-A+B+C+D=0\end{cases}\Rightarrow\begin{cases}A+\quad C\quad=0 & (1)\\A+B-C+D=0 & (2)\\2B-2D=4 & (3)\\2B+2D=0 & (4)\end{cases}\Rightarrow\begin{cases}A+\quad C\quad=0\\A+B-C+D=0\\4B=4\\2B+2D=0\end{cases}\Rightarrow B=1,D=-1,A=0,C=0$

$\frac{4x}{(x^2-1)^2}=\frac{1}{(x-1)^2}-\frac{1}{(x+1)^2}$

60. $\frac{2x+3}{(x^2+1)(x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+1}\Rightarrow 2x+3=A(x^2+1)+(Bx+C)(x+1)\Rightarrow$

$2x+3=(A+B)x^2+(B+C)x+(A+C)\Rightarrow\begin{cases}A+B\quad=0\\B+C=2\\A+\quad C=3\end{cases}\Rightarrow A=\frac{1}{2},B=-\frac{1}{2},C=\frac{5}{2}$

$\frac{2x+3}{(x^2+1)(x+1)}=\frac{1}{2(x+1)}+\frac{-x+5}{2(x^2+1)}$

61. $\frac{x+1}{(x^2+1)(x-1)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{Cx+D}{x^2+1}\Rightarrow x+1=A(x-1)(x^2+1)+B(x^2+1)+(Cx+D)(x-1)^2\Rightarrow$

$x+1=(A+C)x^3+(-A+B-2C+D)x^2+(A+B+C-2D)x+(-A+B+D)\Rightarrow$

$\begin{cases}A+\quad C\quad=0\\-A+B-2C+D=0\\A+B+C-2D=1\\-A+B+\quad D=1\end{cases}$

From the first equation, we have $C=-A$. Substitute this into the last three equations and simplify.

$\begin{cases}A+\quad C\quad=0\\-A+B-2C+D=0\\A+\quad C-2D=1\\-A+B+\quad D=1\end{cases}\Rightarrow\begin{cases}C=-A\\-A+B-2(-A)+D=0\\A+\quad(-A)-2D=1\\-A+B+\quad D=1\end{cases}\Rightarrow\begin{cases}C=-A\\A+B+D=0\\-2D=1\\-A+B+D=1\end{cases}\Rightarrow\begin{cases}C=-A\\A+B+D=0\\D=-\frac{1}{2}\\-A+B+D=1\end{cases}$

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Substitute the value for D into the second and fourth equations, and simplify:

$$\begin{cases} C = -A \\ A + B = \frac{1}{2} \\ D = -\frac{1}{2} \\ -A + B = \frac{3}{2} \end{cases} \Rightarrow B = 1, A = -\frac{1}{2}, C = \frac{1}{2}$$

$$\frac{x+1}{(x^2+1)(x-1)^2} = -\frac{1}{2(x-1)} + \frac{1}{(x-1)^2} + \frac{x-1}{2(x^2+1)}$$

62. $\dfrac{x}{(x^2+1)^2(x-1)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+1} + \dfrac{Dx+E}{(x^2+1)^2} \Rightarrow x = A(x^2+1)^2 + (Bx+C)(x-1)(x^2+1) + (Dx+E)(x-1)$

Letting $x = 1$, we have $1 = 2^2 A \Rightarrow A = \frac{1}{4}$. Expand the equation and substitute $A = \frac{1}{4}$:

$$x = \left(\frac{1}{4} + B\right)x^4 + (-B + C)x^3 + \left(\frac{1}{2} + B - C + D\right)x^2 + (-B + C - D + E)x + \left(\frac{1}{4} - C - E\right) \Rightarrow$$

$$\begin{cases} \frac{1}{4} + B = 0 \\ -B + C = 0 \\ \frac{1}{2} + B - C + D = 0 \\ -B + C - D + E = 1 \\ \frac{1}{4} - C - E = 0 \end{cases} \Rightarrow B = -\frac{1}{4}, C = -\frac{1}{4}, D = -\frac{1}{2}, E = \frac{1}{2}$$

$$\frac{x}{(x^2+1)^2(x-1)} = \frac{1}{4(x-1)} + \frac{-x-1}{4(x^2+1)} + \frac{-x+1}{2(x^2+1)^2}$$

63. $\dfrac{x^3}{(x+1)^2(x+2)^2} = \dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x+2} + \dfrac{D}{(x+2)^2} \Rightarrow$

$$x^3 = A(x+1)(x+2)^2 + B(x+2)^2 + C(x+2)(x+1)^2 + D(x+1)^2$$

Letting $x = -2$, we have $(-2)^3 = D(-1)^2 \Rightarrow D = -8$. Letting $x = -1$, we have $(-1)^3 = B(1)^2 \Rightarrow B = -1$. Expand the equation and substitute the values for B and D:

$$x^3 = (A + C)x^3 + (-9 + 5A + 4C)x^2 + (-20 + 8A + 5C)x + (-12 + 4A + 2C) \Rightarrow$$

$$\begin{cases} A + C = 1 \\ 5A + 4C = 9 \\ 8A + 5C = 20 \\ 4A + 2C = 12 \end{cases} \Rightarrow A = 5, C = -4$$

$$\frac{x^3}{(x+1)^2(x+2)^2} = \frac{5}{x+1} - \frac{1}{(x+1)^2} - \frac{4}{x+2} - \frac{8}{(x+2)^2}$$

64. $\dfrac{2x^3+2x^2-1}{(x^2+x+1)^2}=\dfrac{Ax+B}{x^2+x+1}+\dfrac{Cx+D}{(x^2+x+1)^2}\Rightarrow 2x^3+2x^2-1=(Ax+B)(x^2+x+1)+(Cx+D)\Rightarrow$

$$2x^3+2x^2-1=Ax^3+(A+B)x^2+(A+B+C)x+(B+D)\Rightarrow\begin{cases}A=2\\A+B=2\\A+B+C=0\\B+D=-1\end{cases}\Rightarrow$$

$A=2, B=0, C=-2, D=-1$

$$\frac{2x^3+2x^2-1}{(x^2+x+1)^2}=\frac{2x}{x^2+x+1}+\frac{-2x-1}{(x^2+x+1)^2}$$

65. $\dfrac{15x}{x^3-27}=\dfrac{15x}{(x-3)(x^2+3x+9)}=\dfrac{A}{x-3}+\dfrac{Bx+C}{(x^2+3x+9)}\Rightarrow 15x=A(x^2+3x+9)+(Bx+C)(x-3)$

Letting $x=3$, we have $15(3)=A(3^2+3(3)+9)\Rightarrow A=5/3$.

Expand the equation and substitute the value for A:

$$15x=\left(\frac{5}{3}+B\right)x^2+(5-3B+C)x+(15-3C)\Rightarrow\begin{cases}B=-5/3\\-3B+C=10\\-3C=-15\end{cases}\Rightarrow B=-\frac{5}{3}, C=5$$

$$\frac{15x}{x^3-27}=\frac{5}{3(x-3)}+\frac{-5x+15}{3(x^2+3x+9)}$$

66. $\dfrac{2x^2-9x+10}{x^3+8}=\dfrac{2x^2-9x+10}{(x+2)(x^2-2x+4)}=\dfrac{A}{x+2}+\dfrac{Bx+C}{x^2-2x+4}\Rightarrow$

$2x^2-9x+10=A(x^2-2x+4)+(Bx+C)(x+2)\Rightarrow$

Letting $x=-2$, we have $2(-2)^2-9(-2)+10=A((-2)^2-2(-2)+4)\Rightarrow 36=12A\Rightarrow A=3$.

Expand the equation, substitute the value for A, and simplify:

$$2x^2-9x+10=(3+B)x^2+(-6+2B+C)x+(12+2C)\Rightarrow\begin{cases}B=-1\\2B+C=-3\\2C=-2\end{cases}\Rightarrow B=-1, C=-1$$

$$\frac{2x^2-9x+10}{x^3+8}=\frac{3}{x+2}+\frac{-x-1}{x^2-2x+4}$$

67. Using the hint provided, we have

$$\frac{4x}{x^4+4}=\frac{4x}{(x^2-2x+2)(x^2+2x+2)}=\frac{Ax+B}{x^2-2x+2}+\frac{Cx+D}{x^2+2x+2}\Rightarrow$$

$4x=(Ax+B)(x^2+2x+2)+(Cx+D)(x^2-2x+2)\Rightarrow$

$4x=(A+C)x^3+(2A+B-2C+D)x^2+(2A+2B+2C-2D)x+(2B+2D)\Rightarrow$

$$\begin{cases}A+C=0\\2A+B-2C+D=0\\2A+2B+2C-2D=4\\2B+2D=0\end{cases}\Rightarrow A=0, B=1, C=0, D=-1$$

$$\frac{4x}{x^4+4}=\frac{1}{x^2-2x+2}-\frac{1}{x^2+2x+2}$$

68. $\dfrac{x^2-8x+18}{(x-5)^3}=\dfrac{A}{x-5}+\dfrac{B}{(x-5)^2}+\dfrac{C}{(x-5)^3}\Rightarrow x^2-8x+18=A(x-5)^2+B(x-5)+C\Rightarrow$

$x^2-8x+18=Ax^2+(-10A+B)x+(25A-5B+C)\Rightarrow\begin{cases}A=1\\-10A+B=-8\\25A-5B+C=18\end{cases}\Rightarrow A=1, B=2, C=3$

$\dfrac{x^2-8x+18}{(x-5)^3}=\dfrac{1}{x-5}+\dfrac{2}{(x-5)^2}+\dfrac{3}{(x-5)^3}$

69. $$\begin{array}{r|l} & 1 \\ \hline x^2+3x+2 & x^2+4x+5 \\ & \underline{x^2+3x+2} \\ & x+3 \end{array}$$

$\dfrac{x^2+4x+5}{x^2+3x+2}=1+\dfrac{x+3}{x^2+3x+2}$

Decompose $\dfrac{x+3}{x^2+3x+2}$:

$\dfrac{x+3}{x^2+3x+2}=\dfrac{x+3}{(x+1)(x+2)}=\dfrac{A}{x+1}+\dfrac{B}{x+2}\Rightarrow x+3=A(x+2)+B(x+1)\Rightarrow$

$x+3=(A+B)x+(2A+B)\Rightarrow\begin{cases}A+B=1\\2A+B=3\end{cases}\Rightarrow A=2, B=-1$

$\dfrac{x^2+4x+5}{x^2+3x+2}=1+\dfrac{2}{x+1}-\dfrac{1}{x+2}$

70. $\dfrac{(x-1)(x+2)}{(x+3)(x-4)}=\dfrac{x^2+x-2}{x^2-x-12}$

$$\begin{array}{r|l} & 1 \\ \hline x^2-x-12 & x^2+x-2 \\ & \underline{x^2-x-12} \\ & 2x+10 \end{array}$$

$\dfrac{x^2+x-2}{x^2-x-12}=1+\dfrac{2x+10}{x^2-x-12}$

Decompose $\dfrac{2x+10}{x^2-x-12}$:

$\dfrac{2x+10}{x^2-x-12}=\dfrac{2x+10}{(x+3)(x-4)}=\dfrac{A}{x+3}+\dfrac{B}{x-4}\Rightarrow 2x+10=A(x-4)+B(x+3)\Rightarrow$

$2x+10=(A+B)x+(-4A+3B)\Rightarrow\begin{cases}A+B=2\\-4A+3B=10\end{cases}\Rightarrow A=-\dfrac{4}{7}, B=\dfrac{18}{7}$

$\dfrac{(x-1)(x+2)}{(x+3)(x-4)}=1-\dfrac{4}{7(x+3)}+\dfrac{18}{7(x-4)}$

71. $\dfrac{2x^4+x^3+2x^2-2x-1}{(x-1)(x^2+1)}=\dfrac{2x^4+x^3+2x^2-2x-1}{x^3-x^2+x-1}$

$$\begin{array}{r|l} & 2x+3 \\ \hline x^3-x^2+x-1 & 2x^4+x^3+2x^2-2x-1 \\ & \underline{2x^4-2x^3+2x^2-2x} \\ & 3x^3 \qquad\qquad -1 \\ & \underline{3x^3-3x^2+3x-3} \\ & 3x^2-3x+2 \end{array}$$

$\dfrac{2x^4+x^3+2x^2-2x-1}{(x-1)(x^2+1)}=2x+3+\dfrac{3x^2-3x+2}{(x-1)(x^2+1)}$

(*continued on next page*)

(*continued*)

Decompose $\dfrac{3x^2-3x+2}{(x-1)(x^2+1)}$:

$$\frac{3x^2-3x+2}{(x-1)(x^2+1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}\Rightarrow 3x^2-3x+2=A(x^2+1)+(Bx+C)(x-1)\Rightarrow$$

$$3x^2-3x+2=(A+B)x^2+(-B+C)x+(A-C)\Rightarrow\begin{cases}A+B=3\\ -B+C=-3\\ A-\quad C=2\end{cases}\Rightarrow A=1, B=2, C=-1$$

$$\frac{3x^2-3x+2}{(x-1)(x^2+1)}=\frac{1}{x-1}+\frac{2x-1}{x^2+1}$$

Thus, $\dfrac{2x^4+x^3+2x^2-2x-1}{(x-1)(x^2+1)}=2x+3+\dfrac{1}{x-1}+\dfrac{2x-1}{x^2+1}.$

72. $\dfrac{x^4}{(x-1)(x-2)(x-3)}=\dfrac{x^4}{x^3-6x^2+11x-6}$

$$\begin{array}{r} x+6 \\ x^3-6x^2+11x-6\,\overline{)\,x^4 \qquad\qquad\qquad\qquad} \\ \underline{x^4-6x^3+11x^2-\ \ 6x} \\ 6x^3-11x^2+\ \ 6x \\ \underline{6x^3-36x^2+66x-36} \\ 25x^2-60x+36 \end{array}$$

$$\frac{x^4}{(x-1)(x-2)(x-3)}=x+6+\frac{25x^2-60x+36}{(x-1)(x-2)(x-3)}$$

Decompose $\dfrac{25x^2-60x+36}{(x-1)(x-2)(x-3)}$:

$$\frac{25x^2-60x+36}{(x-1)(x-2)(x-3)}=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3}\Rightarrow$$

$$25x^2-60x+36=A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)\Rightarrow$$

$$\frac{2x^4+x^3+2x^2-2x-1}{(x-1)(x^2+1)}=2x+3+\frac{1}{x-1}+\frac{2x-1}{x^2+1}$$

$$25x^2-60x+36=(A+B+C)x^2+(-5A-4B-3C)x+(6A+3B+2C)\Rightarrow\begin{cases}A+\ \ B+\ \ C=25\\ -5A-4B-3C=-60\\ 6A+3B+2C=36\end{cases}\Rightarrow$$

$$A=\frac{1}{2}, B=-16, C=\frac{81}{2}$$

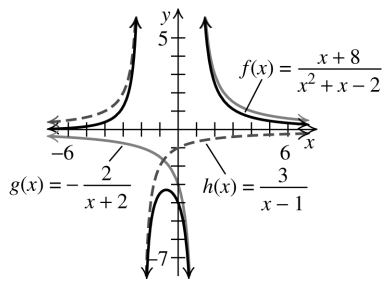
$$\frac{x^4}{(x-1)(x-2)(x-3)}=x+6+\frac{1}{2(x-1)}-\frac{16}{x-2}+\frac{81}{2(x-3)}$$

8.3 Critical Thinking/Discussion/Writing

73. Two equal polynomials have equal corresponding coefficients. See page 755 in the text.

74. $f(x) = \dfrac{x+8}{x^2+x-2} = \dfrac{x+8}{(x+2)(x-1)}$
$= \dfrac{A}{x+2} + \dfrac{B}{x-1} \Rightarrow$
$x+8 = A(x-1) + B(x+2)$
$= (A+B)x + (-A+2B)$
$\begin{cases} A + B = 1 \\ -A + 2B = 8 \end{cases} \Rightarrow 3B = 9 \Rightarrow B = 3,\ A = -2$
$\dfrac{x+8}{x^2+x-2} = -\dfrac{2}{x+2} + \dfrac{3}{x-1}$
$g(x) = -\dfrac{2}{x+2},\quad h(x) = \dfrac{3}{x-1}$

a.



b. The graphs of *f* and *g* have the same vertical asymptote, $x = -2$, and the same horizontal asymptote, $y = 0$, or the *x*-axis. Also *g* is an asymptote of *f* as *x* approaches 1 from the right or from the left.

c. The graphs of *f* and *h* have the same vertical asymptote, $x = 1$, and the same horizontal asymptote, $y = 0$, or the *x*-axis. Also *h* is an asymptote of *f* as *x* approaches −2 from the right or from the left.

8.3 Maintaining Skills

75. $x^2 + 10x + 21 = 0 \Rightarrow (x+7)(x+3) = 0 \Rightarrow$
$x = -7,\ x = -3$

76. $x^2 - 2x - 24 = 0 \Rightarrow (x+4)(x-6) = 0 \Rightarrow$
$x = -4,\ x = 6$

77. $2x^2 + x - 10 = 0 \Rightarrow (2x+5)(x-2) = 0 \Rightarrow$
$x = -\dfrac{5}{2},\ x = 2$

78. $6x^2 + x - 2 = 0 \Rightarrow (3x+2)(2x-1) = 0 \Rightarrow$
$x = -\dfrac{2}{3},\ x = \dfrac{1}{2}$

In exercises 79−86, be sure to check the solution in both equations.

79. $\begin{cases} x + y = 3 & (1) \\ 3x + 2y = 7 & (2) \end{cases}$

From equation (1), we have $y = 3 - x$.
Substituting in equation (2), we have
$3x + 2(3-x) = 7 \Rightarrow x + 6 = 7 \Rightarrow x = 1$
Substituting $x = 1$ in equation (1) gives
$1 + y = 3 \Rightarrow y = 2$.
Solution set: {(1, 2)}

80. $\begin{cases} 3x + y = -3 & (1) \\ 4x + 3y = 1 & (2) \end{cases}$

From equation (1), we have $y = -3x - 3$.
Substituting in equation (2), we have
$4x + 3(-3x-3) = 1 \Rightarrow -5x - 9 = 1 \Rightarrow$
$-5x = 10 \Rightarrow x = -2$
Substituting $x = -2$ in equation (1) gives
$3(-2) + y = -3 \Rightarrow -6 + y = -3 \Rightarrow y = 3$.
Solution set: {(−2, 3)}

81. $\begin{cases} x - 2y = 1 & (1) \\ 2x + 3y = 16 & (2) \end{cases}$

From equation (1), we have $x = 2y + 1$.
Substituting in equation (2), we have
$2(2y+1) + 3y = 16 \Rightarrow 7y + 2 = 16 \Rightarrow$
$7y = 14 \Rightarrow y = 2$
Substituting $y = 2$ in equation (1) gives
$x - 2(2) = 1 \Rightarrow x - 4 = 1 \Rightarrow x = 5$
Solution set: {(5, 2)}

82. $\begin{cases} x - 2y = 10 & (1) \\ \frac{1}{2}x + \frac{1}{3}y = 1 & (2) \end{cases}$

From equation (1), we have $x = 2y + 10$.
Substituting in equation (2), we have
$\dfrac{1}{2}(2y+10) + \dfrac{1}{3}y = 1 \Rightarrow \dfrac{4}{3}y + 5 = 1 \Rightarrow$
$\dfrac{4}{3}y = -4 \Rightarrow y = -3$
Substituting $y = -3$ in equation (1) gives
$x - 2(-3) = 100 \Rightarrow x + 6 = 10 \Rightarrow x = 4$
Solution set: {(4, −3)}

83. $\begin{cases} 2x - y = 4 & (1) \\ 3x + 2y = 13 & (2) \end{cases}$

Multiply equation (1) by 2, then add the two equations and solve for *x*.

$$\begin{array}{r} 4x - 2y = 8 \\ \underline{3x + 2y = 13} \\ 7x = 21 \Rightarrow x = 3 \end{array}$$

Substitute $x = 3$ in equation (1), then solve for *y*.

$2(3) - y = 4 \Rightarrow 6 - y = 4 \Rightarrow y = 2$

Solution set: {(3, 2)}

84. $\begin{cases} 2x + 3y = -1 & (1) \\ 3x - 2y = 5 & (2) \end{cases}$

Multiply equation (1) by 2 and equation (2) by 3 to eliminate *y*. Add the resulting equations and solve for *x*.

$$\begin{array}{r} 4x + 6y = -2 \\ \underline{9x - 6y = 15} \\ 13x = 13 \Rightarrow x = 1 \end{array}$$

Substitute $x = 1$ in equation (1), then solve for *y*.

$2(1) + 3y = -1 \Rightarrow 2 + 3y = -1 \Rightarrow 3y = -3 \Rightarrow$
$y = -1$

Solution set: {(1, −1)}

85. $\begin{cases} 2x + 5y = 1 & (1) \\ 3x - 2y = -8 & (2) \end{cases}$

Multiply equation (1) by 2 and equation (2) by 5 to eliminate *y*. Add the resulting equations and solve for *x*.

$$\begin{array}{r} 4x + 10y = 2 \\ \underline{15x - 10y = -40} \\ 19x = -38 \Rightarrow x = -2 \end{array}$$

Substitute $x = -2$ in equation (1), then solve for *y*.

$2(-2) + 5y = 1 \Rightarrow -4 + 5y = 1 \Rightarrow 5y = 5 \Rightarrow$
$y = 1$

Solution set: {(−2, 1)}

86. $\begin{cases} 2x - 3y = 6 & (1) \\ -3x + 2y = 1 & (2) \end{cases}$

Multiply equation (1) by 3 and equation (2) by 2 to eliminate *x*. Add the resulting equations and solve for *y*.

$$\begin{array}{r} 6x - 9y = 18 \\ \underline{-6x + 4y = 2} \\ -5y = 20 \Rightarrow y = -4 \end{array}$$

Substitute $y = -4$ in equation (1), then solve for *x*.

$2x - 3(-4) = 6 \Rightarrow 2x + 12 = 6 \Rightarrow 2x = -6 \Rightarrow$
$x = -3$

Solution set: {(−3, −4)}

8.4 Systems of Nonlinear Equations

8.4 Practice Problems

1. $\begin{cases} x^2 + y = 2 \\ 2x + y = 3 \end{cases} \Rightarrow \begin{cases} y = 2 - x^2 \\ 2x + y = 3 \end{cases} \Rightarrow$

$2x + (2 - x^2) = 3 \Rightarrow -x^2 + 2x - 1 = 0 \Rightarrow$
$x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0 \Rightarrow x = 1$
$2(1) + y = 3 \Rightarrow y = 1$

The solution is {(1, 1)}.

2. $\begin{cases} x^2 + 2y^2 = 34 \\ x^2 - y^2 = 7 \end{cases} \Rightarrow 3y^2 = 27 \Rightarrow y^2 = 9 \Rightarrow$
$y = \pm 3$
$x^2 - (-3)^2 = 7 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$
$x^2 - (3)^2 = 7 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

The solution is {(−4, −3), (−4, 3), (4, −3), (4, 3)}.

3. Let x = the number of shares received as dividends, and let p = the selling price per share. Then $xp = 1950$. The number of shares she sold at *p* dollars per share is $240 + x$. Thus, revenue = $(240 + x)p$ and the cost of her stock was $(240)(40) + 100 = 9700$. Since revenue − cost = profit, we have

$(240 + x)p - 9700 = 7850 \Rightarrow$
$240p + xp = 17{,}550$

Thus, the system of equations is

$\begin{cases} xp = 1950 \\ 240p + xp = 17{,}550 \end{cases}.$

$\begin{cases} xp = 1950 \\ 240p + xp = 17{,}550 \end{cases} \Rightarrow$
$240p + 1950 = 17{,}550 \Rightarrow 240p = 15{,}600 \Rightarrow$
$p = 65$
$65x = 1950 \Rightarrow x = 30$

Danielle received 30 shares as dividends and sold her stock at $65 per share.

8.3 Basic Concepts and Skills

1. In a system of nonlinear equations, <u>at least one</u> equation must be nonlinear.

2. Both the <u>substitution</u> and <u>elimination</u> methods can also be used to solve systems of nonlinear equations.

3. The solutions of the system
$$\begin{cases} ax+by=c & (1) \\ (x-h)^2+(y-k)^2=r^2 & (2) \end{cases}$$
represent the points of intersection of the graphs of equations (1) and (2).

4. The system of equations in Example 1 has exactly one solution because the graph of equation (1) is a tangent line to the graph of equation (2).

5. True

6. False. The system is linear if all the equations in the system are linear.

7. Substituting each ordered pair into the system
$$\begin{cases} 2x+3y=3 \\ x-y^2=2 \end{cases}$$
we find that (3, –1) is a solution.
$$\begin{cases} 2(3)+3(-1)=6-3=3 \\ 3-(-1)^2=3-1=2 \end{cases}$$

8. Substituting each ordered pair into the system
$$\begin{cases} x+2y=6 \\ y=x^2 \end{cases}$$
we find that (–2, 4) is a solution.
$$\begin{cases} -2+2(4)=-2+8=6 \\ 4=(-2)^2=4 \end{cases}$$

9. Substituting each ordered pair into the system
$$\begin{cases} 5x-2y=7 \\ x^2+y^2=2 \end{cases},$$
we find that (1, −1) is a solution:
$$\begin{cases} 5(1)-2(-1)=5+2=7 \\ (1)^2+(-1)^2=1+1=2 \end{cases}$$

10. Substituting each ordered pair into the system
$$\begin{cases} x-2y=-5 \\ x^2+y^2=25 \end{cases}$$
we find that (–5, 0) and (3, 4) are solutions.
$$\begin{cases} -5-2(0)=-5 \\ (-5)^2+0^2=25 \end{cases}$$
$$\begin{cases} 3-2(4)=3-8=-5 \\ 3^2+4^2=9+16=25 \end{cases}$$

11. Substituting each ordered pair into the system
$$\begin{cases} 4x^2+5y^2=180 \\ x^2-y^2=9 \end{cases}$$
we find that (5, 4), (–5, 4), and (–5, –4) are solutions.
$$\begin{cases} 4(5)^2+5(4)^2=100+80=180 \\ 5^2-4^2=25-16=9 \end{cases}$$
$$\begin{cases} 4(-5)^2+5(4)^2=100+80=180 \\ (-5)^2-(4)^2=25-16=9 \end{cases}$$
$$\begin{cases} 4(-5)^2+5(-4)^2=100+80=180 \\ (-5)^2-(-4)^2=25-16=9 \end{cases}$$

12. Substituting each ordered pair into the system
$$\begin{cases} x^2-y^2=3 \\ x^2-4x+y^2=-3 \end{cases}$$
we find that (2,1) and (2, −1) are solutions.
$$\begin{cases} 2^2-1^2=4-1=3 \\ 2^2-4(2)+1^2=4-8+1=-3 \end{cases}$$
$$\begin{cases} 2^2-(-1)^2=4-1=3 \\ 2^2-4(2)+(-1)^2=4-8+1=-3 \end{cases}$$

13. Substituting each ordered pair into the system
$$\begin{cases} y=e^{x-1} \\ y=2x-1 \end{cases}$$
we find that (1, 1) is a solution:
$$\begin{cases} y=e^{1-1}=e^0=1 \\ y=2(1)-1=1 \end{cases}$$

14. Substituting each ordered pair into the system
$$\begin{cases} y=\ln(x+1) \\ y=x \end{cases}$$
we find that (0, 0) is a solution:
$$\begin{cases} 0=\ln(0+1)=\ln(1)=0 \\ 0=0 \end{cases}$$

15. $\begin{cases} y=x^2 \\ y=x+2 \end{cases} \Rightarrow x^2=x+2 \Rightarrow x^2-x-2=0 \Rightarrow$
$(x-2)(x+1)=0 \Rightarrow x=2$ or $x=-1$
$y=2^2=4$ or $y=(-1)^2=1$
The solution is {(2, 4), (−1, 1)}.

16. $\begin{cases} y=x^2 \\ x+y=6 \end{cases} \Rightarrow x+x^2=6 \Rightarrow x^2+x-6=0 \Rightarrow$
$(x+3)(x-2)=0 \Rightarrow x=-3$ or $x=2$
$y=(-3)^2=9$ or $y=2^2=4$
The solution is {(−3, 9), (2, 4)}.

17. $\begin{cases} x^2-y=6 \\ x-y=0 \end{cases} \Rightarrow \begin{cases} x^2-y=6 \\ x=y \end{cases} \Rightarrow x^2-x=6 \Rightarrow$
$x^2-x-6=0 \Rightarrow (x-3)(x+2)=0 \Rightarrow$
$x=3$ or $x=-2$
$3-y=0 \Rightarrow y=3$ or $-2-y=0 \Rightarrow y=-2$
The solution is {(3, 3), (−2, −2)}.

18. $\begin{cases} x^2 - y = 6 \\ 5x - y = 0 \end{cases} \Rightarrow \begin{cases} x^2 - y = 6 \\ 5x = y \end{cases} \Rightarrow x^2 - 5x = 6 \Rightarrow$
$x^2 - 5x - 6 = 0 \Rightarrow (x-6)(x+1) = 0 \Rightarrow$
$x = 6$ or $x = -1$
$6^2 - y = 6 \Rightarrow y = 30$ or $(-1)^2 - y = 6 \Rightarrow y = -5$
The solution is $\{(6, 30), (-1, -5)\}$.

19. $\begin{cases} x^2 + y^2 = 9 \\ x = 3 \end{cases} \Rightarrow 3^2 + y^2 = 9 \Rightarrow y = 0$
The solution is $\{(3, 0)\}$.

20. $\begin{cases} x^2 + y^2 = 9 \\ y = 3 \end{cases} \Rightarrow x^2 + 3^2 = 9 \Rightarrow x = 0$
The solution is $\{(0, 3)\}$.

21. $\begin{cases} x^2 + y^2 = 5 \\ x - y = -3 \end{cases} \Rightarrow \begin{cases} x^2 + y^2 = 5 \\ x = y - 3 \end{cases} \Rightarrow$
$(y-3)^2 + y^2 = 5 \Rightarrow 2y^2 - 6y + 4 = 0 \Rightarrow$
$y^2 - 3y + 2 = 0 \Rightarrow (y-2)(y-1) = 0 \Rightarrow$
$y = 2$ or $y = 1$
$x - 2 = -3 \Rightarrow x = -1$
$x - 1 = -3 \Rightarrow x = -2$
The solution is $\{(-2, 1), (-1, 2)\}$.

22. $\begin{cases} x^2 + y^2 = 13 \\ 2x - 3y = 0 \end{cases} \Rightarrow \begin{cases} x^2 + y^2 = 13 \\ x = \frac{3}{2}y \end{cases} \Rightarrow$
$\left(\frac{3}{2}y\right)^2 + y^2 = 13 \Rightarrow \frac{13}{4}y^2 = 13 \Rightarrow$
$y^2 = 4 \Rightarrow y = \pm 2$
$2x - 3(-2) = 0 \Rightarrow 2x = -6 \Rightarrow x = -3$
$2x - 3(2) = 0 \Rightarrow 2x = 6 \Rightarrow x = 3$
The solution is $\{(3, 2), (-3, -2)\}$.

23. $\begin{cases} x^2 - 4x + y^2 = -2 \\ x - y = 2 \end{cases} \Rightarrow \begin{cases} x^2 - 4x + y^2 = -2 \\ x = y + 2 \end{cases} \Rightarrow$
$(y+2)^2 - 4(y+2) + y^2 = -2 \Rightarrow 2y^2 = 2 \Rightarrow$
$y = \pm 1$
$x - (-1) = 2 \Rightarrow x = 1$
$x - 1 = 2 \Rightarrow x = 3$
The solution is $\{(1, -1), (3, 1)\}$.

24. $\begin{cases} x^2 - 8y + y^2 = -6 \\ 2x - y = 1 \end{cases} \Rightarrow \begin{cases} x^2 - 8y + y^2 = -6 \\ y = 2x - 1 \end{cases} \Rightarrow$
$x^2 - 8(2x-1) + (2x-1)^2 = -6 \Rightarrow$
$5x^2 - 20x + 15 = 0 \Rightarrow x^2 - 4x + 3 = 0 \Rightarrow$
$(x-3)(x-1) = 0 \Rightarrow x = 3$ or $x = 1$
$2(3) - y = 1 \Rightarrow y = 5$
$2(1) - y = 1 \Rightarrow y = 1$
The solution is $\{(1, 1), (3, 5)\}$.

25. $\begin{cases} x - y = -2 \\ xy = 3 \end{cases} \Rightarrow \begin{cases} x = y - 2 \\ xy = 3 \end{cases} \Rightarrow y(y-2) = 3 \Rightarrow$
$y^2 - 2y - 3 = 0 \Rightarrow (y-3)(y+1) = 0 \Rightarrow$
$y = 3$ or $y = -1$
$x - 3 = -2 \Rightarrow x = 1$
$x - (-1) = -2 \Rightarrow x = -3$
The solution is $\{(1, 3), (-3, -1)\}$.

26. $\begin{cases} x - 2y = 4 \\ xy = 6 \end{cases} \Rightarrow \begin{cases} x = 2y + 4 \\ xy = 6 \end{cases} \Rightarrow y(2y+4) = 6 \Rightarrow$
$2y^2 + 4y - 6 = 0 \Rightarrow y^2 + 2y - 3 = 0 \Rightarrow$
$(y+3)(y-1) \Rightarrow y = -3$ or $y = 1$
$-3x = 6 \Rightarrow x = -2$
$(1)x = 6 \Rightarrow x = 6$
The solution is $\{(6, 1), (-2, -3)\}$.

27. $\begin{cases} 4x^2 + y^2 = 25 \\ x + y = 5 \end{cases} \Rightarrow \begin{cases} 4x^2 + y^2 = 25 \\ x = 5 - y \end{cases} \Rightarrow$
$4(5-y)^2 + y^2 = 25 \Rightarrow 5y^2 - 40y + 75 = 0 \Rightarrow$
$y^2 - 8y + 15 = 0 \Rightarrow (y-5)(y-3) = 0 \Rightarrow$
$y = 5$ or $y = 3$
$x + 5 = 5 \Rightarrow x = 0$
$x + 3 = 5 \Rightarrow x = 2$
The solution is $\{(0, 5), (2, 3)\}$.

28. $\begin{cases} x^2 + 4y^2 = 16 \\ x + 2y = 4 \end{cases} \Rightarrow \begin{cases} x^2 + 4y^2 = 16 \\ x = 4 - 2y \end{cases} \Rightarrow$
$(4-2y)^2 + 4y^2 = 16 \Rightarrow 8y^2 - 16y = 0 \Rightarrow$
$y^2 - 2y = 0 \Rightarrow y(y-2) = 0 \Rightarrow y = 0$ or $y = 2$
$x + 2(0) = 4 \Rightarrow x = 4$
$x + 2(2) = 4 \Rightarrow x = 0$
The solution is $\{(0, 2), (4, 0)\}$.

29. $\begin{cases} x^2 - y^2 = 24 \\ 5x - 7y = 0 \end{cases} \Rightarrow \begin{cases} x^2 - y^2 = 24 \\ x = \frac{7}{5}y \end{cases} \Rightarrow$
$\left(\frac{7}{5}y\right)^2 - y^2 = 24 \Rightarrow \frac{24}{25}y^2 = 24 \Rightarrow y^2 = 25 \Rightarrow$
$y = \pm 5$
$5x - 7(-5) = 0 \Rightarrow x = -7$
$5x - 7(5) = 0 \Rightarrow x = 7$
The solution is $\{(7, 5), (-7, -5)\}$.

30. $\begin{cases} x^2 - 7y^2 = 9 \\ x - y = 3 \end{cases} \Rightarrow \begin{cases} x^2 - 7y^2 = 9 \\ y = x - 3 \end{cases} \Rightarrow$
$x^2 - 7(x-3)^2 = 9 \Rightarrow -6x^2 + 42x - 72 = 0 \Rightarrow$
$x^2 - 7x + 12 = 0 \Rightarrow (x-3)(x-4) = 0 \Rightarrow$
$x = 3$ or $x = 4$
$3 - y = 3 \Rightarrow y = 0$
$4 - y = 3 \Rightarrow y = 1$
The solution is $\{(4, 1), (3, 0)\}$.

