# Chapter 6 Trigonometric Identities and Equations

## 6.1 Verifying Identities

### 6.1 Practice Problems

**1.** $\sin\theta = \frac{4}{5} \Rightarrow \csc\theta = \frac{5}{4}$

$1+\cot^2\theta = \left(\frac{5}{4}\right)^2 \Rightarrow \cot\theta = \pm\frac{3}{4}$

$\frac{\pi}{2} < \theta < \pi \Rightarrow \cot\theta = -\frac{3}{4}$

**2.** $\tan x = -\frac{1}{2} \Rightarrow 1+\left(-\frac{1}{2}\right)^2 = \sec^2 x \Rightarrow$

$\sec x = \pm\frac{\sqrt{5}}{2} \Rightarrow \cos x = \pm\frac{2\sqrt{5}}{5}$

$\frac{\pi}{2} < x < \pi \Rightarrow \cos x = -\frac{2\sqrt{5}}{5}$

$\tan x = \frac{\sin x}{\cos x} \Rightarrow -\frac{1}{2} = \frac{\sin x}{-2\sqrt{5}/5} \Rightarrow \frac{\sqrt{5}}{5} = \sin x$

$\csc x = \frac{1}{\sin x} = \sqrt{5}; \quad \sec x = \frac{1}{\cos x} = -\frac{\sqrt{5}}{2}$

$\cot x = \frac{1}{\tan x} \Rightarrow \cot x = -2$

**3.** $\frac{\tan x}{\sec x+1} + \frac{\tan x}{\sec x-1}$

$= \frac{\tan x(\sec x-1)+\tan x(\sec x+1)}{(\sec x+1)(\sec x-1)}$

$= \frac{\frac{\sin x}{\cos x}\left(\frac{1}{\cos x}-1\right)+\frac{\sin x}{\cos x}\left(\frac{1}{\cos x}+1\right)}{\sec^2 x-1}$

$= \frac{\frac{\sin x-\sin x\cos x+\sin x+\sin x\cos x}{\cos^2 x}}{\tan^2 x}$

$= \frac{\frac{2\sin x}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}} = \frac{2\sin x}{\cos^2 x}\left(\frac{\cos^2 x}{\sin^2 x}\right) = \frac{2}{\sin x}$

**4.** $\cos x = 1-\sin x$ is not an identity because for $x = \frac{\pi}{3}, \sin x = \frac{\sqrt{3}}{2}$, while $\cos x = \frac{1}{2} \neq 1-\frac{\sqrt{3}}{2}$.

**5.** $\frac{1-\sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x} = \cos x$

**6.** Start with the right side.

$\frac{\tan\theta}{\sec\theta-\cos\theta} = \frac{\tan\theta}{\frac{1}{\cos\theta}-\cos\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1-\cos^2\theta}{\cos\theta}}$

$= \frac{\sin\theta}{1-\cos^2\theta} = \frac{\sin\theta}{\sin^2\theta}$

$= \frac{1}{\sin\theta} = \frac{\csc\theta}{1}$

**7.** $\tan^4 x+\tan^2 x = \frac{\sin^4 x}{\cos^4 x}+\frac{\sin^2 x}{\cos^2 x}$

$= \frac{\sin^4 x+\sin^2 x\cos^2 x}{\cos^4 x}$

$= \frac{\sin^2 x\left(\sin^2 x+\cos^2 x\right)}{\cos^4 x}$

$= \frac{\sin^2 x(1)}{\cos^4 x} = \frac{\sin^2 x}{\cos^2 x}\cdot\frac{1}{\cos^2 x}$

$= \tan^2 x\sec^2 x$

**8.** $\tan\theta+\sec\theta = \frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta} = \frac{\sin\theta+1}{\cos\theta}$

$\frac{\csc\theta+1}{\cot\theta} = \frac{\frac{1}{\sin\theta}+1}{\frac{\cos\theta}{\sin\theta}} = \frac{\frac{1+\sin\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}} = \frac{1+\sin\theta}{\cos\theta}$

Since both sides of the original identity are equal to $\frac{1+\sin\theta}{\cos\theta}$, the identity is verified.

**9.** $\frac{\tan x}{\sec x+1} = \frac{\tan x}{\sec x+1}\cdot\frac{\sec x-1}{\sec x-1}$

$= \frac{\tan x(\sec x-1)}{\sec^2 x-1} = \frac{\tan x(\sec x-1)}{\tan^2 x}$

$= \frac{\sec x-1}{\tan x}$

**10.** $\sqrt{9-(3\sin\theta)^2} = \sqrt{9-9\sin^2\theta} = \sqrt{9\left(1-\sin^2\theta\right)}$

$= \sqrt{9\cos^2\theta} = 3\cos\theta$

### 6.1 Basic Concepts and Skills

**1.** An equation that is true for all values of the variable in its domain is called an <u>identity</u>.

**2.** To show that an equation is not an identity, we must find at least <u>one</u> value of the variable that results in both sides being defined but <u>not equal</u>.

**3.** $\sin^2 x + \underline{\cos^2 x} = 1$; $1 + \underline{\tan^2 x} = \sec^2 x$; $\csc^2 x - \cot^2 x = \underline{1}$

**4.** False. The statement is an identity.
$\tan^2 x \cot x = \tan^2 x \cdot \dfrac{1}{\tan x} = \tan x$

**5.** True

**6.** False. $\theta$ lies in quadrant III, so $\tan\theta > 0$.

**7.** $\sin\theta = -\dfrac{12}{13} \Rightarrow \left(-\dfrac{12}{13}\right)^2 + \cos^2\theta = 1 \Rightarrow$
$\cos\theta = \pm\dfrac{5}{13}$. $\pi < \theta < \dfrac{3\pi}{2} \Rightarrow \cos\theta = -\dfrac{5}{13}$

**8.** $\sec\theta = \dfrac{5}{4} \Rightarrow 1 + \tan^2\theta = \left(\dfrac{5}{4}\right)^2 \Rightarrow \tan\theta = \pm\dfrac{3}{4}$
$\dfrac{3\pi}{2} < \theta < 2\pi \Rightarrow \tan\theta = -\dfrac{3}{4}$

**9.** $\cot\theta = \dfrac{1}{2} \Rightarrow 1 + \left(\dfrac{1}{2}\right)^2 = \csc^2\theta \Rightarrow \csc\theta = \pm\dfrac{\sqrt{5}}{2}$
$\pi < \theta < \dfrac{3\pi}{2} \Rightarrow \csc\theta = -\dfrac{\sqrt{5}}{2}$

**10.** $\cos\theta = -\dfrac{1}{3} \Rightarrow \sec\theta = -3 \Rightarrow 1 + \tan^2\theta = (-3)^2 \Rightarrow$
$\tan\theta = \pm 2\sqrt{2}$. $\dfrac{\pi}{2} < \theta < \pi \Rightarrow \tan\theta = -2\sqrt{2}$

**11.** $\sin x = -\dfrac{2}{3} \Rightarrow \csc x = -\dfrac{3}{2} \Rightarrow$
$1 + \cot^2 x = \left(-\dfrac{3}{2}\right)^2 \Rightarrow \cot x = \pm\dfrac{\sqrt{5}}{2}$
$\pi < x < \dfrac{3\pi}{2} \Rightarrow \cot x = \dfrac{\sqrt{5}}{2}$

**12.** $\tan x = \dfrac{1}{2} \Rightarrow 1 + \left(\dfrac{1}{2}\right)^2 = \sec^2 x \Rightarrow$.
$\sec x = \pm\dfrac{\sqrt{5}}{2} \Rightarrow \cos x = \pm\dfrac{2\sqrt{5}}{5}$
$\pi < x < \dfrac{3\pi}{2} \Rightarrow \cos x = -\dfrac{2\sqrt{5}}{5}$

**13.** $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{4/5}{-3/5} = -\dfrac{4}{3}, \cot x = \dfrac{1}{\tan x} = -\dfrac{3}{4},$
$\sec x = \dfrac{1}{\cos x} = -\dfrac{5}{3}, \csc x = \dfrac{1}{\sin x} = \dfrac{5}{4}$

**14.** $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{-4/5}{3/5} = -\dfrac{4}{3}, \cot x = \dfrac{1}{\tan x} = -\dfrac{3}{4},$
$\sec x = \dfrac{1}{\cos x} = \dfrac{5}{3}, \csc x = \dfrac{1}{\sin x} = -\dfrac{5}{4}$

**15.** $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{1/\sqrt{3}}{\sqrt{2/3}} = \dfrac{\sqrt{2}}{2}, \cot x = \dfrac{1}{\tan x} = \sqrt{2},$
$\sec x = \dfrac{1}{\cos x} = \dfrac{\sqrt{6}}{2}, \csc x = \dfrac{1}{\sin x} = \sqrt{3}$

**16.** $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{1/\sqrt{3}}{-\sqrt{2/3}} = -\dfrac{\sqrt{2}}{2},$
$\cot x = \dfrac{1}{\tan x} = -\sqrt{2},$
$\sec x = \dfrac{1}{\cos x} = -\dfrac{\sqrt{6}}{2}, \csc x = \dfrac{1}{\sin x} = \sqrt{3}$

**17.** $\cos x = \dfrac{1}{\sec x} = -\dfrac{2\sqrt{5}}{5}, \cot x = \dfrac{1}{\tan x} = 2,$
$\tan x = \dfrac{\sin x}{\cos x} \Rightarrow \dfrac{1}{2} = \dfrac{\sin x}{-2\sqrt{5}/5} \Rightarrow \sin x = -\dfrac{\sqrt{5}}{5},$
$\csc x = \dfrac{1}{\sin x} = -\sqrt{5}$

**18.** $\cos x = \dfrac{1}{\sec x} = \dfrac{2\sqrt{5}}{5}, \cot x = \dfrac{1}{\tan x} = 2,$
$\tan x = \dfrac{\sin x}{\cos x} \Rightarrow \dfrac{1}{2} = \dfrac{\sin x}{2\sqrt{5}/5} \Rightarrow \sin x = \dfrac{\sqrt{5}}{5},$
$\csc x = \dfrac{1}{\sin x} = \sqrt{5}$

**19.** $\sin x = \dfrac{1}{\csc x} = \dfrac{1}{3}, \cot x = \dfrac{\cos x}{\sin x} \Rightarrow$
$2\sqrt{2} = \dfrac{\cos x}{1/3} \Rightarrow \cos x = \dfrac{2\sqrt{2}}{3},$
$\tan x = \dfrac{1}{\cot x} = \dfrac{\sqrt{2}}{4}, \sec x = \dfrac{1}{\cos x} = \dfrac{3\sqrt{2}}{4}$

**20.** $\sin x = \dfrac{1}{\csc x} = \dfrac{1}{3}, \cot x = \dfrac{\cos x}{\sin x} \Rightarrow$
$-2\sqrt{2} = \dfrac{\cos x}{1/3} \Rightarrow \cos x = -\dfrac{2\sqrt{2}}{3},$
$\tan x = \dfrac{1}{\cot x} = -\dfrac{\sqrt{2}}{4}, \sec x = \dfrac{1}{\cos x} = -\dfrac{3\sqrt{2}}{4}$

**21.** $\cot x = \dfrac{\cos x}{\sin x} \Rightarrow \dfrac{12}{5} = \dfrac{\cos x}{-5/13} \Rightarrow \cos x = -\dfrac{12}{13},$
$\tan x = \dfrac{1}{\cot x} = \dfrac{5}{12}, \sec x = \dfrac{1}{\cos x} = -\dfrac{13}{12},$
$\csc x = \dfrac{1}{\sin x} = -\dfrac{13}{5}$

**22.** $\cot x = \dfrac{\cos x}{\sin x} \Rightarrow -\dfrac{12}{5} = \dfrac{\cos x}{-5/13} \Rightarrow \cos x = \dfrac{12}{13},$

$\tan x = \dfrac{1}{\cot x} = -\dfrac{5}{12},\ \sec x = \dfrac{1}{\cos x} = \dfrac{13}{12},\ \csc x = \dfrac{1}{\sin x} = -\dfrac{13}{5}$

**23.** $\cos x = \dfrac{1}{\sec x} = \dfrac{1}{3}; \sin^2 x + \left(\dfrac{1}{3}\right)^2 = 1 \Rightarrow \sin x = \pm\dfrac{2\sqrt{2}}{3}; \dfrac{3\pi}{2} < x < 2\pi \Rightarrow \sin x = -\dfrac{2\sqrt{2}}{3}$

$\tan x = \dfrac{\sin x}{\cos x} = \dfrac{-2\sqrt{2}/3}{1/3} = -2\sqrt{2};\ \cot x = \dfrac{1}{\tan x} = -\dfrac{\sqrt{2}}{4}; \csc x = \dfrac{1}{\sin x} = -\dfrac{3\sqrt{2}}{4}$

**24.** $1 + \tan^2 x = \sec^2 x \Rightarrow 1 + (-2)^2 = \sec^2 x \Rightarrow \sec x = \pm\sqrt{5}; \dfrac{\pi}{2} < x < \pi \Rightarrow \sec x = -\sqrt{5};\ \cos x = \dfrac{1}{\sec x} = -\dfrac{\sqrt{5}}{5}$

$\tan x = -2 = \dfrac{1}{\cot x} \Rightarrow \cot x = -\dfrac{1}{2};\quad \tan x = \dfrac{\sin x}{\cos x} \Rightarrow -2 = \dfrac{\sin x}{-\sqrt{5}/5} \Rightarrow \sin x = \dfrac{2\sqrt{5}}{5};\ \csc x = \dfrac{1}{\sin x} = \dfrac{\sqrt{5}}{2}$

**25.** $\dfrac{\cos^2 x + \sin^2 x}{\sin x \cos x} = \dfrac{1}{\sin x \cos x}$

This is choice **c**.

**26.** $\dfrac{1}{1 + \tan^2 x} = \dfrac{1}{\sec^2 x} = \cos^2 x$

This is choice **f**.

**27.** $-\cot x \sin(-x) = -\dfrac{\cos x}{\sin x}(-\sin x) = \cos x$

This is choice **a**.

**28.** $1 + 2\cos x + \cos^2 x = (1 + \cos x)^2$

This is choice **b**.

**29.** $1 - 2\cos^2 x + \cos^4 x = (1 - \cos^2 x)^2$

$= (\sin^2 x)^2 = \sin^4 x$

This is choice **d**.

**30.** $\sin^2 x(1 + \cot^2 x) = \sin^2 x\left(1 + \dfrac{\cos^2 x}{\sin^2 x}\right)$

$= \sin^2 x + \cos^2 x = 1$

This is choice **e**.

**31.** $(1 + \tan x)(1 - \tan x) + \sec^2 x = 1 - \tan^2 x + \sec^2 x = 1 - \tan^2 x + 1 + \tan^2 x = 2$

**32.** $(\sec x - 1)(\sec x + 1) - \tan^2 x = \sec^2 x - 1 - \tan^2 x \Rightarrow \tan^2 x - \tan^2 x = 0$

**33.** $(\sec x + \tan x)(\sec x - \tan x) = \sec^2 x - \tan^2 x = 1 + \tan^2 x - \tan^2 x = 1$

**34.** $\dfrac{\sec^2 x - 4}{\sec x - 2} = \dfrac{(\sec x - 2)(\sec x + 2)}{\sec x - 2} = \sec x + 2$

**35.** $(\csc^4 x - \cot^4 x) = (\csc^2 x - \cot^2 x)(\csc^2 x + \cot^2 x) = \csc^2 x + \cot^2 x$

**36.** $\sin x \cos x(\tan x + \cot x) = \sin x \cos x\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) = \sin x \cos x\left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right) = \sin^2 x + \cos^2 x = 1$

**37.** $\frac{\sec x \csc x(\sin x + \cos x)}{\sec x + \csc x} = \frac{\frac{1}{\cos x \sin x}(\sin x + \cos x)}{\frac{1}{\sin x} + \frac{1}{\cos x}} = \frac{\frac{\sin x + \cos x}{\cos x \sin x}}{\frac{\cos x + \sin x}{\sin x \cos x}} = 1$

**38.** $\frac{1}{\csc x + 1} - \frac{1}{\csc x - 1} = \frac{(\csc x - 1) - (\csc x + 1)}{\csc^2 x - 1} = -\frac{2}{\cot^2 x} = -2\tan^2 x$

**39.** $\frac{\tan^2 x - 2\tan x - 3}{\tan x + 1} = \frac{(\tan x - 3)(\tan x + 1)}{\tan x + 1} = \tan x - 3$

**40.** $\frac{\tan^2 x + \sec x - 1}{\sec x - 1} = \frac{(\sec^2 x - 1) + \sec x - 1}{\sec x - 1} = \frac{\sec^2 x + \sec x - 2}{\sec x - 1} = \frac{(\sec x + 2)(\sec x - 1)}{\sec x - 1} = \sec x + 2$

Answers may vary for exercises 41–46.

**41.** $\sin x = 1 - \cos x$ is not an identity because for $x = \pi$, $\sin x = 0$, while $1 - \cos x = 2$.

**42.** $\tan x = \sec x - 1$ is not an identity because for $x = \pi$, $\tan x = 0$, while $\sec x - 1 = -2$.

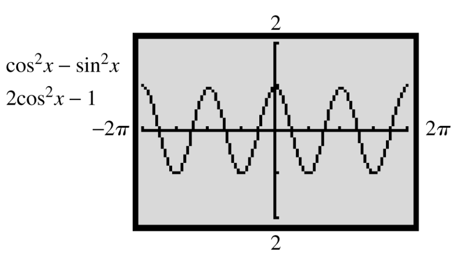
**43.** $\cos x = \sqrt{1 - \sin^2 x}$ is not an identity because for $x = \pi$, $\cos x = -1$, while $\sqrt{1 - \sin^2 x} = 1$.

**44.** $\sec x = \sqrt{1 + \tan^2 x}$ is not an identity because for $x = \pi$, $\sec x = -1$, while $\sqrt{1 + \tan^2 x} = 1$.

**45.** $\sin^2 x = (1 - \cos x)^2$ is not an identity because for $x = \pi$, $\sin^2 x = 0$, while $(1 - \cos x)^2 = 4$.

**46.** $\cot^2 x = (\csc x + 1)^2$ is not an identity because for $x = \frac{\pi}{2}$, $\cot^2 x = 0$, while $(\csc x + 1)^2 = 4$.

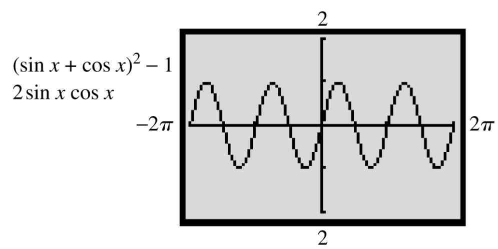
**47.**




This is an identity.

$$\cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x)$$
$$= 2\cos^2 x - 1$$

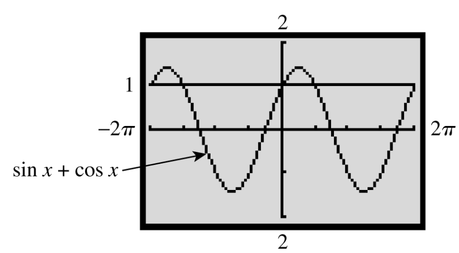
**48.**




This is an identity.

$$(\sin x + \cos x)^2 - 1$$
$$= \sin^2 x + 2\sin x \cos x + \cos^2 x - 1$$
$$= (\sin^2 x + \cos^2 x) + 2\sin x \cos x - 1$$
$$= 1 + 2\sin x \cos x - 1$$
$$= 2\sin x \cos x$$

**49.**

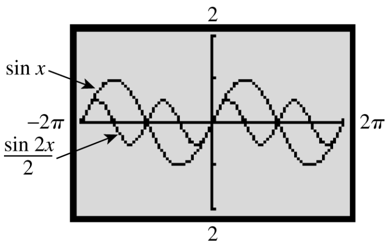



If $x = \frac{\pi}{4}$, then

$$\sin x + \cos x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2} \neq 1.$$

Thus, $\sin x + \cos x = 1$ is not an identity.

**50.**

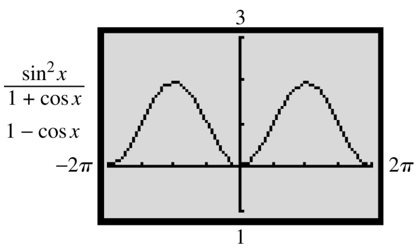



If $x = \frac{\pi}{4}$, then $\sin x = \frac{\sqrt{2}}{2}$ and

$$\frac{\sin 2x}{2} = \frac{\sin \frac{\pi}{2}}{2} = \frac{1}{1} \neq \frac{\sqrt{2}}{2}.$$

Thus, $\frac{\sin 2x}{2} = \sin x$ is not an identity.

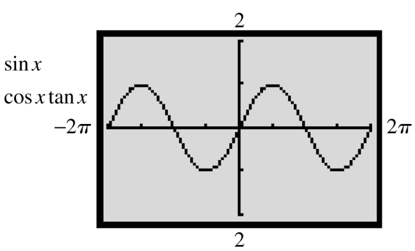
**51.**




This is an identity.

$$\frac{\sin^2 x}{1+\cos x} = \frac{1-\cos^2 x}{1+\cos x} = \frac{(1+\cos x)(1-\cos x)}{1+\cos x}$$
$$= 1-\cos x$$

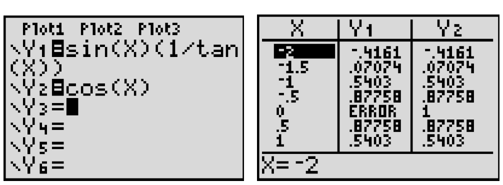
**52.**




This is an identity.

$$\cos x \tan x = \cos x \cdot \frac{\sin x}{\cos x} = \sin x$$

**53.**

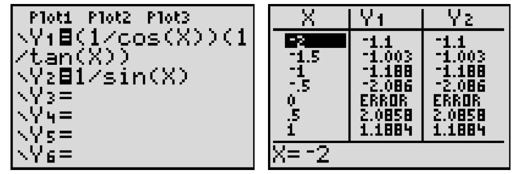



This is an identity.

$$\sin x \cot x = \sin x \cdot \frac{\cos x}{\sin x} = \cos x$$

Note that there is an error in the table for $x = 0$ because cot 0 is undefined.

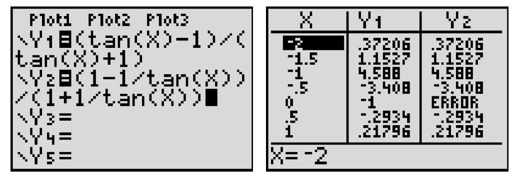
**54.**




This is an identity.

$$\sec x \cot x = \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} = \frac{1}{\sin x} = \csc x$$

**55.**



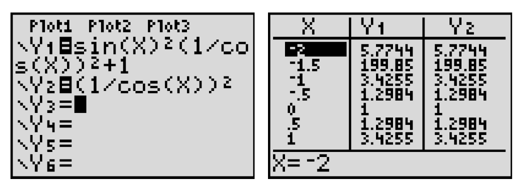

$$\frac{\tan x - 1}{\tan x + 1} = \frac{\frac{\sin x}{\cos x} - 1}{\frac{\sin x}{\cos x} + 1} = \frac{\frac{\sin x - \cos x}{\cos x}}{\frac{\sin x + \cos x}{\cos x}}$$
$$= \frac{\sin x - \cos x}{\sin x + \cos x}$$

$$\frac{1 - \cot x}{1 + \cot x} = \frac{1 - \frac{\cos x}{\sin x}}{1 + \frac{\cos x}{\sin x}} = \frac{\frac{\sin x - \cos x}{\sin x}}{\frac{\sin x + \cos x}{\sin x}}$$
$$= \frac{\sin x - \cos x}{\sin x + \cos x}$$

This is an identity.

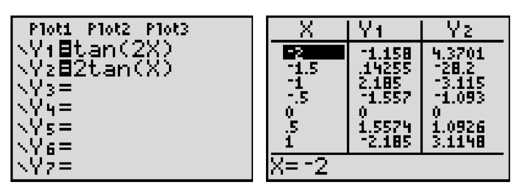
Thus, $\frac{\tan x - 1}{\tan x + 1} = \frac{\sin x - \cos x}{\sin x + \cos x} = \frac{1 - \cot x}{1 + \cot x}$.

**56.**



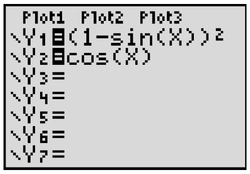

$$\sin^2 x \sec^2 x + 1 = \sin^2 x \cdot \frac{1}{\cos^2 x} + 1$$
$$= \frac{\sin^2 x}{\cos^2 x} + 1 = \tan^2 x + 1$$
$$= \sec^2 x$$

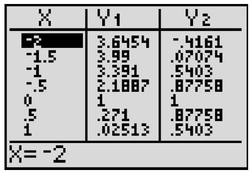
**57.**




Note that the values for $Y_1$ and $Y_2$ in the table are not equal, so $\tan 2x = 2\tan x$ is not an identity.

**58.**




Note that the values for $Y_1$ and $Y_2$ in the table are not equal, so $(1-\sin x)^2 = \cos x$ is not an identity.

**59.** $\sin x \tan x + \cos x$

$$= \sin x \cdot \frac{\sin x}{\cos x} + \cos x = \frac{\sin^2 x}{\cos x} + \cos x$$
$$= \frac{\sin^2 x + \cos^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$$

**60.** $\cos x \cot x + \sin x$

$$= \cos x \cdot \frac{\cos x}{\sin x} + \sin x = \frac{\cos^2 x}{\sin x} + \sin x$$
$$= \frac{\cos^2 x + \sin^2 x}{\sin x} = \frac{1}{\sin x} = \csc x$$

**61.** $$\frac{1-4\cos^2 x}{1-2\cos x} = \frac{(1+2\cos x)(1-2\cos x)}{1-2\cos x}$$
$$= 1 + 2\cos x$$

**62.** $$\frac{9-16\sin^2 x}{3+4\sin x} = \frac{(3+4\sin x)(3-4\sin x)}{3+4\sin x}$$
$$= 3 - 4\sin x$$

**63.** $(\cos x - \sin x)(\cos x + \sin x) = \cos^2 x - \sin^2 x$
$= (1-\sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$

**64.** $(\sin x - \cos x)(\sin x + \cos x) = \sin^2 x - \cos^2 x$
$= (1-\cos^2 x) - \cos^2 x = 1 - 2\cos^2 x$

**65.** $$\sin^2 x \cot^2 x + \sin^2 x = \sin^2 x \cdot \frac{\cos^2 x}{\sin^2 x} + \sin^2 x$$
$$= \cos^2 x + \sin^2 x = 1$$

**66.** $$\tan^2 x - \sin^2 x = \frac{\sin^2 x}{\cos^2 x} - \sin^2 x$$
$$= \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x} = \frac{\sin^2 x(1-\cos^2 x)}{\cos^2 x}$$
$$= \frac{\sin^2 x(\sin^2 x)}{\cos^2 x} = \frac{\sin^4 x}{\cos^2 x} = \sin^4 x \cdot \frac{1}{\cos^2 x}$$
$$= \sin^4 x \sec^2 x$$

**67.** $\sin^3 x - \cos^3 x$
$= (\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)$
$= (\sin x - \cos x)(1 + \sin x \cos x)$

**68.** $\sin^3 x + \cos^3 x$
$= (\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)$
$= (\sin x + \cos x)(1 - \sin x \cos x)$

**69.** $\cos^4 x - \sin^4 x$
$= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$
$= 1\left((1-\sin^2 x) - \sin^2 x\right) = 1 - 2\sin^2 x$

**70.** $\cos^4 x - \sin^4 x$
$= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$
$= 1(\cos^2 x - (1-\cos^2 x)) = 2\cos^2 x - 1$

**71.** $$\frac{1}{1-\sin x} + \frac{1}{1+\sin x} = \frac{(1+\sin x)+(1-\sin x)}{1-\sin^2 x}$$
$$= \frac{2}{\cos^2 x} = 2\sec^2 x$$

**72.** $$\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = \frac{(1+\cos x)+(1-\cos x)}{1-\cos^2 x}$$
$$= \frac{2}{\sin^2 x} = 2\csc^2 x$$

**73.** $$\frac{1}{\csc x - 1} - \frac{1}{\csc x + 1} = \frac{\csc x + 1 - (\csc x - 1)}{\csc^2 x - 1}$$
$$= \frac{2}{\cot^2 x} = 2\tan^2 x$$

**74.** $$\frac{1}{\sec x - 1} + \frac{1}{\sec x + 1} = \frac{\sec x + 1 + \sec x - 1}{\sec^2 x - 1}$$
$$= \frac{2\sec x}{\tan^2 x} = 2\sec x \cot^2 x$$

**75.** $$\sec^2 x + \csc^2 x = \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$$
$$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} = \frac{1}{\sin^2 x \cos^2 x}$$
$$= \sec^2 x \csc^2 x$$

**76.** $$\cot^2 x + \tan^2 x = (\csc^2 x - 1) + (\sec^2 x - 1)$$
$$= (\csc^2 x + \sec^2 x) - 2$$
$$= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} - 2$$
$$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} - 2$$
$$= \frac{1}{\sin^2 x \cos^2 x} - 2$$
$$= \sec^2 x \csc^2 x - 2$$

**77.** $\dfrac{1}{\sec x-\tan x}+\dfrac{1}{\sec x+\tan x}=\dfrac{\sec x+\tan x+\sec x-\tan x}{\sec^2 x-\tan^2 x}=\dfrac{2\sec x}{\sec^2 x-(\sec^2 x-1)}=2\sec x=\dfrac{2}{\cos x}$

**78.** $\dfrac{1}{\csc x+\cot x}+\dfrac{1}{\csc x-\cot x}=\dfrac{\csc x-\cot x+\csc x+\cot x}{\csc^2 x-\cot^2 x}=\dfrac{2\csc x}{\csc^2 x-(\csc^2 x-1)}=2\csc x=\dfrac{2}{\sin x}$

**79.** $\dfrac{\sin x}{1+\cos x}=\dfrac{\sin x}{1+\cos x}\cdot\dfrac{1-\cos x}{1-\cos x}=\dfrac{\sin x(1-\cos x)}{1-\cos^2 x}=\dfrac{\sin x(1-\cos x)}{\sin^2 x}=\dfrac{1-\cos x}{\sin x}=\dfrac{1}{\sin x}-\dfrac{\cos x}{\sin x}=\csc x-\cot x$

**80.** $\dfrac{\sin x}{1-\cos x}=\dfrac{\sin x}{1-\cos x}\cdot\dfrac{1+\cos x}{1+\cos x}=\dfrac{\sin x(1+\cos x)}{1-\cos^2 x}=\dfrac{\sin x(1+\cos x)}{\sin^2 x}=\dfrac{1+\cos x}{\sin x}=\dfrac{1}{\sin x}+\dfrac{\cos x}{\sin x}=\csc x+\cot x$

**81.** $(\sin x+\cos x)^2=\sin^2 x+2\sin x\cos x+\cos^2 x=1+2\sin x\cos x$

**82.** $(\sin x-\cos x)^2=\sin^2 x-2\sin x\cos x+\cos^2 x=1-2\sin x\cos x$

**83.** $(1+\tan x)^2=1+2\tan x+\tan^2 x=\sec^2 x+2\tan x$

**84.** $(1-\cot x)^2=1-2\cot x+\cot^2 x=\csc^2 x-2\cot x$

**85.** $\dfrac{\tan x\sin x}{\tan x+\sin x}=\dfrac{\tan x\sin x}{\tan x+\sin x}\cdot\dfrac{\tan x-\sin x}{\tan x-\sin x}=\dfrac{\tan x\sin x(\tan x-\sin x)}{\tan^2 x-\sin^2 x}=\dfrac{\tan x\sin x(\tan x-\sin x)}{\frac{\sin^2 x}{\cos^2 x}-\sin^2 x}$

$=\dfrac{\tan x\sin x(\tan x-\sin x)}{\frac{\sin^2 x-\sin^2 x\cos^2 x}{\cos^2 x}}=\dfrac{\tan x\sin x(\tan x-\sin x)}{\frac{\sin^2 x(1-\cos^2 x)}{\cos^2 x}}=\dfrac{\tan x\sin x(\tan x-\sin x)}{\frac{\sin^2 x\sin^2 x}{\cos^2 x}}$

$=\dfrac{\tan x\sin x(\tan x-\sin x)}{\tan^2 x\sin^2 x}=\dfrac{\tan x-\sin x}{\tan x\sin x}$

**86.** $\dfrac{\cot x\cos x}{\cot x+\cos x}=\dfrac{\cot x\cos x}{\cot x+\cos x}\cdot\dfrac{\cot x-\cos x}{\cot x-\cos x}=\dfrac{\cot x\cos x(\cot x-\cos x)}{\cot^2 x-\cos^2 x}=\dfrac{\cot x\cos x(\cot x-\cos x)}{\frac{\cos^2 x}{\sin^2 x}-\cos^2 x}$

$=\dfrac{\cot x\cos x(\cot x-\cos x)}{\frac{\cos^2 x-\cos^2 x\sin^2 x}{\sin^2 x}}=\dfrac{\cot x\cos x(\cot x-\cos x)}{\frac{\cos^2 x(1-\sin^2 x)}{\sin^2 x}}=\dfrac{\cot x\cos x(\cot x-\cos x)}{\frac{\cos^2 x(\cos^2 x)}{\sin^2 x}}$

$=\dfrac{\cot x\cos x(\cot x-\cos x)}{\cot^2 x\cos^2 x}=\dfrac{\cot x-\cos x}{\cot x\cos x}$

**87.** $(\tan x+\cot x)^2=\tan^2 x+2\tan x\cot x+\cot^2 x=\tan^2 x+\cot^2 x+2\tan x\left(\dfrac{1}{\tan x}\right)=\tan^2 x+\cot^2 x+2$

$=(\tan^2 x+1)+(\cot^2 x+1)=\sec^2 x+\csc^2 x$

**88.** $(1+\cot^2 x)(1+\tan^2 x)=\left(1+\dfrac{\cos^2 x}{\sin^2 x}\right)\left(1+\dfrac{\sin^2 x}{\cos^2 x}\right)=\left(\dfrac{\sin^2 x+\cos^2 x}{\sin^2 x}\right)\left(\dfrac{\cos^2 x+\sin^2 x}{\cos^2 x}\right)=\dfrac{1}{\sin^2 x\cos^2 x}$

**89.** $\dfrac{\sin^2 x-\cos^2 x}{\sec^2 x-\csc^2 x}=\dfrac{\sin^2 x-\cos^2 x}{\frac{1}{\cos^2 x}-\frac{1}{\sin^2 x}}=\dfrac{\sin^2 x-\cos^2 x}{\frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}}=(\sin^2 x-\cos^2 x)\cdot\dfrac{\sin^2 x\cos^2 x}{\sin^2 x-\cos^2 x}=\sin^2 x\cos^2 x$

**90.** $\left(\tan x+\dfrac{1}{\cot x}\right)\left(\cot x+\dfrac{1}{\tan x}\right)=(2\tan x)(2\cot x)=4(\tan x\cot x)=4(1)=4$

**91.** $$\frac{\tan x}{1+\sec x}+\frac{1+\sec x}{\tan x}=\frac{\tan^2 x+(1+\sec x)^2}{\tan x(1+\sec x)}=\frac{\tan^2 x+1+2\sec x+\sec^2 x}{\tan x(1+\sec x)}=\frac{\sec^2 x+2\sec x+\sec^2 x}{\tan x(1+\sec x)}$$
$$=\frac{2\sec^2 x+2\sec x}{\tan x(1+\sec x)}=\frac{2\sec x(1+\sec x)}{\tan x(1+\sec x)}=\frac{2\sec x}{\tan x}=\frac{\frac{2}{\cos x}}{\frac{\sin x}{\cos x}}=\frac{2}{\sin x}=2\csc x$$

**92.** $$\frac{\cot x}{1+\csc x}+\frac{1+\csc x}{\cot x}=\frac{\cot^2 x+(1+\csc x)^2}{\cot x(1+\csc x)}=\frac{\cot^2 x+1+2\csc x+\csc^2 x}{\cot x(1+\csc x)}=\frac{\csc^2 x+2\csc x+\csc^2 x}{\cot x(1+\csc x)}$$
$$=\frac{2\csc^2 x+2\csc x}{\cot x(1+\csc x)}=\frac{2\csc x(1+\csc x)}{\cot x(1+\csc x)}=\frac{2\csc x}{\cot x}=\frac{\frac{2}{\sin x}}{\frac{\cos x}{\sin x}}=\frac{2}{\cos x}=2\sec x$$

**93.** $$\frac{\sin x+\tan x}{\cos x+1}=\frac{\sin x+\frac{\sin x}{\cos x}}{\cos x+1}=\frac{\frac{\sin x\cos x+\sin x}{\cos x}}{\cos x+1}=\frac{\frac{\sin x(\cos x+1)}{\cos x}}{\cos x+1}=\frac{\sin x}{\cos x}=\tan x$$

**94.** $$\frac{\sin x}{1+\tan x}=\frac{\sin x}{1+\frac{\sin x}{\cos x}}=\frac{\sin x}{\frac{\cos x+\sin x}{\cos x}}=\frac{\sin x\cos x}{\cos x+\sin x}$$
$$\frac{\cos x}{1+\cot x}=\frac{\cos x}{1+\frac{\cos x}{\sin x}}=\frac{\cos x}{\frac{\sin x+\cos x}{\sin x}}=\frac{\sin x\cos x}{\cos x+\sin x}$$

Since both sides of the original identity are equal to $\frac{\sin x\cos x}{\cos x+\sin x}$, the identity is verified.

**95.** $\sqrt{1+\tan^2\theta}=\sec\theta$

**96.** $$\sqrt{4-(2\cos\theta)^2}=\sqrt{4-4\cos^2\theta}=\sqrt{4\left(1-\cos^2\theta\right)}=2\sin\theta$$

**97.** $\sqrt{\sec^2\theta-1}=\tan\theta$

**98.** $$\sqrt{(2\sec\theta)^2-4}=\sqrt{4\sec^2\theta-4}=\sqrt{4\left(\sec^2\theta-1\right)}=2\tan\theta$$

**99.** $$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}=\frac{\cos\theta}{\sin\theta}=\cot\theta$$

**100.** $$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}=\frac{\sec\theta}{\tan\theta}=\frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}=\frac{1}{\sin\theta}=\csc\theta$$

## 6.1 Applying the Concepts

**101.** $x=20\csc\theta$

**102.** $x=60\cot\theta$

**103.**
$$m_1\cos\theta-\sin\theta=m_2\cos\theta+m_1m_2\sin\theta$$
$$m_1\cos\theta-m_2\cos\theta=\sin\theta+m_1m_2\sin\theta$$
$$\cos\theta\left(m_1-m_2\right)=\sin\theta\left(1+m_1m_2\right)$$
$$\frac{m_1-m_2}{1+m_1m_2}=\frac{\sin\theta}{\cos\theta}\quad\text{or}\quad\frac{\cos\theta}{\sin\theta}=\frac{1+m_1m_2}{m_1-m_2}$$
$$\frac{m_1-m_2}{1+m_1m_2}=\tan\theta\quad\text{or}\quad\cot\theta=\frac{1+m_1m_2}{m_1-m_2}$$

**104.**
$$h=\frac{d\sin\alpha\sin\beta}{\cos\alpha\sin\beta-\sin\alpha\cos\beta}=\frac{\frac{d\sin\alpha\sin\beta}{\sin\alpha\sin\beta}}{\frac{\cos\alpha\sin\beta-\sin\alpha\cos\beta}{\sin\alpha\sin\beta}}=\frac{d}{\frac{\cos\alpha}{\sin\alpha}-\frac{\cos\beta}{\sin\beta}}=\frac{d}{\cot\alpha-\cot\beta}$$

### 6.1 Beyond the Basics

**105.** $\frac{1-\sin x}{1-\sec x}-\frac{1+\sin x}{1+\sec x}=\frac{(1-\sin x)(1+\sec x)-(1+\sin x)(1-\sec x)}{1-\sec^2 x}$

$=\frac{1+\sec x-\sin x-\sin x\sec x-1+\sec x-\sin x+\sin x\sec x}{1-\sec^2 x}$

$=\frac{2(\sec x-\sin x)}{-\tan^2 x}=\frac{2\sin x-2\sec x}{\frac{\sin^2 x}{\cos^2 x}}=\frac{2\sin x\cos^2 x-2\sec x\cos^2 x}{\sin^2 x}$

$=\frac{2\sin x\cos^2 x}{\sin^2 x}-\frac{2\sec x\cos^2 x}{\sin^2 x}=\frac{2\cos^2 x}{\sin x}-\frac{\frac{2}{\cos x}\cos^2 x}{\sin^2 x}$

$=2\cos x\cot x-\frac{2}{\sin x}\cot x=2\cos x\cot x-2\csc x\cot x=2\cot x(\cos x-\csc x)$

**106.** $\frac{\sec x+\tan x}{\csc x+\cot x}-\frac{\sec x-\tan x}{\csc x-\cot x}=\frac{(\sec x+\tan x)(\csc x-\cot x)}{\csc^2 x-\cot^2 x}-\frac{(\sec x-\tan x)(\csc x+\cot x)}{\csc^2 x-\cot^2 x}$

$=\frac{\sec x\csc x-\cot x\sec x+\tan x\csc x-\tan x\cot x}{\csc^2 x-\cot^2 x}-$

$\frac{\sec x\csc x+\cot x\sec x-\tan x\csc x-\tan x\cot x}{\csc^2 x-\cot^2 x}$

$=\frac{2\tan x\csc x-2\cot x\sec x}{(1+\cot^2 x)-\cot^2 x}=\frac{2\sin x}{\cos x\sin x}-\frac{2\cos x}{\sin x\cos x}$

$=2\sec x-2\csc x=2(\sec x-\csc x)$

**107.** $(1-\tan x)^2+(1-\cot x)^2=1-2\tan x+\tan^2 x+1-2\cot x+\cot^2 x=\sec^2 x-2(\tan x+\cot x)+\csc^2 x$

$=\sec^2 x-2\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)+\csc^2 x=\sec^2 x-2\left(\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right)+\csc^2 x$

$=\sec^2 x-\frac{2}{\sin x\cos x}+\csc^2 x=\sec^2 x-2\sec x\csc x+\csc^2 x=(\sec x-\csc x)^2$

**108.** $\sec^6 x-\tan^6 x=(\sec^2 x)^3-(\tan^2 x)^3=(\sec^2 x-\tan^2 x)[(\sec^2 x)^2+\sec^2 x\tan^2 x+\tan^4 x]$

$=(\tan^2 x+1-\tan^2 x)[(\tan^2 x+1)^2+(\tan^2 x+1)(\tan^2 x)+\tan^4 x]$

$=(\tan^4 x+2\tan^2 x+1)+(\tan^4 x+\tan^2 x)+\tan^4 x=3\tan^4 x+3\tan^2 x+1$

**109.** $\frac{\cot x+\csc x-1}{\cot x+\csc x+1}=\frac{\frac{\cos x}{\sin x}+\frac{1}{\sin x}-1}{\frac{\cos x}{\sin x}+\frac{1}{\sin x}+1}\cdot\frac{\sin x}{\sin x}=\frac{\cos x+1-\sin x}{\cos x+1+\sin x}=\frac{(\cos x+1)-\sin x}{(\cos x+1)+\sin x}\cdot\frac{(\cos x+1)-\sin x}{(\cos x+1)-\sin x}$

$=\frac{(\cos x+1)^2-2\sin x(\cos x+1)+\sin^2 x}{(\cos x+1)^2-\sin^2 x}=\frac{\cos^2 x+2\cos x+1-2\sin x\cos x-2\sin x+\sin^2 x}{\cos^2 x+2\cos x+1-\sin^2 x}$

$=\frac{2\cos x+2-2\sin x\cos x-2\sin x}{\cos^2 x+2\cos x+1-(1-\cos^2 x)}=\frac{2(\cos x+1)-2\sin x(\cos x+1)}{2\cos^2 x+2\cos x}$

$=\frac{2(1-\sin x)(\cos x+1)}{2\cos(\cos x+1)}=\frac{1-\sin x}{\cos x}$

**110.** $$\frac{\tan x+\sec x-1}{\tan x-\sec x+1}=\frac{\frac{\sin x}{\cos x}+\frac{1}{\cos x}-1}{\frac{\sin x}{\cos x}-\frac{1}{\cos x}+1}\cdot\frac{\cos x}{\cos x}=\frac{\sin x+1-\cos x}{\sin x-1+\cos x}=\frac{\sin x+(1-\cos x)}{\sin x-(1-\cos x)}\cdot\frac{\sin x+(1-\cos x)}{\sin x+(1-\cos x)}$$
$$=\frac{\sin^2 x+2\sin x(1-\cos x)+1-2\cos x+\cos^2 x}{\sin^2 x-(1-2\cos x+\cos^2 x)}=\frac{2+2\sin x(1-\cos x)-2\cos x}{1-\cos^2 x-1+2\cos x-\cos^2 x}$$
$$=\frac{2(1-\cos x)+2\sin x(1-\cos x)}{2\cos x-2\cos^2 x}=\frac{2(1-\cos x)(1+\sin x)}{2\cos x(1-\cos x)}=\frac{1+\sin x}{\cos x}$$

**111.** $$(\sin u\cos v+\cos u\sin v)^2+(\cos u\cos v-\sin u\sin v)^2$$
$$=(\sin^2 u\cos^2 v+2\sin u\cos v\cos u\sin v+\cos^2 u\sin^2 v)+(\cos^2 u\cos^2 v-2\sin u\cos v\cos u\sin v+\sin^2 u\sin^2 v)$$
$$=\sin^2 u\cos^2 v+\cos^2 u\sin^2 v+\cos^2 u\cos^2 v+\sin^2 u\sin^2 v$$
$$=\sin^2 u(\cos^2 v+\sin^2 v)+\cos^2 u(\sin^2 v+\cos^2 v)=(\sin^2 u+\cos^2 u)(\sin^2 v+\cos^2 v)=1$$

**112.** $$(\sin u\cos v-\cos u\sin v)^2+(\cos u\cos v+\sin u\sin v)^2$$
$$=(\sin^2 u\cos^2 v-2\sin u\cos v\cos u\sin v+\cos^2 u\sin^2 v)+(\cos^2 u\cos^2 v+2\sin u\cos v\cos u\sin v+\sin^2 u\sin^2 v)$$
$$=\sin^2 u\cos^2 v+\cos^2 u\sin^2 v+\cos^2 u\cos^2 v+\sin^2 u\sin^2 v$$
$$=\sin^2 u(\sin^2 v+\cos^2 v)+\cos^2 u(\sin^2 v+\cos^2 v)=(\sin^2 u+\cos^2 u)(\sin^2 v+\cos^2 v)=1$$

**113.** $$3(\sin^4 x+\cos^4 x)-2(\sin^6 x+\cos^6 x)=3(\sin^4 x+\cos^4 x)-2\left((\sin^2 x)^3+(\cos^2 x)^3\right)$$
$$=3\sin^4 x+3\cos^4 x-2(\sin^2 x+\cos^2 x)(\sin^4 x-\sin^2 x\cos^2 x+\cos^4 x)$$
$$=3\sin^4 x+3\cos^4 x-2\sin^4 x+2\sin^2 x\cos^2 x-2\cos^4 x$$
$$=\sin^4 x+2\sin^2 x\cos^2 x+\cos^4 x=(\sin^2 x+\cos^2 x)^2=1$$

Remember that $u^3+v^3=(u+v)(u^2-uv+v^2)$

**114.** $$\sin^6 x+\cos^6 x-3\cos^4 x=\left(\left(\sin^2 x\right)^3+\left(\cos^2 x\right)^3\right)-3\cos^4 x$$
$$=\left(\sin^2 x+\cos^2 x\right)\left(\sin^4 x-\sin^2 x\cos^2 x+\cos^4 x\right)-3\cos^4 x$$
$$=\sin^4 x-\sin^2 x\cos^2 x-2\cos^4 x=\left(\sin^2 x+\cos^2 x\right)\left(\sin^2 x-2\cos^2 x\right)$$
$$=\sin^2 x-2\cos^2 x=\left(1-\cos^2 x\right)-2\cos^2 x=1-3\cos^2 x$$

**115.** $$\frac{1-\cos\theta}{1+\cos\theta}+\frac{1+\cos\theta}{1-\cos\theta}=\frac{1-2\cos\theta+\cos^2\theta+1+2\cos\theta+\cos^2\theta}{1-\cos^2\theta}=\frac{2+2\cos^2\theta}{\sin^2\theta}$$
$$=\frac{2}{\sin^2\theta}+\frac{2\cos^2\theta}{\sin^2\theta}=2\csc^2\theta+2\cot^2\theta=2(\csc^2\theta+\cot^2\theta)$$
$$=2\left(1+\cot^2\theta+\cot^2\theta\right)=2+4\cot^2\theta$$

**116.** $$\frac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}+\frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$$
$$=\frac{(\cos\theta+\sin\theta)(\cos^2\theta-\sin\theta\cos\theta+\sin^2\theta)}{\cos\theta+\sin\theta}+\frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin\theta\cos\theta+\sin^2\theta)}{\cos\theta-\sin\theta}$$
$$=\cos^2\theta-\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta+\sin^2\theta=2(\sin^2\theta+\cos^2\theta)=2$$

**117.** $$\frac{\sec^2\theta+2\tan^2\theta}{1+3\tan^2\theta}=\frac{1+\tan^2\theta+2\tan^2\theta}{1+3\tan^2\theta}=\frac{1+3\tan^2\theta}{1+3\tan^2\theta}=1$$

**118.** $\dfrac{\csc^2\alpha+\sec^2\alpha}{\csc^2\alpha-\sec^2\alpha}-\dfrac{1+\tan^2\alpha}{1-\tan^2\alpha}=\dfrac{1+\cot^2\alpha+1+\tan^2\alpha}{1+\cot^2\alpha-1-\tan^2\alpha}-\dfrac{1+\tan^2\alpha}{1-\tan^2\alpha}=\dfrac{2+\dfrac{1}{\tan^2\alpha}+\tan^2\alpha}{\dfrac{1}{\tan^2\alpha}-\tan^2\alpha}-\dfrac{1+\tan^2\alpha}{1-\tan^2\alpha}$

$=\dfrac{\dfrac{2\tan^2\alpha+1+\tan^4\alpha}{\tan^2\alpha}}{\dfrac{1-\tan^4 x}{\tan^2\alpha}}-\dfrac{1+\tan^2\alpha}{1-\tan^2\alpha}=\dfrac{\tan^4\alpha+2\tan^2\alpha+1}{1-\tan^4 x}-\dfrac{1+\tan^2\alpha}{1-\tan^2\alpha}$

$=\dfrac{(\tan^2\alpha+1)^2}{(1-\tan^2\alpha)(1+\tan^2\alpha)}-\dfrac{1+\tan^2\alpha}{1-\tan^2\alpha}=\dfrac{\tan^2\alpha+1}{1-\tan^2\alpha}-\dfrac{1+\tan^2\alpha}{1-\tan^2\alpha}=0$