31. $\begin{cases} x^2 + y^2 = 20 \\ x^2 - y^2 = 12 \end{cases} \Rightarrow 2x^2 = 32 \Rightarrow x = \pm 4$

$(-4)^2 + y^2 = 20 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$

$(4)^2 + y^2 = 20 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$

The solution is {(−4, −2), (−4, 2), (4, −2), (4, 2)}.

32. $\begin{cases} x^2 + 8y^2 = 9 \\ 3x^2 + y^2 = 4 \end{cases} \Rightarrow \begin{cases} x^2 + 8y^2 = 9 \\ -24x^2 - 8y^2 = -32 \end{cases} \Rightarrow$

$-23x^2 = -23 \Rightarrow x = \pm 1$

$3(-1)^2 + y^2 = 4 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1$

$3(1)^2 + y^2 = 4 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1$

The solution is {(−1, −1), (−1, 1), (1, −1), (1, 1)}.

33. $\begin{cases} x^2 + 2y^2 = 12 \\ 7y^2 - 5x^2 = 8 \end{cases} \Rightarrow \begin{cases} 5x^2 + 10y^2 = 60 \\ 7y^2 - 5x^2 = 8 \end{cases} \Rightarrow$

$17y^2 = 68 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$

$x^2 + 2(-2)^2 = 12 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

$x^2 + 2(2)^2 = 12 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

The solution is {(−2, −2), (−2, 2), (2, −2), (2, 2)}.

34. $\begin{cases} 3x^2 + 2y^2 = 77 \\ x^2 - 6y^2 = 19 \end{cases} \Rightarrow \begin{cases} 3x^2 + 2y^2 = 77 \\ -3x^2 + 18y^2 = -57 \end{cases} \Rightarrow$

$20y^2 = 20 \Rightarrow y = \pm 1$

$x^2 - 6(-1)^2 = 19 \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$

$x^2 - 6(1)^2 = 19 \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$

The solution is {(−5, −1), (−5, 1), (5, −1), (5, 1)}.

35. $\begin{cases} x^2 - y = 2 \\ 2x - y = 4 \end{cases} \Rightarrow \begin{cases} x^2 - y = 2 \\ -2x + y = -4 \end{cases} \Rightarrow$

$x^2 - 2x = -2 \Rightarrow x^2 - 2x + 2 = 0 \Rightarrow$

$x = \dfrac{2 \pm \sqrt{4-8}}{2} = 1 \pm i \Rightarrow$ there are no real solutions. Solution set: $\varnothing$

36. $\begin{cases} x^2 + 3y = 0 \\ x - y = -12 \end{cases} \Rightarrow \begin{cases} x^2 + 3y = 0 \\ 3x - 3y = -36 \end{cases} \Rightarrow$

$x^2 + 3x + 36 = 0 \Rightarrow x = \dfrac{-3 \pm \sqrt{9-144}}{2} \Rightarrow$

$x = \dfrac{-3 \pm 3i\sqrt{15}}{2} \Rightarrow$ there are no real solutions.

Solution set: $\varnothing$

37. $\begin{cases} x^2 + y^2 = 5 \\ 3x^2 - 2y^2 = -5 \end{cases} \Rightarrow \begin{cases} 2x^2 + 2y^2 = 10 \\ 3x^2 - 2y^2 = -5 \end{cases} \Rightarrow$

$5x^2 = 5 \Rightarrow x = \pm 1$

$(1)^2 + y^2 = 5 \Rightarrow y = \pm 2$

$(-1)^2 + y^2 = 5 \Rightarrow y = \pm 2$

The solution is {(−1, −2), (−1, 2), (1, −2), (1, 2)}.

38. $\begin{cases} x^2 + 4y^2 = 5 \\ 9x^2 - y^2 = 8 \end{cases} \Rightarrow \begin{cases} x^2 + 4y^2 = 5 \\ 36x^2 - 4y^2 = 32 \end{cases} \Rightarrow$

$37x^2 = 37 \Rightarrow x = \pm 1$

$(1)^2 + 4y^2 = 5 \Rightarrow y = \pm 1$

$(-1)^2 + 4y^2 = 5 \Rightarrow y = \pm 1$

The solution is {(−1, −1), (−1, 1), (1, −1), (1, 1)}.

39. $\begin{cases} x^2 + y^2 + 2x = 9 \\ x^2 + 4y^2 + 3x = 14 \end{cases} \Rightarrow$

$\begin{cases} 4x^2 + 4y^2 + 8x = 36 \\ x^2 + 4y^2 + 3x = 14 \end{cases} \Rightarrow 3x^2 + 5x - 22 = 0 \Rightarrow$

$(x-2)(3x+11) = 0 \Rightarrow x = 2$ or $x = -\dfrac{11}{3}$

$(2)^2 + y^2 + 2(2) = 9 \Rightarrow y = \pm 1$

$\left(-\dfrac{11}{3}\right)^2 + y^2 + 2\left(-\dfrac{11}{3}\right) = 9 \Rightarrow y^2 + \dfrac{55}{9} = 9 \Rightarrow$

$y = \pm\dfrac{\sqrt{26}}{3}$

The solution is $\left\{(2,-1), (2,1), \left(-\dfrac{11}{3}, -\dfrac{\sqrt{26}}{3}\right), \left(-\dfrac{11}{3}, \dfrac{\sqrt{26}}{3}\right)\right\}$.

40. $\begin{cases} x^2 + y^2 = 9 \\ x^2 + y^2 - 18x = 0 \end{cases} \Rightarrow 18x = 9 \Rightarrow x = \dfrac{1}{2}$

$\left(\dfrac{1}{2}\right)^2 + y^2 = 9 \Rightarrow y = \pm\dfrac{\sqrt{35}}{2}$

The solution is $\left\{\left(\dfrac{1}{2}, \dfrac{\sqrt{35}}{2}\right), \left(\dfrac{1}{2}, -\dfrac{\sqrt{35}}{2}\right)\right\}$.

41. Using substitution, we have

$$\begin{cases} x + y = 8 \\ xy = 15 \end{cases} \Rightarrow \begin{cases} x = 8 - y \\ xy = 15 \end{cases} \Rightarrow$$

$y(8 - y) = 15 \Rightarrow -y^2 + 8y - 15 = 0 \Rightarrow$

$y^2 - 8y + 15 = 0 \Rightarrow (y-3)(y-5) = 0 \Rightarrow$

$y = 3$ or $y = 5$

$x + 3 = 8 \Rightarrow x = 5$

$x + 5 = 8 \Rightarrow x = 3$

The solution is $\{(3, 5), (5, 3)\}$.

42. Using substitution, we have

$$\begin{cases} 2x + y = 8 \\ x^2 - y^2 = 5 \end{cases} \Rightarrow \begin{cases} y = 8 - 2x \\ x^2 - y^2 = 5 \end{cases} \Rightarrow$$

$x^2 - (8 - 2x)^2 = 5 \Rightarrow -3x^2 + 32x - 69 = 0 \Rightarrow$

$-(x-3)(3x-23) = 0 \Rightarrow x = 3$ or $x = \frac{23}{3}$

$2(3) + y = 8 \Rightarrow y = 2$

$2\left(\frac{23}{3}\right) + y = 8 \Rightarrow y = -\frac{22}{3}$

The solution is $\left\{(3, 2), \left(\frac{23}{3}, -\frac{22}{3}\right)\right\}$.

43. Using elimination, we have

$$\begin{cases} x^2 + y^2 = 2 \\ 3x^2 + 3y^2 = 9 \end{cases} \Rightarrow \begin{cases} -3x^2 - 3y^2 = -6 \\ 3x^2 + 3y^2 = 9 \end{cases} \Rightarrow$$

$0 = 3 \Rightarrow$ there is no solution.

Solution set: $\varnothing$

44. Using elimination, we have

$$\begin{cases} x^2 + y^2 = 5 \\ 3x^2 - y^2 = -11 \end{cases} \Rightarrow 4x^2 = -6 \Rightarrow x^2 = -\frac{3}{2} \Rightarrow$$

there is no solution. Solution set: $\varnothing$

45. Using substitution, we have

$$\begin{cases} y^2 = 4x + 4 \\ y = 2x - 2 \end{cases} \Rightarrow (2x - 2)^2 = 4x + 4 \Rightarrow$$

$4x^2 - 12x = 0 \Rightarrow 4x(x - 3) = 0 \Rightarrow x = 0$ or $x = 3$

$y = 2(0) - 2 = -2$

$y = 2(3) - 2 = 4$

The solution is $\{(0, -2), (3, 4)\}$.

46. Using substitution, we have

$$\begin{cases} xy = 125 \\ y = x^2 \end{cases} \Rightarrow x^3 = 125 \Rightarrow x = 5$$

$y = 5^2 = 25$

The solution is $\{(5, 25)\}$.

47. Using substitution, we have

$$\begin{cases} x^2 + 4y^2 = 25 \\ x - 2y + 1 = 0 \end{cases} \Rightarrow \begin{cases} x^2 + 4y^2 = 25 \\ x = 2y - 1 \end{cases} \Rightarrow$$

$(2y - 1)^2 + 4y^2 = 25 \Rightarrow 8y^2 - 4y - 24 = 0 \Rightarrow$

$4(y - 2)(2y + 3) = 0 \Rightarrow y = 2$ or $y = -\frac{3}{2}$

$x - 2(2) + 1 = 0 \Rightarrow x = 3$

$x - 2\left(-\frac{3}{2}\right) + 1 = 0 \Rightarrow x = -4$

The solution is $\left\{(3, 2), \left(-4, -\frac{3}{2}\right)\right\}$.

48. Using substitution, we have

$$\begin{cases} y = x^2 - 5x + 4 \\ 3x + y = 3 \end{cases} \Rightarrow 3x + x^2 - 5x + 4 = 3 \Rightarrow$$

$x^2 - 2x + 1 = 0 \Rightarrow (x - 1)^2 = 0 \Rightarrow x = 1$

$3(1) + y = 3 \Rightarrow y = 0$

The solution is $\{(1, 0)\}$.

49. Using elimination, we have

$$\begin{cases} x^2 - 3y^2 = 1 \\ x^2 + 4y^2 = 8 \end{cases} \Rightarrow -7y^2 = -7 \Rightarrow y = \pm 1$$

$x^2 - 3(-1)^2 = 1 \Rightarrow x = \pm 2$

$x^2 - 3(1)^2 = 1 \Rightarrow x = \pm 2$

The solution is $\{(-2, -1), (-2, 1), (2, -1), (2, 1)\}$.

50. Using elimination, we have

$$\begin{cases} 4x^2 - y^2 = 12 \\ 4y^2 - x^2 = 12 \end{cases} \Rightarrow \begin{cases} 4x^2 - y^2 = 12 \\ -4x^2 + 16y^2 = 48 \end{cases} \Rightarrow$$

$15y^2 = 60 \Rightarrow y = \pm 2$

$4(-2)^2 - x^2 = 12 \Rightarrow x = \pm 2$

The solution is $\{(-2, -2), (-2, 2), (2, -2), (2, 2)\}$.

51. Using elimination, we have

$$\begin{cases} x^2 - xy + 5x = 4 \\ 2x^2 - 3xy + 10x = -2 \end{cases} \Rightarrow$$

$$\begin{cases} -3x^2 + 3xy - 15x = -12 \\ 2x^2 - 3xy + 10x = -2 \end{cases} \Rightarrow -x^2 - 5x = -14 \Rightarrow$$

$x^2 + 5x - 14 = 0 \Rightarrow (x + 7)(x - 2) = 0 \Rightarrow$

$x = -7$ or $x = 2$

$(-7)^2 - (-7)y + 5(-7) = 4 \Rightarrow y = -\frac{10}{7}$

$(2)^2 - 2y + 5(2) = 4 \Rightarrow y = 5$

The solution is $\left\{\left(-7, -\frac{10}{7}\right), (2, 5)\right\}$.

52. Using elimination, we have

$$\begin{cases} x^2 - xy + x = -4 \\ 3x^2 - 2xy - 2x = 4 \end{cases} \Rightarrow$$

$$\begin{cases} -2x^2 + 2xy - 2x = 8 \\ 3x^2 - 2xy - 2x = 4 \end{cases} \Rightarrow x^2 - 4x = 12 \Rightarrow$$

$x^2 - 4x - 12 = 0 \Rightarrow (x-6)(x+2) = 0 \Rightarrow$
$x = 6$ or $x = -2$

$(6)^2 - 6y + 6 = -4 \Rightarrow y = \frac{23}{3}$

$(-2)^2 - (-2)y + (-2) = -4 \Rightarrow y = -3$

The solution is $\left\{\left(6, \frac{23}{3}\right), (-2, -3)\right\}$.

53. Using elimination, we have

$$\begin{cases} x^2 + y^2 - 8x = -8 \\ x^2 - 4y^2 + 6x = 0 \end{cases} \Rightarrow$$

$$\begin{cases} 4x^2 + 4y^2 - 32x = -32 \\ x^2 - 4y^2 + 6x = 0 \end{cases} \Rightarrow 5x^2 - 26x = -32 \Rightarrow$$

$5x^2 - 26x + 32 = 0 \Rightarrow (x-2)(5x-16) = 0 \Rightarrow$

$x = 2$ or $x = \frac{16}{5}$

$2^2 + y^2 - 8(2) = -8 \Rightarrow y = \pm 2$

$\left(\frac{16}{5}\right)^2 + y^2 - 8\left(\frac{16}{5}\right) = -8 \Rightarrow y^2 = \frac{184}{25} \Rightarrow$

$y = \pm\frac{2\sqrt{46}}{5}$

The solution is

$\left\{(2,-2), (2,2), \left(\frac{16}{5}, -\frac{2\sqrt{46}}{5}\right), \left(\frac{16}{5}, \frac{2\sqrt{46}}{5}\right)\right\}$.

54. Using elimination, we have

$$\begin{cases} 4x^2 + y^2 - 9y = -4 \\ 4x^2 - y^2 - 3y = 0 \end{cases} \Rightarrow 2y^2 - 6y = -4 \Rightarrow$$

$2y^2 - 6y + 4 = 0 \Rightarrow 2(y-2)(y-1) = 0 \Rightarrow$
$y = 2$ or $y = 1$

$4x^2 - 2^2 - 3(2) = 0 \Rightarrow x^2 = \frac{5}{2} \Rightarrow x = \pm\frac{\sqrt{10}}{2}$

$4x^2 - 1^2 - 3(1) = 0 \Rightarrow x = \pm 1$

The solution is $\left\{(-1,1), (1,1), \left(\frac{\sqrt{10}}{2}, 2\right),\right.$

$\left.\left(-\frac{\sqrt{10}}{2}, 2\right)\right\}$.

8.4 Applying the Concepts

55. Using elimination, we have

$$\begin{cases} p + 2x^2 = 96 \\ p - 13x = 39 \end{cases} \Rightarrow 2x^2 + 13x = 57 \Rightarrow$$

$2x^2 + 13x - 57 = 0 \Rightarrow (x-3)(2x+19) = 0 \Rightarrow$

$x = 3$ or $x = -\frac{19}{2}$ (reject this)

$p - 13(3) = 39 \Rightarrow p = 78$

Market equilibrium occurs when 3 (hundred) units are sold and the price is \$78/unit.

56. Using substitution, we have

$$\begin{cases} p^2 + 6p + 3x = 75 \\ -p + x = 13 \end{cases} \Rightarrow$$

$$\begin{cases} p^2 + 6p + 3x = 75 \\ x = p + 13 \end{cases} \Rightarrow$$

$p^2 + 6p + 3(p+13) = 75 \Rightarrow$

$p^2 + 9p - 36 = 0 \Rightarrow (p+12)(p-3) = 0 \Rightarrow$

$p = -12$ (reject this) or $p = 3$

$-3 + x = 13 \Rightarrow x = 16$

Market equilibrium occurs when 16 (hundred) units are sold and the price is \$3/unit.

57. Let x = the first positive number and let y = the second positive number. Then

$$\begin{cases} x + y = 24 \\ xy = 143 \end{cases} \Rightarrow \begin{cases} y = 24 - x \\ xy = 143 \end{cases} \Rightarrow$$

$x(24 - x) = 143 \Rightarrow -x^2 + 24x - 143 = 0 \Rightarrow$

$-(x-13)(x-11) = 0 \Rightarrow x = 11$ or $x = 13$

$11 + y = 24 \Rightarrow y = 13$

$13 + y = 24 \Rightarrow y = 11$

The numbers are 11 and 13.

58. Let x = the first positive number and let y = the second positive number. Then

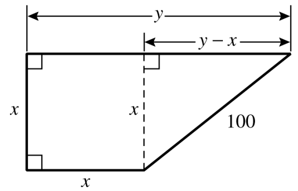
$$\begin{cases} x - y = 13 \\ xy = 114 \end{cases} \Rightarrow \begin{cases} x = y + 13 \\ xy = 114 \end{cases} \Rightarrow$$

$y(y+13) = 114 \Rightarrow y^2 + 13y - 114 = 0 \Rightarrow$

$(y+19)(y-6) = 0 \Rightarrow y = -19$ (reject this) or $y = 6$; $x - 6 = 13 \Rightarrow x = 19$

The numbers are 19 and 6.

59. a. Let x = the length of the two equal sides and let y = the length of the third side.



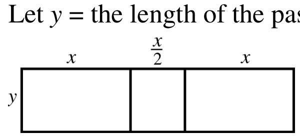
Using the Pythagorean theorem, we have $x^2+(y-x)^2=100^2$. So,

$$\begin{cases} 2x+y+100=360 \\ x^2+(y-x)^2=100^2 \end{cases} \Rightarrow$$
$$\begin{cases} y=260-2x \\ x^2+(y-x)^2=100^2 \end{cases} \Rightarrow$$
$$x^2+((260-2x)-x)^2=10{,}000 \Rightarrow$$
$$10x^2-1560x+57{,}600=0 \Rightarrow$$
$$10(x-60)(x-96)=0 \Rightarrow x=60 \text{ or } x=96$$
$$2(60)+y+100=360 \Rightarrow y=140$$
$$2(96)+y+100=360 \Rightarrow y=68$$

Because $y > x$ (from the diagram), we reject $x = 96$ and $y = 68$.
So, $x = 60$ m and $y = 140$ m

b. The area is $60^2+\frac{1}{2}(60)(80)=6000\text{ m}^2$.

60. Let x = the width of each larger pasture. Then $x/2$ = the width of the smaller pasture.
Let y = the length of the pasture.



Then

$$\begin{cases} \frac{5}{2}xy=8750 \\ x+2y=190 \end{cases} \Rightarrow \begin{cases} \frac{5}{2}xy=8750 \\ x=190-2y \end{cases} \Rightarrow$$
$$\frac{5}{2}y(190-2y)=8750 \Rightarrow$$
$$-5y^2+475y-8750=0 \Rightarrow$$
$$-5(y-70)(y-25)=0 \Rightarrow y=70 \text{ or } y=25$$
$$x+2(70)=190 \Rightarrow x=50$$
$$x+2(25)=190 \Rightarrow x=140$$

If $x = 50$, then the width of the pasture is 125 feet and the length is 70 feet. If $x = 140$, then the width of the pasture is 350 feet and the length is 25 feet. So, the dimensions are 125 feet by 70 feet or 25 feet by 350 feet.

61. Let x = the original number of students in the group and let y = the original cost per student. Then

$$\begin{cases} xy=960 \\ (x+8)(y-6)=960 \end{cases} \Rightarrow$$
$$\begin{cases} y=\dfrac{960}{x} \\ (x+8)(y-6)=960 \end{cases} \Rightarrow$$
$$(x+8)\left(\frac{960}{x}-6\right)=960 \Rightarrow$$
$$-6x+\frac{7680}{x}+912=960 \Rightarrow$$
$$-6x+\frac{7680}{x}-48=0 \Rightarrow$$
$$-6x^2-48x+7680=0 \Rightarrow$$
$$-6(x-32)(x+40)=0 \Rightarrow x=32 \text{ or}$$
$$x=-40 \text{ (reject this)}$$
$$32y=960 \Rightarrow y=30$$

There were originally 32 students at a cost of $30 each.

62. Let x = the width of the rectangular carpet (in feet). Then $x + 6$ = the length of the rectangular carpet (in feet). Let y = the length of a side of the square carpet (in feet). To convert square feet to square yards, divide by 9. Then

$$\begin{cases} x(x+6)+y^2=540 \\ 10\left(\dfrac{x(x+6)}{9}\right)=50+12\left(\dfrac{y^2}{9}\right) \end{cases} \Rightarrow$$
$$\begin{cases} x^2+6x+y^2=540 \\ 10x^2+60x=450+12y^2 \end{cases} \Rightarrow$$
$$\begin{cases} x^2+6x+y^2=540 \\ 10x^2+60x-12y^2=450 \end{cases} \Rightarrow$$
$$\begin{cases} 12x^2+72x+12y^2=6480 \\ 10x^2+60x-12y^2=450 \end{cases} \Rightarrow$$
$$22x^2+132x=6930 \Rightarrow x^2+6x-315=0 \Rightarrow$$
$(x-15)(x+21)=0 \Rightarrow x=15$ or $x=-21$ (reject the negative solution.)
$15(15+6)+y^2=540 \Rightarrow y^2=225 \Rightarrow y=\pm 15$ (reject the negative solution).
The rectangular carpet is 15 feet by 21 feet.
The square carpet is 15 feet by 15 feet.

63. Let x = the number of shares of stock she bought and let y = the original price per share. Then

$$\begin{cases} xy + 100 = 10{,}000 \\ (x+30)(y+3) = 11{,}900 + 100 \end{cases} \Rightarrow$$

$$\begin{cases} y = \dfrac{9900}{x} \\ (x+30)(y+3) = 12{,}000 \end{cases} \Rightarrow$$

$$(x+30)\left(\frac{9900}{x} + 3\right) = 12{,}000 \Rightarrow$$

$$3x + \frac{297{,}000}{x} - 2010 = 0 \Rightarrow$$

$$3x^2 - 2010x + 297{,}000 = 0 \Rightarrow$$

$$3(x-450)(x-220) = 0 \Rightarrow x = 450 \text{ or } x = 220$$

The problem says that she bought more than 400 shares, so reject $x = 220$.

$$450y + 100 = 10{,}000 \Rightarrow y = 22$$

She bought 450 shares at \$22 per share.

64. Let x = the number of shares of stock she bought and let y = the original price per share. Then

$$\begin{cases} xy + 100 = 10{,}000 \\ (x-100)(y+10) = 13{,}900 + 100 \end{cases} \Rightarrow$$

$$\begin{cases} y = \dfrac{9900}{x} \\ (x-100)(y+10) = 14{,}000 \end{cases} \Rightarrow$$

$$(x-100)\left(\frac{9900}{x} + 10\right) = 14{,}000 \Rightarrow$$

$$10x - \frac{990{,}000}{x} - 5100 = 0 \Rightarrow$$

$$10x^2 - 5100x - 990{,}000 = 0 \Rightarrow$$

$$10(x+150)(x-660) = 0 \Rightarrow x = -150 \text{ or } x = 660$$

(Reject the negative solution.)

$660y + 100 = 10{,}000 \Rightarrow y = 15$. She bought 660 shares at \$15 per share.

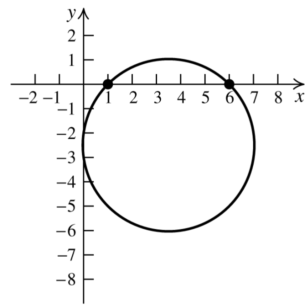
8.4 Beyond the Basics

65. $$\begin{cases} x^2 + y^2 - 7x + 5y + 6 = 0 \\ y = 0 \end{cases} \Rightarrow$$

$$x^2 - 7x + 6 = 0 \Rightarrow (x-6)(x-1) = 0 \Rightarrow$$

$$x = 6 \text{ or } x = 1$$

The circle intersects the x-axis at $A(1, 0)$ and $B(6, 0)$. $d(A, B) = \sqrt{(6-1)^2 + (0-0)^2} = 5.$

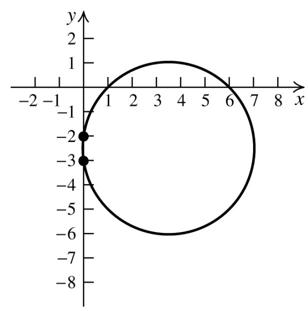


66. $$\begin{cases} x^2 + y^2 - 7x + 5y + 6 = 0 \\ x = 0 \end{cases} \Rightarrow$$

$$y^2 + 5y + 6 = 0 \Rightarrow (y+2)(y+3) = 0 \Rightarrow$$

$$y = -2 \text{ or } y = -3$$

The circle intersects the y-axis at $A(0, -2)$ and $B(0, -3)$. $d(A, B) = 1.$



67. $$\begin{cases} x^2 + y^2 + 2x - 4y - 5 = 0 \\ x - y + 1 = 0 \end{cases} \Rightarrow$$

$$\begin{cases} x^2 + y^2 + 2x - 4y - 5 = 0 \\ x = y - 1 \end{cases} \Rightarrow$$

$$(y-1)^2 + y^2 + 2(y-1) - 4y - 5 = 0 \Rightarrow$$

$$2y^2 - 4y - 6 = 0 \Rightarrow 2(y-3)(y+1) = 0 \Rightarrow$$

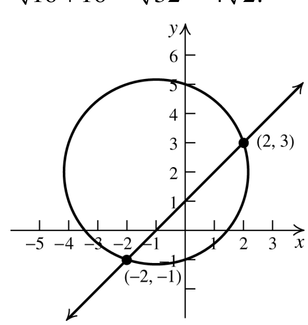
$$y = 3 \text{ or } y = -1$$

$$x - 3 + 1 = 0 \Rightarrow x = 2$$

$$x - (-1) + 1 \Rightarrow x = -2$$

The circle and the line intersect at $A(-2, -1)$ and $B(2, 3)$.

$$d(A, B) = \sqrt{(-2-2)^2 + (-1-3)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}.$$



68. $\begin{cases} x^2+y^2-6x-8y-50=0 \\ 2x+y-5=0 \end{cases} \Rightarrow$

$\begin{cases} x^2+y^2-6x-8y-50=0 \\ y=5-2x \end{cases} \Rightarrow$

$x^2+(5-2x)^2-6x-8(5-2x)-50=0 \Rightarrow$

$5x^2-10x-65=0 \Rightarrow x=\dfrac{10\pm\sqrt{100+1300}}{10} \Rightarrow$

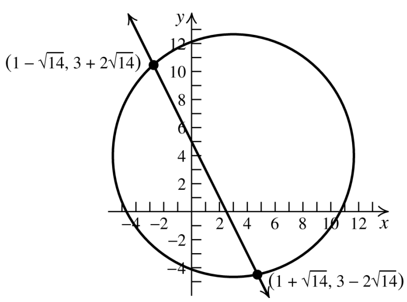
$x=1\pm\sqrt{14}$

$y=5-2(1+\sqrt{14})=3-2\sqrt{14}$

$y=5-2(1-\sqrt{14})=3+2\sqrt{14}$

The circle and the line intersect at $A\left(1+\sqrt{14},3-2\sqrt{14}\right)$ and $B\left(1-\sqrt{14},3+2\sqrt{14}\right)$.

$$d(A,B)=\sqrt{\left(\left(1+\sqrt{14}\right)-\left(1-\sqrt{14}\right)\right)^2+\left(\left(3-2\sqrt{14}\right)-\left(3+2\sqrt{14}\right)\right)^2}$$
$$=\sqrt{\left(2\sqrt{14}\right)^2+\left(-4\sqrt{14}\right)^2}=\sqrt{56+224}$$
$$=\sqrt{280}=2\sqrt{70}$$



69. $\begin{cases} x^2+y^2-2x+2y-3=0 \\ x+2y+6=0 \end{cases} \Rightarrow$

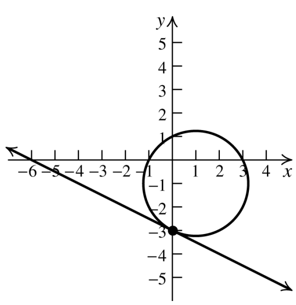
$\begin{cases} x^2+y^2-2x+2y-3=0 \\ x=-2y-6 \end{cases} \Rightarrow$

$(-2y-6)^2+y^2-2(-2y-6)+2y-3=0 \Rightarrow$

$5y^2+30y+45=0 \Rightarrow 5(y+3)^2=0 \Rightarrow y=-3$

$x+2(-3)+6=0 \Rightarrow x=0$

The line and the circle intersect at only one point (0, −3), so the line is tangent to the circle.



70. $\begin{cases} x^2+y^2-2x-4y+4=0 \\ 3x-4y=0 \end{cases} \Rightarrow$

$\begin{cases} x^2+y^2-2x-4y+4=0 \\ y=\dfrac{3}{4}x \end{cases} \Rightarrow$

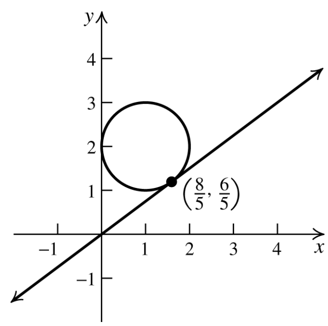
$x^2+\left(\dfrac{3}{4}x\right)^2-2x-4\left(\dfrac{3}{4}x\right)+4=0 \Rightarrow$

$\dfrac{25}{16}x^2-5x+4=0 \Rightarrow 25x^2-80x+64=0 \Rightarrow$

$(5x-8)^2=0 \Rightarrow x=\dfrac{8}{5}$

$3\left(\dfrac{8}{5}\right)-4y=0 \Rightarrow y=\dfrac{6}{5}$

The line and the circle intersect at only one point $(8/5, 6/5)$, so the line is tangent to the circle.



71. $\begin{cases} x+y=2 & (1) \\ x^2+y^2=c^2 & (2) \end{cases}$

From equation (1), we have $y = 2 - x$. Substituting in equation (2) and then solving for x, we have

$x^2+(2-x)^2=c^2 \Rightarrow x^2+4-4x+x^2=c^2 \Rightarrow$

$2x^2-4x+4-c^2=0 \Rightarrow$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)\left(4-c^2\right)}}{2(2)}$$

Since the equations are tangent, there is only one solution. Recall that the value of the discriminant is 0 when there is exactly one real solution, so solve

$(-4)^2-4(2)\left(4-c^2\right)=0 \Rightarrow$

$16-32+8c^2=0 \Rightarrow c^2=2 \Rightarrow c=\pm\sqrt{2}$

72. $\begin{cases} cx - y = 5 & (1) \\ x^2 + y^2 = 16 & (2) \end{cases}$

From equation (1), we have $y = cx - 5$. Substituting in equation (2) and then solving for x, we have

$$x^2 + (cx-5)^2 = 16 \Rightarrow$$
$$x^2 + c^2x^2 - 10cx + 25 = 16 \Rightarrow$$
$$x^2(1+c^2) - 10cx + 9 = 0 \Rightarrow$$
$$x = \frac{-(-10c) \pm \sqrt{(-10c)^2 - 4(1+c^2)(9)}}{2(1+c^2)}$$

Since the equations are tangent, there is only one solution. Recall that the value of the discriminant is 0 when there is exactly one real solution, so solve

$$(-10c)^2 - 4(1+c^2)(9) = 0 \Rightarrow$$
$$100c^2 - 36 - 36c^2 = 0 \Rightarrow 64c^2 - 36 = 0 \Rightarrow$$
$$(8c-6)(8c+6) = 0 \Rightarrow c = \pm\frac{3}{4}$$

73. $\begin{cases} 4x^2 + y^2 = 25 & (1) \\ 8x + 3y = c & (2) \end{cases}$

From equation (2), we have $y = \dfrac{c-8x}{3}$.

Substituting in equation (1) and then solving for x, we have

$$4x^2 + \left(\frac{c-8x}{3}\right)^2 = 25 \Rightarrow$$
$$4x^2 + \frac{c^2 - 16cx + 64x^2}{9} = 25 \Rightarrow$$
$$36x^2 + c^2 - 16cx + 64x^2 = 225 \Rightarrow$$
$$100x^2 - 16cx + c^2 - 225 = 0$$
$$x = \frac{-(-16c) \pm \sqrt{(-16c)^2 - 4(100)(c^2-225)}}{2(100)}$$

Since the equations are tangent, there is only one solution. Recall that the value of the discriminant is 0 when there is exactly one real solution, so solve

$$(-16c)^2 - 4(100)(c^2 - 225) = 0 \Rightarrow$$
$$256c^2 - 400c^2 + 90{,}000 = 0 \Rightarrow$$
$$90{,}000 - 144c^2 = 0 \Rightarrow$$
$$(300-12c)(300+12c) = 0 \Rightarrow c = \pm 25$$

74. $\begin{cases} x^2 + 4y^2 = 9 & (1) \\ cx + 2y = -4 & (2) \end{cases}$

From equation (2), we have $y = \dfrac{-4-cx}{2}$.

Substituting in equation (1) and then solving for x, we have

$$x^2 + 4\left(\frac{-4-cx}{2}\right)^2 = 9 \Rightarrow$$
$$x^2 + c^2x^2 + 8cx + 16 = 9 \Rightarrow$$
$$(1+c^2)x^2 + 8cx + 7 = 0$$
$$x = \frac{-8c \pm \sqrt{(8c)^2 - 4(1+c^2)(7)}}{2(1+c^2)}$$

Since the equations are tangent, there is only one solution. Recall that the value of the discriminant is 0 when there is exactly one real solution, so solve

$$(8c)^2 - 4(1+c^2)(7) = 0 \Rightarrow$$
$$64c^2 - 28 - 28c^2 = 0 \Rightarrow 36c^2 = 28 \Rightarrow$$
$$c^2 = \frac{7}{9} \Rightarrow c = \pm\frac{\sqrt{7}}{3}$$

In exercises 75–78, let $u = \dfrac{1}{x}$ and $v = \dfrac{1}{y}$.

75. $\begin{cases} \dfrac{1}{x} - \dfrac{1}{y} = 5 \\ \dfrac{1}{xy} - 24 = 0 \end{cases} \Rightarrow \begin{cases} u - v = 5 & (1) \\ uv - 24 = 0 & (2) \end{cases}$

From equation (1), we have $u = v + 5$. Substitute this into equation (2) and solve for v.

$$v(v+5) - 24 = 0 \Rightarrow v^2 + 5v - 24 = 0 \Rightarrow$$
$$(v+8)(v-3) = 0 \Rightarrow v = -8,\ 3$$
$$v = -8 \Rightarrow u + 8 = 5 \Rightarrow u = -3 \Rightarrow$$
$$x = -\frac{1}{3},\ y = -\frac{1}{8}$$
$$v = 3 \Rightarrow u - 3 = 5 \Rightarrow u = 8 \Rightarrow x = \frac{1}{8},\ y = \frac{1}{3}$$

Solution set: $\left\{\left(-\frac{1}{3}, -\frac{1}{8}\right), \left(\frac{1}{8}, \frac{1}{3}\right)\right\}$

76. $\begin{cases} \frac{1}{x}+\frac{1}{y}=3 \\ \frac{1}{x^2}-\frac{1}{y^2}=-3 \end{cases} \Rightarrow \begin{cases} u+v=3 & (1) \\ u^2-v^2=-3 & (2) \end{cases}$

From equation (1), we have $v = 3 - u$. Substitute this into equation (2) and solve for u.

$u^2-(3-u)^2=-3 \Rightarrow u^2-(u^2-6u+9)=-3 \Rightarrow$
$6u-9=-3 \Rightarrow 6u=6 \Rightarrow u=1 \Rightarrow x=1$

$1+v=3 \Rightarrow v=2 \Rightarrow y=\frac{1}{2}$

Solution set: $\left\{\left(1, \frac{1}{2}\right)\right\}$

77. $\begin{cases} \frac{5}{x^2}-\frac{3}{y^2}=2 \\ \frac{6}{x^2}+\frac{1}{y^2}=7 \end{cases} \Rightarrow \begin{cases} 5u^2-3v^2=2 & (1) \\ 6u^2+v^2=7 & (2) \end{cases}$

Solve equation (2) for v^2, then substitute this expression in equation (1) and solve for u.