**119.** $\cos^2 x(3-4\cos^2 x)^2+\sin^2 x(3-4\sin^2 x)^2=\cos^2 x(9-24\cos^2 x+16\cos^4 x)+\sin^2 x(9-24\sin^2 x+16\sin^4 x)$

$=16\cos^6 x-24\cos^4 x+9\cos^2 x+16\sin^6 x-24\sin^4 x+9\sin^2 x$

$=16(\sin^6 x+\cos^6 x)-24(\sin^4 x+\cos^4 x)+9(\sin^2 x+\cos^2 x)$

$=16\left((\sin^2 x)^3+(\cos^2 x)^3\right)-24(\sin^4 x+\cos^4 x)+9$

$=16(\sin^2 x+\cos^2 x)(\sin^4 x-\sin^2 x\cos^2 x+\cos^4 x)-24(\sin^4 x+\cos^4 x)+9$

$=16\sin^4 x-16\sin^2 x\cos^2 x+16\cos^4 x-24\sin^4 x-24\cos^4 x+9$

$=-8\sin^4 x-16\sin^2 x\cos^2 x-8\cos^4 x+9=-8(\sin^2 x+\cos^2 x)^2+9=1$

**120.** $(\sin\theta+\csc\theta)^2+(\cos\theta+\sec\theta)^2-\cot^2\theta$

$=\sin^2\theta+2\sin\theta\csc\theta+\csc^2\theta+\cos^2\theta+2\cos\theta\sec\theta+\sec^2\theta-\cot^2\theta$

$=\left(\sin^2\theta+\cos^2\theta\right)+2\sin\theta\left(\dfrac{1}{\sin\theta}\right)+2\cos\theta\left(\dfrac{1}{\cos\theta}\right)+\csc^2\theta+\sec^2\theta-\cot^2\theta$

$=1+2+2+\left(1+\cot^2\theta\right)+\left(1+\tan^2\theta\right)-\cot^2\theta=7+\tan^2\theta$

**121.** $x^2+y^2+z^2=(r\cos u\cos v)^2+(r\cos u\sin v)^2+(r\sin u)^2$

$=r^2\cos^2 u\cos^2 v+r^2\cos^2 u\sin^2 v+r^2\sin^2 u$

$=r^2(\cos^2 u\cos^2 v+\cos^2 u\sin^2 v+\sin^2 u)$

$=r^2(\cos^2 u(\cos^2 v+\sin^2 v)+\sin^2 u)=r^2(\cos^2 u+\sin^2 u)=r^2$

**122. a.** $\tan x+\cot x=2\Rightarrow(\tan x+\cot x)^2=4\Rightarrow\tan^2 x+2\tan x\cot x+\cot^2 x=4\Rightarrow$

$\tan^2 x+2\tan x\left(\dfrac{1}{\tan x}\right)+\cot^2 x=4\Rightarrow\tan^2 x+2+\cot^2 x=4\Rightarrow\tan^2 x+\cot^2 x=2$

**b.** $\tan x+\cot x=2\Rightarrow(\tan x+\cot x)^3=8\Rightarrow\tan^3 x+3\tan^2 x\cot x+3\tan x\cot^2 x+\cot^3 x=8\Rightarrow$

$\tan^3 x+3\tan x+3\cot x+\cot^3 x=8\Rightarrow\tan^3 x+3(\tan x+\cot x)+\cot^3 x=8\Rightarrow$

$\tan^3 x+3(2)+\cot^3 x=8\Rightarrow\tan^3 x+\cot^3 x=2$

**123. a.** $\sec x+\cos x=2\Rightarrow(\sec x+\cos x)^2=4\Rightarrow\sec^2 x+2\sec x\cos x+\cos^2 x=4\Rightarrow$

$\sec^2 x+2(1)+\cos^2 x=4\Rightarrow\sec^2 x+\cos^2 x=2$

$\sec x+\cos x=2\Rightarrow(\sec x+\cos x)^4=16\Rightarrow$

$\sec^4 x+4\sec^3 x\cos x+6\sec^2 x\cos^2 x+4\sec x\cos^3 x+\cos^4 x=16\Rightarrow$

$\sec^4 x+\cos^4 x+4\sec^2 x+6\sec^2 x\cos^2 x+4\cos^2 x=16\Rightarrow$

$\sec^4 x+\cos^4 x+4(\sec^2 x+\cos^2 x)+6=16\Rightarrow\sec^4 x+\cos^4 x+4(2)+6=16\Rightarrow$

$\sec^4 x+\cos^4 x=2$

**b.** $\sec x + \cos x = 2 \Rightarrow (\sec x + \cos x)^3 = 8 \Rightarrow \sec^3 x + 3\sec^2 x\cos x + 3\sec x\cos^2 x + \cos^3 x = 8 \Rightarrow$
$\sec^3 x + \cos^3 x + 3\sec x + 3\cos x = 8 \Rightarrow \sec^3 x + \cos^3 x + 3(\sec x + \cos x) = 8 \Rightarrow$
$\sec^3 x + \cos^3 x + 3(2) = 8 \Rightarrow \sec^3 x + \cos^3 x = 2$

**124.** $\sin x + \sin^2 x = 1 \Rightarrow \sin x = 1 - \sin^2 x \Rightarrow \sin^2 x = 1 - 2\sin^2 x + (\sin^2 x)^2 \Rightarrow$
$1 - \cos^2 x = 1 - 2(1 - \cos^2 x) + (1 - \cos^2 x)^2 \Rightarrow 1 - \cos^2 x = 1 - 2 + 2\cos^2 x + 1 - 2\cos^2 x + \cos^4 x \Rightarrow$
$1 - \cos^2 x = \cos^4 x \Rightarrow 1 = \cos^2 x + \cos^4 x$

### 6.1 Critical Thinking/Discussion/Writing

**125. a.** Answers may vary. Sample answer:
If $x = \frac{3\pi}{2}$, then $\sqrt{\sin^2\left(\frac{3\pi}{2}\right)} = 1$, while $\sin\frac{3\pi}{2} = -1$.

**b.** Answers may vary. Sample answer:
If $x = \frac{\pi}{2}$, then $\sqrt{\sin^2\left(\frac{\pi}{2}\right)} = 1$, and $\sin\frac{\pi}{2} = 1$.

**126.** If $\sec\theta = \frac{xy}{x^2 + y^2}$, we have a right triangle with hypotenuse *xy* and one leg $x^2 + y^2$. Using the Pythagorean theorem to find the length of the other leg, we have

$$\begin{aligned} b &= \sqrt{(xy)^2 - (x^2 + y^2)^2} \\ &= \sqrt{x^2y^2 - (x^4 + 2x^2y^2 + y^4)} \\ &= \sqrt{-x^4 - x^2y^2 - y^4} \\ &= \sqrt{-(x^4 + x^2y^2 + y^4)} \end{aligned}$$

which is the square root of a negative number. Therefore $\sec\theta \neq \frac{xy}{x^2 + y^2}$.

**127.** $-1 \le \sin\theta \le 1 \Rightarrow -1 \le \frac{1+t^2}{1-t^2} \le 1 \Rightarrow$
$-1 + t^2 \le 1 + t^2 \le 1 - t^2 \Rightarrow t^2 \le 0$. The only value of *t* that makes this true is *t* = 0.

**128.** $0 \le \cos^2\theta \le 1 \Rightarrow 0 \le \frac{a^2 + b^2}{2ab} \le 1 \Rightarrow$
$0 \le a^2 + b^2 \le 2ab \Rightarrow a^2 - 2ab + b^2 \le 0 \Rightarrow$
$(a-b)^2 \le 0$. This is true only if $a = b$ for $a, b \neq 0$.

**129.** $\csc^2\theta = \frac{2ab}{a^2 + b^2} \Rightarrow \frac{2ab}{a^2 + b^2} \ge 1 \Rightarrow$
$2ab \ge a^2 + b^2 \Rightarrow 0 \ge a^2 - 2ab + b^2 \Rightarrow$
$0 \ge (a-b)^2$. This is true only if $a = b$ for $a, b \neq 0$.

**130. a.** We know that $(\sin\theta - \csc\theta)^2 \ge 0$ because $a^2 \ge 0$ for any real number *a*. So
$\sin^2\theta - 2\sin\theta\csc\theta + \csc^2\theta \ge 0$.
$-2\sin\theta\csc\theta = -2\sin\theta\left(\frac{1}{\sin\theta}\right) = -2$, so
$\sin^2\theta + \csc^2\theta - 2 \ge 0 \Rightarrow \sin^2\theta + \csc^2\theta \ge 2$

**b.** $(\cos\theta - \sec\theta)^2 \ge 0 \Rightarrow$
$\cos^2\theta - 2\cos\theta\sec\theta + \sec^2\theta \ge 0.$
$2\cos\theta\sec\theta = 2\cos\theta\left(\frac{1}{\cos\theta}\right) = 2,$ so
$\cos^2\theta + \sec^2\theta \ge 2$

**c.** $(\sin\theta - \cos\theta)^2 \ge 0 \Rightarrow$
$\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta \ge 0 \Rightarrow$
$1 \ge 2\sin\theta\cos\theta \Rightarrow \frac{1}{2} \ge \sin\theta\cos\theta \Rightarrow$
$\frac{1}{2} \ge \frac{1}{\sec\theta\csc\theta} \Rightarrow \sec\theta\csc\theta \ge 2$
$(\sec\theta - \csc\theta)^2 \ge 0 \Rightarrow$
$\sec^2\theta - 2\sec\theta\csc\theta + \csc^2\theta \ge 0 \Rightarrow$
$\sec^2\theta + \csc^2\theta \ge 2\sec\theta\csc\theta$
Since $\sec\theta\csc\theta \ge 2,$
$\sec^2\theta + \csc^2\theta \ge 2\sec\theta\csc\theta \Rightarrow$
$\sec^2\theta + \csc^2\theta \ge 4$

### 6.1 Maintaining Skills

**131.** $(2x - y)^2 = 4x^2 - 4xy + y^2$

**132.** $(3x - 5y)^2 = 9x^2 - 30xy + 25y^2$

**133.** $\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$

**134.** $\cos\left(-\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

**135.** $\tan\left(-\frac{\pi}{6}\right)=-\tan\frac{\pi}{6}=-\frac{\sqrt{3}}{3}$

**136.** $\cos^2 x=1-\sin^2 x\Rightarrow \sin^2 x+\cos^2 x=1$

**137.** $\sec^2 x-\tan^2 x=1\Rightarrow \tan^2 x+1=\sec^2 x$

**138.** $\cot^2 x=\csc^2 x-1\Rightarrow 1+\cot^2 x=\csc^2 x$

**139.** $\cos t=-\frac{\sqrt{3}}{4}$, $\sin t>0\Rightarrow t$ lies in quadrant II.

$\cos t=-\frac{\sqrt{3}}{4}=\frac{x}{r};\quad \sin t=\frac{y}{r}$

$r^2=x^2+y^2\Rightarrow 16=\left(-\sqrt{3}\right)^2+y^2\Rightarrow$

$y=\sqrt{13}$

$\sin t=\frac{\sqrt{13}}{4}$

**140.** In quadrant III, $\cos t<0$.

$\tan t=\frac{4}{3}=\frac{-4}{-3}=\frac{y}{x}$

$r^2=x^2+y^2=(-3)^2+(-4)^2\Rightarrow r=5$

$\cos t=-\frac{3}{5}$

## 6.2 Sum and Difference Formulas

### 6.2 Practice Problems

**1.** $$\begin{aligned}\cos 15^\circ&=\cos(45^\circ-30^\circ)\\&=\cos 45^\circ\cos 30^\circ+\sin 45^\circ\sin 30^\circ\\&=\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)\\&=\frac{\sqrt{6}+\sqrt{2}}{4}\end{aligned}$$

**2.** $$\begin{aligned}\cos\frac{7\pi}{12}&=\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)\\&=\cos\frac{\pi}{3}\cos\frac{\pi}{4}-\sin\frac{\pi}{3}\sin\frac{\pi}{4}\\&=\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)-\frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)\\&=\frac{\sqrt{2}-\sqrt{6}}{4}\end{aligned}$$

**3.** $\sec\left(\frac{\pi}{2}-x\right)=\frac{1}{\cos\left(\frac{\pi}{2}-x\right)}=\frac{1}{\sin x}=\csc x$

**4.** $$\begin{aligned}\sin(\pi+x)&=\sin\pi\cos x+\cos\pi\sin x\\&=0\cdot\cos x+(-1)\sin x\\&=-\sin x\end{aligned}$$

**5.** $$\begin{aligned}\sin 43^\circ\cos 13^\circ-\cos 43^\circ\sin 13^\circ&=\sin(43^\circ-13^\circ)\\&=\sin 30^\circ=\frac{1}{2}\end{aligned}$$

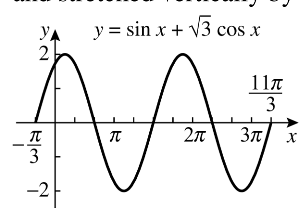
**6.** $$\begin{aligned}\cos(u+v)&=\cos u\cos v-\sin u\sin v\\&=\left(-\frac{4}{5}\right)\left(\frac{12}{13}\right)-\left(-\frac{3}{5}\right)\left(-\frac{5}{13}\right)=-\frac{63}{65}\end{aligned}$$

**7.** $$\begin{aligned}\frac{\cos(x+y)}{\sin x\sin y}&=\frac{\cos x\cos y-\sin x\sin y}{\sin x\sin y}\\&=\frac{\cos x\cos y}{\sin x\sin y}-\frac{\sin x\sin y}{\sin x\sin y}\\&=\cot x\cot y-1\end{aligned}$$

**8.** $y=\sin x+\sqrt{3}\cos x=a\sin x+b\cos x\Rightarrow$
$a=1, b=\sqrt{3}\Rightarrow\sqrt{a^2+b^2}=\sqrt{1+3}=2.$
So, $\theta$ is any angle in standard position that has $\left(1,\sqrt{3}\right)$ on its terminal side $\Rightarrow\theta=\frac{\pi}{3}$.

Thus, $\sin x+\sqrt{3}\cos x=2\sin\left(x+\frac{\pi}{3}\right)$.

**9.** From practice problem 8,
$\sin x+\sqrt{3}\cos x=2\sin\left(x+\frac{\pi}{3}\right)$. This is the graph of $y=\sin x$, shifted $\frac{\pi}{3}$ units to the left and stretched vertically by a factor of 2.



**10.** $y = y_1 + y_2 = 0.1\sin(400t) + 0.2\cos(400t) \Rightarrow$
$a = 0.1,\ b = 0.2 \Rightarrow$
$\sqrt{0.1^2 + 0.2^2} = \sqrt{0.05} = 0.1\sqrt{5} = A$
$\theta = \tan^{-1}\left(\frac{0.2}{0.1}\right) = \tan^{-1} 2$
$y = 0.1\sin(400t) + 0.2\cos(400t) \Rightarrow$
$y = 0.1\sqrt{5}\sin(400t + \theta)$
$= 0.1\sqrt{5}\sin 400\left(t + \frac{\theta}{400}\right)$

Amplitude: $0.1\sqrt{5}$;

phase shift: $-\frac{\theta}{400} = -\frac{\tan^{-1} 2}{400} \approx -0.0028$

period: $\frac{2\pi}{400} = \frac{\pi}{200}$; frequency: $\frac{400}{2\pi} = \frac{200}{\pi}$

**11.** $\tan(\pi + x) = \frac{\tan\pi + \tan x}{1 - \tan\pi\tan x} = \frac{0 + \tan x}{1 - 0} = \tan x$

## 6.2 Basic Concepts and Skills

**1.** $\sin(A + B) = \underline{\sin A\cos B + \cos A\sin B}$

**2.** $\cos A\cos B - \sin A\sin B = \underline{\cos(A + B)}$

**3.** $\tan(A + B) = \underline{\frac{\tan A + \tan B}{1 - \tan A\tan B}}$

**4.** False. $\cos\left(\frac{\pi}{2} + x\right) = \cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x$
$= 0\cdot\cos x - 1\cdot\sin x$
$= -\sin x$

**5.** True. $\sin\left(\frac{\pi}{2} + x\right) = \sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x$
$= 1\cdot\cos x - 0\cdot\sin x$
$= \cos x$

**6.** False. $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$

**7.** $\sin(45° + 30°) = \sin 45°\cos 30° + \cos 45°\sin 30°$
$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$
$= \frac{\sqrt{6} + \sqrt{2}}{4}$

**8.** $\sin(45° - 30°) = \sin 45°\cos 30° - \cos 45°\sin 30°$
$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$
$= \frac{\sqrt{6} - \sqrt{2}}{4}$

**9.** $\sin(60° - 45°) = \sin 60°\cos 45° - \cos 60°\sin 45°$
$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$
$= \frac{\sqrt{6} - \sqrt{2}}{4}$

**10.** $\sin(60° + 45°) = \sin 60°\cos 45° + \cos 60°\sin 45°$
$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$
$= \frac{\sqrt{6} + \sqrt{2}}{4}$

**11.** $\sin(-105°) = -\sin(105°) = -\sin(60° + 45°)$
$= -(\sin 60°\cos 45° + \cos 60°\sin 45°)$
$= -\left[\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\right]$
$= -\frac{\sqrt{6} + \sqrt{2}}{4}$

**12.** $\cos 285° = \cos 75° = \cos(45° + 30°)$
$= \cos 45°\cos 30° - \sin 45°\sin 30°$
$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$
$= \frac{\sqrt{6} - \sqrt{2}}{4}$

**13.** $\tan 225° = \tan(180° + 45°)$
$= \frac{\tan 180° + \tan 45°}{1 - \tan 180°\tan 45°} = 1$

**14.** $\tan(-165°) = \tan 195° = \tan(135° + 60°)$
$= \frac{\tan 135° + \tan 60°}{1 - \tan 135°\tan 60°} = 2 - \sqrt{3}$

**15.** $\sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4}$
$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$
$= \frac{\sqrt{6} + \sqrt{2}}{4}$

**16.** $\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$
$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$
$= \frac{\sqrt{6} + \sqrt{2}}{4}$

**17.** $\tan\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=\frac{\tan(\pi/4)-\tan(\pi/6)}{1+\tan(\pi/4)\tan(\pi/6)}$
$=\frac{1-\sqrt{3}/3}{1+1\left(\sqrt{3}/3\right)}=\frac{3-\sqrt{3}}{3+\sqrt{3}}$
$=2-\sqrt{3}$

**18.** $\cot\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\frac{1}{\tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right)}$
$=\frac{1+\tan(\pi/3)\tan(\pi/4)}{\tan(\pi/3)-\tan(\pi/4)}$
$=\frac{\sqrt{3}+1}{\sqrt{3}-1}=2+\sqrt{3}$

**19.** $\sec\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$
$=\frac{1}{\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)}=\frac{1}{\cos\frac{\pi}{3}\cos\frac{\pi}{4}-\sin\frac{\pi}{3}\sin\frac{\pi}{4}}$
$=\frac{1}{\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}=\frac{1}{\frac{\sqrt{2}-\sqrt{6}}{4}}$
$=\frac{4}{\sqrt{2}-\sqrt{6}}\cdot\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}+\sqrt{6}}=\frac{4\left(\sqrt{2}+\sqrt{6}\right)}{-4}$
$=-\sqrt{2}-\sqrt{6}$

**20.** $\csc\left(\frac{\pi}{4}-\frac{\pi}{3}\right)$
$=\frac{1}{\sin\left(\frac{\pi}{4}-\frac{\pi}{3}\right)}=\frac{1}{\sin\frac{\pi}{4}\cos\frac{\pi}{3}-\cos\frac{\pi}{4}\sin\frac{\pi}{3}}$
$=\frac{1}{\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}=\frac{1}{\frac{\sqrt{2}-\sqrt{6}}{4}}$
$=\frac{4}{\sqrt{2}-\sqrt{6}}=-\sqrt{2}-\sqrt{6}$

**21.** $\cos\left(-\frac{5\pi}{12}\right)=\cos\left(\frac{5\pi}{12}\right)=\cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$
$=\cos\frac{\pi}{6}\cos\frac{\pi}{4}-\sin\frac{\pi}{6}\sin\frac{\pi}{4}$
$=\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)-\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)$
$=\frac{\sqrt{6}-\sqrt{2}}{4}$

**22.** $\sin\frac{7\pi}{12}=\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$
$=\sin\frac{\pi}{3}\cos\frac{\pi}{4}+\cos\frac{\pi}{3}\sin\frac{\pi}{4}$
$=\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{6}+\sqrt{2}}{4}$

**23.** $\tan\frac{19\pi}{12}=\tan\left(\frac{3\pi}{4}+\frac{5\pi}{6}\right)=\frac{\tan\frac{3\pi}{4}+\tan\frac{5\pi}{6}}{1-\tan\frac{3\pi}{4}\tan\frac{5\pi}{6}}$
$=\frac{-1-\sqrt{3}/3}{1-(-1)\left(-\sqrt{3}/3\right)}=\frac{\sqrt{3}+3}{\sqrt{3}-3}=-2-\sqrt{3}$

**24.** $\sec\frac{\pi}{12}=\frac{1}{\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)}$
$=\frac{1}{\cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6}}$
$=\frac{1}{\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)}$
$=\frac{4}{\sqrt{6}+\sqrt{2}}=\sqrt{6}-\sqrt{2}$

**25.** $\tan\left(\frac{17\pi}{12}\right)=\tan\left(\frac{\pi}{4}+\frac{7\pi}{6}\right)=\frac{\tan\frac{\pi}{4}+\tan\frac{7\pi}{6}}{1-\tan\frac{\pi}{4}\tan\frac{7\pi}{6}}$
$=\frac{1+\sqrt{3}/3}{1-(1)\left(\sqrt{3}/3\right)}=\frac{3+\sqrt{3}}{3-\sqrt{3}}=2+\sqrt{3}$

**26.** $\csc\frac{11\pi}{12}=\frac{1}{\sin\left(\frac{\pi}{6}+\frac{3\pi}{4}\right)}$
$=\frac{1}{\sin\frac{\pi}{6}\cos\frac{3\pi}{4}+\cos\frac{\pi}{6}\sin\frac{3\pi}{4}}$
$=\frac{1}{\left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}$
$=\frac{4}{\sqrt{6}-\sqrt{2}}=\sqrt{6}+\sqrt{2}$

**27.** $\sin\left(x+\frac{\pi}{2}\right)=\sin x\cos\frac{\pi}{2}+\cos x\sin\frac{\pi}{2}$
$=0+\cos x(1)=\cos x$

**28.** $\cos\left(x+\frac{\pi}{2}\right)=\cos x\cos\frac{\pi}{2}-\sin x\sin\frac{\pi}{2}$
$=0-(1)\sin x=-\sin x$

**29.** $\sin\left(x-\frac{\pi}{2}\right)=\sin x\cos\frac{\pi}{2}-\cos x\sin\frac{\pi}{2}$
$=0-\cos x(1)=-\cos x$

**30.** $\cos\left(x-\frac{\pi}{2}\right)=\cos x\cos\frac{\pi}{2}+\sin x\sin\frac{\pi}{2}$
$=0+(1)\sin x=\sin x$

**31.** $\tan\left(x+\frac{\pi}{2}\right)=\frac{\sin\left(x+\frac{\pi}{2}\right)}{\cos\left(x+\frac{\pi}{2}\right)}=\frac{\cos x}{-\sin x}=-\cot x$

**32.** $\tan\left(x-\frac{\pi}{2}\right)=\frac{\sin\left(x-\frac{\pi}{2}\right)}{\cos\left(x-\frac{\pi}{2}\right)}=\frac{-\cos x}{\sin x}=-\cot x$

**33.** $\csc(x+\pi)=\frac{1}{\sin(x+\pi)}$
$=\frac{1}{\sin x\cos\pi+\cos x\sin\pi}=\frac{1}{-\sin x}$
$=-\csc x$

**34.** $\sec(x+\pi)=\frac{1}{\cos(x+\pi)}$
$=\frac{1}{\cos x\cos\pi-\sin x\sin\pi}$
$=\frac{1}{-\cos x}=-\sec x$

**35.** $\cos\left(x+\frac{3\pi}{2}\right)=\cos x\cos\frac{3\pi}{2}-\sin x\sin\frac{3\pi}{2}$
$=0-(-1)\sin x=\sin x$

**36.** $\cos\left(x-\frac{3\pi}{2}\right)=\cos x\cos\frac{3\pi}{2}+\sin x\sin\frac{3\pi}{2}$
$=0+(-1)\sin x=-\sin x$

**37.** $\tan\left(x-\frac{3\pi}{2}\right)=\frac{\sin\left(x-\frac{3\pi}{2}\right)}{\cos\left(x-\frac{3\pi}{2}\right)}$
$=\frac{\sin x\cos\frac{3\pi}{2}-\sin\frac{3\pi}{2}\cos x}{\cos x\cos\frac{3\pi}{2}+\sin x\sin\frac{3\pi}{2}}$
$=\frac{\cos x}{-\sin x}=-\cot x$

**38.** $\tan\left(x+\frac{3\pi}{2}\right)=\frac{\sin\left(x+\frac{3\pi}{2}\right)}{\cos\left(x+\frac{3\pi}{2}\right)}$
$=\frac{\sin x\cos\frac{3\pi}{2}+\sin\frac{3\pi}{2}\cos x}{\cos x\cos\frac{3\pi}{2}-\sin x\sin\frac{3\pi}{2}}$
$=\frac{-\cos x}{\sin x}=-\cot x$

**39.** $\cot(3\pi-x)=\frac{\cos(3\pi-x)}{\sin(3\pi-x)}$
$=\frac{\cos 3\pi\cos x+\sin 3\pi\sin x}{\sin 3\pi\cos x-\sin x\cos 3\pi}$
$=\frac{-\cos x}{\sin x}=-\cot x$

**40.** $\csc\left(\frac{5\pi}{2}-x\right)=\frac{1}{\sin\left(\frac{5\pi}{2}-x\right)}$
$=\frac{1}{\sin\frac{5\pi}{2}\cos x-\sin x\cos\frac{5\pi}{2}}$
$=\frac{1}{\cos x}=\sec x$

**41.** $\sin 56^\circ\cos 34^\circ+\cos 56^\circ\sin 34^\circ=\sin(56^\circ+34^\circ)$
$=\sin 90^\circ=1$

**42.** $\cos 57^\circ\cos 33^\circ-\sin 57^\circ\cos 33^\circ$
$=\cos(57^\circ+33^\circ)=\cos 90^\circ=0$

**43.** $\cos 331^\circ\cos 61^\circ+\sin 331^\circ\sin 61^\circ$
$=\cos(331^\circ-61^\circ)=\cos 270^\circ=0$

**44.** $\cos 110^\circ\sin 70^\circ+\sin 110^\circ\cos 70^\circ$
$=\sin(110^\circ+70^\circ)=\sin 180^\circ=0$

**45.** $\frac{\tan 129^\circ-\tan 84^\circ}{1+\tan 129^\circ\tan 84^\circ}=\tan(129^\circ-84^\circ)=\tan 45^\circ=1$

**46.** $\frac{\tan 28^\circ+\tan 17^\circ}{1-\tan 28^\circ\tan 17^\circ}=\tan(28^\circ+17^\circ)=\tan 45^\circ=1$

**47.** $\sin\frac{7\pi}{12}\cos\frac{3\pi}{12}-\cos\frac{7\pi}{12}\sin\frac{3\pi}{12}$
$=\sin\left(\frac{7\pi}{12}-\frac{3\pi}{12}\right)=\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$

**48.** $\cos\frac{5\pi}{12}\cos\frac{\pi}{12}-\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$
$=\cos\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)=\cos\frac{\pi}{2}=0$

**49.** $\dfrac{\tan\frac{5\pi}{12}-\tan\frac{2\pi}{12}}{1+\tan\frac{5\pi}{12}\tan\frac{2\pi}{12}}=\tan\left(\frac{5\pi}{12}-\frac{2\pi}{12}\right)=\tan\frac{\pi}{4}=1$

**50.** $\dfrac{\tan\frac{5\pi}{12}+\tan\frac{7\pi}{12}}{1+\tan\frac{5\pi}{12}\tan\frac{7\pi}{12}}=\tan\left(\frac{5\pi}{12}+\frac{7\pi}{12}\right)=\tan\pi=0$

In exercises 51 − 56, $\tan u=\frac{3}{4}$ with $u$ in Quadrant III $\Rightarrow \sin u=-\frac{3}{5}$ and $\cos u=-\frac{4}{5}$; $\sin v=\frac{5}{13}$ with $v$ in Quadrant II $\Rightarrow \cos v=-\frac{12}{13}$ and $\tan v=-\frac{5}{12}$.

**51.** $\sin(u-v)=\sin u\cos v-\cos u\sin v$
$=\left(-\frac{3}{5}\right)\left(-\frac{12}{13}\right)-\left(-\frac{4}{5}\right)\left(\frac{5}{13}\right)=\frac{56}{65}$

**52.** $\sin(u+v)=\sin u\cos v+\cos u\sin v$
$=\left(-\frac{3}{5}\right)\left(-\frac{12}{13}\right)+\left(-\frac{4}{5}\right)\left(\frac{5}{13}\right)=\frac{16}{65}$

**53.** $\cos(u+v)=\cos u\cos v-\sin u\sin v$
$=\left(-\frac{4}{5}\right)\left(-\frac{12}{13}\right)-\left(-\frac{3}{5}\right)\left(\frac{5}{13}\right)=\frac{63}{65}$

**54.** $\cos(u-v)=\cos u\cos v+\sin u\sin v$
$=\left(-\frac{4}{5}\right)\left(-\frac{12}{13}\right)+\left(-\frac{3}{5}\right)\left(\frac{5}{13}\right)=\frac{33}{65}$

**55.** $\tan(u+v)=\dfrac{\tan u+\tan v}{1-\tan u\tan v}=\dfrac{\frac{3}{4}+\left(-\frac{5}{12}\right)}{1-\left(\frac{3}{4}\left(-\frac{5}{12}\right)\right)}$
$=\frac{16}{63}$

**56.** $\tan(u-v)=\dfrac{\tan u-\tan v}{1+\tan u\tan v}=\dfrac{\frac{3}{4}-\left(-\frac{5}{12}\right)}{1+\left(\frac{3}{4}\left(-\frac{5}{12}\right)\right)}=\frac{56}{33}$

In exercises 57 − 62, $\cos\alpha=-\frac{2}{5}$ with $\alpha$ in Quadrant II $\Rightarrow \sin\alpha=\frac{\sqrt{21}}{5}$ and $\tan\alpha=-\frac{\sqrt{21}}{2}$; $\sin\beta=-\frac{3}{7}$ with $\beta$ in Quadrant IV $\Rightarrow \cos\beta=\frac{2\sqrt{10}}{7}$ and $\tan\beta=-\frac{3\sqrt{10}}{20}$.

**57.** $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$
$=\left(\frac{\sqrt{21}}{5}\right)\left(\frac{2\sqrt{10}}{7}\right)-\left(-\frac{2}{5}\right)\left(-\frac{3}{7}\right)$
$=\frac{2\sqrt{210}-6}{35}$

**58.** $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
$=\left(-\frac{2}{5}\right)\left(\frac{2\sqrt{10}}{7}\right)+\left(\frac{\sqrt{21}}{5}\right)\left(-\frac{3}{7}\right)$
$=\frac{-4\sqrt{10}-3\sqrt{21}}{35}$

**59.** $\csc(\alpha+\beta)=\frac{1}{\sin(\alpha+\beta)}$
$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$
$=\left(\frac{\sqrt{21}}{5}\right)\left(\frac{2\sqrt{10}}{7}\right)+\left(-\frac{2}{5}\right)\left(-\frac{3}{7}\right)$
$=\frac{2\sqrt{210}+6}{35}\Rightarrow$
$\csc(\alpha+\beta)=\frac{35}{2\sqrt{210}+6}=\frac{35\sqrt{210}-105}{402}$

**60.** $\sec(\alpha+\beta)=\frac{1}{\cos(\alpha+\beta)}$
$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$
$=\left(-\frac{2}{5}\right)\left(\frac{2\sqrt{10}}{7}\right)-\left(\frac{\sqrt{21}}{5}\right)\left(-\frac{3}{7}\right)$
$=\frac{-4\sqrt{10}+3\sqrt{21}}{35}\Rightarrow$
$\sec(\alpha+\beta)=\frac{35}{-4\sqrt{10}+3\sqrt{21}}$
$=\frac{35\left(3\sqrt{21}+4\sqrt{10}\right)}{29}$

**61.** Use the results from exercises 57 and 58:

$$\cot(\alpha-\beta)=\frac{\cos(\alpha-\beta)}{\sin(\alpha-\beta)}=\frac{\frac{-4\sqrt{10}-3\sqrt{21}}{35}}{\frac{2\sqrt{210}-6}{35}}=\frac{-4\sqrt{10}-3\sqrt{21}}{2\sqrt{210}-6}$$

**62.** Use the middle steps from exercises 59 and 60:

$$\cot(\alpha+\beta)=\frac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)}=\frac{\frac{-4\sqrt{10}+3\sqrt{21}}{35}}{\frac{2\sqrt{210}+6}{35}}=\frac{-4\sqrt{10}+3\sqrt{21}}{2\sqrt{210}+6}$$

**63.** $$\frac{\sin(x+y)}{\cos x\cos y}=\frac{\sin x\cos y+\sin y\cos x}{\cos x\cos y}=\frac{\sin x\cos y}{\cos x\cos y}+\frac{\sin y\cos x}{\cos x\cos y}=\frac{\sin x}{\cos x}+\frac{\sin y}{\cos y}=\tan x+\tan y$$

**64.** $$\frac{\sin(x+y)}{\sin x\sin y}=\frac{\sin x\cos y+\sin y\cos x}{\sin x\sin y}=\frac{\sin x\cos y}{\sin x\sin y}+\frac{\sin y\cos x}{\sin x\sin y}=\frac{\cos y}{\sin y}+\frac{\cos x}{\sin x}=\cot x+\cot y$$

**65.** $$\frac{\cos(x+y)}{\cos x\cos y}=\frac{\cos x\cos y-\sin x\sin y}{\cos x\cos y}=\frac{\cos x\cos y}{\cos x\cos y}-\frac{\sin x\sin y}{\cos x\cos y}=1-\tan x\tan y$$

**66.** $$\frac{\cos(x-y)}{\sin x\sin y}=\frac{\cos x\cos y+\sin x\sin y}{\sin x\sin y}=\frac{\cos x\cos y}{\sin x\sin y}+\frac{\sin x\sin y}{\sin x\sin y}=\cot x\cot y+1$$

**67.** We use the results from exercises 64 and 66 in the first step. Start with the right side.

$$\frac{1+\cot x\cot y}{\cot x+\cot y}=\frac{\cos(x-y)}{\sin x\sin y}\div\frac{\sin(x+y)}{\sin x\sin y}=\frac{\cos(x-y)}{\sin x\sin y}\cdot\frac{\sin x\sin y}{\sin(x+y)}=\frac{\cos(x-y)}{\sin(x+y)}$$

**68.** $$\frac{\cos(x+y)}{\sin(x-y)}=\frac{\frac{\cos(x+y)}{\sin x\sin y}}{\frac{\sin(x-y)}{\sin x\sin y}}=\frac{\frac{\cos x\cos y-\sin x\sin y}{\sin x\sin y}}{\frac{\sin x\cos y-\sin y\cos x}{\sin x\sin y}}=\frac{\frac{\cos x\cos y}{\sin x\sin y}-\frac{\sin x\sin y}{\sin x\sin y}}{\frac{\sin x\cos y}{\sin x\sin y}-\frac{\sin y\cos x}{\sin x\sin y}}=\frac{\cot x\cot y-1}{\cot y-\cot x}=\frac{1-\cot x\cot y}{\cot x-\cot y}$$

**69.** $$\frac{\sin(x-y)}{\sin(x+y)}=\frac{\frac{\sin(x-y)}{\sin x\sin y}}{\frac{\sin(x+y)}{\sin x\sin y}}=\frac{\frac{\sin x\cos y-\sin y\cos x}{\sin x\sin y}}{\frac{\sin x\cos y+\sin y\cos x}{\sin x\sin y}}=\frac{\frac{\sin x\cos y}{\sin x\sin y}-\frac{\sin y\cos x}{\sin x\sin y}}{\frac{\sin x\cos y}{\sin x\sin y}+\frac{\sin y\cos x}{\sin x\sin y}}=\frac{\cot y-\cot x}{\cot y+\cot x}$$

**70.** We use the results from exercises 66 and 68 in the first step.

$$\frac{\cos(x+y)}{\cos(x-y)}=\frac{\frac{\cos(x+y)}{\sin x\sin y}}{\frac{\cos(x-y)}{\cos x\cos y}}=\frac{\cot x\cot y-1}{\cot x\cot y+1}$$

**71.** $y=\sin x+\cos x=a\sin x+b\cos x\Rightarrow$ $a=1,b=1\Rightarrow\sqrt{a^2+b^2}=\sqrt{2}=A.$ So $\theta$ is any angle in standard position that has (1, 1) on its terminal side $\Rightarrow\theta=\frac{\pi}{4}$.

$$y=\sin x+\cos x=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right).$$

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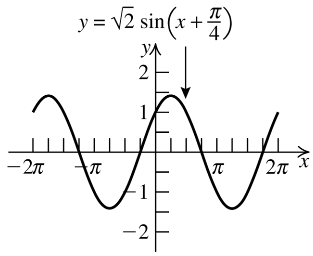
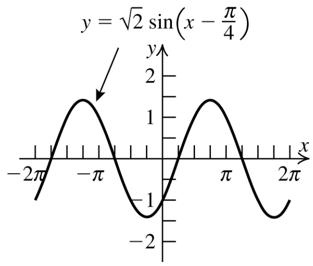



**72.** $y = \sin x - \cos x = a \sin x - b \cos x \Rightarrow$

$a = 1, b = -1 \Rightarrow \sqrt{a^2 + b^2} = \sqrt{2} = A.$ So $\theta$ is any angle in standard position that has (1, −1) on its terminal side $\Rightarrow \theta = -\dfrac{\pi}{4}$.

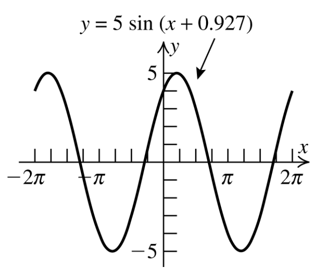
$y = \sin x - \cos x = \sqrt{2}\sin\left(x - \dfrac{\pi}{4}\right)$.




**73.** $y = 3\sin x + 4\cos x = a \sin x + b \cos x \Rightarrow$

$a = 3, b = 4 \Rightarrow \sqrt{a^2 + b^2} = 5 = A.$ So $\theta$ is any angle in standard position that has (3, 4) on its terminal side $\Rightarrow \tan\theta = \dfrac{4}{3} \Rightarrow \theta \approx 0.927$.

$y = 3\sin x + 4\cos x = 5\sin(x + 0.927)$.

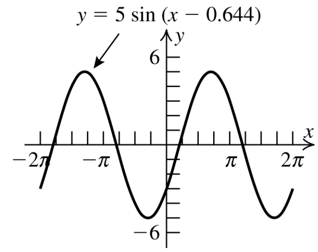



**74.** $y = 4\sin x - 3\cos x = a \sin x + b \cos x \Rightarrow$

$a = 4, b = -3 \Rightarrow \sqrt{a^2 + b^2} = 5 = A.$ So $\theta$ is any angle in standard position that has (4, −3) on its terminal side

$\Rightarrow \tan\theta = -\dfrac{3}{4} \Rightarrow \theta \approx -0.644$.

$y = 4\sin x - 3\cos x = 5\sin(x - 0.644)$.

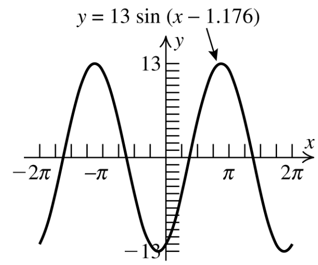



**75.** $y = 5\sin x - 12\cos x = a \sin x + b \cos x \Rightarrow$

$a = 5, b = -12 \Rightarrow \sqrt{a^2 + b^2} = 13 = A.$ So $\theta$ is any angle in standard position that has (5, −12) on its terminal side $\Rightarrow \tan\theta = -\dfrac{12}{5} \Rightarrow$

$\theta \approx -1.176$.

$y = 5\sin x - 12\cos x = 5\sin(x - 1.176)$.

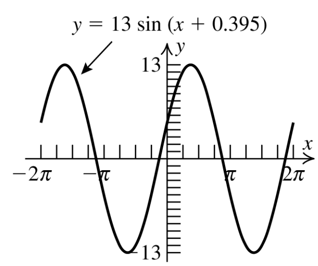



**76.** $y = 12\sin x + 5\cos x = a \sin x + b \cos x \Rightarrow$

$a = 12, b = 5 \Rightarrow \sqrt{a^2 + b^2} = 13 = A.$ So $\theta$ is any angle in standard position that has (12, 5) on its terminal side

$\Rightarrow \tan\theta = \dfrac{5}{12} \Rightarrow \theta \approx 0.395$.

$y = 12\sin x + 5\cos x = 13\sin(x + 0.395)$.




**77.** $y = \cos x - 3\sin x = a \sin x + b \cos x \Rightarrow$

$a = -3, b = 1 \Rightarrow \sqrt{a^2 + b^2} = \sqrt{10} = A.$ So $\theta$ is any angle in standard position that has (−3, 1) on its terminal side $\Rightarrow \tan\theta = -\dfrac{1}{3} \Rightarrow$

$\theta \approx -0.322 + \pi = 2.820$.

$y = \cos x - 3\sin x = \sqrt{10}\sin(x + 2.820)$.

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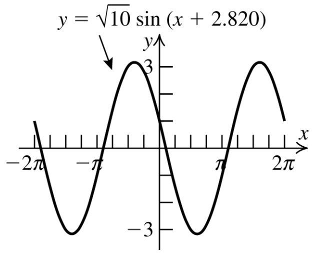
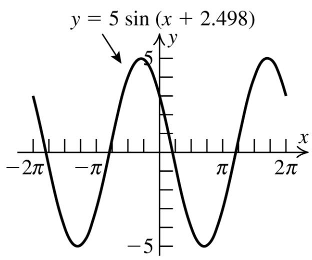



**78.** $y = 3\cos x - 4\sin x = a\sin x + b\cos x \Rightarrow a = -4, b = 3 \Rightarrow \sqrt{a^2 + b^2} = 5 = A.$ So $\theta$ is any angle in standard position that has (–4, 3) on its terminal side $\Rightarrow \tan\theta = -\frac{3}{4} \Rightarrow \theta \approx -0.644 + \pi = 2.498$.

$y = 3\cos x - 4\sin x = 5\sin(x + 2.498)$.




## 6.2 Applying the Concepts

**79.** $\alpha + \theta_1 + (180° - \theta_2) = 180° \Rightarrow \alpha = -(\theta_1 + \theta_2)$. If $\alpha = \theta_2 - \theta_1, \tan\alpha = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_1 \tan\theta_2} = \frac{m_2 - m_1}{1 + m_1 m_2}$.

If $\alpha = \theta_2 - \theta_1 = \pi,\ \tan\alpha = -\frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_1 \tan\theta_2} = -\frac{m_2 - m_1}{1 + m_1 m_2}$. So, $\tan\alpha = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$.

**80.** $\ell_1 : y = x + 5 \Rightarrow m_1 = 1$
$\ell_2 : y = 4x + 2 \Rightarrow m_2 = 4$
$\tan\alpha = \left|\frac{4-1}{1+4}\right| = \frac{3}{5} \Rightarrow \alpha = \tan^{-1}\left(\frac{3}{5}\right) \approx 0.5404$

**81.** $\ell_1 : y = 2x + 5 \Rightarrow m_1 = 2$
$\ell_2 : 6x - 3y = 21 \Rightarrow y = 2x - 7 \Rightarrow m_2 = 2$
$\tan\alpha = \left|\frac{2-2}{1+(2)(2)}\right| = 0 \Rightarrow \alpha = \tan^{-1}(0) = 0$

**82.** $\ell_1 : y = 2x - 4 \Rightarrow m_1 = 2$
$\ell_2 : y = x + 2y = 21 \Rightarrow y = -\frac{1}{2}x + \frac{7}{2} \Rightarrow m_2 = -\frac{1}{2}$
$\tan\alpha = \left|\frac{2-(-1/2)}{1+(2)(-1/2)}\right| = \frac{5/2}{0} \Rightarrow \tan\alpha$ is undefined $\Rightarrow \alpha = \frac{\pi}{2}$

**83.** $y = 0.4\sin(400t) + 0.3\cos(400t) \Rightarrow a = 0.4, b = 0.3 \Rightarrow \sqrt{0.4^2 + 0.3^2} = 0.5 = A$

$\theta = \tan^{-1}\left(\frac{0.3}{0.4}\right)$

$y = 0.4\sin(400t) + 0.3\cos(400t) \Rightarrow y = 0.5\sin(400t + \theta) = 0.5\sin\left[400\left(t + \frac{\theta}{400}\right)\right] \Rightarrow$

amplitude = 0.5; phase shift $= -\frac{\theta}{400} = -\frac{\tan^{-1}(0.3/0.4)}{400} \approx -0.00161$

**84.** $y = 0.05\sin(600t) + 0.12\cos(600t) \Rightarrow a = 0.05, b = 0.12 \Rightarrow \sqrt{0.05^2 + 0.12^2} = 0.13 = A;$

$\theta = \tan^{-1}(0.12/0.05)$

$y = 0.05\sin(600t) + 0.12\cos(600t) \Rightarrow y = 0.13\sin(600t + \theta) = 0.13\sin\left[600\left(t + \frac{\theta}{600}\right)\right] \Rightarrow$ amplitude = 0.13

phase shift $= -\frac{\theta}{600} = -\frac{\tan^{-1}(0.12/0.05)}{600} \approx -0.00196$

**85. a.** $x = 0.12\sin 2t + 0.5\cos 2t \Rightarrow a = 0.12,$

$b = 0.5 \Rightarrow A = \sqrt{0.12^2 + 0.5^2} = 0.5142$

**b.** $\theta = \tan^{-1}\left(\frac{0.5}{0.12}\right) = \tan^{-1}\left(\frac{25}{6}\right)$

$y = 0.12\sin 2t + 0.5\cos 2t \approx 0.5142\sin(2t + \theta) \approx 0.5142\sin 2\left(t + \frac{\theta}{2}\right) \approx 0.5142\sin\left[2\left(t + \frac{\tan^{-1}(25/6)}{2}\right)\right]$

$\approx 0.5142\sin 2(t + 0.6676) \Rightarrow$ phase shift $= -0.6676.$

Period = $2\pi/2 = \pi \Rightarrow$ frequency = $1/\pi$ .

**86. a.** $x = \frac{1}{2}\sin 3t + \frac{1}{3}\cos 3t \Rightarrow a = \frac{1}{2}, b = \frac{1}{3} \Rightarrow A = \sqrt{(1/2)^2 + (1/3)^2} = \frac{\sqrt{13}}{6} \approx 0.6009$

**b.** $\theta = \tan^{-1}\left(\frac{1/3}{1/2}\right) = \tan^{-1}\left(\frac{2}{3}\right)$

$y = \frac{1}{2}\sin 3t + \frac{1}{3}\cos 3t \approx 0.6009\sin(3t + \theta) \approx 0.6009\sin\left[3\left(t + \frac{\theta}{3}\right)\right] \approx 0.6009\sin\left[3\left(t + \frac{\tan^{-1}(2/3)}{3}\right)\right]$

$\approx 0.6009\sin 3(t + 0.196) \Rightarrow$ phase shift $= -0.196.$

Period = $\frac{2\pi}{3} \Rightarrow$ frequency = $\frac{3}{2\pi}$.