$v^2=7-6u^2$

$5u^2-3(7-6u^2)=2 \Rightarrow 23u^2-21=2 \Rightarrow$
$u^2=1 \Rightarrow u=\pm1 \Rightarrow x=\pm1$

If $u = 1$, then

$6(1^2)+v^2=7 \Rightarrow v^2=1 \Rightarrow v=\pm1 \Rightarrow y=\pm1$

If $u = -1$, then

$6(-1^2)+v^2=7 \Rightarrow v^2=1 \Rightarrow v=\pm1 \Rightarrow y=\pm1$

Solution set: {(−1, −1), (−1, 1), (1, −1), (1, 1)}

78. $\begin{cases} \frac{3}{x^2}-\frac{2}{y^2}=1 \\ \frac{6}{x^2}+\frac{5}{y^2}=11 \end{cases} \Rightarrow \begin{cases} 3u^2-2v^2=1 & (1) \\ 6u^2+5v^2=11 & (2) \end{cases}$

Multiply equation (1) by −2, then add the resulting equation to equation (2) and solve for v.

$\begin{cases} -6u^2+4v^2=-2 \\ 6u^2+5v^2=11 \end{cases} \Rightarrow 9v^2=9 \Rightarrow v^2=1 \Rightarrow$
$v=\pm1 \Rightarrow y=\pm1$

If $v = 1$, then

$3u^2-2(1^2)=1 \Rightarrow 3u^2=3 \Rightarrow u^2=1 \Rightarrow$
$u=\pm1 \Rightarrow x=\pm1$

If $v = -1$, then

$3u^2-2(-1^2)=1 \Rightarrow 3u^2=3 \Rightarrow u^2=1 \Rightarrow$
$u=\pm1 \Rightarrow x=\pm1$

Solution set: {(−1, −1), (−1, 1), (1, −1), (1, 1)}

79. $(a+bi)^2=-5+12i \Rightarrow$
$a^2+2abi-b^2=-5+12i \Rightarrow$
$\begin{cases} a^2-b^2=-5 \\ 2ab=12 \end{cases} \Rightarrow \begin{cases} a^2-b^2=-5 & (1) \\ ab=6 & (2) \end{cases}$

From equation (2), we have $b=\frac{6}{a}$. Substitute this into equation (1) and solve for a.

$a^2-\left(\frac{6}{a}\right)^2=-5 \Rightarrow a^4+5a^2-36=0 \Rightarrow$
$(a^2+9)(a^2-4)=0 \Rightarrow$
$(a^2+9)(a-2)(a+2)=0 \Rightarrow a=\pm2, \pm3i$

Since a is real, reject $\pm3i$. Using equation (2), if $a = -2$, then $b = -3$. If $a = 2$, then $b = 3$. Thus, the square roots of $w = -5 + 12i$ are $-2 - 3i$ and $2 + 3i$.

80. $(a+bi)^2=-16-30i \Rightarrow$
$a^2+2abi-b^2=-16-30i \Rightarrow$
$\begin{cases} a^2-b^2=-16 \\ 2ab=-30 \end{cases} \Rightarrow \begin{cases} a^2-b^2=-16 & (1) \\ ab=-15 & (2) \end{cases}$

From equation (2), we have $b=-\frac{15}{a}$.

Substitute this into equation (1) and solve for a.

$a^2-\left(-\frac{15}{a}\right)^2=-16 \Rightarrow a^4+16a^2-225=0 \Rightarrow$
$(a^2+25)(a^2-9)=0 \Rightarrow$
$(a^2+25)(a-3)(a+3)=0 \Rightarrow a=\pm3, \pm5i$

Since a is real, reject $\pm5i$. Using equation (2), if $a = -3$, then $b = 5$. If $a = 3$, then $b = -5$. Thus, the square roots of $w = -16 - 30i$ are $-3 + 5i$ and $3 - 5i$.

81. $(a+bi)^2=7-24i \Rightarrow$
$a^2+2abi-b^2=7-24i \Rightarrow$
$\begin{cases} a^2-b^2=7 \\ 2ab=-24 \end{cases} \Rightarrow \begin{cases} a^2-b^2=7 & (1) \\ ab=-12 & (2) \end{cases}$

From equation (2), we have $b=-\frac{12}{a}$.

Substitute this into equation (1) and solve for a.

$a^2-\left(-\frac{12}{a}\right)^2=7 \Rightarrow a^4-7a^2-144=0 \Rightarrow$
$(a^2-16)(a^2+9)=0 \Rightarrow$
$(a^2+9)(a-4)(a+4)=0 \Rightarrow a=\pm4, \pm3i$

Since a is real, reject $\pm3i$.

(*continued on next page*)

(*continued*)

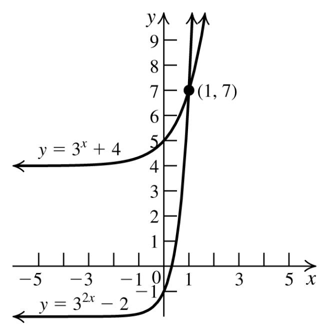
Using equation (2), if $a = -4$, then $b = 3$. If $a = 4$, then $b = -3$. Thus, the square roots of $w = 7 - 24i$ are $-4 + 3i$ and $4 - 3i$.

82. $(a+bi)^2 = 3-4i \Rightarrow$
$a^2 + 2abi - b^2 = 3 - 4i \Rightarrow$
$$\begin{cases} a^2 - b^2 = 3 \\ 2ab = -4 \end{cases} \Rightarrow \begin{cases} a^2 - b^2 = 3 & (1) \\ ab = -2 & (2) \end{cases}$$
From equation (2), we have $b = -\dfrac{2}{a}$.
Substitute this into equation (1) and solve for a.
$$a^2 - \left(-\frac{2}{a}\right)^2 = 3 \Rightarrow a^4 - 3a^2 - 4 = 0 \Rightarrow$$
$$\left(a^2-4\right)\left(a^2+1\right) = 0 \Rightarrow$$
$$\left(a^2+1\right)(a-2)(a+2) = 0 \Rightarrow a = \pm 2,\ \pm i$$
Since a is real, reject $\pm i$. Using equation (2), if $a = -2$, then $b = 1$. If $a = 2$, then $b = -1$. Thus, the square roots of $w = 3 - 4i$ are $-2 + i$ and $2 - i$.

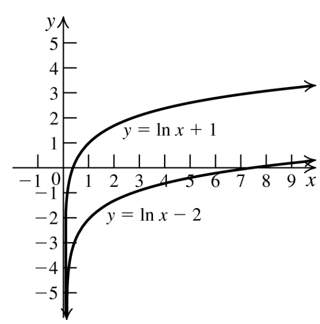
83. $(a+bi)^2 = 13 + 8\sqrt{3}i \Rightarrow$
$a^2 + 2abi - b^2 = 13 + 8\sqrt{3}i \Rightarrow$
$$\begin{cases} a^2 - b^2 = 13 \\ 2ab = 8\sqrt{3} \end{cases} \Rightarrow \begin{cases} a^2 - b^2 = 13 & (1) \\ ab = 4\sqrt{3} & (2) \end{cases}$$
From equation (2), we have $b = \dfrac{4\sqrt{3}}{a}$.
Substitute this into equation (1) and solve for a.
$$a^2 - \left(\frac{4\sqrt{3}}{a}\right)^2 = 13 \Rightarrow a^4 - 13a^2 - 48 = 0 \Rightarrow$$
$$\left(a^2-16\right)\left(a^2+3\right) = 0 \Rightarrow$$
$$\left(a^2+3\right)(a-4)(a+4) = 0 \Rightarrow a = \pm 4,\ \pm\sqrt{3}i$$
Since a is real, reject $\pm\sqrt{3}i$. Using equation (2), if $a = -4$, then $b = -\sqrt{3}$. If $a = 4$, then $b = \sqrt{3}$. Thus, the square roots of $w = 13 + 8\sqrt{3}i$ are $-4 - \sqrt{3}i$ and $4 + \sqrt{3}i$.

84. $(a+bi)^2 = -1 - 4\sqrt{5}i \Rightarrow$
$a^2 + 2abi - b^2 = -1 - 4\sqrt{5}i \Rightarrow$
$$\begin{cases} a^2 - b^2 = -1 \\ 2ab = -4\sqrt{5} \end{cases} \Rightarrow \begin{cases} a^2 - b^2 = -1 & (1) \\ ab = -2\sqrt{5} & (2) \end{cases}$$
From equation (2), we have $b = -\dfrac{2\sqrt{5}}{a}$.
Substitute this into equation (1) and solve for a.
$$a^2 - \left(-\frac{2\sqrt{5}}{a}\right)^2 = -1 \Rightarrow a^4 + a^2 - 20 = 0 \Rightarrow$$
$$\left(a^2-4\right)\left(a^2+5\right) = 0 \Rightarrow$$
$$\left(a^2+5\right)(a-2)(a+2) = 0 \Rightarrow a = \pm 2,\ \pm\sqrt{5}i$$
Since a is real, reject $\pm\sqrt{5}i$. Using equation (2), if $a = -2$, then $b = \sqrt{5}$. If $a = 2$, then $b = -\sqrt{5}$. Thus, the square roots of $w = -1 - 4\sqrt{5}i$ are $2 - \sqrt{5}i$ and $-2 + \sqrt{5}i$.

85. Using substitution, we have
$$\begin{cases} y = 3^x + 4 \\ y = 3^{2x} - 2 \end{cases} \Rightarrow 3^{2x} - 2 = 3^x + 4 \Rightarrow$$
$$3^{2x} - 3^x - 6 = 0$$
Let $u = 3^x$. Then $u^2 - u - 6 = 0 \Rightarrow$
$(u-3)(u+2) = 0 \Rightarrow u = 3$ or $u = -2$
(Reject the negative solution.)
$3 = 3^x \Rightarrow x = 1$
$y = 3^1 + 4 = 7$
The solution is $\{(1, 7)\}$.



86. Using substitution, we have
$$\begin{cases} y = \ln x + 1 \\ y = \ln x - 2 \end{cases} \Rightarrow \ln x + 1 = \ln x - 2 \Rightarrow 0 = -3 \Rightarrow$$
there is no solution.



87. Using substitution, we have

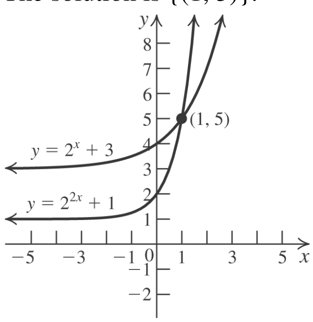
$$\begin{cases} y = 2^x + 3 \\ y = 2^{2x} + 1 \end{cases} \Rightarrow 2^x + 3 = 2^{2x} + 1 \Rightarrow$$

$2^{2x} - 2^x - 2 = 0.$

Let $u = 2^x$. Then $u^2 - u - 2 = 0 \Rightarrow$
$(u-2)(u+1) = 0 \Rightarrow u = 2$ or $u = -1$
(reject the negative solution.)

$2 = 2^x \Rightarrow x = 1; \quad y = 2^1 + 3 = 5$

The solution is $\{(1, 5)\}$.



88. Using substitution, we have

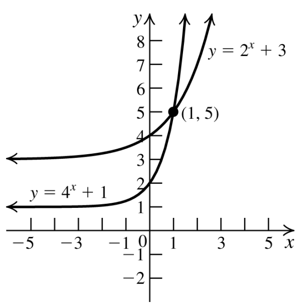
$$\begin{cases} y = 2^x + 3 \\ y = 4^x + 1 \end{cases} \Rightarrow 2^x + 3 = 4^x + 1 \Rightarrow$$

$4^x - 2^x - 2 = 0 \Rightarrow 2^{2x} - 2^x - 2 = 0$. Let $u = 2^x$.

Then, $u^2 - u - 2 = 0 \Rightarrow (u-2)(u+1) = 0 \Rightarrow$
$u = 2$ or $u = -1$. (Reject the negative solution.)

$2 = 2^x \Rightarrow x = 1; \quad y = 2^1 + 3 = 5$

The solution is $\{(1, 5)\}$.



89. $\begin{cases} x = 3^y \\ 3^{2y} = 3x - 2 \end{cases} \Rightarrow 3^{2y} = 3 \cdot 3^y - 2 \Rightarrow$

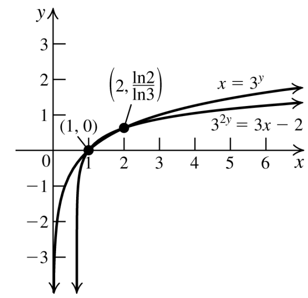
$3^{2y} - 3 \cdot 3^y + 2 = 0$. Let $u = 3^y$. Then
$u^2 - 3u + 2 = 0 \Rightarrow (u-2)(u-1) = 0 \Rightarrow$
$u = 2$ or $u = 1$

$2 = 3^y \Rightarrow \ln 2 = y \ln 3 \Rightarrow \dfrac{\ln 2}{\ln 3} = y;$

$x = 3^{\ln 2/\ln 3} = 2$ or $1 = 3^y \Rightarrow y = 0$

$x = 3^0 = 1$

The solution is $\left\{(1,0), \left(2, \dfrac{\ln 2}{\ln 3}\right)\right\}$.



90. $\begin{cases} x = 3^y \\ x^2 = 3^{y+1} - 2 \end{cases} \Rightarrow 3^{2y} = 3^{y+1} - 2 \Rightarrow$

$3^{2y} - 3^{y+1} + 2 = 0$. Let $u = 3^y$. Then
$u^2 - 3u + 2 = 0 \Rightarrow (u-2)(u-1) = 0 \Rightarrow$
$u = 2$ or $u = 1$.

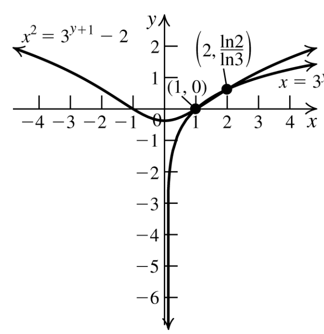
$2 = 3^y \Rightarrow \ln 2 = y \ln 3 \Rightarrow \dfrac{\ln 2}{\ln 3} = y$

$x = 3^{\ln 2/\ln 3} = 2$ or

$1 = 3^y \Rightarrow y = 0$

$x = 3^0 = 1$

The solution is $\left\{(1,0), \left(2, \dfrac{\ln 2}{\ln 3}\right)\right\}$.



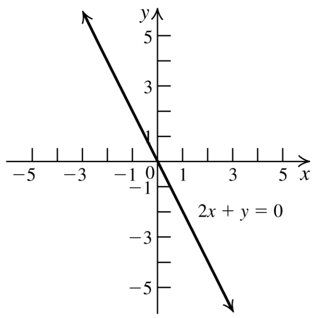
8.4 Critical Thinking/Discussion/Writing

91. a. Not possible
b. Possible
c. Possible
d. Possible
e. Not possible
f. Not possible.

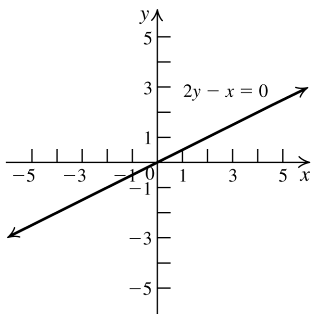
92. a. Possible
b. Possible
c. Possible
d. Possible
e. Possible
f. Not possible.

8.4 Maintaining Skills

93.



94.



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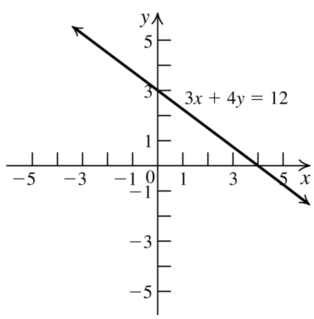

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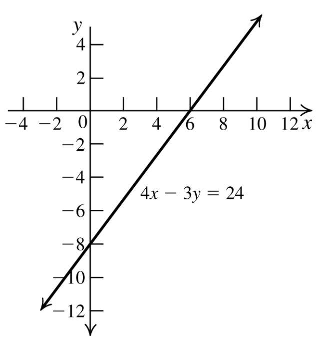

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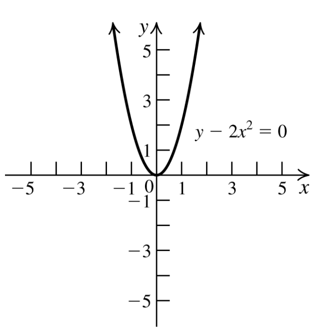

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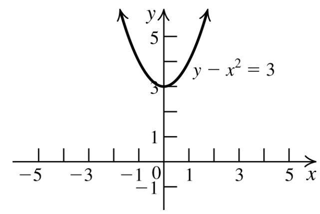

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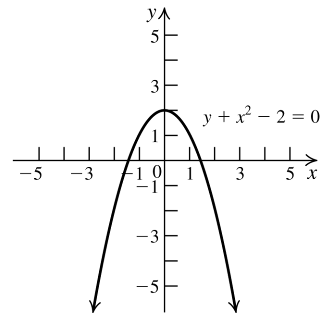
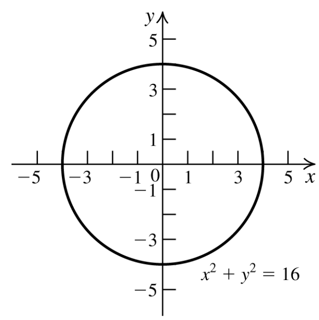


100.



101.

Point	**$5x + 3y = 0$?**
(0, 0)	$5(0)+3(0)\stackrel{?}{=}0$ $0=0$ ✓
(−3, 5)	$5(-3)+3(5)\stackrel{?}{=}0$ $0=0$ ✓
(1, −1)	$5(-1)+3(1)\stackrel{?}{=}0$ $-2=0$ ×
(−1, 1)	$5(1)+3(-1)\stackrel{?}{=}0$ $2=0$ ×

(0, 0) and (−3, 5) lie on the graph of the equation $5x + 3y = 0$.

102.

Point	$3x + 4y = 12$?
(0, 0)	$3(0)+4(0)\stackrel{?}{=}12$ $0=12$ ×
(4, 0)	$3(4)+4(0)\stackrel{?}{=}12$ $12=12$ ✓
(0, 3)	$3(0)+4(3)\stackrel{?}{=}12$ $12=12$ ✓
(3, 4)	$3(3)+4(4)\stackrel{?}{=}12$ $25=12$ ×

(4, 0) and (0, 3) lie on the graph of the equation $3x + 4y = 12$.

103.

Point	$y = x^2 + 3$?
(0, 0)	$0\stackrel{?}{=}0^2+3$ $0=3$ ×
(1, 4)	$4\stackrel{?}{=}1^2+3$ $3=3$ ✓
(−1, 4)	$4\stackrel{?}{=}(-1)^2+3$ $4=4$ ✓
(2, 5)	$5\stackrel{?}{=}2^2+3$ $5=7$ ×

(1, 4) and (−1, 4) lie on the graph of the equation $y = x^2 + 3$.

104.

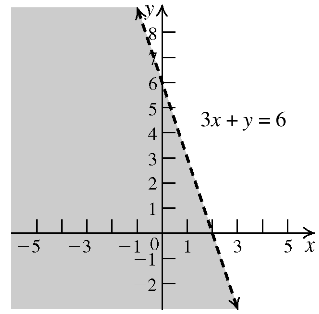
Point	$y + x^2 = 2$?
(0, 0)	$0+0^2\stackrel{?}{=}2$ $0=2$ ×
(0, 2)	$2+0^2\stackrel{?}{=}2$ $2=2$ ✓
(2, −2)	$-2+(-2)^2\stackrel{?}{=}2$ $2=2$ ✓
(1, 1)	$1+1^2\stackrel{?}{=}2$ $2=2$ ✓

(0, 2), (2, −2), and (1, 1) lie on the graph of the equation $y + x^2 = 2$.

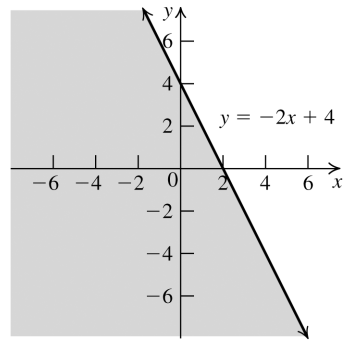
8.5 Systems of Inequalities

8.5 Practice Problems

1.



2.



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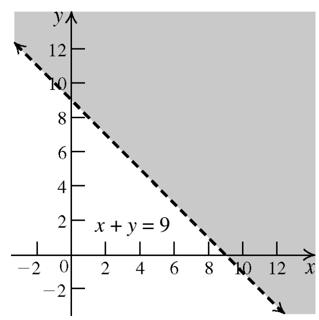
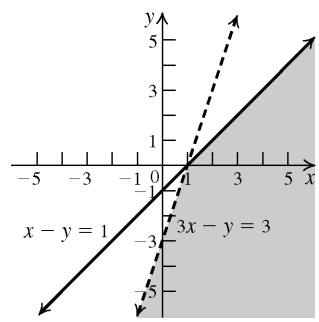
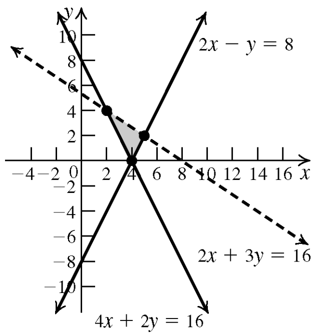


4.



5. Solve the systems $\begin{cases} 2x + 3y = 16 \\ 4x + 2y = 16 \end{cases}$, $\begin{cases} 4x + 2y = 16 \\ 2x - \quad y = 8 \end{cases}$, and $\begin{cases} 2x + 3y = 16 \\ 2x - \quad y = 8 \end{cases}$ to find the corner points: (4, 0), (5, 2), and (2, 4).



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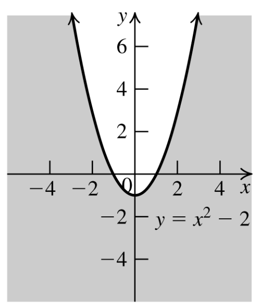
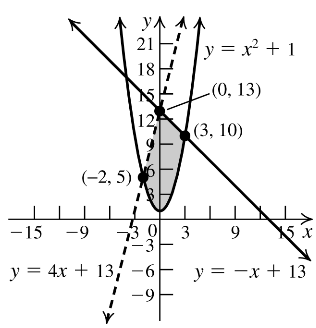


7.



8.5 Basic Concepts and Skills

1. In the graph of $x - 3y > 1$, the corresponding equation $x - 3y = 1$ is graphed as a <u>dashed</u> line.

2. If a test point from one of the two regions determined by an inequality's corresponding equation satisfies the inequality, then <u>all points</u> in that region satisfy the inequality.

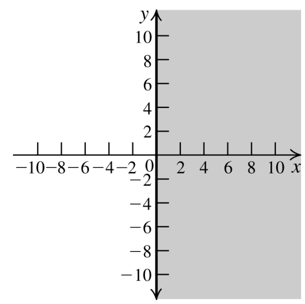
3. In a system of inequalities containing both $2x + y > 5$ and $2x - y \le 3$, the point of intersection of the lines $2x + y = 5$ and $2x - y = 3$ is <u>not a solution of the system</u>.

4. In a nonlinear system of equations or inequalities, <u>at least one</u> equation or inequality must be nonlinear.

5. True

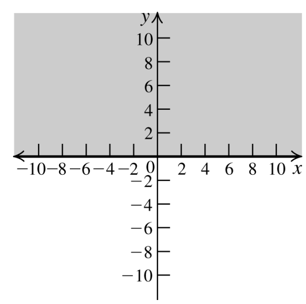
6. False. The point (2, 1) is not on the graph of $x^2 + 2y = 5$.

7.



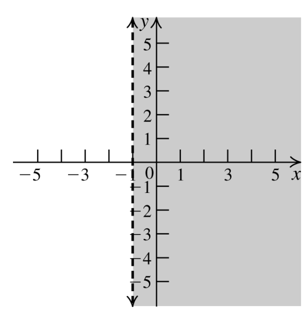
$x \ge 0$

8.



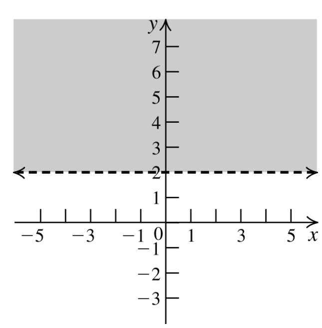
$y \ge 0$

9.



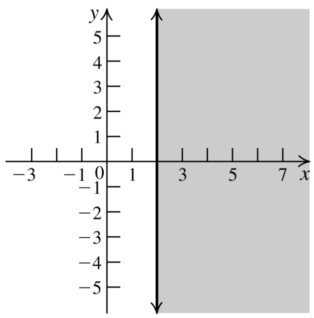
$x > -1$

10.



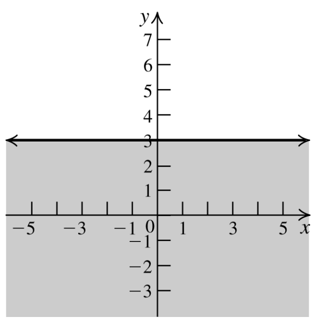
$y > 2$

11.



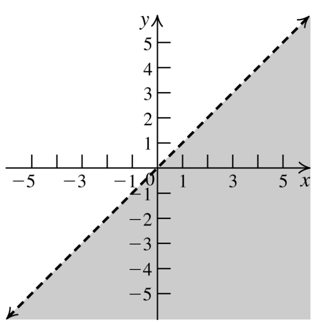
$x \geq 2$

12.



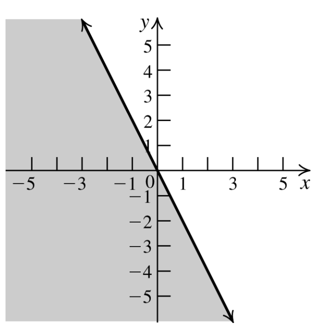
$y \leq 3$

13.



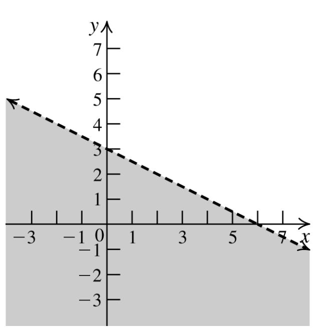
$y - x < 0$

14.



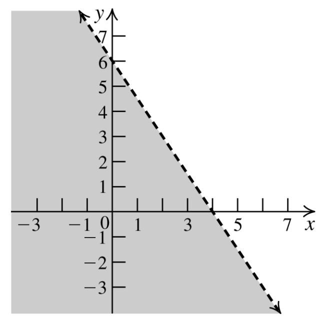
$y + 2x \leq 0$

15.



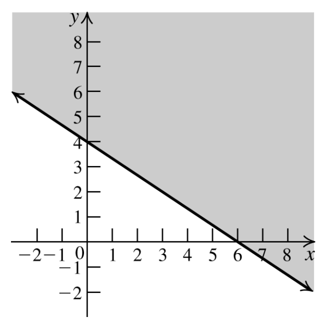
$x + 2y < 6$

16.



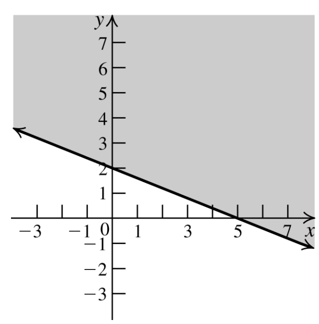
$3x + 2y < 12$

17.



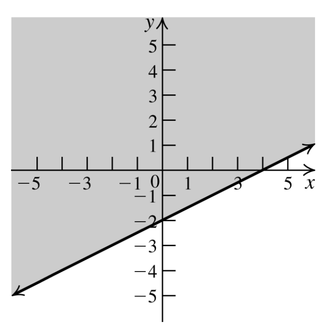
$2x + 3y \geq 12$

18.



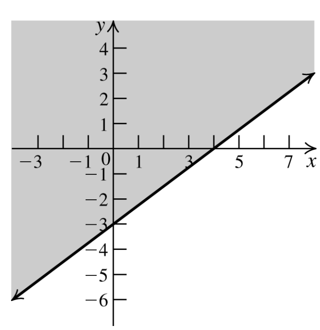
$2x + 5y \geq 10$

19.



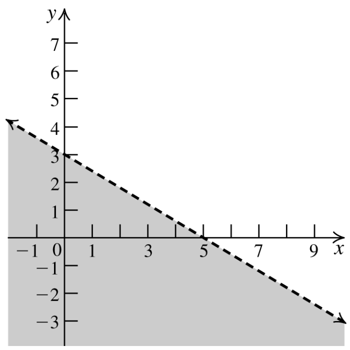
$x - 2y \leq 4$

20.



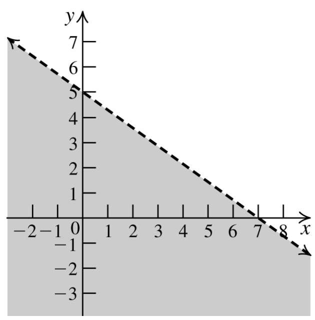
$3x - 4y \leq 12$

21.



$3x + 5y < 15$

22.



$5x + 7y < 35$

23. Substitute each ordered pair into the system:

$\begin{cases} 0+0<2 \\ 2(0)+0\ge 6 \end{cases}$ $\begin{cases} -4-1<2 \\ 2(-4)-1\ge 6 \end{cases}$

$\begin{cases} 3+0<2 \\ 2(3)+0\ge 6 \end{cases}$ $\begin{cases} 0+3<2 \\ 2(0)+3\ge 6 \end{cases}$

None of the ordered pairs are a solution.

24. Substitute each ordered pair into the system:

$\begin{cases} 0-0<2 \\ 3(0)+4(0)\ge 12 \end{cases}$ $\begin{cases} \dfrac{16}{5}-\dfrac{3}{5}<2 \\ 3\left(\dfrac{16}{5}\right)+4\left(\dfrac{3}{5}\right)\ge 12 \end{cases}$

$\begin{cases} 4-0<2 \\ 3(4)+4(0)\ge 12 \end{cases}$ $\begin{cases} \dfrac{1}{2}-\dfrac{1}{2}<2 \\ 3\left(\dfrac{1}{2}\right)+4\left(\dfrac{1}{2}\right)\ge 12 \end{cases}$

None of the ordered pairs are a solution.

25. Substitute each ordered pair into the system:

$\begin{cases} 0<0+2 \\ 0+0\le 4 \end{cases}$ $\begin{cases} 0<1+2 \\ 1+0\le 4 \end{cases}$ $\begin{cases} 1<0+2 \\ 0+1\le 4 \end{cases}$ $\begin{cases} 1<1+2 \\ 1+1\le 4 \end{cases}$

All of the ordered pairs are solutions.

26. Substitute each ordered pair into the system:

$\begin{cases} 0\le 2-0 \\ 0+0\le 1 \end{cases}$ $\begin{cases} 1\le 2-0 \\ 0+1\le 1 \end{cases}$

$\begin{cases} 2\le 2-1 \\ 1+2\le 1 \end{cases}$ $\begin{cases} -1\le 2-(-1) \\ -1+(-1)\le 1 \end{cases}$

The solutions are (0, 0), (0, 1), and (−1, −1).

27. Substitute each ordered pair into the system:

$\begin{cases} 3(0)-4(0)\le 12 \\ 0+0\le 4 \\ 5(0)-2(0)\ge 6 \end{cases}$ $\begin{cases} 3(2)-4(0)\le 12 \\ 2+0\le 4 \\ 5(2)-2(0)\ge 6 \end{cases}$

$\begin{cases} 3(3)-4(1)\le 12 \\ 3+1\le 4 \\ 5(3)-2(1)\ge 6 \end{cases}$ $\begin{cases} 3(2)-4(2)\le 12 \\ 2+2\le 4 \\ 5(2)-2(2)\ge 6 \end{cases}$

The solutions are (2, 0), (3, 1) and (2, 2).

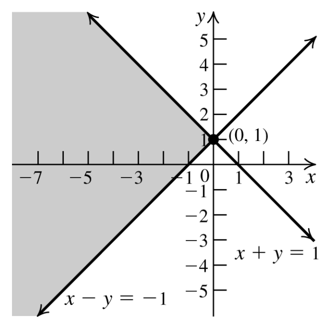
28. Substitute each ordered pair into the system:

$\begin{cases} 2+0\le 3 \\ 0-2(0)\le 3 \\ 5(0)+2(0)\ge 3 \end{cases}$ $\begin{cases} 2+0\le 3 \\ 1-2(0)\le 3 \\ 5(1)+2(0)\ge 3 \end{cases}$

$\begin{cases} 2+2\le 3 \\ 1-2(2)\le 3 \\ 5(1)+2(2)\ge 3 \end{cases}$ $\begin{cases} 2+1\le 3 \\ 2-2(1)\le 3 \\ 5(2)+2(1)\ge 3 \end{cases}$

The solutions are (1, 0) and (2, 1).

29.



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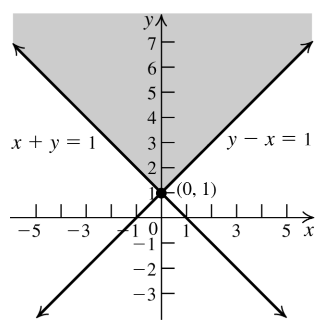

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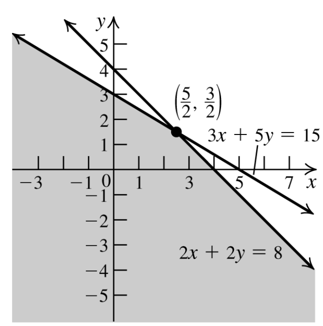

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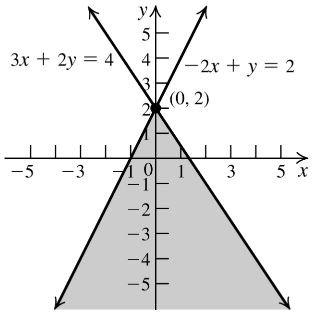

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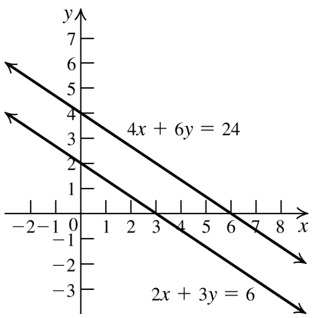
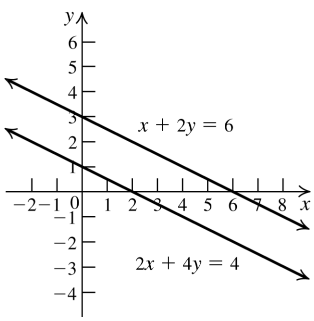


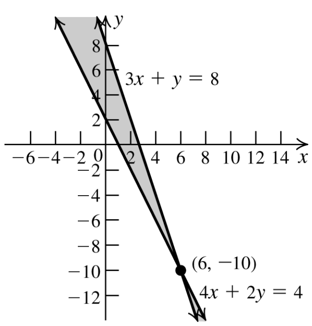
The system is inconsistent. There are no vertices of the solution set.

34.



The system is inconsistent. There are no vertices of the solution set.

35.



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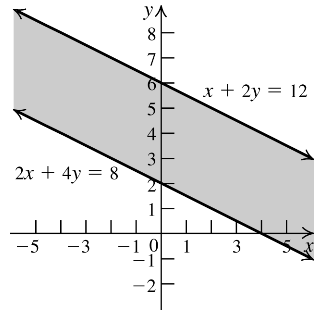
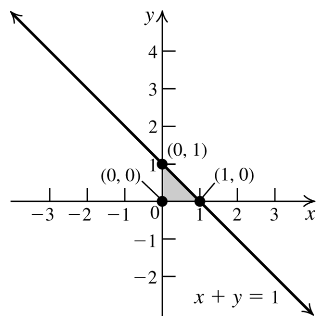


There are no vertices of the solution set.

37.



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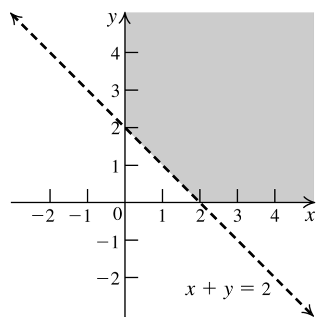
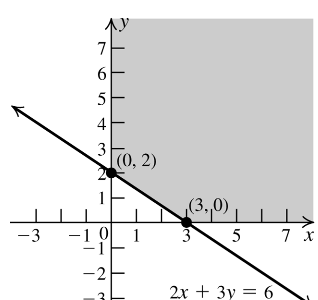


There are no vertices of the solution set.

39.



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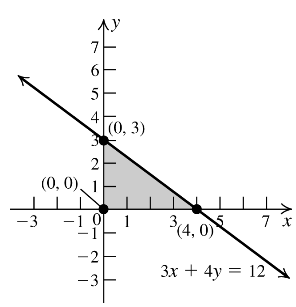

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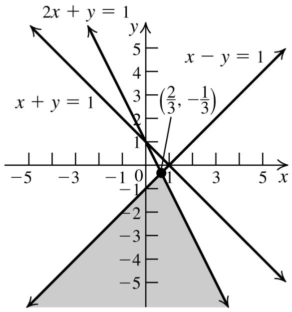

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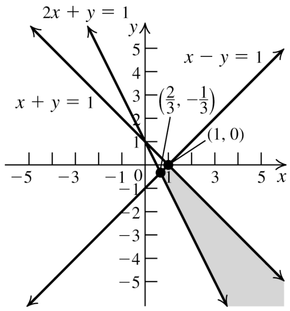

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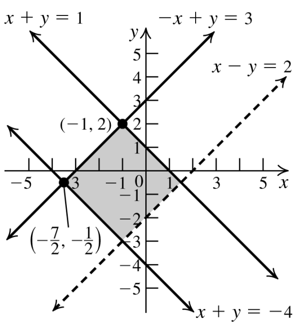

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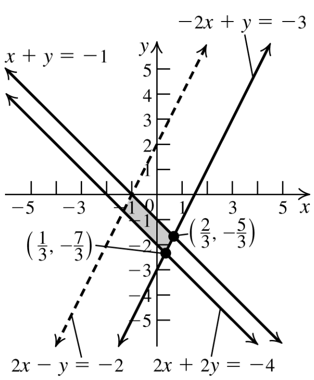
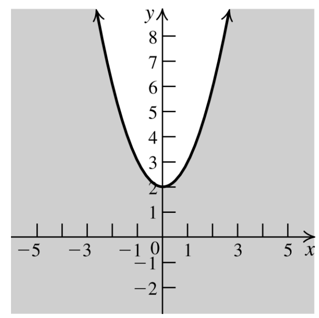


45. A **46.** C

47. B **48.** D

49. E **50.** F

51.



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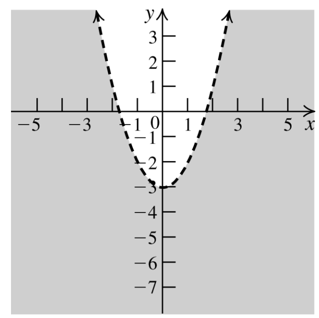

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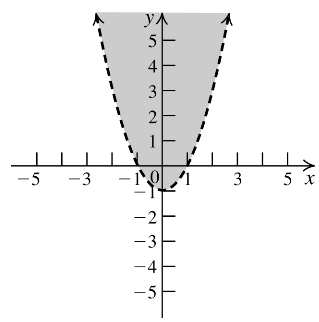

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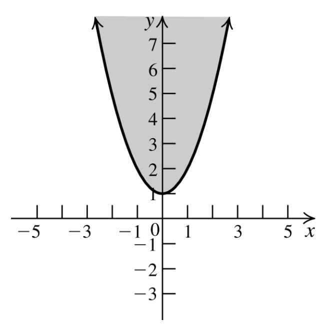

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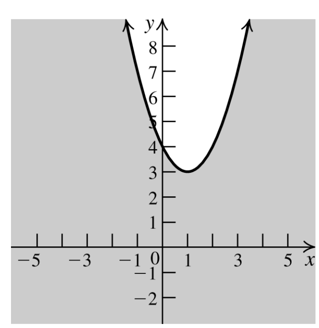

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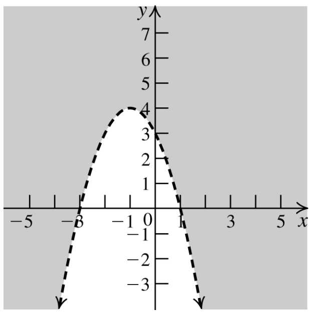

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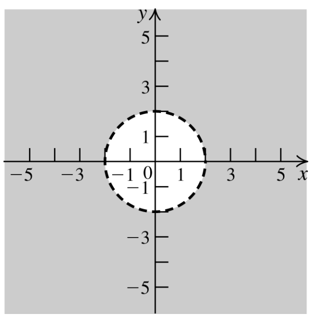

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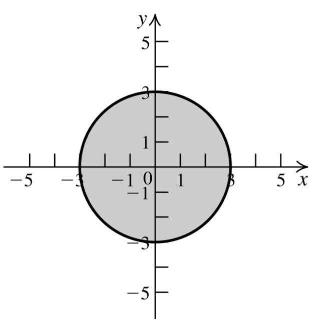

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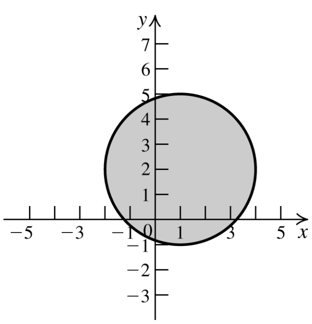

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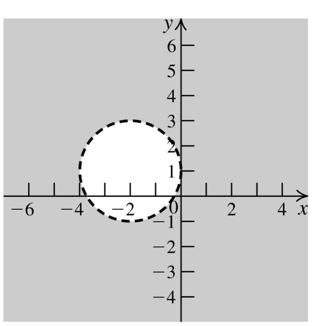

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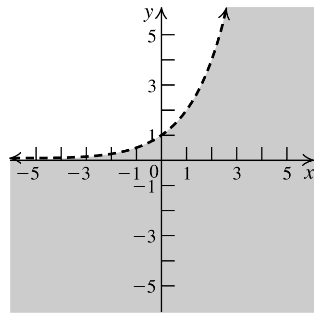

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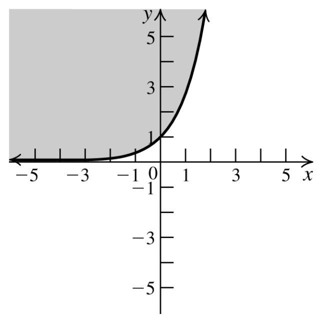

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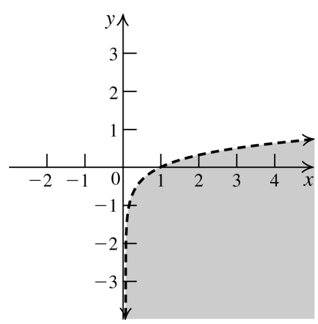

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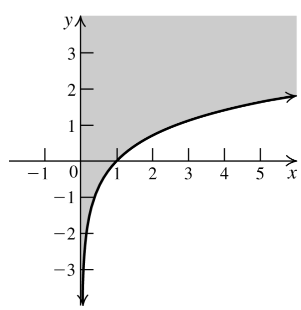
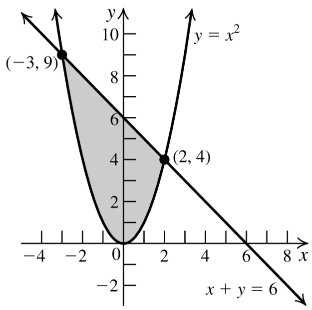


65. D, K   **66.** H

67. E, L   **68.** A, F

69. E, B   **70.** C, D

71.



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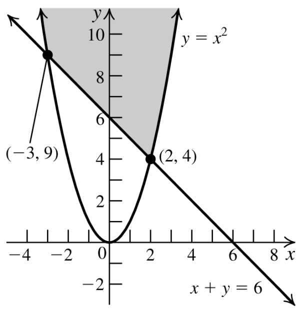

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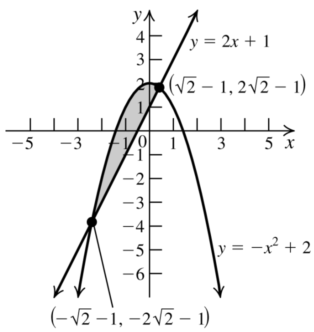

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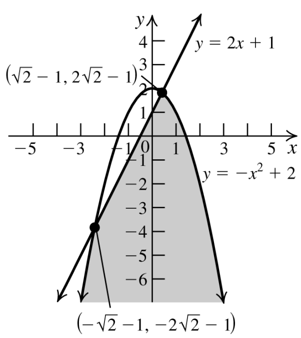

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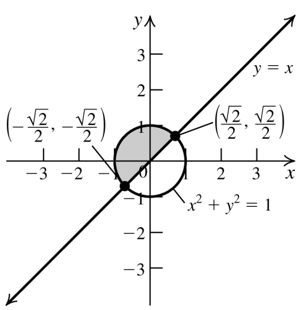
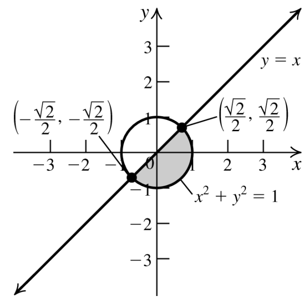


76.



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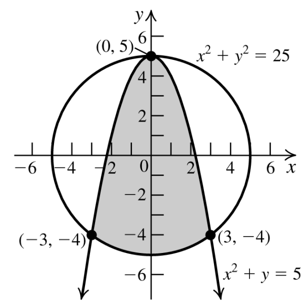
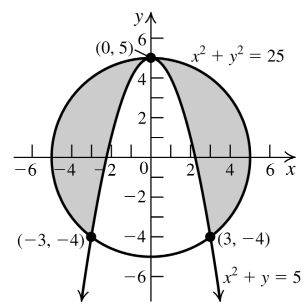


78.



8.5 Applying the Concepts

79. Let x = the number of cases of Coke and y = the number of cases of Sprite. Then we have

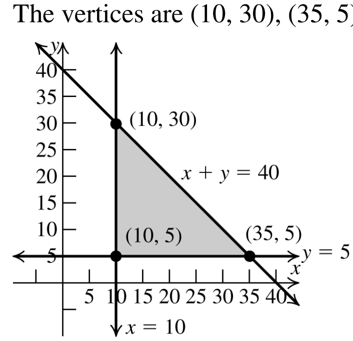
$$\begin{cases} x + y \le 40 & (1) \\ x \ge 10 & (2) \\ y \ge 5 & (3) \end{cases}$$

To find the vertices, solve the systems

$$\begin{cases} x + y = 40 & (1) \\ x = 10 & (2) \end{cases}, \begin{cases} x + y = 40 & (1) \\ y = 5 & (3) \end{cases}, \text{ and}$$

$$\begin{cases} x = 10 & (2) \\ y = 5 & (3) \end{cases}.$$

The vertices are (10, 30), (35, 5), and (10, 5).



80. Let x = the number of type A DVD players and y = the number of type B DVD players. Then we have

$$\begin{cases} x + y \geq 20 \\ 60x + 80y \leq 6000 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

To find the vertices, solve the systems

$$\begin{cases} x + y = 20 \\ 60x + 80y = 6000 \end{cases}, \begin{cases} x + y = 20 \\ x = 0 \end{cases}, \begin{cases} x + y = 20 \\ y = 0 \end{cases},$$

$$\begin{cases} 60x + 80y = 6000 \\ x = 0 \end{cases}, \begin{cases} 60x + 80y = 6000 \\ y = 0 \end{cases},$$

and $\begin{cases} x = 0 \\ y = 0 \end{cases}$.

$$\begin{cases} x + y = 20 \\ 60x + 80y = 6000 \end{cases} \Rightarrow \begin{cases} -80x - 80y = -1600 \\ 60x + 80y = 6000 \end{cases} \Rightarrow$$

$-20x = 4400 \Rightarrow x = -220$

$-220 + y = 20 \Rightarrow y = 240$

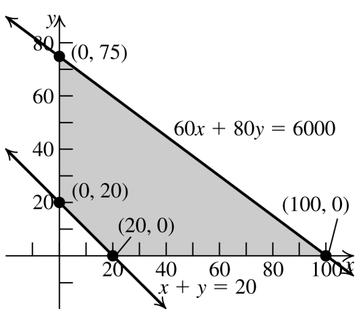
A vertex is (–220, 240), but this makes no sense in the problem.

$$\begin{cases} x + y = 20 \\ x = 0 \end{cases} \Rightarrow y = 20; \begin{cases} x + y = 20 \\ y = 0 \end{cases} \Rightarrow x = 20$$

$$\begin{cases} 60x + 80y = 6000 \\ x = 0 \end{cases} \Rightarrow y = 75$$

$$\begin{cases} 60x + 80y = 6000 \\ y = 0 \end{cases} \Rightarrow x = 100$$

The vertices are (0, 20), (20, 0), (0, 75), and (100, 0).



81. Let x = the number of two story houses and y = the number of one story houses. Then we have

$$\begin{cases} 7x + 5y \leq 43 \\ 4x + 3y \leq 25 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

To find the vertices, solve the systems

$$\begin{cases} 7x + 5y = 43 \\ 4x + 3y = 25 \end{cases}, \begin{cases} 7x + 5y = 43 \\ y = 0 \end{cases}, \begin{cases} 4x + 3y = 25 \\ x = 0 \end{cases},$$

and $\begin{cases} x = 0 \\ y = 0 \end{cases}$.

$$\begin{cases} 7x + 5y = 43 \\ 4x + 3y = 25 \end{cases} \Rightarrow \begin{cases} 28x + 20y = 172 \\ -28x - 21y = -175 \end{cases} \Rightarrow$$

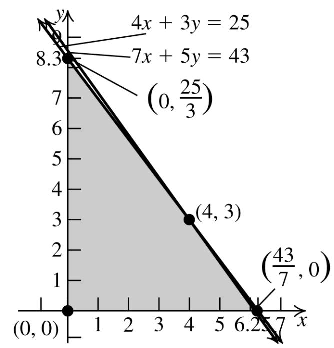
$-y = -3 \Rightarrow y = 3$

$7x + 15 = 43 \Rightarrow x = 4$

$$\begin{cases} 7x + 5y = 43 \\ y \geq 0 \end{cases} \Rightarrow x = \frac{43}{7}$$

$$\begin{cases} 4x + 3y = 25 \\ x = 0 \end{cases} \Rightarrow y = \frac{25}{3}$$

The vertices are (0, 0), $\left(0, \frac{25}{3}\right)$, $\left(\frac{43}{7}, 0\right)$, and (4, 3).



82. Let x = the amount to be invested in stocks and y = the amount to be invested in bonds. Then we have

$$\begin{cases} x + y = 100,000 \\ 0.04x + 0.07y \geq 5600 \\ y \geq 60,000 \\ x \geq 0 \end{cases}$$

To find the vertices, solve the systems

$$\begin{cases} x + y = 100,000 \\ y = 60,000 \end{cases}, \begin{cases} 0.04x + 0.07y = 5600 \\ y = 60,000 \end{cases},$$

$$\begin{cases} 0.04x + 0.07y = 5600 \\ x = 0 \end{cases}, \text{ and } \begin{cases} x + y = 100,000 \\ x = 0 \end{cases}$$

$$\begin{cases} x + y = 100,000 \\ y = 60,000 \end{cases} \Rightarrow x = 40,000$$

$$\begin{cases} 0.04x + 0.07y = 5600 \\ y = 60,000 \end{cases} \Rightarrow$$

$$\begin{cases} 0.04x + 0.07y = 5600 \\ -0.07y = -4200 \end{cases} \Rightarrow 0.04x = 1400 \Rightarrow$$

$x = 35,000$

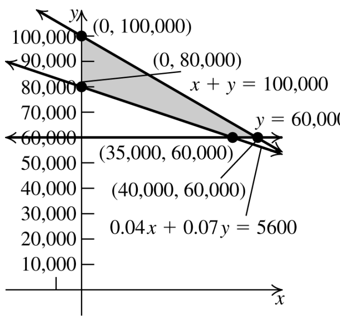
(*continued on next page*)

(*continued*)

$$\begin{cases} 0.04x + 0.07y = 5600 \\ x = 0 \end{cases} \Rightarrow 0.07y = 5600 \Rightarrow$$

$y = 80,000$

The vertices are (40,000, 60,000), (35,000, 60,000), (0, 100,000), and (0, 80,000).



83. Let x = number of 3 hp engines and y = the number of 5 hp engines. Then we have

$$\begin{cases} 3x + 4.5y \le 360 & (1) \\ 2x + y \le 200 & (2) \\ 0.5x + 0.75y \le 60 & (3) \\ x \ge 0 \\ y \ge 0 \end{cases}$$

Equations (1) and (3) coincide. To find the vertices, solve the systems

$$\begin{cases} 3x + 4.5y = 360 \\ 2x + y = 200 \end{cases}, \begin{cases} 3x + 4.5y = 360 \\ x = 0 \end{cases}, \text{ and } \begin{cases} 2x + y = 200. \\ y = 0 \end{cases}$$

$$\begin{cases} 3x + 4.5y = 360 \\ 2x + y = 200 \end{cases} \Rightarrow$$

$3x + 4.5(200 - 2x) = 360 \Rightarrow 900 - 6x = 360 \Rightarrow$

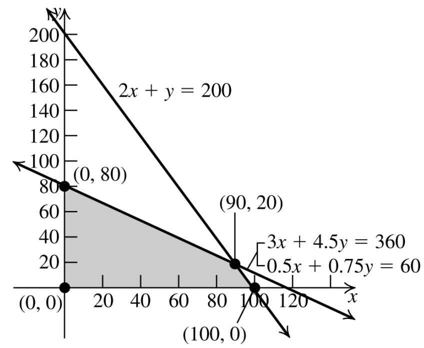
$x = 90$

$2(90) + y = 200 \Rightarrow y = 20$

$$\begin{cases} 3x + 4.5y = 360 \\ x = 0 \end{cases} \Rightarrow 4.5y = 360 \Rightarrow y = 80$$

$$\begin{cases} 2x + y = 200 \\ y = 0 \end{cases} \Rightarrow 2x = 200 \Rightarrow x = 100$$

The vertices are (0, 0) (100, 0), (0, 80), and (90, 20).



84. Let x = the number of pounds of Vegies and y = the number of pounds of Yummies. Then we have

$$\begin{cases} 3x + 1.5y \le 20,000 & \text{(vegetables)} \\ 2x + 3.5y \le 27,000 & \text{(cereal)} \\ x \ge 0 \\ y \ge 0 \end{cases}$$

To find the vertices, solve the systems

$$\begin{cases} 3x + 1.5y \le 20,000 \\ 2x + 3.5y \le 27,000 \end{cases}, \begin{cases} 3x + 1.5y \le 20,000 \\ y = 0 \end{cases},$$

and $\begin{cases} 2x + 3.5y \le 27,000 \\ x = 0 \end{cases}$.

$$\begin{cases} 3x + 1.5y = 20,000 \\ 2x + 3.5y = 27,000 \end{cases} \Rightarrow$$

$$\begin{cases} 6x + 3y = 40,000 \\ -6x - 10.5y = -81,000 \end{cases} \Rightarrow$$

$-7.5y = -41,000 \Rightarrow y \approx 5466.67$

$3x + 1.5(5466.67) = 20,000 \Rightarrow$

$3x = 11,800 \Rightarrow x \approx 3933.33$

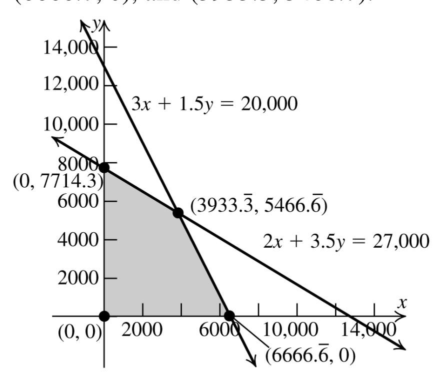
$$\begin{cases} 3x + 1.5y \le 20,000 \\ y = 0 \end{cases} \Rightarrow 3x = 20,000 \Rightarrow$$

$x \approx 6666.67$

$$\begin{cases} 2x + 3.5y \le 27,000 \\ x = 0 \end{cases} \Rightarrow 3.5y = 27,000 \Rightarrow$$

$y \approx 7714.3$

The vertices are (0, 0), (0, 7714.3), (6666.7, 0), and (3933.3, 5466.7).



8.5 Beyond the Basics

85. The equation of the line connecting (0, 2) and (3, 0) is $y = -\frac{2}{3}x + 2$. The system is

$$\begin{cases} y \le -\frac{2}{3}x + 2 \\ x \ge 0 \\ y \ge 0 \end{cases}.$$

86. The equation of the line connecting (0, 3) and (2, 0) is $y = -\frac{3}{2}x + 3$. The system is

$$\begin{cases} x \ge 0 \\ x \le 2 \\ y \ge 0 \\ y \le 5 \\ y \ge -\frac{3}{2}x + 3 \end{cases}.$$

87. $\begin{cases} x \ge -1 \\ x \le 3 \end{cases}$

88. $\begin{cases} x \ge 0 \\ x < 2 \end{cases}$

89. The equation of the line connecting (0, 4) and (2, 3) is $y = -\frac{1}{2}x + 4$. The equation of the line connecting (2, 3) and (4, 0) is $y = -\frac{3}{2}x + 6$. The system is

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ y \le -\frac{1}{2}x + 4 \\ y \le -\frac{3}{2}x + 6 \end{cases}.$$

90. The equation of the line connecting (2, 4) and (4, 0) is $y = -2x + 8$. The system is

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ y \le 4 \\ y \le -2x + 8 \end{cases}.$$

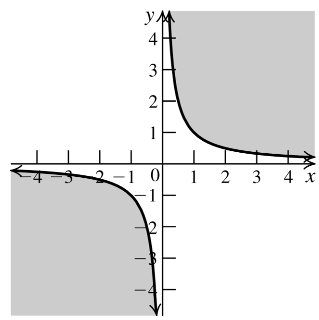
91. The equation of the line connecting (0, 16) and (10, 6) is $y = -x + 16$. The equation of the line connecting (10, 6) and (5, 1) is $y = x - 4$. The equation of the line connecting (5, 1) and (0, 6) is $y = -x + 6$. The system is

$$\begin{cases} x \ge 0 \\ y \le -x + 16 \\ y \ge x - 4 \\ y \ge -x + 6 \end{cases}.$$

92. The equation of the line connecting (0, 16) and (13, 3) is $y = -x + 16$. The equation of the line connecting (13, 3) and (10, 0) is $y = x - 10$. The equation of the line connecting (6, 0) and (0, 6) is $y = -x + 6$. The system is

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ y \le -x + 16 \\ y \ge x - 10 \\ y \ge -x + 6 \end{cases}.$$

93.



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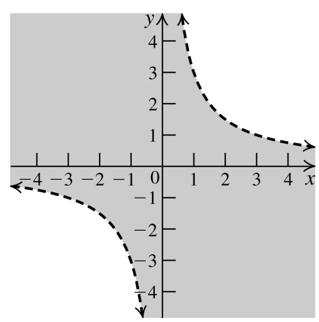

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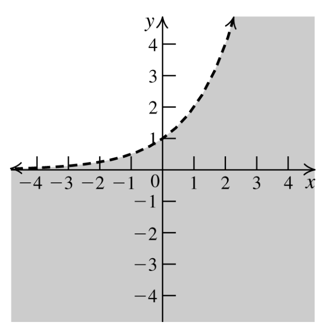

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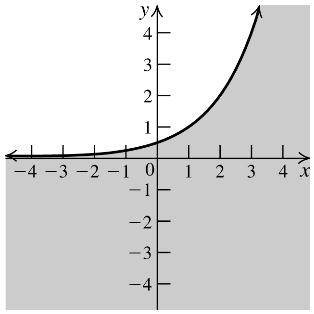

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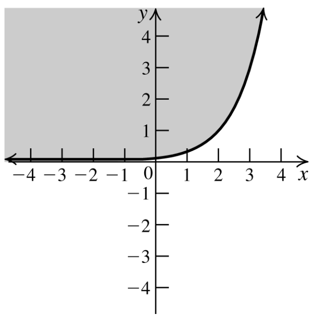

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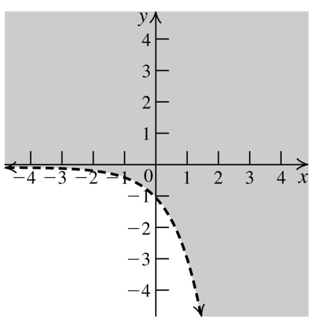

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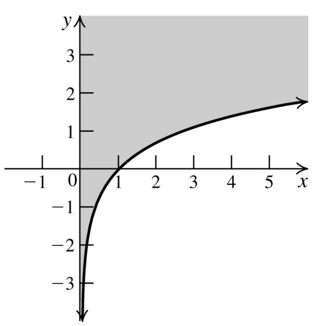

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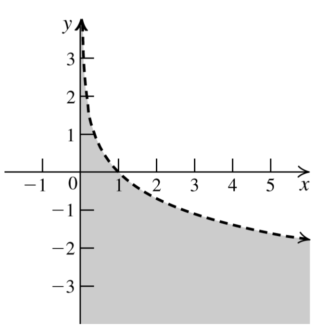

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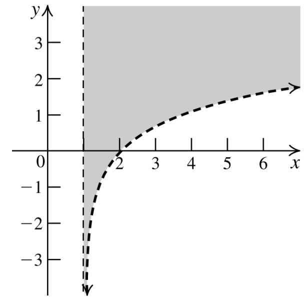

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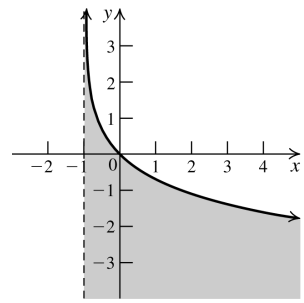
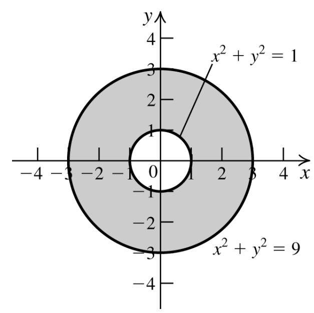


103.



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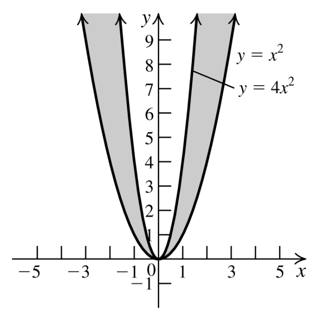
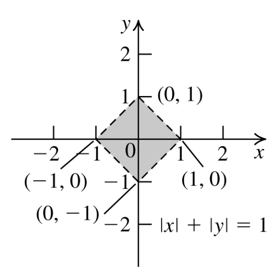
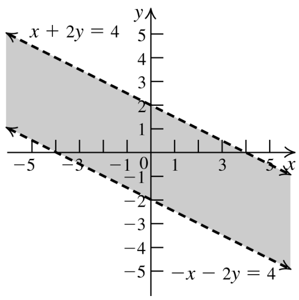


8.5 Critical Thinking/Discussion/Writing

105.



106.



8.5 Maintaining Skills

107. To find the point of intersection, solve the system

$$\begin{cases} x+2y=40 & (1) \\ 3x+y=30 & (2) \end{cases}$$

Solve equation (2) for y in terms of x, then substitute the expression in equation (1) and solve for x.

$3x+y=30 \Rightarrow y=-3x+30$

$x+2(-3x+30)=40 \Rightarrow -5x+60=40 \Rightarrow$

$-5x=-20 \Rightarrow x=4$

Substitute $x = 4$ into equation (1) and solve for y.

$4+2y=40 \Rightarrow 2y=36 \Rightarrow y=18$

The point of intersection is (4, 18).

108. To find the point of intersection, solve the system

$$\begin{cases} 3x+y=30 & (1) \\ 4x+3y=60 & (2) \end{cases}$$

Solve equation (1) for y in terms of x, then substitute the expression in equation (2) and solve for x.

$3x+y=30 \Rightarrow y=-3x+30$

$4x+3(-3x+30)=60 \Rightarrow -5x+90=60 \Rightarrow$

$-5x=-30 \Rightarrow x=6$

Substitute $x=6$ into equation (1) and solve for y.

$6+2y=30 \Rightarrow 2y=24 \Rightarrow y=12$

The point of intersection is $(6, 12)$.

109. To find the point of intersection, solve the system

$$\begin{cases} x-2y=2 & (1) \\ 3x+2y=12 & (2) \end{cases}$$

Add the two equations, then solve for x.

$$\begin{array}{r} x-2y=2 \\ \underline{3x+2y=12} \\ 4x=14 \Rightarrow x=\dfrac{14}{4}=\dfrac{7}{2} \end{array}$$

Substitute $x=\dfrac{7}{2}$ into equation (1) and solve for y.

$\dfrac{7}{2}-2y=2 \Rightarrow -2y=-\dfrac{3}{2} \Rightarrow y=\dfrac{3}{4}$

The point of intersection is $\left(\dfrac{7}{2}, \dfrac{3}{4}\right)$.

110. To find the point of intersection, solve the system

$$\begin{cases} 3x+2y=12 & (1) \\ -3x+2y=3 & (2) \end{cases}$$

Add the two equations, then solve for y.

$$\begin{array}{r} 3x+2y=12 \\ \underline{-3x+2y=3} \\ 4y=15 \Rightarrow y=\dfrac{15}{4} \end{array}$$

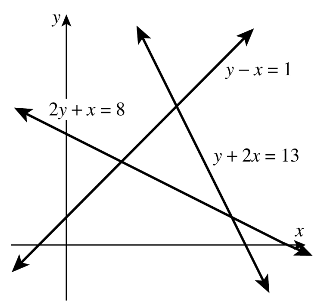
Substitute $y=\dfrac{15}{4}$ into equation (1) and solve for x.

$3x+2\left(\dfrac{15}{4}\right)=12 \Rightarrow 3x+\dfrac{15}{2}=12 \Rightarrow$

$3x=\dfrac{9}{2} \Rightarrow x=\dfrac{3}{2}$

The point of intersection is $\left(\dfrac{3}{2}, \dfrac{15}{4}\right)$.

111.



Solve each pair of equations to find the points of intersection.

I: $\begin{cases} 2y+x=8 \\ y-x=1 \end{cases}$

II: $\begin{cases} y-x=1 \\ y+2x=13 \end{cases}$

III: $\begin{cases} y+2x=13 \\ 2y+x=8 \end{cases}$

For system I, we will use substitution.

$y = x + 1$

$2(x+1)+x=8 \Rightarrow 3x+2=8 \Rightarrow 3x=6 \Rightarrow$

$x=2$

$y-2=1 \Rightarrow y=3$

The point of intersection is (2, 3).

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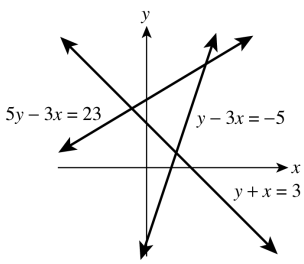
For system II, we will use substitution.
$y = x + 1$
$(x+1)+2x=13 \Rightarrow 3x+1=13 \Rightarrow 3x=12 \Rightarrow$
$x=4; \quad y-4=1 \Rightarrow y=5$
The point of intersection is (4, 5).
For system III, we will use substitution.
$y = -2x + 13$
$2(-2x+13)+x=8 \Rightarrow -3x+26=8 \Rightarrow$
$-3x=-18 \Rightarrow x=6$
$y+2(6)=13 \Rightarrow y+12=13 \Rightarrow y=1$
The point of intersection is (6, 1).
The vertices of the triangle are (2, 3), (4, 5), and (6, 1).

112.



Solve each pair of equations to find the points of intersection.

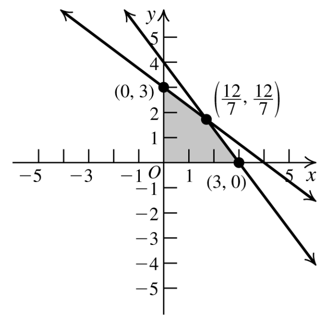
I: $\begin{cases} 5y-3x=23 \\ y-3x=-5 \end{cases}$

II: $\begin{cases} y-3x=-5 \\ y+x=3 \end{cases}$

III: $\begin{cases} y+x=3 \\ 5y-3x=23 \end{cases}$

For system I, we will use substitution.
$y = 3x - 5$
$5(3x-5)-3x=23 \Rightarrow 12x-25=23 \Rightarrow$
$12x=48 \Rightarrow x=4$
$y-3(4)=-5 \Rightarrow y-12=-5 \Rightarrow y=7$
The point of intersection is (4, 7).
For system II, we will use substitution.
$y = 3x - 5$
$(3x-5)+x=3 \Rightarrow 4x-5=3 \Rightarrow 4x=8 \Rightarrow$
$x=2$
$y-3(2)=-5 \Rightarrow y-6=-5 \Rightarrow y=1$
The point of intersection is (2, 1).
For system III, we will use substitution.
$y = -x + 3$
$5(-x+3)-3x=23 \Rightarrow -8x+15=23 \Rightarrow$
$-8x=8 \Rightarrow x=-1$
$y+(-1)=3 \Rightarrow y=4$
The point of intersection is (−1, 4).
The vertices of the triangle are (4, 7), (2, 1), and (−1, 4).

113. a.



b. Solve the systems
$\begin{cases} x=0 \\ y=0 \end{cases}$, $\begin{cases} 3x+4y=12 \\ y=0 \end{cases}$, $\begin{cases} 3x+4y=12 \\ 4x+3y=12 \end{cases}$,
and $\begin{cases} 4x+3y=12 \\ x=0 \end{cases}$

to find the vertices (0, 0), (0, 3), $\left(\frac{12}{7}, \frac{12}{7}\right)$, and (3, 0).

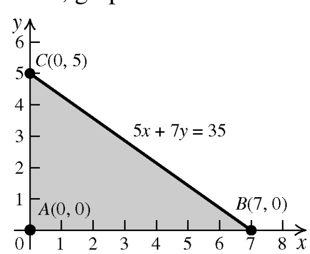
8.6 Linear Programming

8.6 Practice Problems

1. Maximize $f = 4x + 5y$ subject to the constraints

$\begin{cases} 5x+7y \le 35 \\ x \ge 0 \\ y \ge 0 \end{cases}$.

First, graph the solution set of the constraints.



Solve the systems $\begin{cases} x=0 \\ y=0 \end{cases}$, $\begin{cases} 5x+7y=15 \\ y=0 \end{cases}$, and

$\begin{cases} 5x+7y=15 \\ x=0 \end{cases}$ to find the vertices: (0, 0),

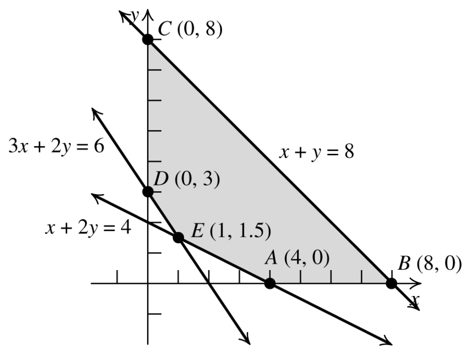
(7, 0) and (0, 5). Now find the values of the objective function at each vertex:

Ordered pair	$f = 4x+5y$
(0,0)	0
(7, 0)	28
(0, 5)	25

The maximum is 28 at (7, 0).

2. Minimize $f = \frac{1}{2}x + y$ subject to the constraints

$$\begin{cases} x + y \le 8 \\ x + 2y \ge 4 \\ 3x + 2y \ge 6. \\ x \ge 0 \\ y \ge 0 \end{cases}$$



The graph and vertices of the set of feasible solutions are as given in Example 2. The vertices are: (0, 8), (0, 3), (1, 1.5), (4, 0), and (8, 0). Now find the values of the objective function at each vertex:

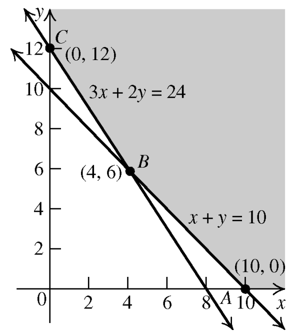
Ordered pair	$f = \frac{1}{2}x + y$
(0, 8)	8
(0, 3)	3
(1, 1.5)	2
(4, 0)	2
(8, 0)	4

The minimum is 2 at (1, 1.5) and (4, 0).