## 6.2 Beyond the Basics

**87.** $\frac{\sin(x+h) - \sin x}{h} = \frac{\sin x\cos h + \sin h\cos x - \sin x}{h} = \frac{\sin x(\cos h - 1) + \sin h\cos x}{h} = \sin x\frac{(\cos h - 1)}{h} + \cos x\frac{\sin h}{h}$

**88.** $\frac{\cos(x+h) - \cos x}{h} = \frac{\cos x\cos h - \sin x\sin h - \cos x}{h} = \frac{\cos x(\cos h - 1) - \sin x\sin h}{h} = \cos x\frac{(\cos h - 1)}{h} - \sin x\frac{\sin h}{h}$

**89.** $\cos u\cos(u+v) + \sin u\sin(u+v) = \cos u(\cos u\cos v - \sin u\sin v) + \sin u(\sin u\cos v + \sin v\cos u)$

$= \cos^2 u\cos v - \cos u\sin u\sin v + \sin^2 u\cos v + \sin u\sin v\cos u$

$= \cos v(\cos^2 u + \sin^2 u) = \cos v$

**90.** $\sin(x+y)\cos y - \cos(x+y)\sin y = \cos y(\sin x\cos y + \sin y\cos x) - \sin y(\cos x\cos y - \sin x\sin y)$
$= \sin x\cos^2 y + \sin y\cos x\cos y - \sin y\cos x\cos y + \sin x\sin^2 y$
$= \sin x(\cos^2 y + \sin^2 y) = \sin x$

**91.** $\sin 5x\cos 3x - \cos 5x\sin 3x = \sin(3x+2x)\cos 3x - \cos(3x+2x)\sin 3x$
$= (\cos 3x)(\sin 3x\cos 2x + \sin 2x\cos 3x) - \sin 3x(\cos 3x\cos 2x - \sin 3x\sin 2x)$
$= \sin 3x\cos 3x\cos 2x + \sin 2x\cos^2 3x - (\sin 3x\cos 3x\cos 2x - \sin^2 3x\sin 2x)$
$= \sin 2x(\cos^2 3x + \sin^2 3x) = \sin 2x$

**92.** $\sin(2x-y)\cos y + \cos(2x-y)\sin y = \cos y(\sin 2x\cos y - \sin y\cos 2x) + \sin y(\cos 2x\cos y + \sin 2x\sin y)$
$= \sin 2x\cos^2 y - \sin y\cos y\cos 2x + \sin y\cos y\cos 2x + \sin 2x\sin^2 y$
$= \sin 2x(\cos^2 y + \sin^2 y) = \sin 2x$

**93.** $\sin\left(\frac{\pi}{2} + x - y\right) = \sin\left(\frac{\pi}{2} - (-x+y)\right) = \cos(-x+y) = \cos(y-x) = \cos x\cos y + \sin x\sin y$

**94.** $\cos\left(\frac{\pi}{2} + x - y\right) = \cos\left(\frac{\pi}{2} - (-x+y)\right) = \sin(-x+y) = \sin(y-x) = \sin y\cos x - \sin x\cos y$

**95.** $\sin(x+y)\sin(x-y) = (\sin x\cos y + \sin y\cos x)(\sin x\cos y - \sin y\cos x) = (\sin^2 x\cos^2 y - \sin^2 y\cos^2 x)$
$= \sin^2 x(1-\sin^2 y) - \sin^2 y(1-\sin^2 x) = \sin^2 x - \sin^2 x\sin^2 y - \sin^2 y + \sin^2 x\sin^2 y$
$= \sin^2 x - \sin^2 y$

**96.** $\cos(\alpha+\beta)\cos(\alpha-\beta) = (\cos\alpha\cos\beta - \sin\beta\sin\alpha)(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = \cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta$
$= \cos^2\alpha(1-\sin^2\beta) - (1-\cos^2\alpha)\sin^2\beta$
$= \cos^2\alpha - \cos^2\alpha\sin^2\beta - \sin^2\beta + \cos^2\alpha\sin^2\beta = \cos^2\alpha - \sin^2\beta$

**97.** $\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right) - \sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right) = \left(\sin\frac{\pi}{4}\cos\frac{x}{2} + \sin\frac{x}{2}\cos\frac{\pi}{4}\right)^2 - \left(\sin\frac{\pi}{4}\cos\frac{x}{2} - \sin\frac{x}{2}\cos\frac{\pi}{4}\right)^2$
$= \left(\frac{\sqrt{2}}{2}\cos\frac{x}{2} + \frac{\sqrt{2}}{2}\sin\frac{x}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\cos\frac{x}{2} - \frac{\sqrt{2}}{2}\sin\frac{x}{2}\right)^2$
$= \left(\frac{1}{2}\cos^2\frac{x}{2} + \cos\frac{x}{2}\sin\frac{x}{2} + \frac{1}{2}\sin^2\frac{x}{2}\right) - \left(\frac{1}{2}\cos^2\frac{x}{2} - \cos\frac{x}{2}\sin\frac{x}{2} + \frac{1}{2}\sin^2\frac{x}{2}\right)$
$= 2\cos\frac{x}{2}\sin\frac{x}{2} = \sin x$

**98.** $\cos^2\left(\frac{\pi}{4}+\frac{x}{2}\right) - \sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right) = \left(\cos\frac{\pi}{4}\cos\frac{x}{2} - \sin\frac{\pi}{4}\sin\frac{x}{2}\right)^2 - \left(\sin\frac{\pi}{4}\cos\frac{x}{2} - \sin\frac{x}{2}\cos\frac{\pi}{4}\right)^2$
$= \left(\frac{\sqrt{2}}{2}\cos\frac{x}{2} - \frac{\sqrt{2}}{2}\sin\frac{x}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\cos\frac{x}{2} - \frac{\sqrt{2}}{2}\sin\frac{x}{2}\right)^2 = 0$

**99.** $\frac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta} + \frac{\sin(\beta-\gamma)}{\sin\beta\sin\gamma} + \frac{\sin(\gamma-\alpha)}{\sin\gamma\sin\alpha}$
$= \frac{\sin\gamma(\sin\alpha\cos\beta - \sin\beta\cos\alpha) + \sin\alpha(\sin\beta\cos\gamma - \cos\beta\sin\gamma) + \sin\beta(\sin\gamma\cos\alpha - \sin\alpha\cos\gamma)}{\sin\alpha\sin\beta\sin\gamma}$
$= \frac{\sin\alpha\sin\gamma\cos\beta - \sin\beta\sin\gamma\cos\alpha + \sin\alpha\sin\beta\cos\gamma - \sin\alpha\sin\gamma\cos\beta + \sin\beta\sin\gamma\cos\alpha - \sin\alpha\sin\beta\cos\gamma}{\sin\alpha\sin\beta\sin\gamma}$
$= 0$

**100.** $\dfrac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}+\dfrac{\sin(\beta-\gamma)}{\cos\beta\cos\gamma}+\dfrac{\sin(\gamma-\alpha)}{\cos\gamma\cos\alpha}$

$=\dfrac{\cos\gamma(\sin\alpha\cos\beta-\sin\beta\cos\alpha)+\cos\alpha(\sin\beta\cos\gamma-\cos\beta\sin\gamma)+\cos\beta(\sin\gamma\cos\alpha-\sin\alpha\cos\gamma)}{\cos\alpha\cos\beta\cos\gamma}$

$=\dfrac{\sin\alpha\cos\gamma\cos\beta-\sin\beta\cos\alpha\cos\gamma+\sin\beta\cos\alpha\cos\gamma-\cos\alpha\sin\gamma\cos\beta+\cos\beta\sin\gamma\cos\alpha-\cos\beta\sin\alpha\cos\gamma}{\cos\alpha\cos\beta\cos\gamma}$

$=0$

**101.** Using the reduction formula, we have $\sin x+\cos x=\sqrt{1^2+1^2}\sin(x+\theta)=\sqrt{2}\sin(x+\theta)$

Since $\theta=\tan^{-1}\left(\dfrac{b}{a}\right)=\tan^{-1}(1)=\dfrac{\pi}{4}$ has the point (1, 1) on its terminal side,

$\sin x+\cos x=\sqrt{2}\sin\left(x+\dfrac{\pi}{4}\right)=1.$

$\sqrt{2}\sin\left(x+\dfrac{\pi}{4}\right)=1\Rightarrow\sin\left(x+\dfrac{\pi}{4}\right)=\dfrac{\sqrt{2}}{2}\Rightarrow x+\dfrac{\pi}{4}=\dfrac{\pi}{4}$ or $x+\dfrac{\pi}{4}=\dfrac{3\pi}{4}\Rightarrow x=0$ or $x=\dfrac{\pi}{2}$

**102.** Using the reduction formula, we have $\sin x-\cos x=\sqrt{1^2+(-1)^2}\sin(x+\theta)=\sqrt{2}\sin(x+\theta)$

Since $\theta=\tan^{-1}\left(\dfrac{b}{a}\right)=\tan^{-1}(-1)=-\dfrac{\pi}{4}$ has the point (1, −1) on its terminal side,

$\sin x-\cos x=\sqrt{2}\sin\left(x-\dfrac{\pi}{4}\right)=1.$

$\sqrt{2}\sin\left(x-\dfrac{\pi}{4}\right)=1\Rightarrow\sin\left(x-\dfrac{\pi}{4}\right)=\dfrac{\sqrt{2}}{2}\Rightarrow x-\dfrac{\pi}{4}=\dfrac{\pi}{4}$ or $x-\dfrac{\pi}{4}=\dfrac{3\pi}{4}\Rightarrow x=\dfrac{\pi}{2}$ or $x=\pi$

**103.** Using the reduction formula, we have $\sin x+\sqrt{3}\cos x=\sqrt{1^2+\left(\sqrt{3}\right)^2}\sin(x+\theta)=2\sin(x+\theta)$

Since $\theta=\tan^{-1}\left(\dfrac{b}{a}\right)=\tan^{-1}\left(\sqrt{3}\right)=\dfrac{\pi}{3}$ has the point $\left(1,\sqrt{3}\right)$ on its terminal side,

$\sin x+\sqrt{3}\cos x=2\sin\left(x+\dfrac{\pi}{3}\right)=-1.$

$2\sin\left(x+\dfrac{\pi}{3}\right)=-1\Rightarrow\sin\left(x+\dfrac{\pi}{3}\right)=-\dfrac{1}{2}\Rightarrow x+\dfrac{\pi}{3}=\dfrac{7\pi}{6}$ or $x+\dfrac{\pi}{3}=\dfrac{11\pi}{6}\Rightarrow x=\dfrac{5\pi}{6}$ or $x=\dfrac{3\pi}{2}$

**104.** Using the reduction formula, we have $\sqrt{3}\sin x-\cos x=\sqrt{\left(\sqrt{3}\right)^2+(-1)^2}\sin(x+\theta)=2\sin(x+\theta)$

Since $\theta=\tan^{-1}\left(\dfrac{b}{a}\right)=\tan^{-1}\left(-\dfrac{1}{\sqrt{3}}\right)=-\dfrac{\pi}{6}$ has the point $\left(\sqrt{3},-1\right)$ on its terminal side,

$\sqrt{3}\sin x-\cos x=2\sin\left(x-\dfrac{\pi}{6}\right)=1.$

$2\sin\left(x-\dfrac{\pi}{6}\right)=1\Rightarrow\sin\left(x-\dfrac{\pi}{6}\right)=\dfrac{1}{2}\Rightarrow x-\dfrac{\pi}{6}=\dfrac{\pi}{6}$ or $x-\dfrac{\pi}{6}=\dfrac{5\pi}{6}\Rightarrow x=\dfrac{\pi}{3}$ or $x=\pi$

**105.** Using the reduction formula, we have $\sin x - \sqrt{3}\cos x = \sqrt{1^2 + \left(-\sqrt{3}\right)^2}\sin\left(x+\theta\right) = 2\sin\left(x+\theta\right)$

Since $\theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$ has the point $\left(1, -\sqrt{3}\right)$ on its terminal side,

$\sin x - \sqrt{3}\cos x = 2\sin\left(x - \frac{\pi}{3}\right) = 2.$

$2\sin\left(x-\frac{\pi}{3}\right) = 2 \Rightarrow \sin\left(x-\frac{\pi}{3}\right) = 1 \Rightarrow x - \frac{\pi}{3} = \frac{\pi}{2} \Rightarrow x = \frac{5\pi}{6}$

**106.** Using the reduction formula, we have $5\sin x + 12\cos x = \sqrt{5^2 + 12^2}\sin\left(x+\theta\right) = 13\sin\left(x+\theta\right)$

Since $\theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{12}{5}\right) \approx 1.1760$ has the point $\left(5, 12\right)$ on its terminal side,

$5\sin x + 12\cos x = 2\sin\left(x + 1.1760\right) = 14.$

$2\sin\left(x+1.1760\right) = 14 \Rightarrow \sin\left(x+1.1760\right) = 7,$ which is impossible. Thus, the solution set is $\varnothing$.

**107.** $\cos^2 15° = \cos^2(45° - 30°) = \left(\cos 45° \cos 30° + \sin 45° \sin 30°\right)^2$

$$= \left(\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)\right)^2 = \left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)^2 = \frac{2+\sqrt{3}}{4}$$

$$\cos^2 75° = \cos^2(45° + 30°) = \left(\cos 45° \cos 30° - \sin 45° \sin 30°\right)^2 = \left(\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)\right)^2$$

$$= \left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)^2 = \frac{2-\sqrt{3}}{4}$$

$$\cos^2 15° - \cos^2 30° + \cos^2 45° - \cos^2 60° + \cos^2 75° = \frac{2+\sqrt{3}}{4} - \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + \frac{2-\sqrt{3}}{4} = \frac{1}{2}$$

**108.** $\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{5\pi}{8} + \cos^2\frac{7\pi}{8} = \cos^2\frac{\pi}{8} + \cos^2\left(\frac{\pi}{2} - \frac{\pi}{8}\right) + \cos^2\left(\frac{\pi}{2} + \frac{\pi}{8}\right) + \cos^2\left(2\pi - \frac{\pi}{8}\right)$

$$= \cos^2\frac{\pi}{8} + \left(\cos\frac{\pi}{2}\cos\frac{\pi}{8} + \sin\frac{\pi}{8}\sin\frac{\pi}{2}\right)^2 + \left(\cos\frac{\pi}{2}\cos\frac{\pi}{8} + \sin\frac{\pi}{8}\sin\frac{\pi}{2}\right)^2 + \left(\cos 2\pi\cos\frac{\pi}{8} + \sin\frac{\pi}{8}\,cins 2\pi\right)^2$$

$\left(\text{Note that}\cos\frac{\pi}{2} = \sin 2\pi = 0 \text{ and } \sin\frac{\pi}{2} = \cos 2\pi = 1\right)$

$$= \cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8} + \sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8} = 1 + 1 = 2$$

**109.** $\cos 60° + \cos 80° + \cos 100° = \cos 60° + \cos(90° - 10°) + \cos(90° + 10°)$

$$= \cos 60° + (\cos 90° \cos 10° + \sin 90° \sin 10°) + (\cos 90° \cos 10° - \sin 90° \sin 10°) = \frac{1}{2}$$

**110.** $\sin 30° - \sin 70° + \sin 110° = \sin 30° - \sin(90° - 20°) + \sin(90° + 20°)$

$$= \sin 30° - (\sin 90° \cos 20° - \cos 90° 20°) + (\sin 90° \cos 20° + \cos 90° 20°) = \frac{1}{2}$$

**111.** $\sin\left[\tan^{-1}\left(-\frac{3}{4}\right) + \cos^{-1}\left(\frac{4}{5}\right)\right] = \sin\left(\tan^{-1}\left(-\frac{3}{4}\right)\right)\cos\left(\cos^{-1}\left(\frac{4}{5}\right)\right) + \cos\left(\tan^{-1}\left(-\frac{3}{4}\right)\right)\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right)$

$$= \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = 0$$

**112.** $\cos\left[\sin^{-1}\left(-\frac{3}{5}\right)+\cos^{-1}\left(\frac{3}{5}\right)\right]=\cos\left(\sin^{-1}\left(-\frac{3}{5}\right)\right)\cos\left(\cos^{-1}\left(\frac{3}{5}\right)\right)-\sin\left(\sin^{-1}\left(-\frac{3}{5}\right)\right)\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$
$=\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)-\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)=\frac{24}{25}$

**113.** $\sin\left[\sin^{-1}\left(\frac{3}{5}\right)-\cos^{-1}\left(\frac{4}{5}\right)\right]=\sin\left(\sin^{-1}\left(\frac{3}{5}\right)\right)\cos\left(\cos^{-1}\left(\frac{4}{5}\right)\right)-\cos\left(\sin^{-1}\left(\frac{3}{5}\right)\right)\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right)$
$=\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)-\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)=0$

**114.** $\cos\left[\sin^{-1}\left(\frac{3}{5}\right)+\tan^{-1}\left(-\frac{4}{3}\right)\right]=\cos\left(\sin^{-1}\left(\frac{3}{5}\right)\right)\cos\left(\tan^{-1}\left(-\frac{4}{3}\right)\right)-\sin\left(\sin^{-1}\left(\frac{3}{5}\right)\right)\sin\left(\tan^{-1}\left(-\frac{4}{3}\right)\right)$
$=\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)-\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)=\frac{24}{25}$

**115.** $\tan\left[\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right]=\dfrac{\tan\left(\cos^{-1}\left(\frac{4}{5}\right)\right)+\tan\left(\tan^{-1}\left(\frac{2}{3}\right)\right)}{1-\tan\left(\cos^{-1}\left(\frac{4}{5}\right)\right)\tan\left(\tan^{-1}\left(\frac{2}{3}\right)\right)}=\dfrac{\frac{3}{4}+\frac{2}{3}}{1-\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)}=\frac{17}{6}$

**116.** $\tan\left[\sin^{-1}\left(-\frac{3}{5}\right)+\cos^{-1}\left(\frac{4}{5}\right)\right]=\dfrac{\tan\left(\sin^{-1}\left(-\frac{3}{5}\right)\right)+\tan\left(\cos^{-1}\left(\frac{4}{5}\right)\right)}{1-\tan\left(\sin^{-1}\left(-\frac{3}{5}\right)\right)\tan\left(\cos^{-1}\left(\frac{4}{5}\right)\right)}=\dfrac{-\frac{3}{4}+\frac{3}{4}}{1-\left(-\frac{3}{4}\right)\left(\frac{3}{4}\right)}=0$

**117.** $2\tan 50^\circ=2\tan(70^\circ-20^\circ)=2\left(\dfrac{\tan 70^\circ-\tan 20^\circ}{1+\tan 70^\circ\tan 20^\circ}\right)=2\left(\dfrac{\tan 70^\circ-\tan 20^\circ}{1+\tan(90^\circ-20^\circ)\tan 20^\circ}\right)=2\left(\dfrac{\tan 70^\circ-\tan 20^\circ}{1+\cot 20^\circ\tan 20^\circ}\right)$
$=2\left(\dfrac{\tan 70^\circ-\tan 20^\circ}{1+1}\right)=\tan 70^\circ-\tan 20^\circ$

**118.** $\sin(\theta+n\pi)=\sin\theta\cos n\pi+\sin n\pi\cos\theta=\sin\theta\cos n\pi$ (because $\sin n\pi=0$)
$\cos n\pi=1$ for $n$ even; $\cos n\pi=-1$ for $n$ odd. So, $\sin\theta\cos n\pi=(-1)^n\sin\theta$

**119.** Pair the factors as follows:
$\tan 1^\circ\tan 2^\circ\tan 3^\circ\cdots\tan 87^\circ\tan 88^\circ\tan 89^\circ=(\tan 1^\circ\tan 89^\circ)(\tan 2^\circ\tan 88^\circ)(\tan 3^\circ\tan 87^\circ)\cdots\tan 1^\circ\tan 89^\circ$
$=\tan(90^\circ-89^\circ)\tan 89^\circ=\cot 89^\circ\tan 89^\circ=\dfrac{1}{\tan 89^\circ}\cdot\tan 89^\circ=1$

Similarly, we can show that $(\tan 2^\circ\tan 88^\circ)=1$, etc. Therefore, the product of the tangents is 1.

**120.** Let $u=\sin^{-1}x$ and $v=\sin^{-1}y$. Then $\sin(u+v)=\sin u\cos v+\sin v\cos u$.
$\cos u=\sqrt{1-x^2}$ and $\cos v=\sqrt{1-y^2}$, so $\sin(u+v)=x\sqrt{1-y^2}+y\sqrt{1-x^2}\Rightarrow$
$\sin^{-1}(\sin(u+v))=\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)\Rightarrow u+v=\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)\Rightarrow$
$\sin^{-1}x+\sin^{-1}y=\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)$.

Similarly, we can show that $\sin^{-1}x-\sin^{-1}y=\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right)$.

**121.** Let $u = \cos^{-1} x$ and let $v = \cos^{-1} y$. Then $\cos(u+v) = \cos u \cos v - \sin u \sin v$.
$\sin u = \sqrt{1-x^2}$ and $\sin v = \sqrt{1-y^2}$, so
$\cos(u+v) = \cos u \cos v - \sqrt{1-x^2}\sqrt{1-y^2} \Rightarrow$
$\cos^{-1}(\cos(u+v))$
$= \cos^{-1}\left(\cos u \cos v - \sqrt{1-x^2}\sqrt{1-y^2}\right) \Rightarrow$
$u + v = \cos^{-1}\left(\cos u \cos v - \sqrt{1-x^2}\sqrt{1-y^2}\right) \Rightarrow$
$\cos^{-1} x + \cos^{-1} y$
$= \cos^{-1}\left(\cos u \cos v - \sqrt{1-x^2}\sqrt{1-y^2}\right)$
$= \cos^{-1}\left(\cos(\cos^{-1} x)\cos(\cos^{-1} y) - \sqrt{1-x^2}\sqrt{1-y^2}\right)$
$= \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$

**122.** Let $u = \sin^{-1} x$ and let $v = \cos^{-1} x$.
$\sin\left(\frac{\pi}{2} - u\right) = \cos u \Rightarrow$
$\sin^{-1}\left(\sin\left(\frac{\pi}{2} - u\right)\right) = \sin^{-1}(\cos u) \Rightarrow$
$\frac{\pi}{2} - u = v \Rightarrow \frac{\pi}{2} = u + v \Rightarrow$
$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.

## 6.2 Critical Thinking/Discussion/Writing

**123.** Answers may vary. Sample answer: $u = 0$, $v = 0$.
$\cos(0+0) = \cos 0 = 1$, while
$\cos 0 + \cos 0 = 1 + 1 = 2$.

**124.** Answers may vary. Sample answer: $u = \frac{\pi}{2}$, $v = 0$.
$\cos\left(\frac{\pi}{2} - 0\right) = \cos\frac{\pi}{2} = 0$, while
$\cos\frac{\pi}{2} - \cos 0 = 0 - 1 = -1$.

**125.** $\tan\left(\frac{\pi}{2} - x\right) = \dfrac{\tan\frac{\pi}{2} - \tan x}{1 + \tan\frac{\pi}{2}\tan x}$.

However, $\tan\frac{\pi}{2}$ is undefined, so the fraction is also undefined.

**126.**
$$\cot(x-y) = \frac{\cos(x-y)}{\sin(x-y)} = \frac{\cos x \cos y + \sin x \sin y}{\sin x \cos y - \cos x \sin y} = \frac{\frac{\cos x \cos y + \sin x \sin y}{\sin x \sin y}}{\frac{\sin x \cos y - \cos x \sin y}{\sin x \sin y}} = \frac{\frac{\cos x \cos y}{\sin x \sin y} + \frac{\sin x \sin y}{\sin x \sin y}}{\frac{\sin x \cos y}{\sin x \sin y} - \frac{\cos x \sin y}{\sin x \sin y}} = \frac{\cot x \cot y + 1}{\cot y - \cot x}$$

**127.** Answers may vary. Sample answer: $u = \pi$, $v = \pi$.
$\cos(\pi + \pi) = \cos(2\pi) = 1$, and
$\cos\pi\cos\pi = (-1)(-1) = 1$

## 6.2 Maintaining Skills

**128.** $\theta$ is in quadrant IV, so $\sin\theta < 0$.
$\cos\theta = \frac{4}{5} = \frac{x}{r}$
$r^2 = x^2 + y^2 \Rightarrow 5^2 = 4^2 + y^2 \Rightarrow y = -3$
$\sin\theta = -\frac{3}{5}$

**129.** $\theta$ is in quadrant III, so $\tan\theta > 0$.
$\sin\theta = -\frac{1}{\sqrt{17}} = \frac{y}{r}$
$r^2 = x^2 + y^2 \Rightarrow \left(\sqrt{17}\right)^2 = x^2 + (-1)^2 \Rightarrow$
$x = -4$
$\tan\theta = \frac{-1}{-4} = \frac{1}{4}$

**130.** In quadrant III, $\sin\theta < 0$ and $\cos\theta < 0$, so $\sin\theta\cos\theta$ is positive.

**131.** In quadrant IV, $\tan\theta$ is negative and $\csc\theta$ is negative, so $\tan\theta\sec\theta$ is positive.

**132.** In quadrant III, $\tan\theta > 0$, $\cot\theta > 0$, and $\sin\theta < 0$, so $\frac{\cot\theta\tan\theta}{\sin\theta}$ is negative.

**133.** In quadrant III, $\cos\theta < 0$, $\sin\theta < 0$, and $\sec\theta < 0$, so $\frac{\cos\theta\sin\theta}{\sec\theta}$ is negative.

**134.** $\theta = \frac{\pi}{6} \Rightarrow \sin 2\theta = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

**135.** $\theta = -\frac{13\pi}{12} \Rightarrow 2\theta = -\frac{13\pi}{6}$
$\tan\left(-\frac{13\pi}{6}\right) = -\tan\left(\frac{13\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$

**136.** $\theta = -\frac{\pi}{3} \Rightarrow \frac{\theta}{2} = -\frac{\pi}{6}$
$\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$

**137.** $\theta = \frac{39\pi}{2} \Rightarrow \frac{\theta}{2} = \frac{39\pi}{4}$
$\cos\frac{\theta}{2} = \cos\frac{39\pi}{4} = \cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

## 6.3 Double-Angle and Half-Angle Formulas

### 6.3 Practice Problems

**1.** $\sin x = \frac{12}{13}, x$ in Quadrant II $\Rightarrow$
$\cos x = -\frac{5}{13}, \tan x = -\frac{12}{5}$

**a.** $\sin 2x = 2\sin x\cos x = 2\left(\frac{12}{13}\right)\left(-\frac{5}{13}\right) = -\frac{120}{169}$

**b.** $\cos 2x = \cos^2 x - \sin^2 x = \left(-\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2$
$= -\frac{119}{169}$

**c.** $\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\left(-\frac{12}{5}\right)}{1-\left(-\frac{12}{5}\right)^2} = \frac{-\frac{24}{5}}{-\frac{119}{25}}$
$= \frac{120}{119}$

**2. a.** $2\cos^2\left(\frac{\pi}{12}\right) - 1 = \cos\left(2\cdot\frac{\pi}{12}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

**b.** $\cos^2 22.5° - \sin^2 22.5° = \cos(2\cdot 22.5°)$
$= \cos 45° = \frac{\sqrt{2}}{2}$

**3.**
$$\begin{aligned}\cos 3x &= \cos(2x+x) = \cos 2x\cos x - \sin 2x\sin x\\ &= (2\cos^2 x - 1)\cos x - (2\sin x\cos x)\sin x\\ &= 2\cos^3 x - \cos x - 2\sin^2 x\cos x\\ &= 2\cos^3 x - \cos x - 2\cos x(1-\cos^2 x)\\ &= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x\\ &= 4\cos^3 x - 3\cos x\end{aligned}$$

**4.**
$$\begin{aligned}\sin^4 x &= (\sin^2 x)^2 = \left(\frac{1-\cos 2x}{2}\right)^2\\ &= \frac{1}{4}(1 - 2\cos 2x + \cos^2 2x)\\ &= \frac{1}{4}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)\\ &= \frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{8} + \frac{1}{8}\cos 4x\\ &= \frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x\end{aligned}$$

**5.** $P = VI = 170\sin(120\pi t)\cdot 0.83\sin(120\pi t)$
$= 141.1\sin^2(120\pi t)$

The maximum value of $\sin^2(120\pi t)$ is 1, so $P_{max} = 141.1$ watts $\Rightarrow$
wattage rating $= \frac{141.1}{\sqrt{2}} \approx 99.8$ watts

**6.** $\sin(112.5°) = \sin\left(\frac{225°}{2}\right) = \sqrt{\frac{1-\cos(225°)}{2}}$
$= \sqrt{\frac{1-\left(-\sqrt{2}/2\right)}{2}} = \frac{\sqrt{2+\sqrt{2}}}{2}$

**7.** $\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$

From Example 7, $\cos\theta = -\frac{12}{13}$, and $\frac{\theta}{2}$ lies in quadrant II. Therefore, $\cos\frac{\theta}{2}$ is negative.

$\cos\frac{\theta}{2} = -\sqrt{\frac{1+\left(-\frac{12}{13}\right)}{2}} = -\frac{\sqrt{26}}{26}$

**8.**
$$\begin{aligned}\sin\frac{x}{2}\sin x &= \sin\frac{x}{2}\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)\\ &= 2\sin^2\frac{x}{2}\cos\frac{x}{2}\\ &= 2\cos\frac{x}{2}\left(\frac{1-\cos x}{2}\right)\\ &= \cos\frac{x}{2}(1-\cos x)\end{aligned}$$

## 6.3 Basic Concepts and Skills

1. The double-angle formula for $\sin 2x = \underline{2 \sin x \cos x}$.

2. In the double-angle formula $\cos 2x = \cos^2 x - \sin^2 x$, replace $\cos^2 x$ by $1 - \sin^2 x$ to obtain a double-angle formula $\cos 2x = \underline{1 - 2\sin^2 x}$ in terms of $\sin^2 x$. Solve this formula for $\sin^2 x$ to obtain the power-reducing formula $\sin^2 x = \underline{\frac{1 - \cos 2x}{2}}$.

3. The formula for cos 2*x* in terms of $\cos^2 x$ is $\cos 2x = \underline{2\cos^2 x - 1}$. Solve this formula for $\cos^2 x$ to obtain the power-reducing formula $\cos^2 x = \underline{\frac{1 + \cos 2x}{2}}$.

4. False. $\frac{1 - \cos 2x}{1 + \cos 2x} = \frac{1 - \left(1 - 2\sin^2 x\right)}{1 + \left(2\cos^2 x - 1\right)} = \frac{2\sin^2 x}{2\cos^2 x}$
$= \tan^2 x \neq \tan 2x$

5. False.
$\frac{1}{2}\tan 2x = \frac{1}{2}\left(\frac{2\tan x}{1 - \tan^2 x}\right) = \frac{\tan x}{1 - \tan^2 x} \neq \tan x$

6. True

7. $\sin\theta = \frac{3}{5}, \theta$ in Quadrant II $\Rightarrow \cos\theta = -\frac{4}{5}$,
$\tan\theta = -\frac{3}{4}$

   **a.** $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = -\frac{24}{25}$

   **b.** $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(-\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$

   **c.** $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2(-3/4)}{1 - (-3/4)^2} = -\frac{24}{7}$

8. $\cos\theta = -\frac{5}{13}, \theta$ in Quadrant III $\Rightarrow$
$\sin\theta = -\frac{12}{13}, \tan\theta = \frac{12}{5}$

   **a.** $\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(-\frac{12}{13}\right)\left(-\frac{5}{13}\right) = \frac{120}{169}$

   **b.** $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(-\frac{5}{13}\right)^2 - \left(-\frac{12}{13}\right)^2 = -\frac{119}{169}$

   **c.** $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2(12/5)}{1 - (12/5)^2} = -\frac{120}{119}$

9. $\tan\theta = 4, \sin\theta < 0 \Rightarrow \theta$ is in Quadrant III $\Rightarrow$
$\sin\theta = -\frac{4}{\sqrt{17}}, \cos\theta = -\frac{1}{\sqrt{17}}$

   **a.** $\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(-\frac{4}{\sqrt{17}}\right)\left(-\frac{1}{\sqrt{17}}\right) = \frac{8}{17}$

   **b.** $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(-\frac{1}{\sqrt{17}}\right)^2 - \left(-\frac{4}{\sqrt{17}}\right)^2 = -\frac{15}{17}$

   **c.** $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2(4)}{1 - (4)^2} = -\frac{8}{15}$

10. $\sec\theta = -\sqrt{3}, \sin\theta > 0 \Rightarrow \theta$ is in Quadrant II $\Rightarrow$
$\cos\theta = -\frac{1}{\sqrt{3}}, \sin\theta = \frac{\sqrt{2}}{\sqrt{3}}, \tan\theta = -\sqrt{2}$

   **a.** $\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(\frac{\sqrt{2}}{\sqrt{3}}\right)\left(-\frac{1}{\sqrt{3}}\right) = -\frac{2\sqrt{2}}{3}$

   **b.** $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(-\frac{1}{\sqrt{3}}\right)^2 - \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 = -\frac{1}{3}$

   **c.** $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{-2\sqrt{2}}{1 - \left(-\sqrt{2}\right)^2} = 2\sqrt{2}$

11. $\tan\theta = -2, \frac{\pi}{2} < \theta < \pi \Rightarrow \theta$ is in Quadrant II $\Rightarrow$
$\sin\theta = \frac{2}{\sqrt{5}}, \cos\theta = -\frac{1}{\sqrt{5}}$

   **a.** $\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(\frac{2}{\sqrt{5}}\right)\left(-\frac{1}{\sqrt{5}}\right) = -\frac{4}{5}$

   **b.** $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(-\frac{1}{\sqrt{5}}\right)^2 - \left(\frac{2}{\sqrt{5}}\right)^2 = -\frac{3}{5}$

**c.** $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2(-2)}{1-(-2)^2} = \dfrac{4}{3}$

**12.** $\cot\theta = -7, \dfrac{3\pi}{2} < \theta < 2\pi \Rightarrow$
$\theta$ is in Quadrant IV $\Rightarrow$
$\sin\theta = -\dfrac{1}{5\sqrt{2}}, \cos\theta = \dfrac{7}{5\sqrt{2}}, \tan\theta = -\dfrac{1}{7}$

**a.** $\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(-\dfrac{1}{5\sqrt{2}}\right)\left(\dfrac{7}{5\sqrt{2}}\right) = -\dfrac{7}{25}$

**b.** $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(\dfrac{7}{5\sqrt{2}}\right)^2 - \left(-\dfrac{1}{5\sqrt{2}}\right)^2 = \dfrac{24}{25}$

**c.** $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2(-1/7)}{1-(-1/7)^2} = -\dfrac{7}{24}$

**13.** $1-2\sin^2 75° = \cos(2\cdot 75°) = \cos 150° = -\dfrac{\sqrt{3}}{2}$

**14.** $\dfrac{2\tan 75°}{1-\tan^2 75°} = \tan 150° = -\dfrac{\sqrt{3}}{3}$

**15.** $2\cos^2 105° - 1 = \cos 210° = -\dfrac{\sqrt{3}}{2}$

**16.** $1-2\sin^2 165° = \cos(2\cdot 165°) = \cos 330° = \dfrac{\sqrt{3}}{2}$

**17.** $\dfrac{2\tan 165°}{1-\tan^2 165°} = \tan(2\cdot 165°) = \tan 330° = -\dfrac{\sqrt{3}}{3}$

**18.** $2\cos^2 165° - 1 = \cos 330° = \dfrac{\sqrt{3}}{2}$

**19.** $1-2\sin^2\dfrac{\pi}{8} = \cos\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

**20.** $2\cos^2\left(-\dfrac{\pi}{8}\right) - 1 = \cos\left(-\dfrac{\pi}{4}\right) = \cos\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

**21.** $\dfrac{2\tan\left(-\dfrac{5\pi}{12}\right)}{1-\tan^2\left(-\dfrac{5\pi}{12}\right)} = \tan\left(-\dfrac{5\pi}{6}\right) = \tan\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{3}$

**22.** $1-2\sin^2\left(-\dfrac{7\pi}{12}\right) = \cos\left(-\dfrac{7\pi}{6}\right)$
$= \cos\dfrac{7\pi}{6} = -\dfrac{\sqrt{3}}{2}$

**23.** $\sin 4\theta = \sin[2(2\theta)] = 2\sin 2\theta\cos 2\theta$
$= 2(2\sin\theta\cos\theta)(1-2\sin^2\theta)$
$= \cos\theta(4\sin\theta - 8\sin^3\theta)$

**24.** $\cos 4\theta = \cos(2\theta + 2\theta)$
$= \cos 2\theta\cos 2\theta - \sin 2\theta\sin 2\theta$
$= (2\cos^2\theta - 1)^2 - 4\sin^2\theta\cos^2\theta$
$= 4\cos^4\theta - 4\cos^2\theta + 1$
$\quad - 4(1-\cos^2\theta)\cos^2\theta$
$= 4\cos^4\theta - 4\cos^2\theta + 1$
$\quad - 4\cos^2\theta + 4\cos^4\theta$
$= 8\cos^4\theta - 8\cos^2\theta + 1$

**25.** $\cos^4 x - \sin^4 x$
$= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$
$= \cos^2 x - \sin^2 x = \cos 2x$

**26.** $1 + \cos 2x + 2\sin^2 x$
$= 1 + (1 - 2\sin^2 x) + 2\sin^2 x = 2$

**27.** We start with the right side.
$\dfrac{\cos x + \sin x}{\cos x - \sin x} = \dfrac{\cos x + \sin x}{\cos x - \sin x}\cdot\dfrac{\cos x + \sin x}{\cos x + \sin x}$
$= \dfrac{\cos^2 x + 2\sin x\cos x + \sin^2 x}{\cos^2 x - \sin^2 x}$
$= \dfrac{1 + 2\sin x\cos x}{\cos 2x} = \dfrac{1 + \sin 2x}{\cos 2x}$

**28.** $\dfrac{\cos 2x}{\sin 2x} + \dfrac{\sin x}{\cos x} = \dfrac{1-2\sin^2 x}{2\sin x\cos x} + \dfrac{\sin x}{\cos x}$
$= \dfrac{1-2\sin^2 x + 2\sin^2 x}{2\sin x\cos x}$
$= \dfrac{1}{\sin 2x} = \csc 2x$

**29.** $\dfrac{\sin 2x}{\sin x} - \dfrac{\cos 2x}{\cos x}$
$= \dfrac{2\sin x\cos x}{\sin x} - \dfrac{2\cos^2 x - 1}{\cos x}$
$= \dfrac{2\sin x\cos^2 x - 2\cos^2 x\sin x + \sin x}{\sin x\cos x}$
$= \dfrac{1}{\cos x} = \sec x$

**30.** $\frac{\cos 3x}{\sin 3x}+\frac{\sin x}{\cos x}=\frac{\cos x(4\cos^3 x-3\cos x)+\sin x(3\sin x-4\sin^3 x)}{\sin 3x\cos x}=\frac{4\cos^4 x-3\cos^2 x+3\sin^2 x-4\sin^4 x}{\sin 3x\cos x}$
$=\frac{4(\cos^4 x-\sin^4 x)-3(\cos^2 x-\sin^2 x)}{\sin 3x\cos x}$
$=\frac{4(\cos^2 x-\sin^2 x)(\cos^2 x+\sin^2 x)-3(\cos^2 x-\sin^2 x)}{\sin 3x\cos x}=\frac{4\cos 2x-3\cos 2x}{\sin 3x\cos x}=\frac{\cos 2x}{\sin 3x\cos x}$

**31.** $\tan 2x+\tan x=\frac{\sin 2x}{\cos 2x}+\frac{\sin x}{\cos x}=\frac{2\sin x\cos^2 x+(1-2\sin^2 x)\sin x}{\cos 2x\cos x}=\frac{2\sin x\cos^2 x+\sin x-2\sin^3 x}{\cos 2x\cos x}$
$=\frac{2\sin x(1-\sin^2 x)+\sin x-2\sin^3 x}{\cos 2x\cos x}=\frac{2\sin x-2\sin^3 x+\sin x-2\sin^3 x}{\cos 2x\cos x}$
$=\frac{3\sin x-4\sin^3 x}{\cos 2x\cos x}=\frac{\sin 3x}{\cos 2x\cos x}$

**32.** $\tan 2x-\tan x=\frac{\sin 2x}{\cos 2x}-\frac{\sin x}{\cos x}=\frac{2\sin x\cos^2 x-\sin x(2\cos^2 x-1)}{\cos 2x\cos x}=\frac{2\sin x\cos^2 x-2\cos^2 x\sin x+\sin x}{\cos 2x\cos x}$
$=\frac{\sin x}{\cos 2x\cos x}=\tan x\sec 2x$

**33.** $4\sin^2 x\cos^2 x=4\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)$
$=1-\cos^2 2x=1-\frac{1+\cos 4x}{2}$
$=\frac{1-\cos 4x}{2}$

**34.** $\sin^2 x\cos^2 x=\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)$
$=\frac{1-\cos^2 2x}{4}=\frac{1-\frac{1+\cos 4x}{2}}{4}$
$=\frac{1-\cos 4x}{8}$

**35.** $4\sin x\cos x(1-2\sin^2 x)=4\sin x\cos x(\cos 2x)$
$=2\sin 2x\cos 2x$
$=\sin 4x$

**36.** $4\sin x\cos x(2\cos^2 x-1)=4\sin x\cos x(\cos 2x)$
$=2\sin 2x\cos 2x$
$=\sin 4x$

**37.** $2\sin 3x\cos 3x(2\cos^2 3x-1)=\sin 6x\cos 6x$
$=\frac{\sin 12x}{2}$

**38.** $\sin 8x(1-2\sin^2 4x)=\sin 8x\left(1-2\left(\frac{1-\cos 8x}{2}\right)\right)$
$=\sin 8x\cos 8x=\frac{\sin 16x}{2}$

**39.** $\sin\frac{x}{2}\cos\frac{x}{2}\left(1-2\sin^2\frac{x}{2}\right)$
$=\sin\frac{x}{2}\cos\frac{x}{2}\left(1-2\left(\frac{1-\cos x}{2}\right)\right)$
$=\frac{\sin x}{2}(\cos x)=\frac{\sin x\cos x}{2}=\frac{\sin 2x}{4}$

**40.** $\sin x\left(2\cos^2\frac{x}{2}-1\right)=\sin x\cos x=\frac{\sin 2x}{2}$

**41.** $8\sin^4\frac{x}{2}=8\left(\sin^2\frac{x}{2}\right)^2=8\left(\frac{1-\cos x}{2}\right)^2$
$=8\left(\frac{1-2\cos x+\cos^2 x}{4}\right)$
$=2-4\cos x+2\cos^2 x$
$=2-4\cos x+2\left(\frac{1+\cos 2x}{2}\right)$
$=\cos 2x-4\cos x+3$

**42.** $8\cos^4\frac{x}{2}=8\left(\cos^2\frac{x}{2}\right)^2=8\left(\frac{1+\cos x}{2}\right)^2$
$=8\left(\frac{1+2\cos x+\cos^2 x}{4}\right)$
$=2+4\cos x+2\cos^2 x$
$=2+4\cos x+2\left(\frac{1+\cos 2x}{2}\right)$
$=\cos 2x+4\cos x+3$

**43.** $\sin\left(\frac{\pi}{12}\right)=\sin\left(\frac{\pi/6}{2}\right)=\sqrt{\frac{1-\cos(\pi/6)}{2}}$
$=\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}=\frac{\sqrt{2-\sqrt{3}}}{2}$

**44.** $\sin\left(\frac{\pi}{8}\right)=\sin\left(\frac{\pi/4}{2}\right)=\sqrt{\frac{1-\cos(\pi/4)}{2}}$
$=\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}=\frac{\sqrt{2-\sqrt{2}}}{2}$

**45.** $\cos\frac{\pi}{8}=\cos\left(\frac{\pi/4}{2}\right)=\sqrt{\frac{1+\cos(\pi/4)}{2}}$
$=\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}=\frac{\sqrt{2+\sqrt{2}}}{2}$

**46.** $\tan\frac{\pi}{8}=\tan\left(\frac{\pi/4}{2}\right)=\sqrt{\frac{1-\cos(\pi/4)}{1+\cos(\pi/4)}}$
$=\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}}=\frac{\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}$

**47.** $\sin\left(-\frac{3\pi}{8}\right)=-\sin\left(\frac{3\pi/4}{2}\right)=-\sqrt{\frac{1-\cos(3\pi/4)}{2}}$
$=-\sqrt{\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{2}}=-\frac{\sqrt{2+\sqrt{2}}}{2}$

**48.** $\cos\left(-\frac{3\pi}{8}\right)=\cos\left(\frac{3\pi/4}{2}\right)=\sqrt{\frac{1+\cos(3\pi/4)}{2}}$
$=-\sqrt{\frac{1+\left(-\frac{\sqrt{2}}{2}\right)}{2}}=\frac{\sqrt{2-\sqrt{2}}}{2}$

**49.** $\tan\left(\frac{7\pi}{8}\right)=\tan\left(\frac{7\pi/4}{2}\right)=-\sqrt{\frac{1-\cos(7\pi/4)}{1+\cos(7\pi/4)}}$
$=-\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}}=-\frac{\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}$

**50.** $\sec\left(-\frac{7\pi}{8}\right)=\frac{1}{\cos(-7\pi/8)}=\frac{1}{\cos(7\pi/8)}$
$=\frac{1}{\cos\left(\frac{7\pi/4}{2}\right)}=\frac{1}{-\sqrt{\frac{1+\cos(7\pi/4)}{2}}}$
$=-\frac{\sqrt{2}}{\sqrt{1+\cos(\pi/4)}}=-\frac{\sqrt{2}}{\sqrt{1+\frac{\sqrt{2}}{2}}}=-\frac{2}{\sqrt{2+\sqrt{2}}}$
$=-\frac{2}{\sqrt{2+\sqrt{2}}}\cdot\frac{\sqrt{2-\sqrt{2}}}{\sqrt{2-\sqrt{2}}}=\frac{2\sqrt{2-\sqrt{2}}}{\sqrt{2}}$
$=-\sqrt{2}\sqrt{2-\sqrt{2}}$

**51.** $\tan 112.5°=\tan\left(\frac{225°}{2}\right)=-\sqrt{\frac{1-\cos 225°}{1+\cos 225°}}$
$=-\sqrt{\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{1+\left(-\frac{\sqrt{2}}{2}\right)}}=-\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}$
$=-1-\sqrt{2}$

**52.** $\cos 112.5°=\cos\left(\frac{225°}{2}\right)=-\sqrt{\frac{1+\cos 225°}{2}}$
$=-\sqrt{\frac{1+\left(-\frac{\sqrt{2}}{2}\right)}{2}}=-\frac{\sqrt{2-\sqrt{2}}}{2}$

**53.** $\sin(-75°)=-\sin\left(\frac{150°}{2}\right)=-\sqrt{\frac{1-\cos(150°)}{2}}$
$=-\sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}=-\frac{\sqrt{2+\sqrt{3}}}{2}$

**54.** $\tan(-105°)=-\tan\left(\frac{210°}{2}\right)=\sqrt{\frac{1-\cos 210°}{1+\cos 210°}}$
$=\sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{1+\left(-\frac{\sqrt{3}}{2}\right)}}=\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}$

**55.** $\sin\theta=\frac{4}{5},\frac{\pi}{2}<\theta<\pi\Rightarrow\cos\theta=-\frac{3}{5}$
$\frac{\pi}{4}<\frac{\theta}{2}<\frac{\pi}{2}\Rightarrow\sin\frac{\theta}{2}>0,\cos\frac{\theta}{2}>0,\tan\frac{\theta}{2}>0$

**a.** $\sin\frac{\theta}{2}=\sqrt{\frac{1-(-3/5)}{2}}=\frac{2\sqrt{5}}{5}$

**b.** $\cos\frac{\theta}{2}=\sqrt{\frac{1+(-3/5)}{2}}=\frac{\sqrt{5}}{5}$

**c.** $\tan\frac{\theta}{2}=\sqrt{\frac{1-(-3/5)}{1+(-3/5)}}=2$

**56.** $\cos\theta = -\frac{12}{13}, \pi < \theta < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4} \Rightarrow$
$\sin\frac{\theta}{2} > 0, \cos\frac{\theta}{2} < 0, \tan\frac{\theta}{2} < 0$

**a.** $\sin\frac{\theta}{2} = \sqrt{\frac{1-(-12/13)}{2}} = \frac{5\sqrt{26}}{26}$

**b.** $\cos\frac{\theta}{2} = -\sqrt{\frac{1+(-12/13)}{2}} = -\frac{\sqrt{26}}{26}$

**c.** $\tan\frac{\theta}{2} = -\sqrt{\frac{1-(-12/13)}{1+(-12/13)}} = -5$

**57.** $\tan\theta = -\frac{2}{3}, \frac{\pi}{2} < \theta < \pi \Rightarrow \cos\theta = -\frac{3\sqrt{13}}{13}$
$\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{2} \Rightarrow \sin\frac{\theta}{2} > 0, \cos\frac{\theta}{2} > 0, \tan\frac{\theta}{2} > 0$

**a.** $\sin\frac{\theta}{2} = \sqrt{\frac{1-\left(-3\sqrt{13}/13\right)}{2}} = \sqrt{\frac{13+3\sqrt{13}}{26}}$

**b.** $\cos\frac{\theta}{2} = \sqrt{\frac{1+\left(-3\sqrt{13}/13\right)}{2}} = \sqrt{\frac{13-3\sqrt{13}}{26}}$

**c.** $\tan\frac{\theta}{2} = -\sqrt{\frac{1-\left(-3\sqrt{13}/13\right)}{1+\left(-3\sqrt{13}/13\right)}} = \sqrt{\frac{13+3\sqrt{13}}{13-3\sqrt{13}}}$

**58.** $\cot\theta = \frac{3}{4}, \pi < \theta < \frac{3\pi}{2} \Rightarrow \cos\theta = -\frac{3}{5}$
$\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4} \Rightarrow \sin\frac{\theta}{2} > 0, \cos\frac{\theta}{2} < 0, \tan\frac{\theta}{2} < 0$

**a.** $\sin\frac{\theta}{2} = \sqrt{\frac{1-(-3/5)}{2}} = \frac{2\sqrt{5}}{5}$

**b.** $\cos\frac{\theta}{2} = -\sqrt{\frac{1+(-3/5)}{2}} = -\frac{\sqrt{5}}{5}$

**c.** $\tan\frac{\theta}{2} = -\sqrt{\frac{1-(-3/5)}{1+(-3/5)}} = -2$

**59.** $\sin\theta = \frac{1}{5}, \cos\theta < 0 \Rightarrow \cos\theta = -\frac{2\sqrt{6}}{5}$ and
$\frac{\pi}{2} < \theta < \pi \Rightarrow \frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{2} \Rightarrow \sin\frac{\theta}{2} > 0,$
$\cos\frac{\theta}{2} > 0, \tan\frac{\theta}{2} > 0$

**a.** $\sin\frac{\theta}{2} = \sqrt{\frac{1-\left(-2\sqrt{6}/5\right)}{2}} = \sqrt{\frac{5+2\sqrt{6}}{10}}$

**b.** $\cos\frac{\theta}{2} = \sqrt{\frac{1+\left(-2\sqrt{6}/5\right)}{2}} = \sqrt{\frac{5-2\sqrt{6}}{10}}$

**c.** $\tan\frac{\theta}{2} = \sqrt{\frac{1-\left(-2\sqrt{6}/5\right)}{1+\left(-2\sqrt{6}/5\right)}} = \sqrt{\frac{5+2\sqrt{6}}{5-2\sqrt{6}}}$

**60.** $\cos\theta = \frac{2}{3}, \sin\theta < 0 \Rightarrow \frac{3\pi}{2} < \theta < 2\pi$
$\frac{3\pi}{4} < \frac{\theta}{2} < \pi \Rightarrow \sin\frac{\theta}{2} > 0, \cos\frac{\theta}{2} < 0, \tan\frac{\theta}{2} < 0$

**a.** $\sin\frac{\theta}{2} = \sqrt{\frac{1-(2/3)}{2}} = \frac{\sqrt{6}}{6}$

**b.** $\cos\frac{\theta}{2} = -\sqrt{\frac{1+(2/3)}{2}} = -\frac{\sqrt{30}}{6}$

**c.** $\tan\frac{\theta}{2} = \sqrt{\frac{1-(2/3)}{1+(2/3)}} = -\frac{\sqrt{5}}{5}$

**61.** $\sec\theta = \sqrt{5} \Rightarrow \cos\theta = \frac{\sqrt{5}}{5}, \sin\theta > 0 \Rightarrow 0 < \theta < \frac{\pi}{2}$
$0 < \frac{\theta}{2} < \frac{\pi}{4} \Rightarrow \sin\frac{\theta}{2} > 0, \cos\frac{\theta}{2} > 0, \tan\frac{\theta}{2} > 0$

**a.** $\sin\frac{\theta}{2} = \sqrt{\frac{1-\left(\sqrt{5}/5\right)}{2}} = \sqrt{\frac{5-\sqrt{5}}{10}}$

**b.** $\cos\frac{\theta}{2} = \sqrt{\frac{1+\left(\sqrt{5}/5\right)}{2}} = \sqrt{\frac{5+\sqrt{5}}{10}}$

**c.** $\tan\frac{\theta}{2} = \sqrt{\frac{1-\left(\sqrt{5}/5\right)}{1+\left(\sqrt{5}/5\right)}} = \sqrt{\frac{5-\sqrt{5}}{5+\sqrt{5}}}$

**62.** $\csc\theta = \sqrt{7} \Rightarrow \sin\theta = \frac{\sqrt{7}}{7}$ and
$\cos\theta = -\frac{\sqrt{6}}{\sqrt{7}} = -\frac{\sqrt{42}}{7}, \tan\theta < 0 \Rightarrow \frac{\pi}{2} < \theta < \pi$
$\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{2} \Rightarrow \sin\frac{\theta}{2} > 0, \cos\frac{\theta}{2} > 0, \tan\frac{\theta}{2} > 0$

**a.** $\sin\frac{\theta}{2} = \sqrt{\frac{1-\left(-\sqrt{42}/7\right)}{2}} = \sqrt{\frac{7+\sqrt{42}}{14}}$

**b.** $\cos\frac{\theta}{2} = \sqrt{\frac{1+\left(-\sqrt{42}/7\right)}{2}} = \sqrt{\frac{7-\sqrt{42}}{14}}$

**c.** $\tan\frac{\theta}{2} = \sqrt{\frac{1-\left(-\sqrt{42}/7\right)}{1+\left(-\sqrt{42}/7\right)}} = \sqrt{\frac{7+\sqrt{42}}{7-\sqrt{42}}}$