3. Let x = the number of ounces of soup and let y = the number of ounces of salad. The number of calories in the two items is $f = 30x + 60y$. (This is the objective function.) The constraints are as given in Example 3.

$$\begin{cases} x + y \ge 10 \\ 3x + 2y \ge 24 \\ x \ge 0 \\ y \ge 0 \end{cases}.$$

The graph and vertices of the set of feasible solutions are as given in Example 3. The vertices are (0, 12), (4, 6), and (10, 0).



Now find the values of the objective function at each vertex:

Ordered pair	$f = 30x + 60y$
(0,12)	720
(4, 6)	480
(10, 0)	300

The minimum is 300 at (10, 0). This means that the lunch menu for Fat Albert should contain 10 ounces of soup and 0 ounces of salad.

8.6 Basic Concepts and Skills

1. The process of finding the maximum or minimum value of a quantity is called optimization.

2. In a linear programming problem, the linear expression f that is to be maximized or minimized is called an objective function.

3. The inequalities that determine the region S in a linear programming problem are called constraints, and S is called the set of feasible solutions.

4. In linear programming, the expression for the objective function is linear, and all the constraint inequalities are also linear.

5. True

6. False. A linear programming problem may have many solutions.

7.

Ordered pair	$f = x + y$
(10, 0)	10
(3, 2)	5
(1, 4)	5
(0, 8)	8
(10, 8)	18

Maximum: 18; minimum: 5

8.

Ordered pair	$f = 2x + y$
(10, 0)	20
(3, 2)	8
(1, 4)	6
(0, 8)	8
(10, 8)	28

Maximum: 28; minimum: 6

9.

Ordered pair	$f = x + 2y$
(10, 0)	10
(3, 2)	7
(1, 4)	9
(0, 8)	16
(10, 8)	26

Maximum: 26; minimum: 7

10.

Ordered pair	$f = 2x + 5y$
(10, 0)	20
(3, 2)	16
(1, 4)	22
(0, 8)	40
(10, 8)	60

Maximum: 60; minimum: 16

11.

Ordered pair	$f = 5x + 2y$
(10, 0)	50
(3, 2)	19
(1, 4)	13
(0, 8)	16
(10, 8)	66

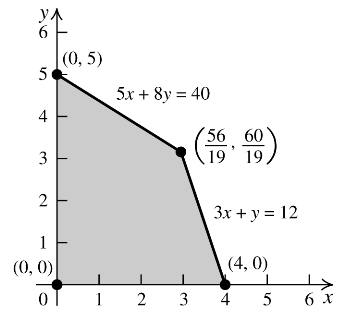
Maximum: 66; minimum: 13

12.

Ordered pair	$f = 8x + 5y$
(10, 0)	80
(3, 2)	34
(1, 4)	28
(0, 8)	40
(10, 8)	120

Maximum: 120; minimum: 28

13.



Solve the systems

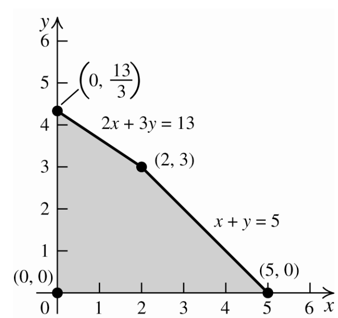
$\begin{cases} x = 0 \\ 5x + 8y = 40 \end{cases}$, $\begin{cases} 5x + 8y = 40 \\ 3x + y = 12 \end{cases}$, $\begin{cases} 3x + y = 12 \\ y = 0 \end{cases}$,

and $\begin{cases} x = 0 \\ y = 0 \end{cases}$ to find the vertices: (0, 5), $\left(\frac{56}{19}, \frac{60}{19}\right)$, (4, 0, and (0, 0). Now find the values of the objective function at each vertex:

Ordered pair	$f = 9x + 13y$
(0, 5)	65
$\left(\frac{56}{19}, \frac{60}{19}\right)$	$\frac{1284}{19}$
(4, 0)	36
(0, 0)	0

The maximum is $\frac{1284}{19}$ at $\left(\frac{56}{19}, \frac{60}{19}\right)$.

14.



Solve the systems

$\begin{cases} x = 0 \\ 2x + 3y = 13 \end{cases}$, $\begin{cases} 2x + 3y = 13 \\ x + y = 5 \end{cases}$, $\begin{cases} x + y = 5 \\ y = 0 \end{cases}$, and

$\begin{cases} x = 0 \\ y = 0 \end{cases}$ to find the vertices: $(0, 13/3)$, (2, 3), (5, 0) and (0, 0). Now find the values of the objective function at each vertex.

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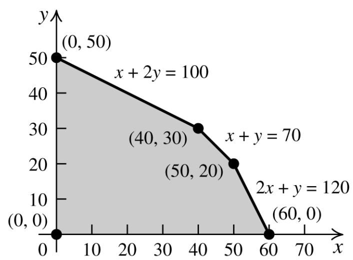
Ordered pair	$f = 7x + 6y$
$\left(0, \frac{13}{3}\right)$	26
(2, 3)	32
(5, 0)	35
(0, 0)	0

The maximum is 35 at (5, 0).

15.



Solve the systems

$\begin{cases} x = 0 \\ x + 2y = 100 \end{cases}$, $\begin{cases} x + 2y = 100 \\ x + y = 70 \end{cases}$, $\begin{cases} x + y = 70 \\ 2x + y = 120 \end{cases}$,

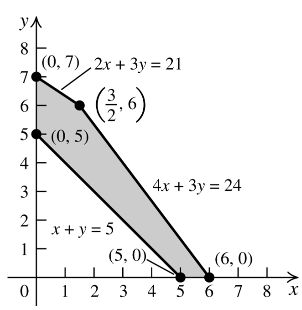
$\begin{cases} 2x + y = 120 \\ y = 0 \end{cases}$, and $\begin{cases} x = 0 \\ y = 0 \end{cases}$ to find the

vertices: (0, 50), (40, 30), (50, 20), (60, 0), and (0, 0). Now find the values of the objective function at each vertex:

Ordered pair	$f = 5x + 7y$
(0, 50)	350
(40, 30)	410
(50, 20)	390
(60, 0)	300
(0, 0)	0

The maximum is 410 at (40, 30).

16.



Solve the systems

$\begin{cases} x = 0 \\ 2x + 3y = 21 \end{cases}$, $\begin{cases} 2x + 3y = 21 \\ 4x + 3y = 24 \end{cases}$, $\begin{cases} 4x + 3y = 24 \\ y = 0 \end{cases}$,

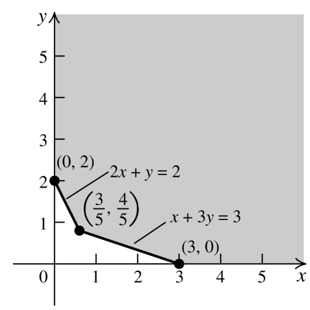
$\begin{cases} x + y = 5 \\ y = 0 \end{cases}$, and $\begin{cases} y = 0 \\ 2x + 3y = 21 \end{cases}$ to find the

vertices (0, 7), $(3/2, 6)$, (6, 0), (5, 0), and (0, 5). Now find the values of the objective function at each vertex:

Ordered pair	$f = 2x + y$
(0, 7)	7
$\left(\frac{3}{2}, 6\right)$	9
(6, 0)	12
(5, 0)	10
(0, 5)	5

The maximum is 12 at (6, 0).

17.



Solve the systems

$\begin{cases} x = 0 \\ 2x + y = 2 \end{cases}$, $\begin{cases} 2x + y = 2 \\ x + 3y = 3 \end{cases}$, and $\begin{cases} x + 3y = 3 \\ y = 0 \end{cases}$ to

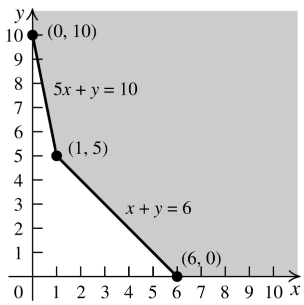
find the vertices: (0, 2), $\left(\frac{3}{5}, \frac{4}{5}\right)$, and (3, 0).

Now find the values of the objective function at each vertex:

Ordered pair	$f = x + 4y$
(0, 2)	8
$\left(\frac{3}{5}, \frac{4}{5}\right)$	$\frac{19}{5}$
(3, 0)	3

The minimum is 3 at (3, 0).

18.



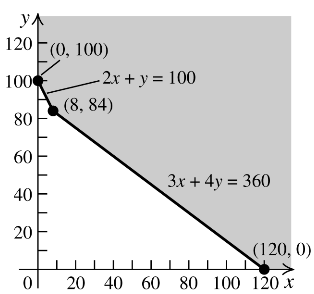
Solve the systems $\begin{cases} x = 0 \\ 5x + y = 10 \end{cases}$, $\begin{cases} 5x + y = 10 \\ x + y = 6 \end{cases}$,

and $\begin{cases} x + y = 6 \\ y = 0 \end{cases}$ to find the vertices: (0, 10), (1, 5), and (6, 0). Now find the values of the objective function at each vertex:

Ordered pair	$f = 5x + 2y$
(0, 10)	20
(1, 5)	15
(6, 0)	30

The minimum is 15 at (1, 5).

19.



Solve the systems

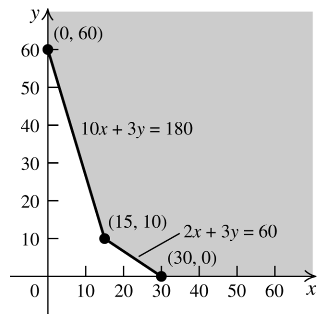
$\begin{cases} x = 0 \\ 2x + y = 100 \end{cases}$, $\begin{cases} 2x + y = 100 \\ 3x + 4y = 360 \end{cases}$, and

$\begin{cases} 3x + 4y = 360 \\ y = 0 \end{cases}$ to find the vertices: (0, 100), (8, 84), and (120, 0). Now find the values of the objective function at each vertex.

Ordered pair	$f = 13x + 15y$
(0,100)	1500
(8, 84)	1364
(120, 0)	1560

The minimum is 1364 at (8, 84).

20.



Solve the systems

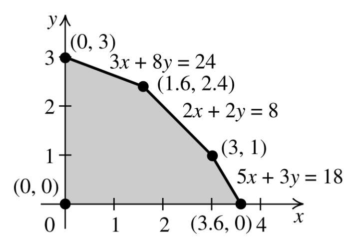
$\begin{cases} x = 0 \\ 10x + 3y = 180 \end{cases}$, $\begin{cases} 10x + 3y = 180 \\ 2x + 3y = 60 \end{cases}$, and

$\begin{cases} 2x + 3y = 60 \\ y = 0 \end{cases}$ to find the vertices: (0, 60), (15, 10), and (30, 0). Now find the values of the objective function at each vertex:

Ordered pair	$f = 40x + 37y$
(0, 60)	2220
(15, 10)	970
(30, 0)	1200

The minimum is 970 at (15, 10).

21.



Solve the systems $\begin{cases} x = 0 \\ y = 0 \end{cases}$, $\begin{cases} 5x + 3y = 18 \\ y = 0 \end{cases}$,

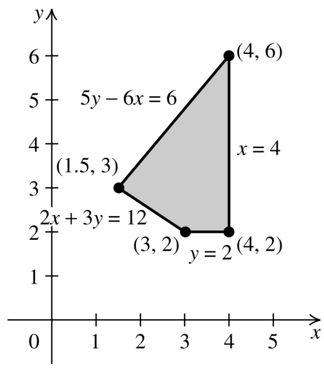
$\begin{cases} 2x + 2y = 8 \\ 5x + 3y = 18 \end{cases}$, $\begin{cases} 3x + 8y = 24 \\ 2x + 2y = 8 \end{cases}$, and

$\begin{cases} 3x + 8y = 24 \\ x = 0 \end{cases}$ to find the vertices: (0, 0), (3.6, 0), (3, 1), (1.6, 2.4), and (0, 3). Now find the values of the objective function at each vertex:

Ordered pair	$f = 15x + 7y$
(0, 0)	0
(3.6, 0)	54
(3, 1)	52
(1.6, 2.4)	40.8
(0, 3)	21

The maximum is 54 at (3.6, 0).

22.

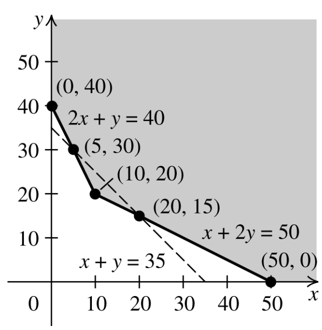


Solve the systems $\begin{cases} x = 4 \\ y = 2 \end{cases}$, $\begin{cases} x = 4 \\ 5y - 6x = 6 \end{cases}$, $\begin{cases} 5y - 6x = 6 \\ 2x + 3y = 12 \end{cases}$, and $\begin{cases} 2x + 3y = 12 \\ y = 2 \end{cases}$ to find the vertices: (4, 2), (4, 6), (1.5, 3), and (3, 2). Now find the values of the objective function at each vertex:

Ordered pair	$f = 17x + 10y$
(4, 2)	88
(4, 6)	128
(1.5, 3)	55.5
(3, 2)	71

The maximum is 128 at (4, 6).

23.

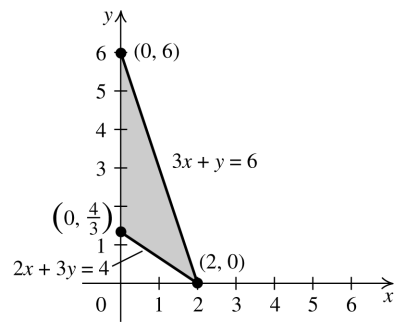


Solve the systems $\begin{cases} y = 0 \\ x + 2y = 50 \end{cases}$, $\begin{cases} x + 2y = 50 \\ x + y = 35 \end{cases}$, $\begin{cases} x + y = 35 \\ 2x + y = 40 \end{cases}$, and $\begin{cases} 2x + y = 40 \\ x = 0 \end{cases}$ to find the vertices: (50, 0), (20, 15), (30, 5), and (0, 40). Now find the values of the objective function at each vertex:

Ordered pair	$f = 8x + 16y$
(50, 0)	400
(20, 15)	400
(10, 20)	400
(5, 30)	520
(0, 40)	640

The minimum is 400 for all (*x*, *y*) on the line segments between (10, 20) and (20, 15), and (20, 15) and (50, 0).

24.



Solve the systems $\begin{cases} 2x + 3y = 4 \\ 3x + y = 6 \end{cases}$, $\begin{cases} 3x + y = 6 \\ x = 0 \end{cases}$, and $\begin{cases} 2x + 3y = 4 \\ x = 0 \end{cases}$ to find the vertices: (2, 0), (0, 6), and $\left(0, \frac{4}{3}\right)$. Now find the values of the objective function at each vertex:

Ordered pair	$f = 12x + 12y$
(2, 0)	24
(0, 6)	72
$\left(0, \frac{4}{3}\right)$	16

The minimum is 16 at $\left(0, \frac{4}{3}\right)$.

8.6 Applying the Concepts

25. Let *x* = the number of corn acres, and let *y* = the number of soybean acres. Then, the profit $p = 50x + 40y$. The constraints are $x \geq 0$, $y \geq 0$, $x + y \leq 240$ (number of acres) and $2x + y \leq 320$ (number of labor hours).

Solve the systems $\begin{cases} x = 0 \\ x + y = 240 \end{cases}$, $\begin{cases} x + y = 240 \\ 2x + y = 320 \end{cases}$, and $\begin{cases} 2x + y = 320 \\ y = 0 \end{cases}$ to find the vertices: (0, 240), (80, 160), and (160, 0). Note that (0, 0) is also a vertex.

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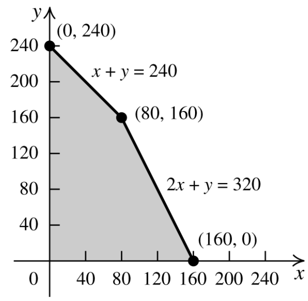
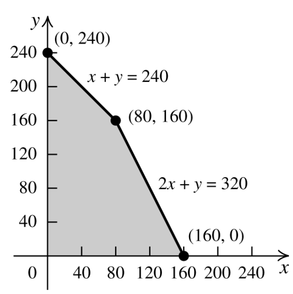


Now find the values of the profit function at each vertex:

Ordered pair	$p = 50x + 40y$
(0, 240)	9600
(80, 160)	10,400
(160, 0)	8000

The maximum profit is \$10,400 when 80 acres of corn and 160 acres of soybeans are planted.

26. Let x = the number of corn acres, and let y = the number of soybean acres. Then, the profit is $p = 30x + 40y$. The constraints are $x \ge 0$, $y \ge 0$, $x + y \le 240$ (number of acres) and $2x + y \le 320$ (number of labor hours). Solve the systems $\begin{cases} x = 0 \\ x + y = 240 \end{cases}$, $\begin{cases} x + y = 240 \\ 2x + y = 320 \end{cases}$, and $\begin{cases} 2x + y = 320 \\ y = 0 \end{cases}$ to find the vertices: (0, 240), (80, 160), and (160, 0). Note that (0, 0) is also a vertex.

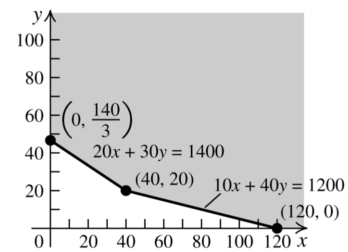


Now find the values of the profit function at each vertex:

Ordered pair	$p = 30x + 40y$
(0, 240)	9600
(80, 160)	8800
(160, 0)	4800

The maximum profit is \$9600 when 0 acres of corn and 240 acres of soybeans are planted.

27. Let x = the number of hours machine I operates, and let y = the number hours machine II operates. Then, the cost is $c = 50x + 80y$. The constraints are $x \ge 0, y \ge 0, 20x + 30y \ge 1400$ (number of units of Grade A plywood) and $10x + 40y \ge 1200$ (number of units of Grade B plywood.) Solve the systems $\begin{cases} x = 0 \\ 20x + 30y = 1400 \end{cases}$, $\begin{cases} 20x + 30y = 1400 \\ 10x + 40y = 1200 \end{cases}$, and $\begin{cases} 10x + 40y = 1200 \\ y = 0 \end{cases}$ to find the vertices $\left(0, \frac{140}{3}\right)$, (40, 20), and (120, 0).



Now find the values of the cost function at each vertex:

Ordered pair	$c = 50x + 80y$
$\left(0, \frac{140}{3}\right)$	≈3733.33
(40, 20)	3600
(120, 0)	6000

The minimum cost is \$3600 when machine I operates for 40 hours and machine II operates for 20 hours.

28. Let x = the number of hours machine I operates, and let y = the number of hours machine II operates. Then, the cost $c = 70x + 90y$. The constraints are $x \ge 0, y \ge 0, 20x + 30y \ge 1400$ (number of units of Grade A plywood) and $10x + 40y \ge 1200$ (number of units of Grade B plywood.) Solve the systems $\begin{cases} x = 0 \\ 20x + 30y = 1400 \end{cases}$, $\begin{cases} 20x + 30y = 1400 \\ 10x + 40y = 1200 \end{cases}$, and $\begin{cases} 10x + 40y = 1200 \\ y = 0 \end{cases}$ to find the vertices $\left(0, \frac{140}{3}\right)$, (40, 20), and (120, 0).

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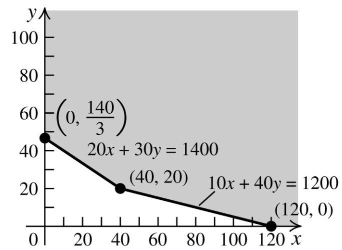
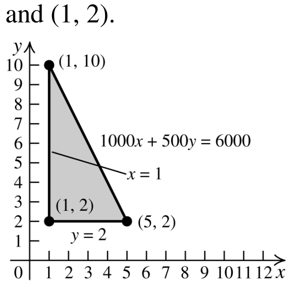


Now find the values of the cost function at each vertex:

Ordered pair	$c = 70x + 90y$
$\left(0, \frac{140}{3}\right)$	4200
(40, 20)	4600
(120, 0)	8400

The minimum cost is \$4200 when machine I operates for 0 hours and machine II operates for approximately 46.7 hours.

29. Let x = the number of minutes of television time, and let y = the number of pages of newspaper advertising. Then, the exposure is $f = 60{,}000x + 20{,}000y$. The constraints are $x \ge 1, y \ge 2,$ and $1000x + 500y \le 6000$ (budget.) Solve the systems $\begin{cases} x = 1 \\ 1000x + 500y = 6000 \end{cases}$, $\begin{cases} 1000x + 500y = 6000 \\ y = 2 \end{cases}$, and $\begin{cases} x = 1 \\ y = 2 \end{cases}$ to find the vertices (1, 10), (5, 2), and (1, 2).

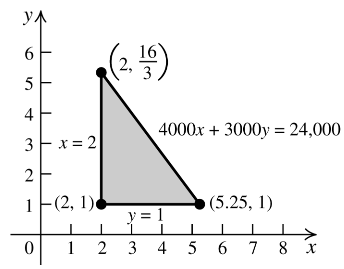


Now find the values of the exposure function at each vertex:

Ordered pair	$f = 60{,}000x + 20{,}000y$
(1, 10)	260,000
(5, 2)	340,000
(1, 2)	100,000

There are a maximum of 340,000 viewers if the company buys 5 minutes of television time and 2 pages of newspaper advertising.

30. Let x = the number of minutes of television time, and let y = the number of pages of newspaper advertising. Then, the exposure is $f = 120{,}000x + 80{,}000y$. The constraints are $x \ge 2, y \ge 1,$ and $4000x + 3000y \le 24{,}000$ (budget.) Solve the systems $\begin{cases} x = 2 \\ 4000x + 3000y = 24{,}000 \end{cases}$, $\begin{cases} 4000x + 3000y = 24{,}000 \\ y = 1 \end{cases}$, and $\begin{cases} x = 2 \\ y = 1 \end{cases}$ to find the vertices $\left(2, \frac{16}{3}\right)$, $\left(\frac{21}{4}, 1\right)$, and (2, 1).



Now find the values of the exposure function at each vertex:

Ordered pair	$f = 120{,}000x + 80{,}000y$
$\left(2, \frac{16}{3}\right)$	≈666,667
$\left(\frac{21}{4}, 1\right)$	710,000
(2, 1)	320,000

There are a maximum of 710,000 viewers if the company buys 5.25 minutes of television time and 1 page of newspaper advertising.

31. Let x = the number of orange acres, and let y = the number of grapefruit acres. Then, the profit is $p = 40x + 30y - 3000$. The constraints are $x \ge 0,\ \ y \ge 0,\ \ x + y \le 480$ (number of acres) and $2x + y \le 800$ (number of labor hours). Solve the systems $\begin{cases} x = 0 \\ x + y = 480 \end{cases}$, $\begin{cases} x + y = 480 \\ 2x + y = 800 \end{cases}$, and $\begin{cases} 2x + y = 800 \\ y = 0 \end{cases}$ to find the vertices: (0, 480), (320, 160), and (400, 0). Note that (0, 0) is also a vertex.

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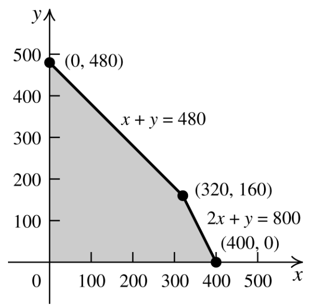
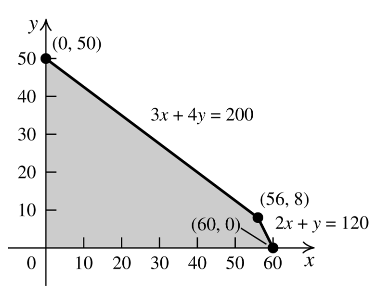


Now find the values of the profit function at each vertex:

Ordered pair	$40x + 30y - 3000$
(0, 480)	11,400
(320, 160)	14,600
(400, 0)	13,000

The maximum profit is $14,600 when 320 acres of oranges and 160 acres of grapefruits are planted.

32. Let x = the number of bottles of Fruity, and let y = the number of bottles of Tangy. Then, the profit is $p = 3x + 2y$. The amount of orange juice in the Fruity = $3x$, and the amount of apple juice in the Fruity = $2x$. The amount of orange juice in the Tangy = $4y$, and the amount of apples juice in the Tangy = y. So, the constraints are $x \geq 0$, $y \geq 0$, $x + y \leq 320$, $3x + 4y \leq 200$, and $2x + y \leq 120$. Solve the systems $\begin{cases} x = 0 \\ 3x + 4y = 200 \end{cases}$, $\begin{cases} 3x + 4y = 200 \\ 2x + y = 120 \end{cases}$, and $\begin{cases} 2x + y = 120 \\ y = 0 \end{cases}$ to find the vertices: (0, 50), (56, 8), and (60, 0). Note that (0, 0) is also a vertex.

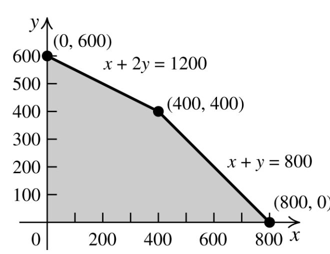


Now find the values of the profit function at each vertex.

Ordered pair	$p = 3x + 2y$
(0, 50)	100
(56, 8)	184
(60, 0)	180

The maximum profit is $184 when 56 bottles of Fruity and 8 bottles of Tangy are sold.

33. Let x = the number of rectangular tables, and let y = the number of circular tables. The profit is $p = 3x + 4y$. The number of hours to assemble the rectangular tables is x and the number of hours to assemble the circular tables is y. The number of hours to finish the rectangular tables is x and the number of hours to finish the circular tables is $2y$. The 20 assemblers work a total of (20)(40) = 800 hours, and the 30 finishers work a total of (30)(40) = 1200 hours. So, the constraints are $x \geq 0$, $y \geq 0$, $x + y \leq 800$, and $x + 2y \leq 1200$. Solve the systems $\begin{cases} x = 0 \\ x + 2y = 1200 \end{cases}$, $\begin{cases} x + 2y = 1200 \\ x + y = 800 \end{cases}$, and $\begin{cases} x + y = 800 \\ y = 0 \end{cases}$ to find the vertices: (0, 600), (400, 400), and (800, 0). Note that (0, 0) is also a vertex.



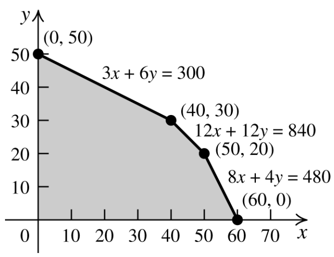
Now find the values of the profit function at each vertex:

Ordered pair	$p = 3x + 4y.$
(0, 600)	2400
(400, 400)	2800
(800, 0)	2400

The maximum profit is $2800 for 400 rectangular tables and 400 circular tables.

34. Let x = the number of GPS units, and let y = the number of DVD units. Then, the profit is $p = 6x + 4y$. The number of hours for Stage I is $12x + 12y$; the number of hours for Stage II is $3x + 6y$; and the number of hours for Stage III is $8x + 4y$. The constraints are $x \ge 0,\ \ y \ge 0,\ \ 12x + 12y \le 840,$ $3x + 6y \le 300,$ and $8x + 4y \le 480$.

Solve the systems
$\begin{cases} x = 0 \\ 3x + 6y = 300 \end{cases}$, $\begin{cases} 3x + 6y = 300 \\ 12x + 12y = 840 \end{cases}$, $\begin{cases} 12x + 12y = 840 \\ 8x + 4y = 480 \end{cases}$, and $\begin{cases} 8x + 4y = 480 \\ y = 0 \end{cases}$ to find the vertices: (0, 50), (40, 30), (50, 20), and (60, 0). Note that (0, 0) is also a vertex.

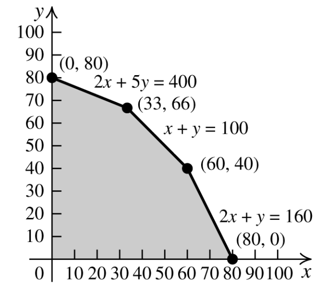


Now find the values of the profit function at each vertex:

Ordered pair	$p = 6x + 4y$
(0, 50)	200
(40, 30)	360
(50, 20)	380
(60, 0)	360

The maximum profit is $380 when 50 GPS units and 20 DVD units are produced.

35. Let x = the number of terraced houses, and let y = the number of cottages. Then, the revenue is $r = 40{,}000x + 45{,}000y$. The number of units of concrete for the two house types is $x + y$; the number of units of wood for the two house types is $2x + y$; and the number of units of glass for the two house types is $2x + 5y$. The constraints are $x \ge 0,\ \ y \ge 0,$ $x + y \le 100,\ \ 2x + y \le 160,$ and $2x + 5y \le 400$. Solve the systems
$\begin{cases} x = 0 \\ 2x + 5y = 400 \end{cases}$, $\begin{cases} 2x + 5y = 400 \\ x + y = 100 \end{cases}$, $\begin{cases} x + y = 100 \\ 2x + y = 160 \end{cases}$, and $\begin{cases} 2x + y = 160 \\ y = 0 \end{cases}$ to find the vertices: (0, 80), $\left(\frac{100}{3}, \frac{200}{3}\right)$, (60, 40), and (80, 0). Note that (0, 0) is also a vertex.



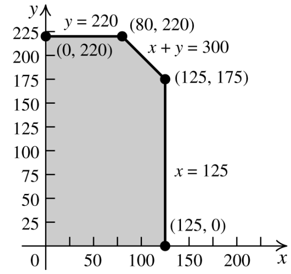
Now find the values of the profit function at each vertex. Note that we cannot use $\left(\frac{100}{3}, \frac{200}{3}\right)$ because there cannot be a fraction of a house. This value becomes (33, 66).

Ordered pair	$r = 40{,}000x + 45{,}000y$
(0, 80)	3,600,000
(33, 66)	4,290,000
(60, 40)	4,200,000
(80, 0)	3,200,000

The maximum profit is $4,290,000 when 33 terraced houses and 66 cottages are built.

36. Let x = the number of female guests, and let y = the number of male guests. Then, the number of guests who eat in the restaurant is $0.5x + 0.3y$, and the number of guests who do not eat in the restaurant is $0.5x + 0.7y$. The profit $p = 18.5(0.5x + 0.3y) + 15(0.5x + 0.7y)$ $= 16.75x + 16.05y$. The constraints are $x \ge 0,$ $y \ge 0,\ \ x \le 125,\ \ y \le 220,$ and $x + y \le 300$.

Solve the systems $\begin{cases} x = 0, \\ y = 220 \end{cases}$, $\begin{cases} y = 220 \\ x + y = 300 \end{cases}$, $\begin{cases} x + y = 300 \\ x = 125 \end{cases}$, and $\begin{cases} x = 125 \\ y = 0 \end{cases}$ to find the vertices: (0, 220), (80, 220), (125, 175), and (125, 0). Note that (0, 0) is also a vertex.



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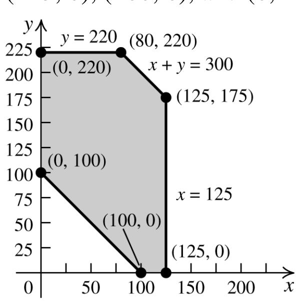
Now find the values of the profit function at each vertex.

Ordered pair	$p = 16.75x + 16.05y$
(0, 220)	3531
(80, 220)	4871
(125, 175)	4902.50
(125, 0)	2093.75

The maximum profit is $4902.50 when there are 125 female guests and 175 male guests.

37. Let x = the number of female guests, and let y = the number of male guests. Then, the number of guests who eat in the restaurant is $0.5x + 0.3y$, and the number of guests who do not eat in the restaurant is $0.5x + 0.7y$. The profit $p = 15(0.5x + 0.3y) - 2(0.5x + 0.7y)$ $= 6.5x + 3.1y$. The constraints are $x \geq 0$, $y \geq 0$, $x \leq 125$, $y \leq 220$, $x + y \geq 100$ and $x + y \leq 300$. Solve the systems $\begin{cases} x = 0, \\ y = 220 \end{cases}$, $\begin{cases} y = 220 \\ x + y = 300 \end{cases}$, $\begin{cases} x + y = 300 \\ x = 125 \end{cases}$, $\begin{cases} x = 125 \\ y = 0 \end{cases}$, $\begin{cases} y = 0 \\ x + y = 100 \end{cases}$, and $\begin{cases} x + y = 100 \\ x = 0 \end{cases}$ to find the vertices: (0, 220), (80, 220), (125, 175), (125, 0), (100, 0), and (0, 100).



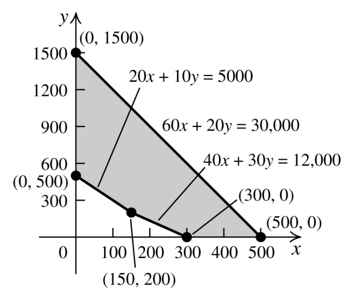
Now find the values of the profit function at each vertex:

Ordered pair	$p = 6.5x + 3.1y$
(0, 220)	682
(80, 220)	1202
(125, 175)	1355
(125, 0)	812.50
(100, 0)	650
(0, 100)	310

The maximum profit is $1355 when there are 125 female guests and 175 male guests.

38. Let x = the number of days the Michigan factory operates, and let y = the number of days the North Carolina plant operates. The cost $c = 60,000x + 40,000y$. The number of luxury cars produced is $20x + 10y$. The number of medium-priced cars is $40x + 30y$. The number of compact cars is $60x + 20y$. The constraints are $x \geq 0$, $y \geq 0$, $20x + 10y \geq 5000$, $40x + 30y \geq 12,000$, and $60x + 20y \leq 30,000$.

Solve the systems $\begin{cases} x = 0, \\ 60x + 20y = 30,000 \end{cases}$, $\begin{cases} 60x + 20y = 30,000 \\ y = 0 \end{cases}$, $\begin{cases} y = 0 \\ 40x + 30y = 12,000 \end{cases}$, $\begin{cases} 40x + 30y = 12,000 \\ 20x + 10y = 5000 \end{cases}$, and $\begin{cases} 20x + 10y = 5000 \\ x = 0 \end{cases}$ to find the vertices: (0, 1500), (500, 0), (300, 0), (150, 200), and (0, 500).



Now find the values of the profit function at each vertex:

Ordered pair	$c = 60,000x + 40,000y$
(0, 1500)	60,000,000
(500, 0)	30,000,000
(300, 0)	18,000,000
(150, 200)	17,000,000
(0, 500)	20,000,000

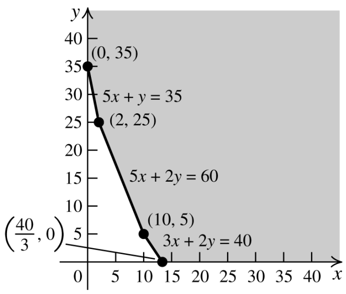
The minimum cost is $17,000,000 when the Michigan plant operates for 150 days and the North Carolina plant operates for 200 days.

39. Let x = the number of pounds of enchiladas, and let y = the number of pounds of vegetable loaf. The cost $c = 3.50x + 2.25y$. The number of units of carbohydrate is $5x + 2y$. The number of units of protein is $3x + 2y$. The number of units of fat is $5x + y$. The constraints are $x \ge 0,\ y \ge 0,\ 5x + 2y \ge 60,\ 3x + 2y \ge 40,$ and $5x + y \ge 35.$ Solve the systems

$$\begin{cases} x = 0, \\ 5x + y = 35 \end{cases}, \begin{cases} 5x + y = 35 \\ 5x + 2y = 60 \end{cases}, \begin{cases} 5x + 2y = 60 \\ 3x + 2y = 40 \end{cases},$$

and $\begin{cases} 3x + 2y = 40 \\ x = 0 \end{cases}$ to find the vertices:

(0, 35), (2, 25), (10, 5), and $(40/3, 0)$.



Now find the values of the profit function at each vertex:

Ordered pair	$c = 3.50x + 2.25y$
(0, 35)	78.75
(2, 25)	63.25
(10, 5)	46.25
$(40/3, 0)$.	46.67

The minimum cost is $46.25 when Elisa buys 10 enchilada meals and 5 vegetable loafs.

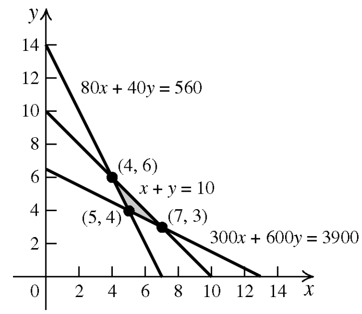
40. Let x = the number of workers from Super Temps, and let y = the number of workers from Ready Aid. The cost $c = 96x + 86y$. The number of units of letters the temps can handle is $300x + 600y$. The number of packages the temps can handle is $80x + 40y$. The constraints are $x \ge 0,\ y \ge 0,\ x + y \le 10,$ $300x + 600y \ge 3900,$ and $80x + 40y \ge 560.$

Solving the systems $\begin{cases} 80x + 40y = 680, \\ x + y = 10 \end{cases}$,

$\begin{cases} x + y = 10 \\ 300x + 400y = 3900 \end{cases}$, and

$\begin{cases} 300x + 400y = 3900 \\ 80x + 40y = 560 \end{cases}$ shows that the corner

points are (4, 6), (7, 3), and (5, 4).



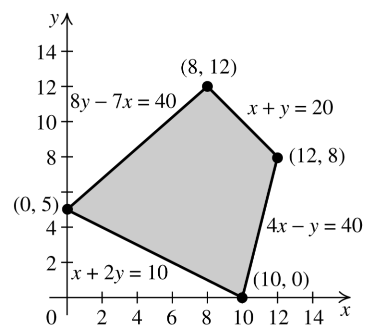
Now find the values of the profit function at each vertex:

Ordered pair	$c = 96x + 86y$
(4, 6)	900
(7, 3)	930
(5, 4)	824

The minimum cost is $824 when five workers are hired from Super Temps and four workers are hired from Ready Aid.

8.6 Beyond the Basics

41.



Solve the systems

$$\begin{cases} x + 2y = 10 \\ 4x - y = 40 \end{cases}, \begin{cases} 4x - y = 40 \\ x + y = 20 \end{cases}, \begin{cases} x + y = 20 \\ 8y - 7x = 40 \end{cases},$$

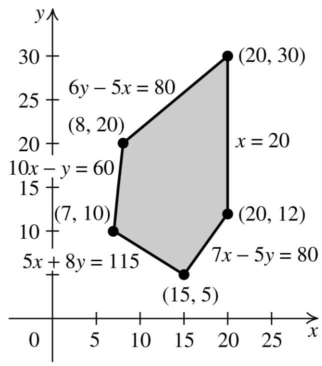
and $\begin{cases} 8y - 7x = 40 \\ x + 2y = 10 \end{cases}$ to find the vertices:

(10, 0), (12, 8), (8, 12), and (0, 5). Now find the values of the objective function at each vertex:

Ordered pair	$f = 8x + 7y$
(10, 0)	80
(12, 8)	152
(8, 12)	148
(0, 5)	35

The maximum is 152 at (12, 8).

42.



Solve the systems

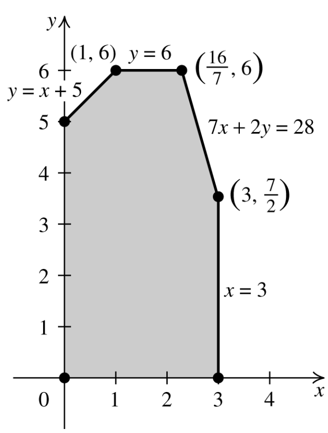
$\begin{cases} 5x+8y=115 \\ 7x-5y=80 \end{cases}$, $\begin{cases} 7x-5y=80 \\ x=20 \end{cases}$, $\begin{cases} x=20 \\ 6y-5x=80 \end{cases}$,

$\begin{cases} 6y-5x=80 \\ 10x-y=60 \end{cases}$, and $\begin{cases} 10x-y=60 \\ 5x+8y=115 \end{cases}$ to find the vertices: (15, 5), (20, 12), (20, 30), (8, 20), and (7, 10). Now find the values of the objective function at each vertex:

Ordered pair	$f=3x+2y$
(15, 5)	55
(20, 12)	84
(20, 30)	120
(8, 20)	64
(7, 10)	41

The minimum is 41 at (7, 10).

43.



Solve the systems

$\begin{cases} x=0 \\ y=0 \end{cases}$, $\begin{cases} y=0 \\ x=3 \end{cases}$, $\begin{cases} x=3 \\ 7x+2y=28 \end{cases}$, $\begin{cases} 7x+2y=28 \\ y=6 \end{cases}$,

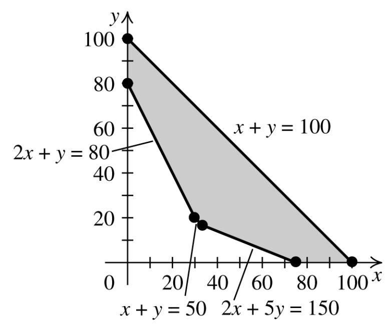
and $\begin{cases} y=6 \\ y=x+5 \end{cases}$ to find the vertices: (0, 0), (3, 0), $(3, 7/2)$, $\left(\frac{16}{7}, 6\right)$, (1, 6), and (0, 5).

Now find the values of the objective function at each vertex:

Ordered pair	$f=5x+6y$
(0, 0)	0
(3, 0)	15
$\left(3, \frac{7}{2}\right)$	36
$\left(\frac{16}{7}, 6\right)$	$\frac{332}{7}$
(1, 6)	41
(0, 5)	30

The minimum is 0 at (0, 0).

44.



Solve the systems

$\begin{cases} 2x+5y=150 \\ y=0 \end{cases}$, $\begin{cases} y=0 \\ x+y=100 \end{cases}$, $\begin{cases} x+y=100 \\ x=0 \end{cases}$,

$\begin{cases} x=0 \\ 2x+y=80 \end{cases}$, $\begin{cases} 2x+y=80 \\ x+y=50 \end{cases}$, and

$\begin{cases} x+y=50 \\ 2x+5y=150 \end{cases}$ to find the vertices: (75, 0), (100, 0), (0, 100), (0, 80), (30, 20) and $\left(\frac{100}{3}, \frac{50}{3}\right)$. Now find the values of the objective function at each vertex:

Ordered pair	$f=7x+3y$
(75, 0)	525
(100, 0)	700
(0, 100)	300
(0, 80)	240
(30, 20)	270
$\left(\frac{100}{3}, \frac{50}{3}\right)$	$\frac{850}{3} \approx 283.3$

The maximum is 700 at (100, 0).

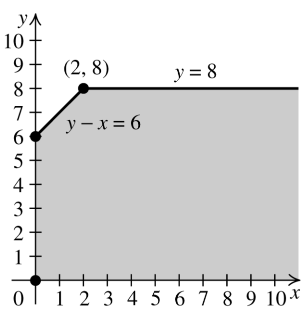
45. $ax_1 + by_1 = M = ax_2 + by_2 \Rightarrow$
$ax_1 - ax_2 = by_2 - by_1 \Rightarrow$
$a(x_1 - x_2) = b(y_2 - y_1) \Rightarrow$
$\frac{a}{b} = \frac{y_2 - y_1}{x_1 - x_2} = -\frac{y_2 - y_1}{x_2 - x_1} = -m,$ where m is the slope of $\overline{PQ}$.

Since the slope is the same between any two points on a given line segment, we can conclude that f has the same value M at every point of the line segment PQ.

46. No

8.6 Critical Thinking/Discussion/Writing

47.



a. S is unbounded because there is no upper limit on the values for x.

b.

Ordered pair	$f = 2x + 3y$
(0, 0)	0
(0, 6)	18
(2, 8)	28
(x, 8)	$2x + 24$

As x increases, the value of the objective function $f = 2x + 3y$ increases. Since S is unbounded, there is no solution.

48. Minimize $f = 3.2x + 2.5y + 1.3z$ subject to the constraints

$$x + 2y + 5 \ge 1$$
$$10x + 5y + 4z \ge 50$$
$$6x + 20y + 7z \ge 10$$
$$x \ge 0,\ y \ge 0,\ z \ge 0$$

8.6 Maintaining Skills

49. $\begin{cases} x - 2y = 4 & (1) \\ -3x + 5y = -7 & (2) \end{cases}$

The first equation already has a leading coefficient of 1. To eliminate x from equation (2), add 3 times equation (1) to equation (2). Then solve for y.

$$\begin{array}{rl} 3x - 6y = 12 & 3(x - 2y = 12) \\ \underline{-3x + 5y = -7} & (2) \\ -y = 5 & \\ y = -5 & (3) \end{array}$$

The equivalent system in triangular form is

$\begin{cases} x - 2y = 4 & (1) \\ y = -5 & (3) \end{cases}$

Back-substitute the value of y into equation (1) and solve for x.

$x - 2(-5) = 4 \Rightarrow x + 10 = 4 \Rightarrow x = 6$

The solution set is $\{(6, -5)\}$. Be sure to check the solution in each of the original equations.

50. $\begin{cases} 3x + 5y = 6 & (1) \\ 2x - y = 17 & (2) \end{cases}$

Interchange equations (1) and (2), then multiply equation (2) by $1/2$ to obtain a leading coefficient of 1.

$\begin{cases} 2x - y = 17 & (2) \\ 3x + 5y = 6 & (1) \end{cases}$

$$\begin{array}{rll} x - \frac{1}{2}y = \frac{17}{2} & \frac{1}{2}(2x - y = 17) & (3) \\ 3x + 5y = 6 & (1) & \end{array}$$

To eliminate x from equation (1), add −3 times equation (3) to equation (1). Then solve for y.

$$\begin{array}{rl} -3x + \frac{3}{2}y = -\frac{51}{2} & -3\left(x - \frac{1}{2}y = \frac{17}{2}\right) \\ \underline{3x + 5y = 6} & (1) \\ \frac{13}{2}y = -\frac{39}{2} & \\ y = -3 & (4) \end{array}$$

The equivalent system in triangular form is

$\begin{cases} x - \frac{1}{2}y = \frac{17}{2} & (3) \\ y = -3 & (4) \end{cases}$

Back-substitute the value of y into equation (3) and solve for x.

$x - \frac{1}{2}(-3) = \frac{17}{2} \Rightarrow x + \frac{3}{2} = \frac{17}{2} \Rightarrow x = \frac{14}{2} = 7$

The solution set is $\{(7, -3)\}$. Be sure to check the solution in each of the original equations.

51. $\begin{cases} x-4y-z=11 & (1) \\ 2x-5y+2z=39 & (2) \\ -3x+2y+z=1 & (3) \end{cases}$

The first equation already has a leading coefficient of 1. To eliminate x from equation (2), add −2 times equation (1) to equation (2).

$$\begin{array}{rl} -2x+8y+2z=-22 & (-2)(x-4y-z=11) \\ 2x-5y+2z=\ \ 39 & (2) \\ \hline 3y+4z=\ \ 17 & (4) \end{array}$$

Multiply equation (4) by $\frac{1}{3}$.

$\begin{cases} x-4y-z=11 & (1) \\ y+\frac{4}{3}z=\frac{17}{4} & (4) \\ -3x+2y+z=1 & (3) \end{cases}$

To eliminate x from equation (3), add 3 times equation (1) to equation (3).

$$\begin{array}{rl} 3x-12y-3z=33 & (3)(x-4y-z=11) \\ -3x+\ 2y+\ z=1 & (3) \\ \hline -10y-2z=34 & (5) \end{array}$$

Multiply equation (5) by $-\frac{1}{10}$.

$\begin{cases} x-4y-z=11 & (1) \\ y+\frac{4}{3}z=\frac{17}{3} & (4) \\ y+\frac{1}{5}z=-\frac{17}{5} & (5) \end{cases}$

Eliminate y in equation (5) by adding −1 times equation (2). Then solve for z.

$$\begin{array}{rl} -y-\frac{4}{3}z=-\frac{17}{3} & (-1)\left(y+\frac{4}{3}z=\frac{17}{4}\right) \\ y+\frac{1}{5}z=-\frac{17}{5} & (5) \\ \hline -\frac{17}{15}z=-\frac{136}{15} & \\ z=8 & (6) \end{array}$$

The equivalent system in triangular form is

$\begin{cases} x-4y-z=11 & (1) \\ y+\frac{4}{3}z=\frac{17}{3} & (4) \\ z=8 & (6) \end{cases}$

Back-substitute the value of z into equation (4) and solve for y.

$y+\frac{4}{3}(8)=\frac{17}{3} \Rightarrow y+\frac{32}{3}=\frac{17}{3} \Rightarrow y=-5$

Back-substitute z = 8 and y = −5 into equation (1) and solve for x.

$x-4(-5)-8=11 \Rightarrow x+12=11 \Rightarrow x=-1$

The solution set is {(−1, −5, 8)}. Be sure to check the solution in each of the original equations.

52. $\begin{cases} x+3y-2z=5 & (1) \\ 2x+y+4z=8 & (2) \\ 6x+y-3z=5 & (3) \end{cases}$

The first equation already has a leading coefficient of 1. To eliminate x from equation (2), add −2 times equation (1) to equation (2).

$$\begin{array}{rl} -2x-6y+4z=-10 & (-2)(x+3y-2z=5) \\ 2x+y+4z=\ \ 8 & (2) \\ \hline -5y+8z=-2 & (4) \end{array}$$

$\begin{cases} x+3y-2z=5 & (1) \\ -5y+8z=-2 & (4) \\ 6x+y-3z=5 & (3) \end{cases}$

To eliminate x from equation (3), add −6 times equation (1) to equation (3).

$$\begin{array}{rl} -6x-18y+12z=-30 & (-6)(x+3y-2z=5) \\ 6x+\ \ y-\ \ 3z=5 & (3) \\ \hline -17y+9z=-25 & (5) \end{array}$$

$\begin{cases} x+3y-2z=5 & (1) \\ -5y+8z=-2 & (4) \\ -17y+9z=-25 & (5) \end{cases}$

Multiply equation (4) by $-\frac{1}{5}$.

$\begin{cases} x+3y-2z=5 & (1) \\ y-\frac{8}{5}z=\frac{2}{5} & (4) \\ -17y+9z=-25 & (5) \end{cases}$

Eliminate y in equation (5) by adding 17 times equation (4). Then solve for z.

$$\begin{array}{rl} 17y-\frac{136}{6}z=\frac{34}{5} & (17)\left(y-\frac{8}{5}z=\frac{2}{5}\right) \\ -17y+9z=-25 & (5) \\ \hline -\frac{91}{5}z=-\frac{91}{5} & \\ z=1 & (6) \end{array}$$

The equivalent system in triangular form is

$\begin{cases} x+3y-2z=5 & (1) \\ y-\frac{8}{5}z=\frac{2}{5} & (4) \\ z=1 & (6) \end{cases}$

Back-substitute the value of z into equation (4) and solve for y.

$y-\frac{8}{5}(1)=\frac{2}{5} \Rightarrow y=\frac{10}{5}=2$

Back-substitute z = 1 and y = 2 into equation (1) and solve for x.

$x+3(2)-2(1)=5 \Rightarrow x+4=5 \Rightarrow x=1$

The solution set is {(1, 2, 1)}. Be sure to check the solution in each of the original equations.

For exercises 53−56, refer to the following array.

$$\begin{bmatrix} 2 & 4 & -6 & 10 \\ 3 & -2 & 4 & 12 \end{bmatrix} \begin{matrix} \leftarrow \text{row 1} \\ \leftarrow \text{row 2} \end{matrix}$$

53. Multiply each number in row 1 by $\frac{1}{2}$.

$$\begin{bmatrix} 1 & 2 & -3 & 5 \\ 3 & -2 & 4 & 12 \end{bmatrix}$$

54. Using the rectangular array in exercise 53, add −3 times each number in row 1 to the corresponding number in row 2.

$-3(1)+3=0 \qquad -3(2)+(-2)=-8$
$-3(-3)+4=13 \qquad -3(5)+12=-3$

The new array is

$$\begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & -8 & 13 & -3 \end{bmatrix}$$

55. Using the rectangular array in exercise 54, multiply each number in row 2 by $-\frac{1}{8}$.

$$\begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & 1 & -\frac{13}{8} & \frac{3}{8} \end{bmatrix}$$

56. Using the rectangular array in exercise 55, add −2 times each number in row 2 to the corresponding number in row 1.

$-2(0)+1=1 \qquad -2(1)+2=0$

$-2\left(-\frac{13}{8}\right)+(-3)=\frac{1}{4} \qquad -2\left(\frac{3}{8}\right)+5=\frac{17}{4}$

$$\begin{bmatrix} 1 & 0 & \frac{1}{4} & \frac{17}{4} \\ 0 & 1 & -\frac{13}{8} & \frac{3}{8} \end{bmatrix}$$

Chapter 8 Review Exercises

Basic Concepts and Skills

1. Using elimination, we have

$$\begin{cases} 3x - y = -5 \\ x + 2y = 3 \end{cases} \Rightarrow \begin{cases} 6x - 2y = -10 \\ x + 2y = 3 \end{cases} \Rightarrow 7x = -7 \Rightarrow$$

$x = -1$

$3(-1) - y = -5 \Rightarrow y = 2$

The solution is {(−1, 2)}.

2. Using substitution, we have

$$\begin{cases} x + 3y + 6 = 0 \\ y = 4x - 2 \end{cases} \Rightarrow x + 3(4x-2) + 6 = 0 \Rightarrow$$

$13x = 0 \Rightarrow x = 0$

$y = 4(0) - 2 = -2$

The solution is {(0, −2)}.

3. Using elimination, we have

$$\begin{cases} 2x + 4y = 3 \\ 3x + 6y = 10 \end{cases} \Rightarrow \begin{cases} -6x - 12y = -9 \\ 6x + 12y = 20 \end{cases} \Rightarrow 0 = 11 \Rightarrow$$

there is no solution. Solution set: ∅

4. Using elimination, we have

$$\begin{cases} x - y = 2 \\ 2x - 2y = 9 \end{cases} \Rightarrow \begin{cases} -2x + 2y = -4 \\ 2x - 2y = 9 \end{cases} \Rightarrow 0 = 5 \Rightarrow$$

there is no solution. Solution set: ∅

5. Using elimination, we have

$$\begin{cases} 3x - y = 3 \\ \frac{1}{2}x + \frac{1}{3}y = 2 \end{cases} \Rightarrow \begin{cases} x - \frac{1}{3}y = 1 \\ \frac{1}{2}x + \frac{1}{3}y = 2 \end{cases} \Rightarrow \frac{3}{2}x = 3 \Rightarrow$$

$x = 2$

$3(2) - y = 3 \Rightarrow y = 3$

The solution is {(2, 3)}.

6. Using elimination, we have

$$\begin{cases} 0.02y - 0.03x = -0.04 \\ 1.5x - 2y = 3 \end{cases} \Rightarrow$$

$$\begin{cases} 2y - 3x = -4 \\ -2y + 1.5x = 3 \end{cases} \Rightarrow -1.5x = -1 \Rightarrow x = \frac{2}{3}$$

$1.5\left(\frac{2}{3}\right) - 2y = 3 \Rightarrow 1 - 2y = 3 \Rightarrow$

$-2y = 2 \Rightarrow y = -1$

Solution set: $\left\{\left(\frac{2}{3}, -1\right)\right\}$

7. Multiply the first equation by −2, add the result to the second equation, and replace the second equation with the new equation:

$$\begin{cases} x + 3y + z = 0 \\ 2x - y + z = 5 \\ 3x - 3y + 2z = 10 \end{cases} \Rightarrow \begin{cases} x + 3y + z = 0 \\ -7y - z = 5 \\ 3x - 3y + 2z = 10 \end{cases}$$

Multiply the first equation by −3, add the result to the third equation, and replace the third equation with the new equation:

$$\begin{cases} x + 3y + z = 0 \\ -7y - z = 5 \\ 3x - 3y + 2z = 10 \end{cases} \Rightarrow \begin{cases} x + 3y + z = 0 \\ -7y - z = 5 \\ -12y - z = 10 \end{cases}$$

Subtract the third equation from the second equation, and replace the second equation with the result:

$$\begin{cases} x + 3y + z = 0 \\ -7y - z = 5 \\ -12y - z = 10 \end{cases} \Rightarrow \begin{cases} x + 3y + z = 0 \\ 5y = -5 \\ -12y - z = 10 \end{cases} \Rightarrow$$

$$\begin{cases} x + 3y + z = 0 \\ y = -1 \\ -12y - z = 10 \end{cases}$$

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Substitute $y = -1$ into the third equation to solve for z and then substitute the values for z and y into the first equation to solve for x:

$$\begin{cases} x + 3y + z = 0 \\ y = -1 \\ -12y - z = 10 \end{cases} \Rightarrow \begin{cases} x + 3y + z = 0 \\ y = -1 \\ -12(-1) - z = 10 \end{cases} \Rightarrow$$

$$\begin{cases} x + 3y + z = 0 \\ y = -1 \\ z = 2 \end{cases} \Rightarrow \begin{cases} x + 3(-1) + 2 = 0 \\ y = -1 \\ z = 2 \end{cases} \Rightarrow$$

$$\begin{cases} x = 1 \\ y = -1 \\ z = 2 \end{cases}$$

The solution is $\{(1, -1, 2)\}$.

8. Switch the second and third equations, multiply the second equation by -2, add the result to the first equation, and replace the second equation with the result:

$$\begin{cases} 2x + y = 11 \\ 3y - z = 5 \\ x + 2z = 1 \end{cases} \Rightarrow \begin{cases} 2x + y = 11 \\ x + 2z = 1 \\ 3y - z = 5 \end{cases} \Rightarrow$$

$$\begin{cases} 2x + y = 11 \\ y - 4z = 9 \\ 3y - z = 5 \end{cases}$$

Multiply the second equation by -3, add the result to the third equation, replace the third equation with the result, and solve for z:

$$\begin{cases} 2x + y = 11 \\ y - 4z = 9 \\ 3y - z = 5 \end{cases} \Rightarrow \begin{cases} 2x + y = 11 \\ y - 4z = 9 \\ 11z = -22 \end{cases} \Rightarrow$$

$$\begin{cases} 2x + y = 11 \\ y - 4z = 9 \\ z = -2 \end{cases}$$

Substitute $z = -2$ into the second equation to solve for y, and then substitute the values for y and z into the first equation to solve for x.

$$\begin{cases} 2x + y = 11 \\ y - 4z = 9 \\ z = -2 \end{cases} \Rightarrow \begin{cases} 2x + y = 11 \\ y - 4(-2) = 9 \\ z = -2 \end{cases} \Rightarrow$$

$$\begin{cases} 2x + y = 11 \\ y = 1 \\ z = -2 \end{cases} \Rightarrow \begin{cases} 2x + 1 = 11 \\ y = 1 \\ z = -2 \end{cases} \Rightarrow \begin{cases} x = 5 \\ y = 1 \\ z = -2 \end{cases}$$

The solution is $\{(5, 1, -2)\}$.

9. Switch the second and third equations, multiply the first equation by -2, add the result to the second equation, and replace the second equation with the result.

$$\begin{cases} x + y = 1 \\ 3y + 2z = 0 \\ 2x - 3z = 7 \end{cases} \Rightarrow \begin{cases} x + y = 1 \\ 2x - 3z = 7 \\ 3y + 2z = 0 \end{cases} \Rightarrow$$

$$\begin{cases} x + y = 1 \\ -2y - 3z = 5 \\ 3y + 2z = 0 \end{cases}$$

Multiply the second equation by 3, multiply the third equation by 2, add the results, replace the third equation with the new equation, and solve for z:

$$\begin{cases} x + y = 1 \\ -2y - 3z = 5 \\ 3y + 2z = 0 \end{cases} \Rightarrow \begin{cases} x + y = 1 \\ -2y - 3z = 5 \\ -5z = 15 \end{cases} \Rightarrow$$

$$\begin{cases} x + y = 1 \\ -2y - 3z = 5 \\ z = -3 \end{cases}$$

Substitute $z = -3$ into the second equation to solve for y, and then substitute the value for y into the first equation to solve for x:

$$\begin{cases} x + y = 1 \\ -2y - 3z = 5 \\ z = -3 \end{cases} \Rightarrow \begin{cases} x + y = 1 \\ -2y - 3(-3) = 5 \\ z = -3 \end{cases} \Rightarrow$$

$$\begin{cases} x + y = 1 \\ y = 2 \\ z = -3 \end{cases} \Rightarrow \begin{cases} x + 2 = 1 \\ y = 2 \\ z = -3 \end{cases} \Rightarrow \begin{cases} x = -1 \\ y = 2 \\ z = -3 \end{cases}$$

The solution is $\{(-1, 2, -3)\}$.

10. Multiply the second equation by -2, add the result to the first equation, and replace the second equation with the result:

$$\begin{cases} 2x - 3y + z = 2 \\ x - 3y + 2z = -1 \\ 2x + 3y + 2z = 3 \end{cases} \Rightarrow \begin{cases} 2x - 3y + z = 2 \\ 3y - 3z = 4 \\ 2x + 3y + 2z = 3 \end{cases}$$

Subtract the third equation from the first equation, and replace the third equation with the result:

$$\begin{cases} 2x - 3y + z = 2 \\ 3y - 3z = 4 \\ 2x + 3y + 2z = 3 \end{cases} \Rightarrow \begin{cases} 2x - 3y + z = 2 \\ 3y - 3z = 4 \\ -6y - z = -1 \end{cases}$$

Multiply the second equation by 2, add the result to the third equation, replace the third equation with the new equation, and solve for z:

$$\begin{cases} 2x - 3y + z = 2 \\ 3y - 3z = 4 \\ -6y - z = -1 \end{cases} \Rightarrow \begin{cases} 2x - 3y + z = 2 \\ 3y - 3z = 4 \\ -7z = 7 \end{cases} \Rightarrow$$

$$\begin{cases} 2x - 3y + z = 2 \\ 3y - 3z = 4 \\ z = -1 \end{cases}$$

Substitute $z = -1$ into the second equation to solve for y, and then substitute the value for y into the first equation to solve for x:

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$$\begin{cases} 2x-3y+z=2 \\ 3y-3z=4 \\ z=-1 \end{cases} \Rightarrow \begin{cases} 2x-3y+z=2 \\ 3y-3(-1)=4 \\ z=-1 \end{cases} \Rightarrow$$

$$\begin{cases} 2x-3y+z=2 \\ y=\frac{1}{3} \\ z=-1 \end{cases} \Rightarrow \begin{cases} 2x-3\left(\frac{1}{3}\right)+(-1)=2 \\ y=\frac{1}{3} \\ z=-1 \end{cases} \Rightarrow$$

$$\begin{cases} x=2 \\ y=\frac{1}{3} \\ z=-1 \end{cases}.$$

The solution is $\left\{\left(2, \frac{1}{3}, -1\right)\right\}$.

11. Multiply the first equation by −5, add the result to the second equation, and replace the second equation with the result, then switch the first and second equations:

$$\begin{cases} x+y+z=1 \\ x+5y+5z=-1 \\ 3x-y-z=4 \end{cases} \Rightarrow \begin{cases} x+y+z=1 \\ -4x=-6 \\ 3x-y-z=4 \end{cases} \Rightarrow$$

$$\begin{cases} x=\frac{3}{2} \\ x+y+z=1 \\ 3x-y-z=4 \end{cases}$$

Add the second and third equations, and replace the second equation with the result.

$$\begin{cases} x=\frac{3}{2} \\ x+y+z=1 \\ 3x-y-z=4 \end{cases} \Rightarrow \begin{cases} x=\frac{3}{2} \\ 4x=5 \\ 3x-y-z=4 \end{cases} \Rightarrow$$

$$\begin{cases} x=3/2 \\ x=5/4 \\ 3x-y-z=4 \end{cases} \Rightarrow \text{There is no solution.}$$

Solution set: $\varnothing$

12. Multiply the first equation by −2, add the result to the second equation, and replace the second equation with the new equation:

$$\begin{cases} x+3y-2z=-4 \\ 2x+6y-4z=3 \\ x+y+z=1 \end{cases} \Rightarrow \begin{cases} x+3y-2z=-4 \\ 0=11 \\ x+y+z=1 \end{cases} \Rightarrow$$

there is no solution. Solution set: $\varnothing$

13. The system is a dependent system because there are two equations in three unknowns. Multiply the first equation by −5, and multiply the second equation by 4, then add the resulting equations and solve for z:

$$\begin{cases} x+4y+3z=1 \\ 2x+5y+4z=4 \end{cases} \Rightarrow \begin{cases} -5x-20y-15z=-5 \\ 8x+20y+16z=16 \end{cases} \Rightarrow$$

$3x+z=11 \Rightarrow z=-3x+11$

Substitute the expression for z into the first equation and solve for y:

$x+4y+3(-3x+11)=1 \Rightarrow -8x+4y+33=1 \Rightarrow$
$y=2x-8.$

The solution is $\{(x, 2x-8, -3x+11)\}$.

14. The system is a dependent system because there are two equations in three unknowns. Multiply the first equation by −2, and add the result to the second equation, then solve for z:

$$\begin{cases} 3x-y+2z=9 \\ x-2y+3z=2 \end{cases} \Rightarrow -5x-z=-16 \Rightarrow$$

$z=16-5x$

Substitute the expression for z into the first equation and solve for y:

$3x-y+2(16-5x)=9 \Rightarrow -7x-y+32=9 \Rightarrow$
$y=23-7x.$

The solution is $\{(x, 23-7x, 16-5x)\}$.

15. Add the first and third equations, and replace the first equation with the result:

$$\begin{cases} 3x-y=2 \\ x+2y=9 \\ 3x+y=10 \end{cases} \Rightarrow \begin{cases} 6x=12 \\ x+2y=9 \\ 3x+y=10 \end{cases} \Rightarrow \begin{cases} x=2 \\ x+2y=9 \\ 3x+y=10 \end{cases}$$

Substitute $x = 2$ into the second equation and solve for y:

$$\begin{cases} x=2 \\ x+2y=9 \\ 3x+y=10 \end{cases} \Rightarrow \begin{cases} x=2 \\ 2+2y=9 \\ 3x+y=10 \end{cases} \Rightarrow \begin{cases} x=2 \\ y=\frac{7}{2} \\ 3x+y=10 \end{cases}$$

$3(2)+\frac{7}{2}=\frac{19}{2}\neq 10 \Rightarrow$ there is no solution.

Solution set: $\varnothing$

16. Add the first and third equations, and replace the third equation with the result:

$$\begin{cases} x-2y=1 \\ 3x+4y=11 \\ 2x+2y=7 \end{cases} \Rightarrow \begin{cases} x-2y=1 \\ 3x+4y=11 \\ 3x=8 \end{cases} \Rightarrow$$

$$\begin{cases} x-2y=1 \\ 3x+4y=11 \\ x=8/3 \end{cases}$$

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Substitute the value for x into the first and second equations to solve for y:

$$\begin{cases} x-2y=1 \\ 3x+4y=11 \\ x=\frac{8}{3} \end{cases} \Rightarrow \begin{cases} \left(\frac{8}{3}\right)-2y=1 \\ 3\left(\frac{8}{3}\right)+4y=11 \Rightarrow \\ x=\frac{8}{3} \end{cases}$$

$$\begin{cases} y=\frac{5}{6} \\ y=\frac{3}{4} \Rightarrow \text{ there is no solution.} \\ x=\frac{8}{3} \end{cases}$$

Solution set: $\varnothing$

17. $\begin{cases} x+y+z=3 & (1) \\ 2x-y+z=4 & (2) \\ x+4y+2z=5 & (3) \end{cases}$

Multiply equation (1) by −2, then add the result to equation (2), and replace equation (2) with the new equation:

$$\begin{cases} x+y+z=3 & (1) \\ 2x-y+z=4 & (2) \Rightarrow \\ x+4y+2z=5 & (3) \end{cases}$$

$$\begin{cases} x+y+z=3 & (1) \\ -3y-z=-2 & (4) \\ x+4y+2z=5 & (3) \end{cases}$$

Multiply equation (1) by −1, then add the result to equation (3), and replace equation (3) with the new equation:

$$\begin{cases} x+y+z=3 & (1) \\ 2x-y+z=4 & (2) \Rightarrow \\ x+4y+2z=5 & (3) \end{cases} \begin{cases} x+y+z=3 & (1) \\ -3y-z=-2 & (4) \\ 3y+z=2 & (5) \end{cases}$$

Add equation (4) to equation (5) to solve for z:

$$\begin{cases} x+y+z=3 & (1) \\ -3y-z=-2 & (4) \Rightarrow 0=0 \\ 3y+z=2 & (5) \end{cases}$$

The equation 0 = 0 is equivalent to $0z = 0$, which is true for every value of z. Solving the second equation for y, we have

$-3y-z=-2 \Rightarrow y=\frac{2}{3}-\frac{1}{3}z$. Substituting this into the first equation, we have $x+y+z=3 \Rightarrow$

$$x+\left(\frac{2}{3}-\frac{1}{3}z\right)-z=3 \Rightarrow x=\frac{7}{3}-\frac{2}{3}z$$

Thus, the solution is $\left\{\left(\frac{7}{3}-\frac{2}{3}z, \frac{2}{3}-\frac{1}{3}z, z\right)\right\}$.

18. $\begin{cases} x-y+z=4 & (1) \\ x+2y+3z=2 & (2) \\ 2x+y+4z=6 & (3) \end{cases}$

Multiply equation (1) by −1, then add the result to equation (2), and replace equation (2) with the new equation:

$$\begin{cases} x-y+z=4 & (1) \\ x+2y+3z=2 & (2) \Rightarrow \\ 2x+y+4z=6 & (3) \end{cases} \begin{cases} x-y+z=4 & (1) \\ 3y+2z=-2 & (4) \\ 2x+y+4z=6 & (3) \end{cases}$$

Multiply equation (1) by −2, then add the result to equation (3), and replace equation (3) with the new equation:

$$\begin{cases} x-y+z=4 & (1) \\ 3y+2z=-2 & (4) \Rightarrow \\ 2x+y+4z=6 & (3) \end{cases} \begin{cases} x-y+z=4 & (1) \\ 3y+2z=-2 & (4) \\ 3y+2z=-2 & (5) \end{cases}$$

Multiply equation (4) by −1, then add the result to equation (5) to solve for z:

$$\begin{cases} x-y+z=4 & (1) \\ 3y+2z=-2 & (4) \Rightarrow 0=0 \\ 3y+2z=-2 & (5) \end{cases}$$

The equation 0 = 0 is equivalent to $0z = 0$, which is true for every value of z. Solving the second equation for y, we have

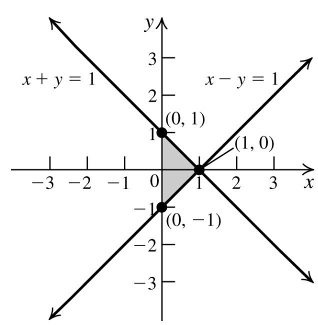
$$3y+2z=-2 \Rightarrow y=-\frac{2}{3}-\frac{2}{3}z.$$

Substituting this into the first equation, we have $x-y+z=4 \Rightarrow$

$$x-\left(-\frac{2}{3}-\frac{2}{3}z\right)+z=4 \Rightarrow x=\frac{10}{3}-\frac{5}{3}z$$

Thus, the solution is $\left\{\left(\frac{10}{3}-\frac{5}{3}z, -\frac{2}{3}-\frac{2}{3}z, z\right)\right\}$.

19.