**63.** $\left(\sin\frac{t}{2}+\cos\frac{t}{2}\right)^2$
$= \sin^2\frac{t}{2} + 2\sin\frac{t}{2}\cos\frac{t}{2} + \cos^2\frac{t}{2} = 1+\sin t$

**64.** $\left(\sin\frac{t}{2}-\cos\frac{t}{2}\right)^2 = \sin^2\frac{t}{2} - 2\sin\frac{t}{2}\cos\frac{t}{2} + \cos^2\frac{t}{2}$
$= 1 - \sin 2\left(\frac{t}{2}\right) = 1-\sin t$

**65.** $2\cos^2\frac{x}{2} = 2\left(\frac{1+\cos 2\left(\frac{x}{2}\right)}{2}\right)$
$= (1+\cos x)\cdot\frac{1-\cos x}{1-\cos x}$
$= \frac{1-\cos^2 x}{1-\cos x} = \frac{\sin^2 x}{1-\cos x}$

**66.** $2\sin^2\frac{x}{2} = 2\left(\frac{1-\cos 2\left(\frac{x}{2}\right)}{2}\right)$
$= (1-\cos x)\cdot\frac{1+\cos x}{1+\cos x}$
$= \frac{1-\cos^2 x}{1+\cos x} = \frac{\sin^2 x}{1+\cos x}$

**67.** $\tan\frac{x}{2} = \sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\sqrt{1-\cos x}}{\sqrt{1+\cos x}}\cdot\frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}}$
$= \frac{\sqrt{1-\cos^2 x}}{1+\cos x} = \frac{\sin x}{1+\cos x}$

**68.** $\tan\frac{x}{2} = \sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\sqrt{1-\cos x}}{\sqrt{1+\cos x}}\cdot\frac{\sqrt{1-\cos x}}{\sqrt{1-\cos x}}$
$= \frac{1-\cos x}{\sqrt{1-\cos^2 x}} = \frac{1-\cos x}{\sin x}$

**69.** $\sin^2\frac{x}{2} + \cos x = \frac{1-\cos 2\left(\frac{x}{2}\right)}{2} + \cos x$
$= \frac{1-\cos x}{2} + \cos x = \frac{1+\cos x}{2}$
$= \frac{1+\cos 2\left(\frac{x}{2}\right)}{2} = \cos^2\frac{x}{2}$

**70.** $2\cos^2\frac{x}{2} - \sin^2 x$
$= 2\left(\frac{1+\cos 2\left(\frac{x}{2}\right)}{2}\right) - (1-\cos^2 x)$
$= 1+\cos x - 1 + \cos^2 x = \cos^2 x + \cos x$

**71.** $\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{\frac{2\sin(x/2)}{\cos(x/2)}}{\sec^2\frac{x}{2}} = \frac{2\sin\frac{x}{2}}{\cos\frac{x}{2}}\cdot\cos^2\frac{x}{2}$
$= 2\sin\left(x/2\right)\cdot\cos\left(x/2\right) = \sin x$

**72.** $\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{1-\frac{1-\cos x}{1+\cos x}}{1+\frac{1-\cos x}{1+\cos x}} = \frac{1+\cos x - 1 + \cos x}{1+\cos x + 1 - \cos x}$
$= \frac{2\cos x}{2} = \cos x$

### 6.3 Applying the Concepts

**73.** $P = VI = 170\sin(120\pi t)\cdot 0.832\sin(120\pi t)$
$= 141.44\sin^2(120\pi t)$

The maximum value of $\sin^2(120\pi t)$ is 1, so
$P_{\max} = 141.44$ watts $\Rightarrow$
wattage rating $= \frac{141.44}{\sqrt{2}} \approx 100$ watts

**74.** $P = VI = 170\sin(120\pi t)\cdot 7.487\sin(120\pi t)$
$= 1272.79\sin^2(120\pi t)$

The maximum value of $\sin^2(120\pi t)$ is 1, so
$P_{\max} = 1272.79$ watts $\Rightarrow$
wattage rating $= \frac{1272.79}{\sqrt{2}} \approx 900$ watts

**75.** $P = VI = 170\sin(120\pi t)\cdot 9.983\sin(120\pi t) = 1697.11\sin^2(120\pi t)$

The maximum value of $\sin^2(120\pi t)$ is 1, so $P_{\max} = 1697.11$ watts $\Rightarrow$ wattage rating $= \dfrac{1697.11}{\sqrt{2}} \approx 1200$ watts.

**76.** $P = VI = 170\sin(120\pi t)\cdot 6.655\sin(120\pi t) = 1131.35\sin^2(120\pi t)$

The maximum value of $\sin^2(120\pi t)$ is 1, so $P_{\max} = 1131.35$ watts $\Rightarrow$ wattage rating $= \dfrac{1131.35}{\sqrt{2}} \approx 800$ watts.

**77.** $P = VI = 170\sin(120\pi t)\cdot 4.991\sin(120\pi t) = 848.47\sin^2(120\pi t)$

The maximum value of $\sin^2(120\pi t)$ is 1, so $P_{\max} = 848.47$ watts $\Rightarrow$ wattage rating $= \dfrac{848.47}{\sqrt{2}} \approx 600$ watts.

**78.** $P = VI = 170\sin(120\pi t)\cdot 2.917\sin(120\pi t) = 495.89\sin^2(120\pi t)$

The maximum value of $\sin^2(120\pi t)$ is 1, so $P_{\max} = 495.89$ watts $\Rightarrow$ wattage rating $= \dfrac{495.89}{\sqrt{2}} \approx 351$ watts.

**79.** $x = \dfrac{{v_0}^2}{16}\sin\theta\cos\theta$. Let $v_0 = \sqrt{32}$. Then $x = \left(\dfrac{\left(\sqrt{32}\right)^2}{16}\right)\sin\theta\cos\theta = 2\sin\theta\cos\theta = \sin 2\theta.$

$\sin 2\theta$ is at its maximum, 1, when $2\theta = \pi/2$, so $\theta = \pi/4$.

## 6.3 Beyond the Basics

**80.** $$\tan 3x = \tan(x+2x) = \frac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = \frac{\tan x + \dfrac{2\tan x}{1-\tan^2 x}}{1 - \tan x\left(\dfrac{2\tan x}{1-\tan^2 x}\right)} = \frac{\dfrac{\tan x - \tan^3 x + 2\tan x}{1-\tan^2 x}}{\dfrac{1-\tan^2 x - 2\tan^2 x}{1-\tan^2 x}} = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

**81.** $$\frac{\sin 3x + \cos 3x}{\cos x - \sin x} = \frac{3\sin x - 4\sin^3 x + 4\cos^3 x - 3\cos x}{\cos x - \sin x} = \frac{4(\cos^3 x - \sin^3 x) - 3(\cos x - \sin x)}{\cos x - \sin x}$$
$$= 4(\cos^2 x + \cos x \sin x + \sin^2 x) - 3 = 4(\cos x \sin x + 1) - 3 = 4\cos x \sin x + 1 = 2\sin 2x + 1$$

**82.** $$\frac{\sin x - \cos x}{\sin x + \cos x} - \frac{\sin x + \cos x}{\sin x - \cos x} = \frac{\sin^2 x - 2\sin x\cos x + \cos^2 x - (\sin^2 x + 2\sin x \cos x + \cos^2 x)}{\sin^2 x - \cos^2 x}$$
$$= \frac{-2\sin 2x}{\sin^2 x - \cos^2 x} = \frac{2\sin 2x}{\cos^2 x - \sin^2 x} = \frac{2\sin 2x}{\cos 2x} = 2\tan 2x$$

**83.** $$\frac{2}{\tan x + \cot x} = \frac{2}{\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}} = \frac{2}{\dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}} = 2\sin x\cos x = \sin 2x$$

**84.** We start with the right side.

$$\tan 3x - \tan x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} - \tan x = \frac{3\tan x - \tan^3 x - \tan x + 3\tan^3 x}{1 - 3\tan^2 x} = \frac{2\tan x + 2\tan^3 x}{1 - 3\tan^2 x}$$
$$= \frac{2\tan x(1+\tan^2 x)}{1 - 3\tan^2 x} = \frac{2\tan x \sec^2 x}{1 - 3\tan^2 x} = \frac{2\tan x}{\cos^2 x(1 - 3\tan^2 x)} = \frac{\dfrac{2\sin x}{\cos x}}{\cos^2 x - 3\sin^2 x}$$
$$= \frac{2\sin x}{\cos^3 x - 3\cos x(1-\cos^2 x)} = \frac{2\sin x}{4\cos^3 x - 3\cos x} = \frac{2\sin x}{\cos 3x}$$

**85.** We start with the right side.

$$2\cot 2x = \frac{2\cos 2x}{\sin 2x} = \frac{2(\cos^2 x - \sin^2 x)}{2\sin x\cos x} = \frac{2\cos^2 x}{2\sin x\cos x} - \frac{2\sin^2 x}{2\sin x\cos x} = \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \cot x - \tan x$$

**86.** Let $u = \frac{\pi}{4} - x$.

$$\frac{1-\tan^2\left(\frac{\pi}{4}-x\right)}{1+\tan^2\left(\frac{\pi}{4}-x\right)} \Rightarrow \frac{1-\tan^2 u}{1+\tan^2 u} = \frac{\frac{2\tan u}{\tan 2u}}{\sec^2 u} = \frac{\frac{2\sin u}{\cos u}}{\frac{\sin 2u}{\cos 2u}}\cdot\cos^2 u = \frac{2\sin u}{\cos u}\cdot\frac{\cos 2u}{\sin 2u}\cdot\cos^2 u = \frac{2\sin u\cos u\cos 2u}{\sin 2u}$$

$$= \cos 2u = \cos\left[2\left(\frac{\pi}{4}-x\right)\right] = \cos\left(\frac{\pi}{2}-2x\right) = \sin 2x$$

**87.** $$\frac{1+\sin 2x-\cos 2x}{1+\sin 2x+\cos 2x} = \frac{1+2\sin x\cos x-\cos^2 x+\sin^2 x}{1+2\sin x\cos x+\cos^2 x-\sin^2 x} = \frac{2\sin x\cos x+2\sin^2 x}{2\sin x\cos x+2\cos^2 x} = \frac{2\sin x(\cos x+\sin x)}{2\cos x(\sin x+\cos x)} = \tan x$$

**88.** Start with the right side.

$$\frac{\sin 8\theta}{8\sin\theta} = \frac{2\sin 4\theta\cos 4\theta}{8\sin\theta} = \frac{2(2\sin 2\theta\cos 2\theta)\cos 4\theta}{8\sin\theta} = \frac{2(2(2\sin\theta\cos\theta)\cos 2\theta)\cos 4\theta}{8\sin\theta} = \cos\theta\cos 2\theta\cos 4\theta$$

**89.** $$\sin^2\left(\frac{\pi}{8}+\frac{x}{2}\right) - \sin^2\left(\frac{\pi}{8}-\frac{x}{2}\right) = \frac{1-\cos 2\left(\frac{\pi}{8}+\frac{x}{2}\right)}{2} - \frac{1-\cos 2\left(\frac{\pi}{8}-\frac{x}{2}\right)}{2} = \frac{1-\cos\left(\frac{\pi}{4}+x\right)-1+\cos\left(\frac{\pi}{4}-x\right)}{2}$$

$$= \frac{1-\cos\frac{\pi}{4}\cos x+\sin\frac{\pi}{4}\sin x-1+\cos\frac{\pi}{4}\cos x+\sin\frac{\pi}{4}\sin x}{2}$$

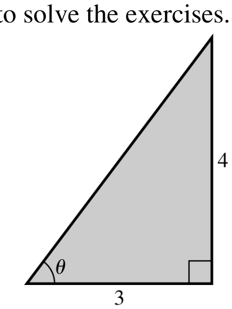
$$= \frac{2\sin\frac{\pi}{4}\sin x}{2} = \frac{1}{\sqrt{2}}\sin x$$

**90.** $$\sqrt{2+\sqrt{2+2\cos 4x}} = \sqrt{2+\sqrt{2+2\cos 2(2x)}} = \sqrt{2+\sqrt{2+2(2\cos^2 2x-1)}} = \sqrt{2+\sqrt{4\cos^2 2x}} = \sqrt{2+2\cos 2x}$$

$$= \sqrt{2+2(2\cos^2 x-1)} = \sqrt{4\cos^2 x} = 2\cos x$$

## 6.3 Critical Thinking/Discussion/Writing

Use the figure to solve the exercises.




**91. a.** $\sin\theta = \frac{4}{5}$

**b.** $\cos\theta = \frac{3}{5}$

**c.** $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \frac{24}{25}$

**d.** $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = -\frac{7}{25}$$

**e.** $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2} = \frac{8/3}{-7/9} = -\frac{24}{7}$

**f.** $\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\left(\frac{3}{5}\right)}{2}} = \frac{\sqrt{5}}{5}$

Note that the answer is positive because $\theta/2$ is an acute angle.

**g.** $\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+\left(\frac{3}{5}\right)}{2}} = \frac{2\sqrt{5}}{5}$

Note that the answer is positive because $\theta/2$ is an acute angle.

**h.** $\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \frac{\frac{\sqrt{5}}{5}}{\frac{2\sqrt{5}}{5}} = \frac{1}{2}$

### 6.3 Maintaining Skills

**92.** $\sin\theta = \sqrt{6}-\sqrt{3} = \cos\left(\frac{\pi}{2}-\theta\right) \Rightarrow$

$\sec\left(\frac{\pi}{2}-\theta\right) = \frac{1}{\sqrt{6}-\sqrt{3}}$

**93.** $\cos\left(\frac{\pi}{2}-\theta\right) = \sqrt{5}-\sqrt{2} = \sin\theta \Rightarrow$

$\csc(\theta) = \frac{1}{\sqrt{5}-\sqrt{2}}$

**94.** $\sin(-\theta) = 0.76 = -\sin\theta \Rightarrow \sin\theta = -0.76$

**95.** $\cos(-\theta) = 0.87 = \cos\theta$

**96.** $\sin(-\theta) = \frac{1}{3} = -\sin\theta \Rightarrow \sin\theta = -\frac{1}{3}$

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \left(-\frac{1}{3}\right)^2 + \cos^2\theta = 1 \Rightarrow$

$\cos^2\theta = \frac{8}{9}$

$\sin\theta\cos^2\theta = -\frac{1}{3}\left(\frac{8}{9}\right) = -\frac{8}{27}$

**97.** $\tan(-\theta) = -2 = -\tan\theta \Rightarrow \tan\theta = 2$

$\tan^2\theta + 1 = \sec^2\theta \Rightarrow 2^2 + 1 = 5 = \sec^2\theta$

$\tan\theta\sec^2\theta = 2\cdot 5 = 10$

**98.** $\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right) = \cos\frac{\pi}{2}\cos\frac{\pi}{6} - \sin\frac{\pi}{2}\sin\frac{\pi}{6}$

$= 0 - \frac{1}{2} = -\frac{1}{2}$

**99.** $\sin\left(\frac{\pi}{6}-\frac{3\pi}{4}\right) = \sin\frac{\pi}{6}\cos\frac{3\pi}{4} - \cos\frac{\pi}{6}\sin\frac{3\pi}{4}$

$= \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{-\sqrt{2}-\sqrt{6}}{4} = -\frac{\sqrt{2}+\sqrt{6}}{4}$

**100.** $\sin\left(\frac{2\pi}{9}\right)\cos\left(\frac{8\pi}{9}\right) - \cos\left(\frac{2\pi}{9}\right)\sin\left(\frac{8\pi}{9}\right)$

$= \sin\left(\frac{2\pi}{9}-\frac{8\pi}{9}\right) = \sin\left(-\frac{2\pi}{3}\right)$

$= -\sin\frac{2\pi}{3} = -\frac{\sqrt{3}}{2}$

**101.** $\cos\frac{\pi}{2}\cos\frac{\pi}{6} + \sin\frac{\pi}{2}\sin\frac{\pi}{6} = \cos\left(\frac{\pi}{2}-\frac{\pi}{6}\right)$

$= \cos\frac{\pi}{3} = \frac{1}{2}$

## 6.4 Product-to-Sum and Sum-to-Product Formulas

### 6.4 Practice Problems

**1.** $\cos 3x\cos x = \frac{1}{2}\left[\cos(3x+x) + \cos(3x-x)\right]$

$= \frac{1}{2}\cos 4x + \frac{1}{2}\cos 2x$

**2.** $\sin 15°\cos 75°$

$= \frac{1}{2}\left[\sin(15°+75°) + \sin(15°-75°)\right]$

$= \frac{1}{2}\sin 90° + \frac{1}{2}\sin(-60°)$

$= \frac{1}{2}(1) + \frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right) = \frac{2-\sqrt{3}}{4}$

**3. a.** $\cos 2x - \cos 4x$

$= -2\sin\left(\frac{2x+4x}{2}\right)\sin\left(\frac{2x-4x}{2}\right)$

$= -2\sin 3x\sin(-x)$

$= 2\sin 3x\sin x$

**b.** $\sin 43° + \sin 17°$

$= 2\sin\left(\frac{43°+17°}{2}\right)\cos\left(\frac{43°-17°}{2}\right)$

$= 2\sin 30°\cos 13°$

$= 2\left(\frac{1}{2}\right)\cos 13° = \cos 13°$

**4.** $\sin 3x - \cos x$

$= \sin 3x - \sin\left(\frac{\pi}{2}-x\right)$

$= 2\cos\left(\frac{3x+\left(\frac{\pi}{2}-x\right)}{2}\right)\sin\left(\frac{3x-\left(\frac{\pi}{2}-x\right)}{2}\right)$

$= 2\cos\left(x+\frac{\pi}{4}\right)\sin\left(2x-\frac{\pi}{4}\right)$

**5.** $\dfrac{\cos 5x - \cos x}{\sin x - \sin 5x}$

$= \dfrac{-2\sin\left(\frac{5x+x}{2}\right)\sin\left(\frac{5x-x}{2}\right)}{2\cos\left(\frac{x+5x}{2}\right)\sin\left(\frac{x-5x}{2}\right)}$

$= \dfrac{-2\sin 3x \sin 2x}{2\cos 3x \sin(-2x)} = \dfrac{-2\sin 3x \sin 2x}{-2\cos 3x \sin 2x}$

$= \dfrac{\sin 3x}{\cos 3x} = \tan 3x$

**6. a.** When you press the "1" key, the sound produced is given by $y = \sin[2\pi \cdot 697t] + \sin[2\pi \cdot 1209t]$.

**b.** $y = \sin[2\pi(697)t] + \sin[2\pi(1209)t]$

$= 2\sin\left[2\pi\left(\frac{697+1209}{2}\right)t\right] \cdot \cos\left[2\pi\left(\frac{697-1209}{2}\right)t\right]$

$= 2\sin[2\pi(953)t]\cos[2\pi(-256)t]$

$= 2\sin[2\pi(953)t]\cos[2512\pi t]$

**c.** The frequency is 953 Hz. The variable amplitude is $2\cos(512\pi t)$.

## 6.4 Basic Concepts and Skills

**1.** We can rewrite the product of two sines as a difference of two cosines by using the formula $\sin x \sin y = \underline{\frac{1}{2}[\cos(x-y) - \cos(x+y)]}$.

**2.** We can rewrite the product of a sine and a cosine as the sum of two sines by using the formula $\sin x \cos y = \underline{\frac{1}{2}[\sin(x+y) + \sin(x-y)]}$.

**3.** We can rewrite the sum of two cosines as a product of two cosines by using the formula $\cos x + \cos y = \underline{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$.

**4.** True

**5.** False.
$\sin(x+y) = \sin x \cos y + \cos x \cos y$
$\neq \sin x + \sin y$

**6.** True

**7.** $\sin x \cos x = \frac{1}{2}(\sin 2x + \sin 0) = \frac{1}{2}\sin 2x$

**8.** $\cos x \cos x = \frac{1}{2}(\cos 0 + \cos 2x) = \frac{1}{2} + \frac{1}{2}\cos 2x$

**9.** $\sin x \sin x = \frac{1}{2}(\cos 0 - \cos 2x) = \frac{1}{2} - \frac{1}{2}\cos 2x$

**10.** $\cos x \sin x = \frac{1}{2}(\sin 2x - \sin 0) = \frac{1}{2}\sin 2x$

**11.** $\sin 25° \cos 5° = \frac{1}{2}(\sin 30° + \sin 20°)$
$= \frac{1}{4} + \frac{1}{2}\sin 20°$

**12.** $\sin 40° \sin 20° = \frac{1}{2}(\cos 20° - \cos 60°)$
$= \frac{1}{2}\cos 20° - \frac{1}{4}$

**13.** $\cos 140° \cos 20° = \frac{1}{2}(\cos 120° + \cos 160°)$
$= -\frac{1}{4} + \frac{1}{2}\cos 160°$
$= -\frac{1}{4} - \frac{1}{2}\cos 20°$

**14.** $\cos 70° \sin 20° = \frac{1}{2}(\sin 90° - \sin 50°)$
$= \frac{1}{2} - \frac{1}{2}\sin 50°$

**15.** $\sin\frac{7\pi}{12}\sin\frac{\pi}{12}$
$= \frac{1}{2}\left[\cos\left(\frac{7\pi}{12} - \frac{\pi}{12}\right) - \cos\left(\frac{7\pi}{12} + \frac{\pi}{12}\right)\right]$
$= \frac{1}{2}\left(\cos\frac{\pi}{2} - \cos\frac{2\pi}{3}\right) = \frac{1}{4}$

**16.** $\sin\frac{3\pi}{8}\cos\frac{\pi}{8}$
$= \frac{1}{2}\left[\sin\left(\frac{3\pi}{8} + \frac{\pi}{8}\right) + \sin\left(\frac{3\pi}{8} - \frac{\pi}{8}\right)\right]$
$= \frac{1}{2}\left(\sin\frac{\pi}{2} + \sin\frac{\pi}{4}\right) = \frac{1}{2} + \frac{\sqrt{2}}{4}$

**17.** $\cos\frac{5\pi}{8}\sin\frac{\pi}{8}$
$= \frac{1}{2}\left[\sin\left(\frac{5\pi}{8} + \frac{\pi}{8}\right) - \sin\left(\frac{5\pi}{8} - \frac{\pi}{8}\right)\right]$
$= \frac{1}{2}\left(\sin\frac{3\pi}{4} - \sin\frac{\pi}{2}\right) = \frac{\sqrt{2}}{4} - \frac{1}{2}$

**18.** $\cos\frac{5\pi}{3}\cos\frac{\pi}{3}$
$= \frac{1}{2}\left[\cos\left(\frac{5\pi}{3}-\frac{\pi}{3}\right)+\cos\left(\frac{5\pi}{3}+\frac{\pi}{3}\right)\right]$
$= \frac{1}{2}\left(\cos\frac{4\pi}{3}+\cos 2\pi\right)=\frac{1}{4}$

**19.** $\sin 5\theta\cos\theta = \frac{1}{2}(\sin 6\theta+\sin 4\theta)$ .
$= \frac{1}{2}\sin 6\theta+\frac{1}{2}\sin 4\theta$

**20.** $\cos 3\theta\sin 2\theta = \frac{1}{2}(\sin 5\theta-\sin\theta)$
$= \frac{1}{2}\sin 5\theta-\frac{1}{2}\sin\theta$

**21.** $\cos 4x\cos 3x = \frac{1}{2}(\cos x+\cos 7x)$
$= \frac{1}{2}\cos x+\frac{1}{2}\cos 7x$

**22.** $\sin 5x\sin 2x = \frac{1}{2}(\cos 3x-\cos 7x)$
$= \frac{1}{2}\cos 3x-\frac{1}{2}\cos 7x$

**23.** $\sin 37.5^\circ\sin 7.5^\circ = \frac{1}{2}(\cos 30^\circ-\cos 45^\circ)$
$= \frac{\sqrt{3}-\sqrt{2}}{4}$

**24.** $\cos 52.5^\circ\cos 7.5^\circ = \frac{1}{2}(\cos 45^\circ+\cos 60^\circ)$
$= \frac{\sqrt{2}+1}{4}$

**25.** $\sin 67.5^\circ\cos 22.5^\circ = \frac{1}{2}(\sin 90^\circ+\sin 45^\circ)$
$= \frac{1}{2}+\frac{\sqrt{2}}{4}$

**26.** $\cos 105^\circ\sin 75^\circ = \frac{1}{2}(\sin 180^\circ-\sin 30^\circ)=-\frac{1}{4}$

**27.** $\sin\frac{5\pi}{24}\cos\frac{\pi}{24}=\frac{1}{2}\left(\sin\frac{\pi}{6}+\sin\frac{\pi}{4}\right)=\frac{1+\sqrt{2}}{4}$

**28.** $\sin\frac{7\pi}{12}\sin\frac{\pi}{12}=\frac{1}{2}\left(\cos\frac{\pi}{2}-\cos\frac{2\pi}{3}\right)=\frac{1}{4}$

**29.** $\cos\frac{13\pi}{24}\cos\frac{5\pi}{24}=\frac{1}{2}\left(\cos\frac{\pi}{3}+\cos\frac{3\pi}{4}\right)=\frac{1-\sqrt{2}}{4}$

**30.** $\cos\frac{7\pi}{24}\sin\frac{\pi}{24}=\frac{1}{2}\left(\sin\frac{\pi}{3}-\sin\frac{\pi}{4}\right)=\frac{\sqrt{3}-\sqrt{2}}{4}$

**31.** $\cos 40^\circ-\cos 20^\circ$
$= -2\sin\left(\frac{40^\circ+20^\circ}{2}\right)\sin\left(\frac{40^\circ-20^\circ}{2}\right)$
$= -2\sin 30^\circ\sin 10^\circ = -\sin 10^\circ$