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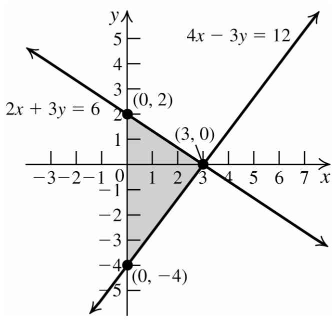

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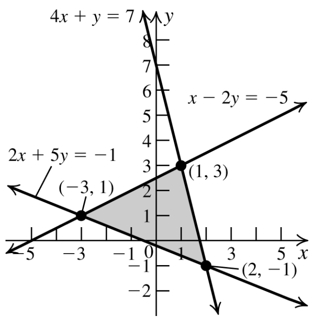

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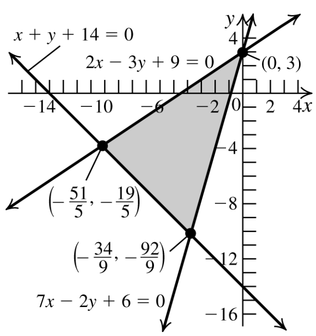

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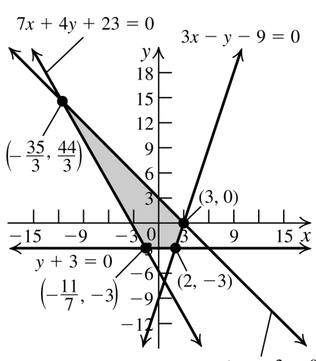

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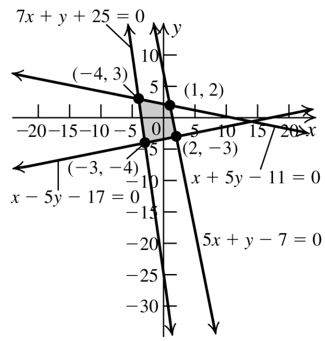

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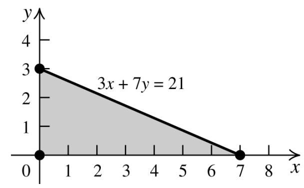


Solve the systems

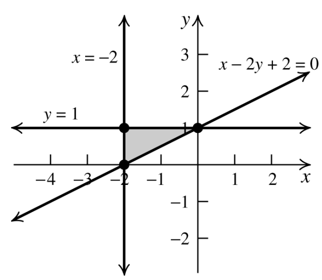
$\begin{cases} x = 0 \\ 3x + 7y = 21 \end{cases}$, $\begin{cases} 3x + 7y = 21 \\ y = 0 \end{cases}$, and $\begin{cases} y = 0 \\ x = 0 \end{cases}$ to

find the vertices: (0, 3), (7, 0),and (0, 0). Now find the values of the objective function at each vertex:

Ordered pair	$z = 2x + 3y$
(0,3)	9
(7, 0)	14
(0, 0)	0

The maximum is 14 at (7, 0).

26.



Solve the systems

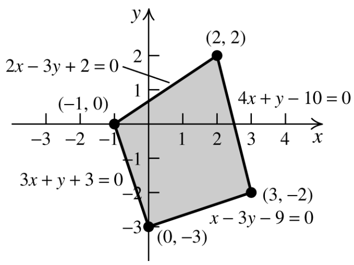
$\begin{cases} x = -2 \\ x - 2y = -2 \end{cases}$, $\begin{cases} x - 2y = -2 \\ y = 1 \end{cases}$, and $\begin{cases} y = 1 \\ x = -2 \end{cases}$ to

find the vertices: (−2, 0), (0, 1),and (−2, 1). Now find the values of the objective function at each vertex:

Ordered pair	$z = -5x + 2y$
(−2, 0)	10
(0, 1)	2
(−2, 1)	12

The minimum is 2 at (0, 1).

27.



Solve the systems

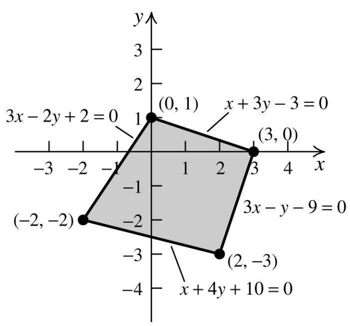
$\begin{cases} 2x-3y=-2 \\ 4x+y=10 \end{cases}, \begin{cases} 4x+y=10 \\ x-3y=9 \end{cases}, \begin{cases} x-3y=9 \\ 3x+y=-3 \end{cases},$

and $\begin{cases} 3x+y=-3 \\ 2x-3y=-2 \end{cases}$ to find the vertices: (2,2), (3, −2), (0, −3), and (−1, 0). Now find the values of the objective function at each vertex:

Ordered pair	$z = x + 3y$
(2, 2)	8
(3, −2)	−3
(0, −3)	−9
(−1, 0)	−1

The minimum is −9 at (0, −3).

28.



Solve the systems

$\begin{cases} x+3y=3 \\ 3x-y=9 \end{cases}, \begin{cases} 3x-y=9 \\ x+4y=-10 \end{cases}, \begin{cases} x+4y=-10 \\ 3x-2y=-2 \end{cases},$

and $\begin{cases} 3x-2y=-2 \\ x+3y=3 \end{cases}$ to find the vertices: (3, 0), (2, −3), (−2, −2), and (0, 1). Now find the values of the objective function at each vertex:

Ordered pair	$z = 2x + 5y$
(3, 0)	6
(2, −3)	−11
(−2, −2)	−14
(0, 1)	5

The maximum is 6 at (3, 0).

29. Using substitution, we have

$\begin{cases} x+3y=1 \\ x^2-3x=7y+3 \end{cases} \Rightarrow \begin{cases} x=1-3y \\ x^2-3x=7y+3 \end{cases} \Rightarrow$

$(1-3y)^2-3(1-3y)=7y+3 \Rightarrow$

$9y^2-4y-5=0 \Rightarrow (y-1)(9y+5)=0 \Rightarrow$

$y=1$ or $y=-\frac{5}{9}$

$x+3(1)=1 \Rightarrow x=-2$

$x+3\left(-\frac{5}{9}\right)=1 \Rightarrow x=\frac{24}{9}=\frac{8}{3}$

The solution is $\left\{(-2,1),\left(\frac{8}{3},-\frac{5}{9}\right)\right\}$.

30. Using substitution, we have

$\begin{cases} 4x-y=3 \\ y^2-2y=x-2 \end{cases} \Rightarrow \begin{cases} y=4x-3 \\ y^2-2y=x-2 \end{cases} \Rightarrow$

$(4x-3)^2-2(4x-3)=x-2 \Rightarrow$

$16x^2-33x+17=0 \Rightarrow (16x-17)(x-1)=0 \Rightarrow$

$x=1$ or $x=17/16$

$4(1)-y=3 \Rightarrow y=1; 4\left(\frac{17}{16}\right)-y=3 \Rightarrow y=\frac{5}{4}$

The solution is $\left\{(1,1),\left(\frac{17}{16},\frac{5}{4}\right)\right\}$.

31. Using substitution, we have

$\begin{cases} x-y=4 \\ 5x^2+y^2=24 \end{cases} \Rightarrow \begin{cases} y=x-4 \\ 5x^2+y^2=24 \end{cases} \Rightarrow$

$5x^2+(x-4)^2=24 \Rightarrow 6x^2-8x-8=0 \Rightarrow$

$2(x-2)(3x+2)=0 \Rightarrow x=2$ or $x=-\frac{2}{3}$

$2-y=4 \Rightarrow y=-2$

$-\frac{2}{3}-y=4 \Rightarrow y=-\frac{14}{3}$

The solution is $\left\{(2,-2),\left(-\frac{2}{3},-\frac{14}{3}\right)\right\}$.

32. Using substitution, we have

$\begin{cases} x-2y=7 \\ 2x^2+3y^2=29 \end{cases} \Rightarrow \begin{cases} x=2y+7 \\ 2x^2+3y^2=29 \end{cases} \Rightarrow$

$2(2y+7)^2+3y^2=29 \Rightarrow 11y^2+56y+69=0 \Rightarrow$

$(y+3)(11y+23)=0 \Rightarrow y=-3$ or $y=-\frac{23}{11}$.

$x-2(-3)=7 \Rightarrow x=1$

$x-2\left(-\frac{23}{11}\right)=7 \Rightarrow x=\frac{31}{11}$

The solution is $\left\{(1,-3),\left(\frac{31}{11},-\frac{23}{11}\right)\right\}$.

33. Using substitution, we have

$$\begin{cases} xy = 2 \\ x^2 + 2y^2 = 9 \end{cases} \Rightarrow \begin{cases} y = \dfrac{2}{x} \\ x^2 + 2y^2 = 9 \end{cases} \Rightarrow$$

$$x^2 + 2\left(\frac{2}{x}\right)^2 = 9 \Rightarrow x^2 - 9 + \frac{8}{x^2} = 0 \Rightarrow$$

$$x^4 - 9x^2 + 8 = 0$$

Let $u = x^2$. Then $u^2 - 9u + 8 = 0 \Rightarrow$

$(u-8)(u-1) = 0 \Rightarrow u = 8 \Rightarrow x^2 = 8 \Rightarrow$

$x = \pm 2\sqrt{2}$ or $u = 1 \Rightarrow x^2 = 1 \Rightarrow x^2 = \pm 1$

$$\left(2\sqrt{2}\right)y = 2 \Rightarrow y = \frac{\sqrt{2}}{2}$$

$$\left(-2\sqrt{2}\right)y = 2 \Rightarrow y = -\frac{\sqrt{2}}{2}$$

$(1)y = 2 \Rightarrow y = 2; (-1)y = 2 \Rightarrow y = -2$

None of the values are extraneous, so the solution is $\left\{(1,2), (-1,-2), \left(2\sqrt{2}, \frac{\sqrt{2}}{2}\right), \left(-2\sqrt{2}, -\frac{\sqrt{2}}{2}\right)\right\}$.

34. Using substitution, we have

$$\begin{cases} xy = 3 \\ 2x^2 + 3y^2 = 21 \end{cases} \Rightarrow \begin{cases} y = \dfrac{3}{x} \\ 2x^2 + 3y^2 = 21 \end{cases} \Rightarrow$$

$$2x^2 + 3\left(\frac{3}{x}\right)^2 = 21 \Rightarrow 2x^2 - 21 + \frac{27}{x^2} = 0 \Rightarrow$$

$2x^4 - 21x^2 + 27 = 0$. Let $u = x^2$. Then

$2u^2 - 21u + 27 = 0 \Rightarrow (u-9)(2u-3) = 0 \Rightarrow$

$u = 9 = x^2 \Rightarrow x = \pm 3$ or $u = \frac{3}{2} = x^2 \Rightarrow x = \pm\frac{\sqrt{6}}{2}$

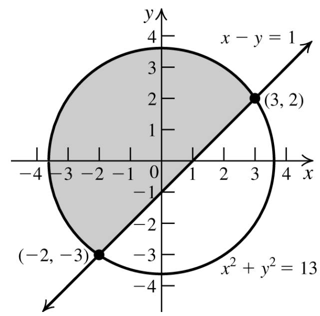
$3y = 3 \Rightarrow y = 1$

$-3y = 3 \Rightarrow y = -1$

$\frac{\sqrt{6}}{2}y = 3 \Rightarrow y = \sqrt{6}; -\frac{\sqrt{6}}{2}y = 3 \Rightarrow y = -\sqrt{6}$

None of the values are extraneous, so the solution is $\left\{(3,1), (-3,-1), \left(\frac{\sqrt{6}}{2}, \sqrt{6}\right), \left(-\frac{\sqrt{6}}{2}, -\sqrt{6}\right)\right\}$.

35.



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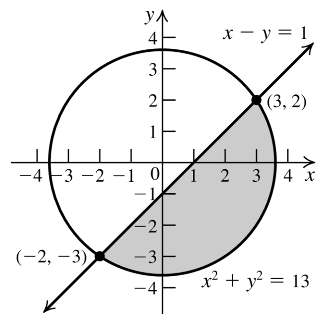

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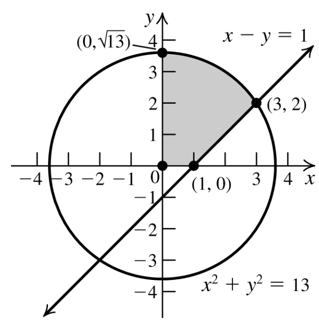

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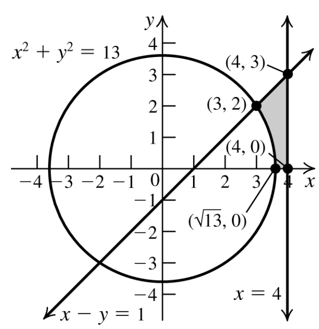

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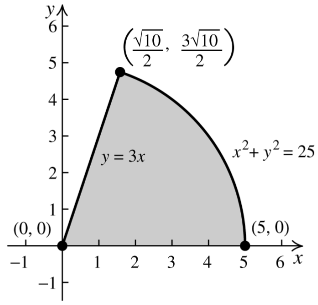

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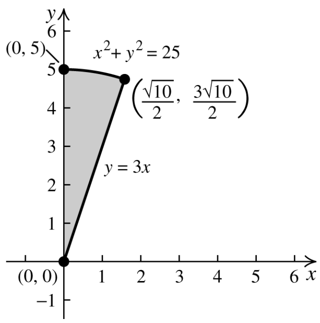


41. $\dfrac{x+4}{x^2+5x+6}=\dfrac{x+4}{(x+2)(x+3)}=\dfrac{A}{x+2}+\dfrac{B}{x+3}\Rightarrow$

$x+4=A(x+3)+B(x+2)\Rightarrow$

$x+4=(A+B)x+(3A+2B)\Rightarrow$

$\begin{cases}A+\ \ B=1\\3A+2B=4\end{cases}\Rightarrow\begin{cases}-2A-2B=-2\\ \ \ 3A+2B=4\end{cases}\Rightarrow$

$A=2, B=-1$

$\dfrac{x+4}{x^2+5x+6}=\dfrac{2}{x+2}-\dfrac{1}{x+3}$

42. $\dfrac{x+14}{x^2+3x-4}=\dfrac{x+14}{(x+4)(x-1)}=\dfrac{A}{x+4}+\dfrac{B}{x-1}\Rightarrow$

$x+14=A(x-1)+B(x+4)\Rightarrow$

$x+14=(A+B)x+(-A+4B)\Rightarrow$

$\begin{cases}A+B=1\\-A+4B=14\end{cases}\Rightarrow 5B=15\Rightarrow B=3, A=-2$

$\dfrac{x+14}{x^2+3x-4}=-\dfrac{2}{x+4}+\dfrac{3}{x-1}$

43. $\dfrac{3x^2+x+1}{x(x-1)^2}=\dfrac{A}{x}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}\Rightarrow$

$3x^2+x+1=A(x-1)^2+Bx(x-1)+Cx\Rightarrow$

$3x^2+x+1$

$\quad=(A+B)x^2+(-2A-B+C)x+A\Rightarrow$

$\begin{cases}A+B\qquad\ \ =3\\-2A-B+C=1\\ A\qquad\qquad\ \ =1\end{cases}\Rightarrow A=1, B=2, C=5$

$\dfrac{3x^2+x+1}{x(x-1)^2}=\dfrac{1}{x}+\dfrac{2}{x-1}+\dfrac{5}{(x-1)^2}$

44. $\dfrac{3x^2+2x+3}{(x^2-1)(x+1)}=\dfrac{3x^2+2x+3}{(x-1)(x+1)^2}$

$\quad=\dfrac{A}{x-1}+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}\Rightarrow$

$3x^2+2x+3$

$\quad=A(x+1)^2+B(x-1)(x+1)+C(x-1)\Rightarrow$

$3x^2+2x+3$

$\quad=(A+B)x^2+(2A+C)x+(A-B-C)\Rightarrow$

$\begin{cases}A+B\qquad\ =3\\2A+\qquad C=2\\A-B-C=3\end{cases}\Rightarrow A=2, B=1, C=-2$

$\dfrac{3x^2+2x+3}{(x^2-1)(x+1)}=\dfrac{2}{x-1}+\dfrac{1}{x+1}-\dfrac{2}{(x+1)^2}$

45. $\dfrac{x^2+2x+3}{(x^2+4)^2}=\dfrac{Ax+B}{x^2+4}+\dfrac{Cx+D}{(x^2+4)^2}\Rightarrow$

$x^2+2x+3=(Ax+B)(x^2+4)+Cx+D\Rightarrow$

x^2+2x+3

$\quad=Ax^3+Bx^2+(4A+C)x+(4B+D)\Rightarrow$

$\begin{cases}\qquad A=0\\ \qquad B=1\\4A+C=2\\4B+D=3\end{cases}\Rightarrow A=0, B=1, C=2, D=-1$

$\dfrac{x^2+2x+3}{(x^2+4)^2}=\dfrac{1}{x^2+4}+\dfrac{2x-1}{(x^2+4)^2}$

46. $\dfrac{2x}{x^4-1}=\dfrac{2x}{(x-1)(x+1)(x^2+1)}$

$\quad=\dfrac{A}{x-1}+\dfrac{B}{x+1}+\dfrac{Cx+D}{x^2+1}\Rightarrow$

$2x=A(x+1)(x^2+1)+B(x-1)(x^2+1)$

$\qquad+(Cx+D)(x-1)(x+1)$

Letting $x=-1$, we have $-2=-4B\Rightarrow B=\dfrac{1}{2}$.

Letting $x=1$, we have $2=4A\Rightarrow A=\dfrac{1}{2}$.

Substitute the values for A and B:

$2x=\dfrac{1}{2}(x+1)(x^2+1)+\dfrac{1}{2}(x-1)(x^2+1)$

$\qquad+(Cx+D)(x-1)(x+1)\Rightarrow$

$2x=(C+1)x^3+Dx^2+(1-C)x-D\Rightarrow$

$\begin{cases}C+1=0\\ \quad D=0\\1-C=2\\ \ -D=0\end{cases}\Rightarrow C=-1, D=0$

$\dfrac{2x}{x^4-1}=\dfrac{1}{2(x-1)}+\dfrac{1}{2(x+1)}-\dfrac{x}{x^2+1}$

Applying the Concepts

47. Let x = the amount invested at high-risk. Let y = the amount invested at 4% interest. Then

$$\begin{cases} x + \quad y = 15,000 \\ 0.12x + 0.04y = 1300 \end{cases} \Rightarrow$$

$$\begin{cases} -0.04x - 0.04y = -600 \\ 0.12x + 0.04y = 1300 \end{cases} \Rightarrow 0.08x = 700 \Rightarrow$$

$x = 8750$

$8750 + y = 15,000 \Rightarrow y = 6250$

The speculator invested \$8750 at 12% and \$6250 at 4%.

48. Let x = the number of acres of tomatoes. Let y = the number of acres of soybeans. Then

$$\begin{cases} y = 5 + 2x \\ 525x + 475y = 24,500 \end{cases} \Rightarrow$$

$525x + 475(5 + 2x) = 24,500 \Rightarrow$

$1475x + 2375 = 24,500 \Rightarrow x = 15$

$y = 5 + 2(15) = 35$

There are 15 acres of tomatoes and 35 acres of soybeans.

49. Let x = the length of the rectangle. Let y = the width of the rectangle. Then

$$\begin{cases} xy = 63 \\ 2x + 2y = 33 \end{cases} \Rightarrow \begin{cases} y = \dfrac{63}{x} \\ 2x + 2y = 33 \end{cases} \Rightarrow$$

$2x + 2\left(\dfrac{63}{x}\right) = 33 \Rightarrow 2x - 33 + \dfrac{126}{x} = 0 \Rightarrow$

$2x^2 - 33x + 126 = 0 \Rightarrow (x - 6)(2x - 21) = 0 \Rightarrow$

$x = 6$ or $x = \dfrac{21}{2} = 10.5;\ 6y = 63 \Rightarrow y = \dfrac{21}{2} = 10.5$

$\dfrac{21}{2}y = 63 \Rightarrow y = 6$

The rectangle is 10.5 feet by 6 feet.

50. Let x and y = the numbers. Then

$$\begin{cases} 2\left(\dfrac{1}{x} + \dfrac{1}{y}\right) = 13 \\ xy = \dfrac{1}{9} \end{cases} \Rightarrow \begin{cases} \dfrac{2}{x} + \dfrac{2}{y} = 13 \\ y = \dfrac{1}{9x} \end{cases} \Rightarrow$$

$\dfrac{2}{x} + \dfrac{2}{\frac{1}{9x}} = 13 \Rightarrow \dfrac{2}{x} + 18x = 13 \Rightarrow$

$18x^2 - 13x + 2 = 0 \Rightarrow (2x - 1)(9x - 2) = 0 \Rightarrow$

$x = \dfrac{1}{2}$ or $x = \dfrac{2}{9}$

$\dfrac{1}{2}y = \dfrac{1}{9} \Rightarrow y = \dfrac{2}{9};\ \dfrac{2}{9}y = \dfrac{1}{9} \Rightarrow y = \dfrac{1}{2}$

The numbers are $\dfrac{1}{2}$ and $\dfrac{2}{9}$.

51. Let x = the length of one leg. Let y = the length of the other leg. Then

$$\begin{cases} x^2 + y^2 = 17^2 \\ (x + 1)^2 + (y + 4)^2 = 20^2 \end{cases} \Rightarrow$$

$$\begin{cases} y^2 = 289 - x^2 \\ x^2 + 2x + y^2 + 8y - 383 = 0 \end{cases} \Rightarrow$$

$x^2 + 2x + 289 - x^2 + 8\sqrt{289 - x^2} - 383 = 0 \Rightarrow$

$8\sqrt{289 - x^2} = -2x + 94 \Rightarrow$

$64(289 - x^2) = (-2x + 94)^2 \Rightarrow$

$18,496 - 64x^2 = 4x^2 - 376x + 8836 \Rightarrow$

$68x^2 - 376x - 9660 = 0 \Rightarrow$

$4(x - 15)(17x + 161) = 0 \Rightarrow x = 15$ or $x = -\dfrac{161}{7}$

(Reject the negative solution.)

$15^2 + y^2 = 17^2 \Rightarrow y = 8$

The legs of the triangle are 15 and 8.

52. Let x = the rate Chris travels. Let y = the time Chris travels. Then

$$\begin{cases} xy = 21 \\ (x + 1)(y - 0.5) = 21 \end{cases} \Rightarrow \begin{cases} y = \dfrac{21}{x} \\ (x + 1)(y - 0.5) = 21 \end{cases} \Rightarrow$$

$(x + 1)\left(\dfrac{21}{x} - 0.5\right) = 21 \Rightarrow$

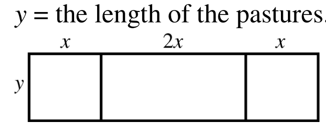
$-0.5x - 0.5 + \dfrac{21}{x} = 0 \Rightarrow -5x^2 - 5x + 210 = 0 \Rightarrow$

$-5(x - 6)(x + 7) = 0 \Rightarrow x = 6$ or $x = -7$

(Reject the negative solution.)

$6y = 21 \Rightarrow y = 3.5$ hours. Chris starts 3.5 hours before 7:30 A.M. or 4:00 A.M.

53. Let x = the width of the smaller pastures. Let y = the length of the pastures.



Then,

$$\begin{cases} 4xy = 6400 \\ 4x + 2y = 240 \end{cases} \Rightarrow \begin{cases} y = \dfrac{1600}{x} \\ 4x + 2y = 240 \end{cases} \Rightarrow$$

$4x + 2\left(\dfrac{1600}{x}\right) = 240 \Rightarrow 4x - 240 + \dfrac{3200}{x} = 0 \Rightarrow$

$4x^2 - 240x + 3200 = 0 \Rightarrow$

$4(x - 40)(x - 20) = 0 \Rightarrow x = 40$ or $x = 20$

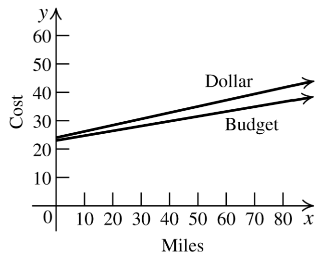
$4(40)y = 6400 \Rightarrow y = 40$

$4(20)y = 6400 \Rightarrow y = 80$

The dimensions of the pasture are 40 meters by 160 meters.

54. a. Let x = the number of miles. For Budget Rentals, the cost function is $C(x) = 23 + 0.17x$. For Dollar Rentals, the cost function is $C(x) = 24 + 0.22x$.

b.



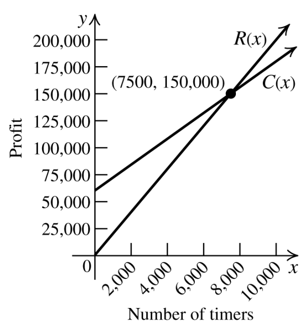
c.

Budget Rentals $C(x) = 23 + 0.17x$	Dollar Rentals $C(x) = 24 + 0.22x$
$C(50) = 23 + 0.17(50) = 31.50$	$C(50) = 24 + 0.22(50) = 35$
$C(60) = 23 + 0.17(60) = 33.20$	$C(60) = 24 + 0.22(60) = 37.20$
$C(70) = 23 + 0.17(70) = 34.90$	$C(70) = 24 + 0.22(70) = 39.40$

Budget Rentals is less expensive for all three scenarios.

55. a. $C(x) = 12x + 60{,}000;\ R(x) = 20x$

b.



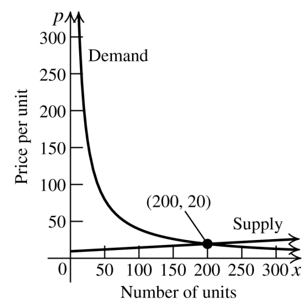
c. $C(x) = R(x) \Rightarrow 12x + 60{,}000 = 20x \Rightarrow$
$x = 7500$
$R(12) = 20(7500) = 150{,}000$
The breakeven point is (7500, 150,000).

d. $P(x) = R(x) - C(x) = 0.15C(x) \Rightarrow$
$20x - (12x + 60{,}000)$
$= 0.15(12x + 60{,}000) \Rightarrow$
$8x - 60{,}000 = 1.8x + 9000 \Rightarrow$
$6.2x = 69{,}000 \Rightarrow x \approx 11{,}129$
11,129 timers should be sold.

56. a. $\dfrac{4000}{x} = \dfrac{x}{20} + 10 \Rightarrow 80{,}000 = x^2 + 200x \Rightarrow$
$x^2 + 200x - 80{,}000 = 0 \Rightarrow$
$(x - 200)(x + 400) = 0 \Rightarrow x = 200$ or
$x = -400$ (reject the negative solution.)
The equilibrium quantity is 200 units.

b. $p = \dfrac{4000}{200} = 20$
The equilibrium price is \$20 per unit.

c.



57. Let x = the double occupancy rate. Let y = the single occupancy rate. Each pays half of the double occupancy rate, so Alisha paid
$6\left(\dfrac{x}{2}\right) + 3(x - y) = 6x - 3y$ for the nine
months, and Sunita paid $6\left(\dfrac{x}{2}\right) + 3y = 3x + 3y$
for the nine months.

$$\begin{cases} 6x - 3y = 2250 \\ 3x + 3y = 3150 \end{cases} \Rightarrow 9x = 5400 \Rightarrow x = 600$$

$3(600) + 3y = 3150 \Rightarrow y = 450$
The double occupancy rate is \$600 per month, and the single occupancy rate is \$450 per month.

58. Let x = the number of chairs in each row. Let y = the number of rows. Then

$$\begin{cases} xy = 600 \\ (x+5)(y-4) = 600 \end{cases} \Rightarrow$$

$$\begin{cases} y = \dfrac{600}{x} \\ (x+5)(y-4) = 600 \end{cases} \Rightarrow$$

$$(x+5)\left(\frac{600}{x} - 4\right) = 600$$

$$-4x - 20 + \frac{3000}{x} = 0$$

$-4x^2 - 20x + 3000 = 0$
$-4(x - 25)(x + 30) = 0 \Rightarrow x = 25$ or $x = -30$
(Reject the negative solution). $25y = 600 \Rightarrow$
$y = 24$. There are 25 chairs in 24 rows.

59. Let x = the number of students who passed the exam. Let y = the number of students who failed the exam. The total number of points scored for the class is 26 × 72 = 1872. Then

$$\begin{cases} x + y = 26 \\ 78x + 26y = 1872 \end{cases} \Rightarrow \begin{cases} -26x - 26y = -676 \\ 78x + 26y = 1872 \end{cases} \Rightarrow$$

$52x = 1196 \Rightarrow x = 23, y = 3$

Three students failed the exam.

60. $$\begin{cases} \dfrac{a+b}{2} = 2.5 \\ \dfrac{b+c}{2} = 3.8 \\ \dfrac{a+c}{2} = 3.1 \end{cases} \Rightarrow \begin{cases} a + b = 5 \\ b + c = 7.6 \\ a + c = 6.2 \end{cases}$$

Subtract the third equation from the first and replace the third equation with the new equation: $\begin{cases} a + b = 5 \\ b + c = 7.6 \\ a + c = 6.2 \end{cases} \Rightarrow \begin{cases} a + b = 5 \\ b + c = 7.6 \\ b - c = -1.2 \end{cases}$

Subtract the third equation from the second equation, replace the third equation with the new equation, and solve for c:

$$\begin{cases} a + b = 5 \\ b + c = 7.6 \\ b - c = -1.2 \end{cases} \Rightarrow \begin{cases} a + b = 5 \\ b + c = 7.6 \\ 2c = 8.8 \end{cases} \Rightarrow$$

$$\begin{cases} a + b = 5 \\ b + c = 7.6 \\ c = 4.4 \end{cases}$$

Substitute $c = 4.4$ into the second equation, solve for b, then substitute the value for b into the first equation to solve for a:

$$\begin{cases} a + b = 5 \\ b + c = 7.6 \\ c = 4.4 \end{cases} \Rightarrow \begin{cases} a + b = 5 \\ b + 4.4 = 7.6 \\ c = 4.4 \end{cases} \Rightarrow$$

$$\begin{cases} a + b = 5 \\ b = 3.2 \\ c = 4.4 \end{cases} \Rightarrow \begin{cases} a + 3.2 = 5 \\ b = 3.2 \\ c = 4.4 \end{cases} \Rightarrow \begin{cases} a = 1.8 \\ b = 3.2 \\ c = 4.4 \end{cases}$$

61. Let x = Steve's age now. Let y = Janet's age now. Then $x - y =$ the difference in their ages.

$$\begin{cases} y = 2(y - (x - y)) \\ x + ((x - y) + x) = 119 \end{cases} \Rightarrow \begin{cases} y = 4y - 2x \\ 3x - y = 119 \end{cases} \Rightarrow$$

$$\begin{cases} -2x + 3y = 0 \\ 3x - y = 119 \end{cases} \Rightarrow \begin{cases} -2x + 3y = 0 \\ 9x - 3y = 357 \end{cases} \Rightarrow$$

$7x = 357 \Rightarrow x = 51$

$51 + (51 - y) + 51 = 119 \Rightarrow y = 34$

So, Steve is 51 years old now and Janet is 34 years old now.

62. $$\begin{cases} x\left(1 + \dfrac{y}{100}\right) = 46 \\ y\left(1 + \dfrac{x}{100}\right) = 21 \end{cases} \Rightarrow \begin{cases} 100x + xy = 4600 \\ 100y + xy = 2100 \end{cases} \Rightarrow$$

$$\begin{cases} 100x + xy = 4600 \\ y = \dfrac{2100}{x + 100} \end{cases} \Rightarrow$$

$$100x + x\left(\frac{2100}{x + 100}\right) = 4600 \Rightarrow$$

$100x^2 + 7500x - 460,000 = 0 \Rightarrow$

$100(x - 40)(x + 115) = 0 \Rightarrow x = 40$ or $x = -115$

Reject the negative solution.

$y\left(1 + \dfrac{40}{100}\right) = 21 \Rightarrow y = 15$.

The solution is {(40, 15)}.

63. Let x = Butch's amount. Let y = Sundance's amount. Let z = Billy's amount. Then

$$\begin{cases} x = 0.75(y + z) \\ y = 0.5(x + z) + 500 \\ z = 3(x - y) - 1000 \end{cases} \Rightarrow$$

$$\begin{cases} x - 0.75y - 0.75z = 0 \\ -0.5x + y - 0.5z = 500 \\ -3x + 3y + z = -1000 \end{cases}$$

Multiply the first equation by 0.5, add the result to the second equation, and replace the second equation with the new equation. Then, multiply the first equation by 3, add the result to the third equation, and replace the third equation with the new equation.

$$\begin{cases} x - 0.75y - 0.75z = 0 \\ -0.5x + y - 0.5z = 500 \\ -3x + 3y + z = -1000 \end{cases} \Rightarrow$$

$$\begin{cases} x - 0.75y - 0.75z = 0 \\ 0.625y - 0.875z = 500 \\ 0.75y - 1.25z = -1000 \end{cases}$$

Multiply the second equation by −1.2, add the result to the third equation, replace the third equation with the new equation, then solve for z:

$$\begin{cases} x - 0.75y - 0.75z = 0 \\ 0.625y - 0.875z = 500 \\ 0.75y - 1.25z = -1000 \end{cases} \Rightarrow$$

$$\begin{cases} x - 0.75y - 0.75z = 0 \\ 0.625y - 0.875z = 500 \\ -0.2z = -1600 \end{cases} \Rightarrow$$

$$\begin{cases} x - 0.75y - 0.75z = 0 \\ 0.625y - 0.875z = 500 \\ z = 8000 \end{cases}$$

(*continued on next page*)

(*continued*)

Substitute $z = 8000$ into the second equation to solve for y, then substitute the values for y and z into the first equation to solve for x:

$$\begin{cases} x-0.75y-0.75z=0 \\ 0.625y-0.875z=500 \\ z=8000 \end{cases} \Rightarrow \begin{cases} x-0.75y-0.75z=0 \\ 0.625y-0.875(8000)=500 \\ z=8000 \end{cases} \Rightarrow \begin{cases} x-0.75y-0.75z=0 \\ y=12,000 \\ z=8000 \end{cases} \Rightarrow$$

$$\begin{cases} x-0.75(12,000)-0.75(8000)=0 \\ y=12,000 \\ z=8000 \end{cases} \Rightarrow \begin{cases} x=15,000 \\ y=12,000 \\ z=8000 \end{cases}$$

So, they stole $35,000. Butch received $15,000, Sundance received $12,000, and Billy received $8000.

64. $$\begin{cases} a(0)^2+b(0)+c=1 \\ a(1)^2+b(1)+c=0 \\ a(-1)^2+b(-1)+c=6 \end{cases} \Rightarrow \begin{cases} c=1 \\ a+b+c=0 \\ a-b+c=6 \end{cases}$$

Subtract the second equation from the third equation, and replace the second equation with the new equation. Solve for b, then substitute the values for b and c into the third equation to solve for a: $\begin{cases} c=1 \\ a+b+c=0 \\ a-b+c=6 \end{cases} \Rightarrow \begin{cases} c=1 \\ -2b=6 \\ a-b+c=6 \end{cases} \Rightarrow$

$$\begin{cases} c=1 \\ b=-3 \\ a-b+c=6 \end{cases} \Rightarrow \begin{cases} c=1 \\ b=-3 \\ a+3+1=6 \end{cases} \Rightarrow \begin{cases} c=1 \\ b=-3 \\ a=2 \end{cases}$$

The equation is $y=2x^2-3x+1$.

65. $2^x=8^y \Rightarrow 2^x=2^{3y} \Rightarrow x=3y$

$9^y=3^{x-2} \Rightarrow 3^{2y}=3^{x-2} \Rightarrow 2y=x-2$

$$\begin{cases} x=3y \\ 2y=x-2 \end{cases} \Rightarrow 2y=3y-2 \Rightarrow y=2, x=6$$

66. Using the Pythagorean theorem, we have $2x^2=y^2$. The length of the side of the square is $2x+y=8$. So

$$\begin{cases} 2x+y=8 \\ 2x^2=y^2 \end{cases} \Rightarrow \begin{cases} y=-2x+8 \\ 2x^2=y^2 \end{cases} \Rightarrow$$

$2x^2=(-2x+8)^2 \Rightarrow -2x^2+32x-64=0 \Rightarrow$

$x^2-16x+32=0 \Rightarrow x=\dfrac{16\pm\sqrt{16^2-4(32)}}{2} \Rightarrow$

$x=8\pm4\sqrt{2}$

$2\left(8+4\sqrt{2}\right)+y=8 \Rightarrow y=-8-8\sqrt{2}$

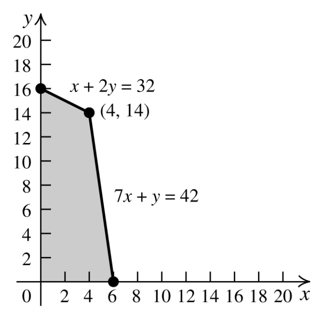
Reject this solution because length cannot be negative. $2\left(8-4\sqrt{2}\right)+y=8 \Rightarrow y=-8+8\sqrt{2}$.

The area of the octagon = the area of the square − 4 × the area of a corner triangle.

$$A_o=8^2-4\left(\frac{1}{2}\left(8-4\sqrt{2}\right)^2\right) = 128\sqrt{2}-128 \approx 53.02 \text{ square units}$$

67. Let x = the number of two-story houses. Let y = the number of one-story houses. The profit $p=10,000x+4000y$. The two-story houses use $7x$ units of material, while the one-story houses use y units of material. The two-story houses use x units of labor, while the one-story houses use $2y$ units of labor. So, the constraints are $x \ge 0, y \ge 0, 7x+y \le 42$, and $x+2y \le 32$.

Solve the systems $\begin{cases} x=0 \\ x+2y=32 \end{cases}$, $\begin{cases} x+2y=32 \\ 7x+y=42 \end{cases}$, and $\begin{cases} 7x+y=42 \\ y=0 \end{cases}$ to find the vertices: (0, 16), (4, 14), and (6, 0).

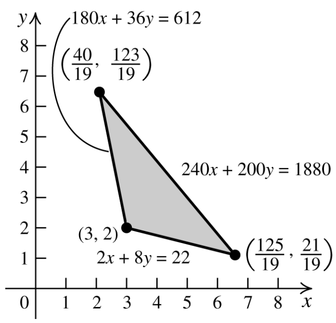


Now find the values of the profit function at each vertex:

Ordered pair	$p=10,000x+4000y.$
(0, 16)	64,000
(4, 14)	96,000
(6, 0)	60,000

The builder should build 4 two-story houses and 14 one-story houses for a maximum profit of $96,000.

68. Let x = the number of pounds of product X. Let y = the number of pounds of product Y. The cost $c = 0.75x + 0.56y$. The number of protein grams is $180x + 36y$. The number of fat grams is $2x + 8y$. The number of carbohydrate grams is $240x + 200y$. The constraints are $x \ge 0$, $y \ge 0$, $180x + 36y \ge 612$, $2x + 8y \ge 22$, and $240x + 200y \le 1880$.



Solve the systems $\begin{cases} 180x + 36y = 612 \\ 240x + 200y = 1880 \end{cases}$, $\begin{cases} 240x + 200y = 1880 \\ 2x + 8y = 22 \end{cases}$, $\begin{cases} 2x + 8y = 22 \\ 240x + 200y = 1880 \end{cases}$ to find the vertices $\left(\frac{40}{19}, \frac{123}{19}\right)$, $\left(\frac{125}{19}, \frac{21}{19}\right)$, and (3,2). Now find the values of the cost function at each vertex:

Ordered pair	$c = 0.75x + 0.56y.$
$\left(\frac{40}{19}, \frac{123}{19}\right)$	≈\$5.20
$\left(\frac{125}{19}, \frac{21}{19}\right)$	≈\$5.55
(3, 2)	\$3.37

The cost is at a minimum if 3 pounds of product X and 2 pounds of product Y are used.

69. Let x = the cost of the small lobster, y = the cost of the medium lobster, and z = the cost of the large lobster. Then we have

$$\begin{cases} 4x + 2y + z = 344 & (1) \\ 3x + 2z = 255 & (2) \\ 5x + 2y + 2z = 449 & (3) \end{cases}$$

Interchange equations (2) and (3).

$$\begin{cases} 4x + 2y + z = 344 & (1) \\ 5x + 2y + 2z = 449 & (3) \\ 3x + 2z = 255 & (2) \end{cases}$$

Multiply equation (1) by −1, add the result to equation (3), then replace equation (1) with the result (4).

$$\begin{cases} 4x + 2y + z = 344 \\ 5x + 2y + 2z = 449 \Rightarrow \\ 3x + 2z = 255 \end{cases}$$

$$\begin{cases} -4x - 2y - z = -344 \\ 5x + 2y + 2z = 449 \Rightarrow \\ 3x + 2z = 255 \end{cases}$$

$$\begin{cases} x + z = 105 & (4) \\ 5x + 2y + 2z = 449 & (3) \\ 3x + 2z = 255 & (2) \end{cases}$$

Multiply equation (4) by −3, add the result to equation (2), then replace equation (2) with the result (5).

$$\begin{cases} x + z = 105 & (4) \\ 5x + 2y + 2z = 499 & (3) \Rightarrow \\ 3x + 2z = 255 & (2) \end{cases}$$

$$\begin{cases} -3x - 3z = -315 \\ 5x + 2y + 2z = 449 \Rightarrow \\ 3x + 2z = 255 \end{cases}$$

$$\begin{cases} x + z = 105 & (4) \\ 5x + 2y + 2z = 449 & (3) \\ -z = -60 \Rightarrow z = 60 & (5) \end{cases}$$

Substitute $z = 60$ in equation (4) and solve for x: $x + 60 = 105 \Rightarrow x = 45$

Now substitute $x = 45$ and $z = 60$ into equation (3) and solve for y:

$5(45) + 2y + 2(60) = 449 \Rightarrow 2y = 104 \Rightarrow$
$y = 52$

A small lobster costs \$45, a medium lobster costs \$52, and a large lobster costs \$60.

70. Let x = the price received for a good crop, y = the price received for a mediocre crop, and z = the price received for a bad crop. Then

$$\begin{cases} 3x + 2y + z = 39 & (1) \\ 2x + 3y + z = 34 & (2) \\ x + 2y + 3z = 26 & (3) \end{cases}$$

Interchange equations (1) and (3).

$$\begin{cases} x + 2y + 3z = 26 & (3) \\ 2x + 3y + z = 34 & (2) \\ 3x + 2y + z = 39 & (1) \end{cases}$$

Multiply equation (3) by −2, add the result to equation (2), and replace equation (2) with the result (4).

$$\begin{cases} -2x - 4y - 6z = -52 \\ 2x + 3y + z = 34 \Rightarrow \\ 3x + 2y + z = 39 \end{cases}$$

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$$\begin{cases} x+2y+3z=26 & (3) \\ -y-5z=-18 & (4) \\ 3x+2y+z=39 & (1) \end{cases}$$

Multiply equation (3) by −3, add the result to equation (1), and replace equation (1) with the result (5).

$$\begin{cases} -3x-6y-9z=-78 \\ -y-5z=-18 \Rightarrow \\ 3x+2y+z=39 \end{cases}$$

$$\begin{cases} x+2y+3z=26 & (3) \\ -y-5z=-18 & (4) \\ -4y-8z=-39 & (5) \end{cases}$$

Multiply equation (4) by −4, add the result to equation (5), replace equation (5) with the result (6), and solve for z.

$$\begin{cases} x+2y+3z=26 \\ 4y+20z=72 \quad \Rightarrow \\ -4y-8z=-39 \end{cases}$$

$$\begin{cases} x+2y+3z=26 & (3) \\ -y-5z=-18 & (4) \\ 12z=33 \Rightarrow z=2.75 & (6) \end{cases}$$

Substitute $z = 2.75$ into equation (4) and solve for y. $-y-5(2.75)=-18 \Rightarrow y=4.25$

Substitute $y = 4.25$ and $z = 2.75$ into equation (3) and solve for x.

$x+2(4.25)+3(2.75)=26 \Rightarrow x=9.25$

A good crop earns 9.25 dou, a mediocre crop earns 4.25 dou, and a bad crop earns 2.75 dou.

Chapter 8 Practice Test A

1. Using elimination, we have

$$\begin{cases} 2x-y=4 \\ 2x+y=4 \end{cases} \Rightarrow 4x=8 \Rightarrow x=2$$

$2(2)+y=4 \Rightarrow y=0$

The solution is $\{(2, 0)\}$.

2. Using elimination, we have

$$\begin{cases} x+2y=8 \\ 3x+6y=24 \end{cases} \Rightarrow \begin{cases} 3x+6y=24 \\ 3x+6y=24 \end{cases} \Rightarrow \text{ the system}$$

is consistent. The solution is $\left\{\left(x,\ 4-\dfrac{x}{2}\right)\right\}$.

3. Using substitution, we have

$$\begin{cases} -2x+y=4 \\ 4x-2y=4 \end{cases} \Rightarrow \begin{cases} y=2x+4 \\ 4x-2y=4 \end{cases} \Rightarrow$$

$4x-2(2x+4)=4 \Rightarrow -8=4 \Rightarrow$ there is no solution.

Solution set: $\varnothing$

4. $$\begin{cases} 3x+3y=-15 \\ 2x-2y=-10 \end{cases} \Rightarrow \begin{cases} x+y=-5 \\ x-y=-5 \end{cases} \Rightarrow 2x=-10 \Rightarrow$$

$x=-5$

$3(-5)+3y=-15 \Rightarrow y=0$

The solution is $\{(-5, 0)\}$.

5. Using elimination, we have

$$\begin{cases} \dfrac{5}{3}x+\dfrac{y}{2}=14 \\ \dfrac{2}{3}x-\dfrac{y}{8}=3 \end{cases} \Rightarrow \begin{cases} 10x+3y=84 \\ 16x-3y=72 \end{cases} \Rightarrow$$

$26x=156 \Rightarrow x=6$

$\dfrac{5}{3}(6)+\dfrac{y}{2}=14 \Rightarrow y=8$

The solution is $\{(6, 8)\}$.

6. Using substitution, we have

$$\begin{cases} y=x^2 \\ 3x-y+4=0 \end{cases} \Rightarrow 3x-x^2+4=0 \Rightarrow$$

$x^2-3x-4=0 \Rightarrow (x-4)(x+1)=0 \Rightarrow$

$x=4$ or $x=-1$

$y=4^2=16$

$y=(-1)^2=1$

The solution is $\{(4, 16), (-1, 1)\}$.

7. Using elimination, we have

$$\begin{cases} x-3y=-4 \\ 2x^2+3x-3y=8 \end{cases} \Rightarrow -2x^2-2x=-12 \Rightarrow$$

$x^2+x-6=0 \Rightarrow (x+3)(x-2)=0 \Rightarrow$

$x=-3$ or $x=2$

$-3-3y=-4 \Rightarrow y=\dfrac{1}{3}$

$2-3y=-4 \Rightarrow y=2$

The solution is $\left\{\left(-3, \dfrac{1}{3}\right), (2, 2)\right\}$.

8. a. Let x = the weight of one bar. Let y = the weight of the other bar. Then

$$\begin{cases} x+y=485 \\ x-y=15 \end{cases}$$

b. $$\begin{cases} x+y=485 \\ x-y=15 \end{cases} \Rightarrow 2x=500 \Rightarrow x=250$$

$250+y=485 \Rightarrow y=235$

One bar weight 250 pounds and the other weighs 235 pounds.

9. Multiply the first equation by −2, add the result to the second equation, and replace the second equation with the new equation. Then divide the second equation by 4.

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$$\begin{cases}2x+\ \ y+2z=4\\4x+6y+\ z\ =15\\2x+2y+7z=-1\end{cases}\Rightarrow\begin{cases}2x+\ \ y+2z=4\\ \quad y-\frac{3}{4}z=\frac{7}{4}\\2x+2y+7z=-1\end{cases}$$

Subtract the third equation from the first equation and replace the third equation with the new equation:

$$\begin{cases}2x+\ \ y+2z=4\\ \quad y-\frac{3}{4}z=\frac{7}{4}\\2x+2y+7z=-1\end{cases}\Rightarrow\begin{cases}2x+\ \ y+2z=4\\ \quad y-\frac{3}{4}z=\frac{7}{4}\\ \quad -y-5z=5\end{cases}$$

Add the second and third equations, replace the third equation with the new equation

$$\begin{cases}2x+\ \ y+2z=4\\ \quad y-\frac{3}{4}z=\frac{7}{4}\\ \quad -y-5z=5\end{cases}\Rightarrow\begin{cases}2x+\ \ y+2z=4\\ \quad y-\frac{3}{4}z=\frac{7}{4}\\ \quad -\frac{23}{4}z=\frac{27}{4}\end{cases}\Rightarrow$$

Solve for the third equation for z. Divide the first equation by 2:

$$\begin{cases}x+\frac{1}{2}y+\ \ z=2\\ \quad y-\frac{3}{4}z=\frac{7}{4}\\ \quad z=-\frac{27}{23}\end{cases}$$

10. $$\begin{cases}x-6y+3z=-2\\ \quad 9y-5z=2\\ \quad 2z=10\end{cases}\Rightarrow\begin{cases}x-6y+3z=-2\\ \quad 9y-5z=2\\ \quad z=5\end{cases}$$

$9y-5(5)=2\Rightarrow y=3$

$x-6(3)+3(5)=-2\Rightarrow x=1$

The solution is $\{(1, 3, 5)\}$.

11. Subtract the third equation from the first equation, replace the third equation with the new equation, and solve for z.

$$\begin{cases}x+\ \ y+\ z=8\\2x-2y+2z=4\\x+\ \ y-\ z=12\end{cases}\Rightarrow\begin{cases}x+\ \ y+\ z=8\\2x-2y+2z=4\\ \quad z=-2\end{cases}$$

Multiply the first equation by −2, add the result to the second equation, and replace the second equation with the new equation. Solve for y:

$$\begin{cases}x+\ \ y+\ z=8\\2x-2y+2z=4\\ \quad z=-2\end{cases}\Rightarrow\begin{cases}x+y+z=8\\ \quad y\ \ =3\\ \quad z=-2\end{cases}$$

Substitute the values for y and z into the first equation to solve for x:

$$\begin{cases}x+3-2=8\\ \quad y\ \ =3\\ \quad z=-2\end{cases}\Rightarrow\begin{cases}x=7\\y=3\\z=-2\end{cases}$$

The solution is $\{(7, 3, -2)\}$.

12. Add the second and third equations, and replace the third equation with the result:

$$\begin{cases}x+\quad\ \ z=-1\\ \quad 3y+2z=5\\3x-3y+\ z=-8\end{cases}\Rightarrow\begin{cases}x+\quad\ \ z=-1\\ \quad 3y+2z=5\\3x\quad +3z=-3\end{cases}\Rightarrow$$

$$\begin{cases}x+\quad\ \ z=-1\\ \quad 3y+2z=5\\x\quad +\ z=-1\end{cases}$$

The first and third equations are the same, so we have two equations in three unknowns. Solve the first equation for z, then substitute that expression into the second equation to solve for y:

$z=-x-1;\ 3y+2(-x-1)=5\Rightarrow$

$3y=2x+7\Rightarrow\ y=\frac{2}{3}x+\frac{7}{3}$

The solution is $\left\{\left(x, \frac{2}{3}x+\frac{7}{3}, -x-1\right)\right\}$.

13. Add the first and second equations, replace the first equation with the solution, then solve for x:

$$\begin{cases}2x-\ \ y+z=2\\x+\ y-z=-1\\x-5y+5z=7\end{cases}\Rightarrow\begin{cases}x\quad\quad =\frac{1}{3}\\x+\ y-z=-1\\x-5y+5z=7\end{cases}$$

Substitute $x=1/3$ into the second and third equations, then solve the system

$$\begin{cases}\frac{1}{3}+\ \ y-\ \ z=-1\\ \frac{1}{3}-5y+5z=7\end{cases}\Rightarrow\begin{cases}5y-5z=-\frac{20}{3}\\-5y+5z=\frac{20}{3}\end{cases}\Rightarrow 0=0$$

So, solve the original second equation for y:

$\frac{1}{3}+y-z=-1\Rightarrow z=z-\frac{4}{3}$.

The solution is $\left\{\left(\frac{1}{3}, -\frac{4}{3}+z, z\right)\right\}$.

14. Let n = the number of nickels. Let d = the number of dimes. Let q = the number of quarters. Then

$$\begin{cases}n+d+q=300\\d=3(n+q)\\0.05n+0.1d+0.25q=30.65\end{cases}\Rightarrow$$

$$\begin{cases}n+\ \ d+\ \ q=300\\-3n+\ \ d-\ 3q=0\\5n+10d+25q=3065\end{cases}$$

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Multiply the first equation by 3, add the result to the second equation, and replace the second equation with the new equation

$$\begin{cases} n + d + q = 300 \\ -3n + d - 3q = 0 \\ 5n + 10d + 25q = 3065 \end{cases} \Rightarrow$$

$$\begin{cases} n + d + q = 300 \\ 4d = 900 \\ 5n + 10d + 25q = 3065 \end{cases} \Rightarrow$$

$$\begin{cases} n + d + q = 300 \\ d = 225 \\ 5n + 10d + 25q = 3065 \end{cases}$$

Multiply the first equation by −5, add the result to the third equation, and replace the third equation with the new equation:

$$\begin{cases} n + d + q = 300 \\ d = 225 \\ 5n + 10d + 25q = 3065 \end{cases} \Rightarrow$$

$$\begin{cases} n + d + q = 300 \\ d = 225 \\ 5d + 20q = 1565 \end{cases}$$

Substitute $d = 225$ into the third equation, solve for q, then substitute the values for d and q into the first equation to solve for n:

$5(225) + 20q = 1565 \Rightarrow q = 22$

$n + 225 + 22 = 300 \Rightarrow n = 53$

There are 53 nickels, 225 dimes, and 22 quarters.

15. $\dfrac{2x}{(x-5)(x+1)} = \dfrac{A}{x-5} + \dfrac{B}{x+1}$

16. $\dfrac{-5x^2 + x - 8}{(x-2)(x^2+1)^2} = \dfrac{A}{x-2} + \dfrac{Bx + C}{x^2+1} + \dfrac{Dx + E}{(x^2+1)^2}$

17. $\dfrac{x+3}{(x+4)^2(x-7)} = \dfrac{A}{x-7} + \dfrac{B}{x+4} + \dfrac{C}{(x+4)^2} \Rightarrow$

$x + 3 = A(x+4)^2 + B(x-7)(x+4) + C(x-7)$

Letting $x = 7$, we have $10 = 121A \Rightarrow A = \dfrac{10}{121}$.

Letting $x = -4$, we have $-1 = -11C \Rightarrow \dfrac{1}{11} = C$.

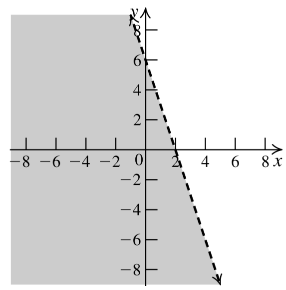
Substitute the values for A and C into the equation and expand.

$$x + 3 = \left(B + \frac{10}{121}\right)x^2 + \left(\frac{91}{121} - 3B\right)x + \left(\frac{83}{121} - 28B\right) \Rightarrow$$

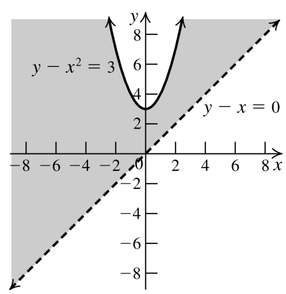
$B + \dfrac{10}{121} = 0 \Rightarrow B = -\dfrac{10}{121}$

$$\frac{x+3}{(x+4)^2(x-7)} = \frac{10}{121(x-7)} - \frac{10}{121(x+4)} + \frac{1}{11(x+4)^2}$$

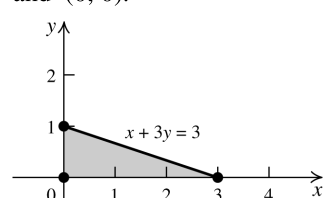
18.



19.



20. Solve the systems $\begin{cases} x = 0 \\ x + 3y = 3 \end{cases}$, $\begin{cases} x + 3y = 3 \\ y = 0 \end{cases}$, and $\begin{cases} y = 0 \\ x = 0 \end{cases}$ to find the vertices: (0, 1), (3, 0), and (0, 0).



Now find the values of the objective function at each vertex:

Ordered pair	$z = 2x + y$
(0, 1)	1
(3, 0)	6
(0, 0)	0

The maximum value is 6 at (3, 0).

Chapter 8 Practice Test B

1. Substitute each of the ordered pairs into the system.

$\begin{cases} 2(5)-3(-2)=16 \\ 5-(-2)=7 \end{cases}$ The answer is A.

2. Substitute each of the ordered pairs into the system.

$$\begin{cases} 6\left(\frac{44}{3}\right)-9(10)=-2 \\ 3\left(\frac{44}{3}\right)-5(10)=-6 \end{cases}$$

The answer is C.

3. Substitute each of the ordered pairs into the system.

$$\begin{cases} 2\left(\frac{5}{2}y+\frac{9}{2}\right)-5y=9 \\ 4\left(\frac{5}{2}y+\frac{9}{2}\right)-10y=18 \end{cases}$$

The answer is B.

4. Substituting each of the ordered pairs into the system, we find that none are solutions. The answer is C.

5. Substitute each of the ordered pairs into the system.

$$\begin{cases} \frac{1}{5}(1)+\frac{2}{5}(2)=1 \\ \frac{1}{4}(1)-\frac{1}{3}(2)=-\frac{5}{12} \end{cases}$$

The answer is B.

6. Substitute each of the ordered pairs into the system.

$$\begin{cases} \left(3y+\frac{1}{2}\right)-3y=\frac{1}{2} \\ -2\left(3y+\frac{1}{2}\right)+6y=-1 \end{cases}$$

The answer is D.

7. Substitute each of the ordered pairs into the system.

$$\begin{cases} 3(4)+4(0)=12 \\ 3(4)^2+16(0)^2=48 \end{cases} \quad \begin{cases} 3(2)+4\left(\frac{3}{2}\right)=12 \\ 3(2)^2+16\left(\frac{3}{2}\right)^2=48 \end{cases}$$

The answer is D.

8. Let x = the number of student tickets and y = the number of nonstudent tickets. Then

$$\begin{cases} x+y=300 \\ 2x+5y=975 \end{cases} \Rightarrow \begin{cases} -2x-2y=-600 \\ 2x+5y=975 \end{cases} \Rightarrow$$

$3y=375 \Rightarrow y=125$

$x+125=300 \Rightarrow x=175$

The answer is D.

9. Multiply the first equation by 2, subtract the second equation from that result, then replace the second equation with the new equation:

$$\begin{cases} x+3y+3z=4 \\ 2x+5y+4x=5 \\ x+2y+2z=6 \end{cases} \Rightarrow \begin{cases} x+3y+3z=4 \\ y+2z=3 \\ x+2y+2z=6 \end{cases}$$

Subtract the third equation from the first equation, then replace the third equation with the new equation:

$$\begin{cases} x+3y+3z=4 \\ y+2z=3 \\ x+2y+2z=6 \end{cases} \Rightarrow \begin{cases} x+3y+3z=4 \\ y+2z=3 \\ y+z=-2 \end{cases}$$

Subtract the third equation from the second equation and replace the third equation with the new equation:

$$\begin{cases} x+3y+3z=4 \\ y+2z=3 \\ y+z=-2 \end{cases} \Rightarrow \begin{cases} x+3y+3z=4 \\ y+2z=3 \\ z=5 \end{cases}$$

The answer is C.

10. From the third equation, $z=3$.

$10y-2(3)=4 \Rightarrow y=1; 2x+1+3=0 \Rightarrow$
$x=-2$

The answer is A.

11. Substitute each of the ordered pairs into the system.

$$\begin{cases} 2(1)+13(-1)+6(2)=1 \\ 3(1)+10(-1)+11(2)=15 \\ 2(1)+10(-1)+8(2)=8 \end{cases}$$

The answer is B.

12. Substituting each of the ordered pairs into the system, we find that none are solutions. The answer is D.

13. Substitute each of the ordered pairs into the system.

$$\begin{cases} x-2(5x+1)+3(3x+2)=4 \\ 2x-(5x+1)+(3x+2)=1 \\ x+(5x+1)-2(3x+2)=-3 \end{cases}$$

The answer is C.

14. Let x = the number of students going to France. Let y = the number of students going to Italy. Let z = the number of students going to Spain. Then

$$\begin{cases} x+y+z=46 \\ x+y=4+z \\ x=z-2 \end{cases} \Rightarrow \begin{cases} x+y+z=46 \\ x+y-z=4 \\ x-z=-2 \end{cases}$$ Subtract the second equation from the first equation, replace the second equation with the new equation, and solve for z:

$$\begin{cases} x+y+z=46 \\ x+y-z=4 \\ x-\quad z=-2 \end{cases} \Rightarrow \begin{cases} x+y+z=46 \\ 2z=42 \\ x-\quad z=-2 \end{cases} \Rightarrow$$

$$\begin{cases} x+y+z=46 \\ z=21 \\ x-\quad z=-2 \end{cases}$$

Substitute $z = 21$ into the third equation to solve for x. Then substitute the values for x and z into the first equation to solve for y:

$x-21=-2 \Rightarrow x=19;\ 19+y+21=46 \Rightarrow y=6$

The answer is B.

15. C **16.** A

17. $$\frac{x^2+15x+18}{x^3-9x} = \frac{x^2+15x+18}{x(x-3)(x+3)} = \frac{A}{x}+\frac{B}{x-3}+\frac{C}{x+3} \Rightarrow$$

$$x^2+15x+18 = A(x^2-9)+Bx(x+3)+Cx(x-3)$$

Letting $x = -3$, we have

$(-3)^2+15(-3)+18 = -3C(-3-3) \Rightarrow$
$-18 = 18C \Rightarrow -1 = C$

Letting $x = 3$, we have

$(3)^2+15(3)+18 = 3B(3+3) \Rightarrow$
$72 = 18B \Rightarrow 4 = B$

Substituting the values for B and C and expanding the equation, we have

$$x^2+15x+18 = A(x^2-9)+4x(x+3)-x(x-3) = (A+3)x^2+15x-9A \Rightarrow$$

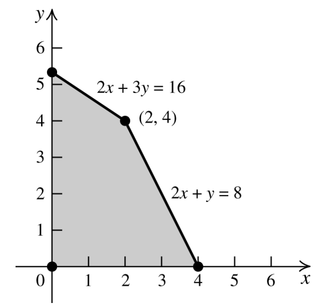
$$\begin{cases} A+3=1 \\ -9A=18 \end{cases} \Rightarrow A=-2$$

$$\frac{x^2+15x+18}{x^3-9x} = -\frac{2}{x}+\frac{4}{x-3}-\frac{1}{x+3}$$

The answer is C.

18. B **19.** A

20. Solve the systems
$\begin{cases} x=0 \\ 2x+3y=16 \end{cases}$, $\begin{cases} 2x+3y=16 \\ 2x+y=8 \end{cases}$, $\begin{cases} 2x+y=8 \\ y=0 \end{cases}$ and $\begin{cases} y=0 \\ x=0 \end{cases}$ to find the vertices: $\left(0, \frac{16}{3}\right)$, $(2, 4)$, $(4, 0)$, and $(0, 0)$.



Now find the values of the objective function at each vertex:

Ordered pair	$z = 3x + 21y$
$\left(0, \frac{16}{3}\right)$	112
(2, 4)	90
(4, 0)	12
(0, 0)	0

The maximum value is 112 at $\left(0, \frac{16}{3}\right)$.

The answer is B.

Cumulative Review Exercises (Chapters P–8)

1. $$\frac{1}{x-1}+\frac{4}{x-4}=\frac{5}{x-5} \Rightarrow$$
$$(x-4)(x-5)+4(x-1)(x-5) = 5(x-1)(x-4) \Rightarrow$$
$$5x^2-33x+40 = 5x^2-25x+20 \Rightarrow$$
$$-8x=-20 \Rightarrow x=\frac{5}{2}$$

2. Let $u = x+\frac{1}{x}$. Then

$2u^2-7u+5=0 \Rightarrow (u-1)(2u-5)=0 \Rightarrow$
$u=1$ or $u=5/2$

$$x+\frac{1}{x}=1 \Rightarrow x^2-x+1=0 \Rightarrow$$
$$x=\frac{1\pm\sqrt{1-4}}{2}=\frac{1\pm i\sqrt{3}}{2}=\frac{1}{2}\pm\frac{i\sqrt{3}}{2}$$

(*continued on next page*)

(*continued*)

$x+\frac{1}{x}=\frac{5}{2}\Rightarrow 2x^2-5x+2=0\Rightarrow$
$(x-2)(2x-1)=0\Rightarrow x=2$ or $x=1/2$
The solution is $\left\{\frac{1}{2}\pm\frac{i\sqrt{3}}{2},2,\frac{1}{2}\right\}$.

3. $\sqrt{3x-5}=x-3\Rightarrow 3x-5=(x-3)^2\Rightarrow$
$x^2-9x+14=0\Rightarrow(x-7)(x-2)=0\Rightarrow$
$x=7$ or $x=2$.
Check each answer to see if either is extraneous:
$\sqrt{3(7)-5}=\sqrt{16}=4=7-3$
$\sqrt{3(2)-5}=\sqrt{1}=1\neq 2-3\Rightarrow 2$ is extraneous.
The solution is $\{7\}$.

4. $\frac{x-1}{x+3}=0\Rightarrow x=1$ is the x-intercept. The vertical asymptote is $x=-3$. The intervals to be tested are $(-\infty,-3)$, $(-3,1]$, and $[1,\infty)$.

Interval	Test Point	Value of $\frac{x-1}{x+3}$	Result
$(-\infty,-3)$	–4	5	+
$(-3,1]$	0	$-\frac{1}{3}$	–
$[1,\infty)$	2	$\frac{1}{5}$	+

The solution is $(-3,1]$.

5. Solve the associated equation to find the test intervals:
$x^2-9x+20=0\Rightarrow(x-4)(x-5)=0\Rightarrow$
$x=4$ or $x=5$.
The intervals to be tested are $(-\infty,4),(4,5)$, and $(5,\infty)$.

Interval	Test Point	Value of $x^2-9x+20$	Result
$(-\infty,4)$	0	20	+
(4, 5)	4.5	–0.25	–
$(5,\infty)$	10	30	+

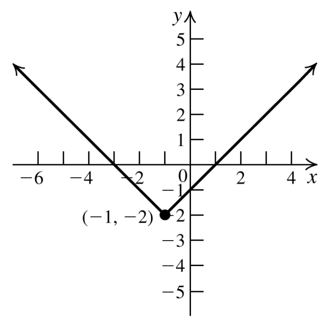
The solution is $(-\infty,4)\cup(5,\infty)$.

6. $2^{x-1}=5\Rightarrow(x-1)\ln 2=\ln 5\Rightarrow$
$x\ln 2-\ln 2=\ln 5\Rightarrow x=\frac{\ln 5+\ln 2}{\ln 2}$

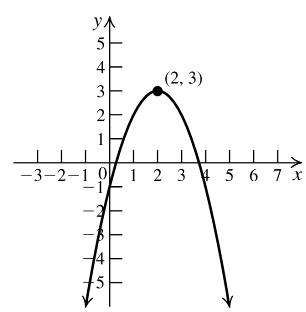
7. $\log_x 16=4\Rightarrow x^4=16\Rightarrow x^4=2^4\Rightarrow x=2$

8. $\log(x-3)+\log(x-1)=\log(2x-5)\Rightarrow$
$\log\left((x-3)(x-1)\right)=\log(2x-5)\Rightarrow$
$x^2-4x+3=2x-5\Rightarrow x^2-6x+8=0\Rightarrow$
$(x-4)(x-2)=0\Rightarrow x=4$ or $x=2$
Reject $x=2$ because it makes $\log(2-3)=\log(-1)$, which is not possible.
The solution is $\{4\}$.

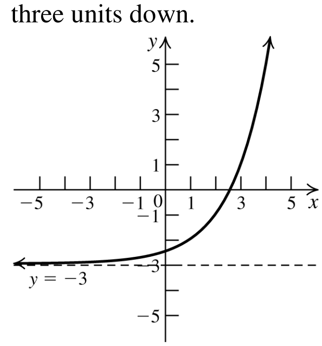
9. Shift the graph of $y=|x|$ one unit left and two units down.



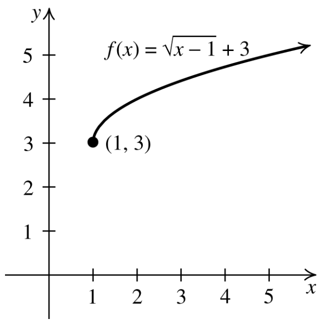
10. Shift the graph of $y=x^2$ two units right, reflect it across the x-axis, then shift it three units up.



11. Shift the graph of $y=2^x$ one unit right and three units down.



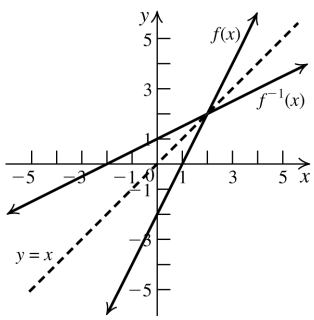
12. Shift the graph of $y = \sqrt{x}$ one unit right and three units up.



13. a. Switch the variables and solve for *y*:

$y = 2x - 2 \Rightarrow x = 2y - 2 \Rightarrow y = f^{-1} = \dfrac{x+2}{2}$

b.



14. a. The factors of the constant term are $\{\pm 1, \pm 2, \pm 3, \pm 6\}$. The factors of the leading coefficient are $\{\pm 1\}$. The possible rational zeros are $\{\pm 1, \pm 2, \pm 3, \pm 6\}$.

b.
$$\begin{array}{r|rrrr} 2 & 1 & -1 & 1 & -6 \\ & & 2 & 2 & 6 \\ \hline & 1 & 1 & 3 & 0 \end{array}$$
A real zero is 2.

c. $x^3 - x^2 + x - 6 = (x-2)(x^2 + x + 3)$

$x^2 + x + 3 = 0 \Rightarrow x = \dfrac{-1 \pm \sqrt{1-12}}{2} \Rightarrow$

$x = \dfrac{-1 \pm i\sqrt{11}}{2}$

The zeros are $\left\{2, -\dfrac{1}{2} \pm \dfrac{i\sqrt{11}}{2}\right\}$.

15. $\log_3(9x^4) = \log_3 9 + \log_3 x^4 = 2 + 4\log_3 x$

16. a. $0.5 = e^{-0.05t} \Rightarrow \ln 0.5 = -0.05t \Rightarrow$
$t = -\dfrac{\ln 0.5}{0.05} \approx 13.86$ years

b. $0.5 = e^{-0.0002t} \Rightarrow \ln 0.5 = -0.0002t \Rightarrow$
$t = -\dfrac{\ln 0.5}{0.0002} \approx 3465.74$ years

17. $f^{-1}(9) = 2$

18. $A = Pe^{rt} \Rightarrow 2P = Pe^{0.075t} \Rightarrow 2 = e^{0.075t} \Rightarrow$
$\ln 2 = 0.075t \Rightarrow t = \dfrac{\ln 2}{0.075} \approx 9.24$
It will take about 9.24 years to double the money.

19. $\begin{cases} 5x - 2y + 25 = 0 \\ 4y - 3x - 29 = 0 \end{cases} \Rightarrow \begin{cases} 10x - 4y = -50 \\ -3x + 4y = 29 \end{cases} \Rightarrow$
$7x = -21 \Rightarrow x = -3$
$4y - 3(-3) - 29 = 0 \Rightarrow y = 5$
The solution is $\{(-3, 5)\}$.

20. Multiply the second equation by −2, add the result to the first equation, and replace the second equation with the new equation:

$\begin{cases} 2x - y + z = 3 \\ x + 3y - 2z = 11 \\ 3x - 2y + z = 4 \end{cases} \Rightarrow \begin{cases} 2x - y + z = 3 \\ -7y + 5z = -19 \\ 3x - 2y + z = 4 \end{cases}$

Multiply the first equation by 3, multiply the third equation by −2, add the two and replace the third equation with the new equation:

$\begin{cases} 2x - y + z = 3 \\ -7y + 5z = -19 \\ 3x - 2y + z = 4 \end{cases} \Rightarrow \begin{cases} 2x - y + z = 3 \\ -7y + 5z = -19 \\ y + z = 1 \end{cases}$

Multiply the third equation by 7, add the result to the second equation, replace the third equation and solve for *z*:

$\begin{cases} 2x - y + z = 3 \\ -7y + 5z = -19 \\ y + z = 1 \end{cases} \Rightarrow \begin{cases} 2x - y + z = 3 \\ -7y + 5z = -19 \\ 12z = -12 \end{cases} \Rightarrow$

$z = -1$
$-7y + 5(-1) = -19 \Rightarrow y = 2$
$2x - 2 - 1 = 3 \Rightarrow x = 3$
The solution is $\{(3, 2, -1)\}$.