**32.** $\sin 22^\circ+\sin 8^\circ$
$= 2\sin\left(\frac{22^\circ+8^\circ}{2}\right)\cos\left(\frac{22^\circ-8^\circ}{2}\right)$
$= 2\sin 15^\circ\cos 7^\circ$

**33.** $\sin 32^\circ-\sin 16^\circ$
$= 2\sin\left(\frac{32^\circ-16^\circ}{2}\right)\cos\left(\frac{32^\circ+16^\circ}{2}\right)$
$= 2\sin 8^\circ\cos 24^\circ$

**34.** $\cos 47^\circ+\cos 13^\circ$
$= 2\cos\left(\frac{47^\circ+13^\circ}{2}\right)\cos\left(\frac{47^\circ-13^\circ}{2}\right)$
$= 2\cos 30^\circ\cos 17^\circ = \sqrt{3}\cos 17^\circ$

**35.** $\sin\frac{\pi}{5}+\sin\frac{2\pi}{5}=2\sin\frac{3\pi}{10}\cos\left(-\frac{\pi}{10}\right)$
$= 2\sin\frac{3\pi}{10}\cos\frac{\pi}{10}$

**36.** $\cos\frac{\pi}{12}+\cos\frac{\pi}{3}=2\cos\frac{5\pi}{24}\cos\left(-\frac{3\pi}{24}\right)$
$= 2\cos\frac{5\pi}{24}\cos\frac{\pi}{8}$

**37.** $\cos\frac{1}{2}+\cos\frac{1}{3}=2\cos\frac{\frac{1}{2}+\frac{1}{3}}{2}\cos\frac{\frac{1}{2}-\frac{1}{3}}{2}$
$= 2\cos\frac{5}{12}\cos\frac{1}{12}$

**38.** $\sin\frac{2}{3}-\sin\frac{1}{4}=2\sin\frac{\frac{2}{3}-\frac{1}{4}}{2}\cos\frac{\frac{2}{3}+\frac{1}{4}}{2}$
$= 2\sin\frac{5}{24}\cos\frac{11}{24}$

**39.** $\cos 3x+\cos 5x = 2\cos 4x\cos(-x)$
$= 2\cos 4x\cos x$

**40.** $\sin 5x-\sin 3x = 2\sin x\cos 4x$

**41.** $\sin 7x+\sin(-x) = 2\sin 3x\cos 4x$

**42.** $\cos 7x-\cos 3x = -2\sin 5x\sin 2x$

**43.** $$\sin x+\cos x=\sin x+\sin\left(\frac{\pi}{2}-x\right)=2\sin\frac{\pi}{4}\cos\left(x-\frac{\pi}{4}\right)=\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$$

**44.** $$\cos x-\sin x=\cos x-\cos\left(\frac{\pi}{2}-x\right)=-2\sin\frac{\pi}{4}\sin\left(x-\frac{\pi}{4}\right)=-\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$$

**45.** $$\sin 2x-\cos 2x=\sin 2x-\sin\left(\frac{\pi}{2}-2x\right)=2\sin\left(2x-\frac{\pi}{4}\right)\cos\frac{\pi}{4}=\sqrt{2}\sin\left(2x-\frac{\pi}{4}\right)$$

**46.** $$\cos 3x+\sin 3x=\cos 3x+\cos\left(\frac{\pi}{2}-3x\right)=2\cos\frac{\pi}{4}\cos\left(3x-\frac{\pi}{4}\right)=\sqrt{2}\cos\left(3x-\frac{\pi}{4}\right)$$

**47.** $$\sin 3x+\cos 5x=\sin 3x+\sin\left(\frac{\pi}{2}-5x\right)=2\sin\left(\frac{\pi}{4}-x\right)\cos\left(4x-\frac{\pi}{4}\right)$$

**48.** $$\sin 5x-\cos x=\sin 5x-\sin\left(\frac{\pi}{2}-x\right)=2\sin\left(3x-\frac{\pi}{4}\right)\cos\left(2x+\frac{\pi}{4}\right)$$

**49.** $$a(\sin x+\cos x)=a\left(\sin x+\sin\left(\frac{\pi}{2}-x\right)\right)=2a\sin\frac{\pi}{4}\cos\left(x-\frac{\pi}{4}\right)=a\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$$

**50.** $$a(\sin bx+\cos bx)=a\left(\sin bx+\sin\left(\frac{\pi}{2}-bx\right)\right)=a\left(2\sin\frac{\pi}{4}\cos\left(bx-\frac{\pi}{4}\right)\right)=a\sqrt{2}\cos\left(bx-\frac{\pi}{4}\right)$$

**51.** $$\frac{\sin x+\sin 3x}{\cos x+\cos 3x}=\frac{2\sin 2x\cos(-x)}{2\cos 2x\cos(-x)}=\tan 2x$$

**52.** $$\frac{\sin 2x+\sin 4x}{\cos 2x+\cos 4x}=\frac{2\sin 3x\cos(-x)}{2\cos 3x\cos(-x)}=\tan 3x$$

**53.** $$\frac{\cos 3x-\cos 7x}{\sin 7x+\sin 3x}=\frac{-2\sin 5x\sin(-2x)}{2\sin 5x\cos 2x}=\frac{\sin 2x}{\cos 2x}=\tan 2x$$

**54.** $$\frac{\cos 12x-\cos 4x}{\sin 4x-\sin 12x}=\frac{-2\sin 8x\sin 4x}{2\sin(-4x)\cos 8x}=\frac{-2\sin 8x\sin 4x}{-2\sin 4x\cos 8x}=\tan 8x$$

**55.** $$\frac{\cos 2x+\cos 2y}{\cos 2x-\cos 2y}=\frac{2\cos\left(\frac{2x+2y}{2}\right)\cos\left(\frac{2x-2y}{2}\right)}{-2\sin\left(\frac{2x+2y}{2}\right)\sin\left(\frac{2x-2y}{2}\right)}=\frac{2\cos(x+y)\cos(x-y)}{-2\sin(x+y)\sin(x-y)}=-\cot(x+y)\cot(x-y)=\cot(y+x)\cot(y-x)$$

**56.** $$\frac{\sin 2x+\sin 2y}{\sin 2x-\sin 2y}=\frac{2\sin(x+y)\cos(x-y)}{2\sin(x-y)\cos(x+y)}=\tan(x+y)\cot(x-y)=\frac{\tan(x+y)}{\tan(x-y)}$$

**57.** $$\sin x+\sin 2x+\sin 3x=\sin 2x+(\sin x+\sin 3x)=\sin 2x+2\sin 2x\cos(-x)=\sin 2x(1+2\cos x)$$

**58.** $$\cos x+\cos 2x+\cos 3x=\cos 2x+(\cos x+\cos 3x)=\cos 2x+2\cos 2x\cos(-x)=\cos 2x(1+2\cos x)$$

**59.** $\sin 2x + \sin 4x + \sin 6x$
$= \sin 0x + \sin 2x + \sin 4x + \sin 6x$
$= (\sin 2x + \sin 4x) + (\sin 0x + \sin 6x)$
$= 2\sin 3x \cos x + 2\sin 3x \cos 3x$
$= 2\sin 3x(\cos x + \cos 3x)$
$= 2\sin 3x(2\cos 2x \cos(-x))$
$= 4\cos x \cos 2x \sin 3x$

**60.** $\cos x + \cos 3x + \cos 5x + \cos 7x$
$= (\cos x + \cos 3x) + (\cos 5x + \cos 7x)$
$= 2\cos 2x \cos(-x) + 2\cos 6x \cos(-x)$
$= 2\cos x(\cos 2x + \cos 6x)$
$= 2\cos x(2\cos 4x \cos(-2x))$
$= 4\cos x \cos 2x \cos 4x$

## 6.4 Applying the Concepts

**61. a.** $y = \sin\left(2\pi(852)t\right) + \sin\left(2\pi(1209)t\right)$

**b.** $y = \sin\left(2\pi(852)t\right) + \sin\left(2\pi(1209)t\right)$
$= 2\sin\left[2\pi\left(\frac{852+1209}{2}\right)t\right] \cdot \cos\left[2\pi\left(\frac{852-1209}{2}\right)t\right]$
$= 2\sin\left(2\pi(1030.5)t\right)\cos\left(2\pi(-357)t\right)$
$= 2\sin\left(2\pi(1030.5)t\right)\cos\left(357\pi t\right)$

**c.** frequency = 1030.5 Hz,
amplitude = $2\cos\left(357\pi t\right)$.

**62. a.** $y = \sin\left(2\pi(941)t\right) + \sin\left(2\pi(1477)t\right)$

**b.** $y = \sin\left(2\pi(941)t\right) + \sin\left(2\pi(1477)t\right)$
$= 2\sin\left[2\pi\left(\frac{941+1477}{2}\right)t\right] \cdot \cos\left[2\pi\left(\frac{941-1477}{2}\right)t\right]$
$= 2\sin\left(2\pi(1209)t\right)\cos\left(2\pi(-536)t\right)$
$= 2\sin\left(2\pi(1209)t\right)\cos(536\pi t)$

**c.** frequency = 1209 Hz
amplitude = $2\cos\left(536\pi t\right)$.

**63. a.** $y = 0.05\cos(112\pi t) + 0.05\cos(120\pi t)$
$= 0.05\left(\cos(112\pi t) + \cos(120\pi t)\right)$
$= 0.05\left(2\cos(116\pi t)\cos(4\pi t)\right)$
$= 0.1\cos(116\pi t)\cos(4\pi t)$

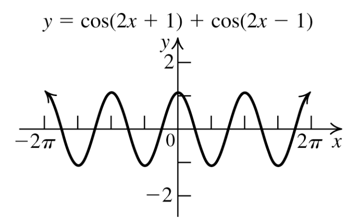
**b.** The frequency of $y_1 = \frac{112\pi}{2\pi} = 56$.
The frequency of $y_2 = \frac{120\pi}{2\pi} = 60$.
The beat frequency is $|56 - 60| = 4$.

**64. a.** $y = 0.04\sin(110\pi t) + 0.04\sin(114\pi t)$
$= 0.04\left(\sin(110\pi t) + \sin(114\pi t)\right)$
$= 0.04\left(2\sin(112\pi t)\cos(2\pi t)\right)$
$= 0.08\sin(112\pi t)\cos(2\pi t)$

**b.** The frequency of $y_1 = \frac{110\pi}{2\pi} = 55$.
The frequency of $y_2 = \frac{114\pi}{2\pi} = 57$.
The beat frequency is $|55 - 57| = 2$.

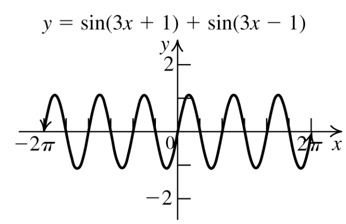
## 6.4 Beyond the Basics

**65.**




$\cos\left(2x+1\right) + \cos\left(2x-1\right)$
$= 2\cos 2x \cos 1 = 2\cos 1 \cos 2x$
Amplitude: $2\cos 1 \approx 1.0806$
Period: $\frac{2\pi}{2} = \pi$

**66.**




$\sin\left(3x+1\right) + \sin\left(3x-1\right)$
$= 2\sin 3x \cos 1 = 2\cos 1 \sin 3x$
Amplitude: $2\cos 1 \approx 1.0806$
Period: $\frac{2\pi}{3}$

**67.** $$\cos 40° + \cos 50° + \cos 70° + \cos 80° = (\cos 40° + \cos 80°) + (\cos 50° + \cos 70°)$$
$$= 2\cos 60° \cos(-20°) + 2\cos 60° \cos(-10°)$$
$$= 2\cos 60°(\cos(-20°) + \cos(-10°)) = \cos 10° + \cos 20°$$

**68.** $$\sin 10° + \sin 20° + \sin 40° + \sin 50° = 2\sin 15° \cos 5° + 2\sin 45° \cos 5° = 2\cos 5°(\sin 15° + \sin 45°)$$
$$= 2\cos 5°(2\sin 30° \cos 15°) = 2\cos 5° \cos 15°$$

**69.** $$\cos 4\theta \cos\theta - \cos 6\theta \cos 9\theta = \frac{1}{2}(\cos 3\theta + \cos 5\theta) - \frac{1}{2}(\cos 3\theta + \cos 15\theta)$$
$$= \frac{1}{2}\cos 3\theta + \frac{1}{2}\cos 5\theta - \frac{1}{2}\cos 3\theta - \frac{1}{2}\cos 15\theta$$
$$= \frac{1}{2}\cos 5\theta - \frac{1}{2}\cos 15\theta = \frac{1}{2}(\cos 5\theta - \cos 15\theta)$$
$$= \frac{1}{2}(-2\sin 10\theta \sin(-5\theta)) = \sin 5\theta \sin 10\theta$$

**70.** $$\cos 38° \cos 46° - \sin 14° \sin 22° = \frac{1}{2}\left(\cos(-8°) + \cos 84°\right) - \frac{1}{2}\left(\cos(-8°) - \cos 36°\right)$$
$$= \frac{1}{2}\cos(-8°) + \frac{1}{2}\cos 84° - \frac{1}{2}\cos(-8°) + \frac{1}{2}\cos 36° = \frac{1}{2}(\cos 84° + \cos 36°)$$
$$= \frac{1}{2}(2\cos 60° \cos 24°) = \frac{1}{2}\cos 24°$$

**71.** $$\sin 25° \sin 35° - \sin 25° \sin 85° - \sin 35° \sin 85°$$
$$= \frac{1}{2}(\cos 10° - \cos 60°) - \frac{1}{2}(\cos 60° - \cos 110°) - \frac{1}{2}(\cos 50° - \cos 120°)$$
$$= \frac{1}{2}\cos 10° - \frac{1}{4} - \frac{1}{4} + \frac{1}{2}\cos 110° - \frac{1}{2}\cos 50° - \frac{1}{4} = -\frac{3}{4} + \frac{1}{2}(\cos 10° + \cos 110° - \cos 50°)$$
$$= -\frac{3}{4} + \frac{1}{2}(2\cos 60° \cos(-50°) - \cos 50°) = -\frac{3}{4} + \frac{1}{2}\cos 50° - \frac{1}{2}\cos 50° = -\frac{3}{4}$$

**72.** $$\cos 20° \cos 40° \cos 60° \cos 80° = (\cos 20° \cos 40°)\cos 60° \cos 80° = \frac{1}{2}\left[\cos(-20°) + \cos 60°\right]\cos 60° \cos 80°$$
$$= \frac{1}{2}\left(\cos 20° + \frac{1}{2}\right)\left(\frac{1}{2}\cos 80°\right) = \frac{1}{4}\cos 80°\left(\cos 20° + \frac{1}{2}\right)$$
$$= \frac{1}{4}\cos 20° \cos 80° + \frac{1}{8}\cos 80° = \frac{1}{4}\left(\frac{1}{2}(\cos(-60°) + \cos 100°)\right) + \frac{1}{8}\cos 80°$$
$$= \frac{1}{16} + \frac{1}{8}\cos 100° + \frac{1}{8}\cos 80° = \frac{1}{16} + \frac{1}{8}(\cos 100° + \cos 80°)$$
$$= \frac{1}{16} + \frac{1}{8}(2\cos 90° \cos 10°) = \frac{1}{16}$$

**73.** $$\frac{\sin 3x \cos 5x - \sin x \cos 7x}{\sin x \sin 7x + \cos 3x \cos 5x} = \frac{\frac{1}{2}(\sin 8x + \sin(-2x)) - \frac{1}{2}(\sin 8x + \sin(-6x))}{\frac{1}{2}(\cos(-6x) - \cos 8x) + \frac{1}{2}(\cos(-2x) + \cos 8x)}$$
$$= \frac{-\sin 2x + \sin 6x}{\cos 6x + \cos 2x} = \frac{2\sin 2x \cos 4x}{2\cos 4x \cos 2x} = \frac{2\sin 2x \cos 4x}{2\cos 4x \cos 2x} = \frac{\sin 2x}{\cos 2x} = \tan 2x$$

**74.** $$\frac{\sin 11x \sin x + \sin 7x \sin 3x}{\cos 11x \sin x + \cos 7x \sin 3x} = \frac{\frac{1}{2}(\cos 10x - \cos 12x) + \frac{1}{2}(\cos 4x - \cos 10x)}{\frac{1}{2}(\sin 12x - \sin 10x) + \frac{1}{2}(\sin 10x - \sin 4x)}$$
$$= \frac{\cos 4x - \cos 12x}{\sin 12x - \sin 4x} = \frac{-2\sin 8x \sin(-4x)}{2\sin 4x \cos 8x} = \frac{\sin 8x}{\cos 8x} = \tan 8x$$

**75.** $\dfrac{\cos x+\cos 3x+\cos 5x+\cos 7x}{\sin x+\sin 3x+\sin 5x+\sin 7x}=\dfrac{4\cos x\cos 2x\cos 4x}{2\sin 2x\cos x+2\sin 6x\cos x}$ (Use the result from exercise 60.)

$$=\frac{4\cos x\cos 2x\cos 4x}{2\cos x(\sin 2x+\sin 6x)}=\frac{4\cos x\cos 2x\cos 4x}{2\cos x(2\sin 4x\cos(-2x))}$$
$$=\frac{4\cos x\cos 2x\cos 4x}{4\cos x\cos 2x\sin 4x}=\cot 4x$$

**76.** $\dfrac{\sin 3x+\sin 5x+\sin 7x+\sin 9x}{\cos 3x+\cos 5x+\cos 7x+\cos 9x}=\dfrac{2\sin 4x\cos x+2\sin 8x\cos x}{2\cos 4x\cos x+2\cos 8x\cos x}=\dfrac{2\cos x(\sin 4x+\sin 8x)}{2\cos x(\cos 4x+\cos 8x)}$

$$=\frac{2\sin 6x\cos 2x}{2\cos 6x\cos 2x}=\tan 6x$$

**77.** $2\cos 6x\cos x-2\cos 4x\cos x+2\cos 2x\cos x-\cos x$

$$=2\left[\frac{1}{2}(\cos 5x+\cos 7x)\right]-2\left[\frac{1}{2}(\cos 3x+\cos 5x)\right]+2\left[\frac{1}{2}(\cos x+\cos 3x)\right]-\cos x$$
$$=\cos 5x+\cos 7x-\cos 3x-\cos 5x+\cos x+\cos 3x-\cos x=\cos 7x$$

**78. a.** $4\sin\theta\sin\left(\dfrac{\pi}{3}+\theta\right)\sin\left(\dfrac{\pi}{3}-\theta\right)=4\sin\theta\left(\sin\dfrac{\pi}{3}\cos\theta+\cos\dfrac{\pi}{3}\sin\theta\right)\left(\sin\dfrac{\pi}{3}\cos\theta-\cos\dfrac{\pi}{3}\sin\theta\right)$

$$=4\sin\theta\left(\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta\right)\left(\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta\right)$$
$$=4\sin\theta\left(\frac{3}{4}\cos^2\theta-\frac{1}{4}\sin^2\theta\right)=3\cos^2\theta\sin\theta-\sin^3\theta$$
$$=3(1-\sin^2\theta)\sin\theta-\sin^3\theta=3\sin\theta-3\sin^3\theta-\sin^3\theta$$
$$=3\sin\theta-4\sin^3\theta=\sin 3\theta$$

**b.** Let $\theta=20°$. Then $4\sin 20°\cdot\sin(60°+20°)\cdot\sin(60°-20°)=\sin(3\cdot 20°)=\sin 60°=\dfrac{\sqrt{3}}{2}\Rightarrow$

$$\sin 20°\cdot\sin(60°+20°)\cdot\sin(60°-20°)=\frac{\sqrt{3}}{8}\Rightarrow[\sin 20°\cdot\sin(60°+20°)\cdot\sin(60°-20°)]\cdot\sin 60°=\frac{\sqrt{3}}{8}\cdot\frac{\sqrt{3}}{2}=\frac{3}{16}$$

**79.** $\dfrac{\sin(x+3y)+\sin(3x+y)}{\sin 2x+\sin 2y}=\dfrac{2\sin(2x+2y)\cos(-x+y)}{\sin 2x+\sin 2y}=\dfrac{2\sin(2x+2y)\cos(-(x-y))}{2\sin(x+y)\cos(x-y)}$

$$=\frac{2\sin 2(x+y)\cos(x-y)}{2\sin(x+y)\cos(x-y)}=\frac{4\sin(x+y)\cos(x+y)\cos(x-y)}{2\sin(x+y)\cos(x-y)}=2\cos(x+y)$$

**80.** $\sin^3 x\sin 3x+\cos^3 x\cos 3x=\sin^3 x(3\sin x-4\sin^3 x)+\cos^3 x(4\cos^3 x-3\cos x)$

$$=3\sin^4 x-4\sin^6 x+4\cos^6 x-3\cos^4 x=4(\cos^6 x-\sin^6 x)-3(\cos^4 x-\sin^4 x)$$
$$=4[(\cos^2 x-\sin^2 x)(\cos^4 x+\cos^2 x\sin^2 x+\sin^4 x]-3[(\cos^2 x)^2-(\sin^2 x)^2]$$
$$=4[(\cos^2 x-\sin^2 x)((\cos^2 x+\sin^2 x)^2-\cos^2 x\sin^2 x]$$
$$-3[(\cos^2 x-\sin^2 x)(\cos^2 x+\sin^2 x)]$$
$$=4[(\cos^2 x-\sin^2 x)-(\cos^2 x\sin^2 x)]-3(\cos^2 x-\sin^2 x)$$
$$=(\cos^2 x-\sin^2 x)(4-4\cos^2 x\sin^2 x-3)=\cos 2x(1-4\cos^2 x\sin^2 x)$$
$$=\cos 2x(1-(2\cos x\sin x)^2)=\cos 2x(1-\sin^2 2x)=\cos 2x(\cos^2 2x)=\cos^3 2x$$

**81.** Start with the right side in parts a–c.

**a.** $-2\sin\dfrac{x+y}{2}\sin\dfrac{x-y}{2}=-2\cdot\dfrac{1}{2}\left[\cos\left(\dfrac{x+y}{2}-\dfrac{x-y}{2}\right)-\cos\left(\dfrac{x+y}{2}+\dfrac{x-y}{2}\right)\right]=-1[\cos y-\cos x]=\cos x-\cos y$

**b.** $2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}=2\cdot\dfrac{1}{2}\left[\sin\left(\dfrac{x+y}{2}+\dfrac{x-y}{2}\right)+\sin\left(\dfrac{x+y}{2}-\dfrac{x-y}{2}\right)\right]=\sin x+\sin y$

**c.** $2\sin\frac{x-y}{2}\cos\frac{x+y}{2} = 2\cdot\frac{1}{2}\left[\sin\left(\frac{x-y}{2}+\frac{x+y}{2}\right)+\sin\left(\frac{x-y}{2}-\frac{x+y}{2}\right)\right] = \sin x + \sin(-y) = \sin x - \sin y$

## 6.4 Critical Thinking/Discussion/Writing

**82.**

| Step | Reason |
|---|---|
| $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14} = \cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}$ | $\sin x = \cos\left(\frac{\pi}{2}-x\right)$ |
| $= \frac{8\sin(\pi/7)\cos(\pi/7)\cos(2\pi/7)\cos(3\pi/7)}{8\sin(\pi/7)}$ | Multiply the numerator and denominator by $8\sin\frac{\pi}{7}$ |
| $= \frac{2\sin(4\pi/7)\cos(3\pi/7)}{8\sin(\pi/7)}$ | $\sin 2\theta = 2\sin\theta\cos\theta$ (applied twice) |
| $= \frac{\sin\pi + \sin(\pi/7)}{8\sin(\pi/7)}$ | Product-to-sum formula: $\sin x\cos y = \frac{1}{2}[\sin(x+y)+\sin(x-y)]$ |
| $= 1/8$ | $\sin\pi = 0$; simplify |

**83.**
$$\sin 2A + \sin 2B + \sin 2C = (\sin 2A + \sin 2B) + \sin 2C = 2\sin(A+B)\cos(A-B) + 2\sin C\cos C$$
$$A + B + C = 180° \Rightarrow C = 180° - (A+B), \text{ so}$$
$$\begin{aligned}2\sin(A+B)\cos(A-B) + 2\sin C\cos C &= 2\sin(180° - (A+B))\cos(A-B) + 2\sin C\cos C\\ &= 2\sin C\cos(A-B) + 2\sin C\cos C = 2\sin C\left(\cos(A-B)+\cos C\right)\\ &= 2\sin C\left[\cos(A-B) + \cos(180° - (A+B))\right]\\ &= 2\sin C\left(\cos(A-B) - \cos(A+B)\right)\\ &= 2\sin C\left(-2\sin A\sin(-2B)\right) = 4\sin A\sin B\sin C\end{aligned}$$

**84.**
$$\cos 2A + \cos 2B + \cos 2C = (\cos 2A + \cos 2B) + \cos 2C = 2\cos(A+B)\cos(A-B) + \cos 2C$$
$$A + B + C = 180° \Rightarrow A + B = 180° - C, \text{ so}$$
$$\begin{aligned}2\cos(A+B)\cos(A-B) + \cos 2C &= 2\cos(180° - C)\cos(A-B) + \cos 2C = -2\cos C\cos(A-B) + 2\cos^2 C - 1\\ &= -1 - 2\cos C[\cos(A-B) - \cos C]\\ &= -1 - 2\cos C[\cos(A-B) - \cos(180° - (A+B))]\\ &= -1 - 2\cos C[\cos(A-B) + \cos(A+B)]\\ &= -1 - 2\cos C[2\cos A\cos B] = -1 - 4\cos A\cos B\cos C\end{aligned}$$

**85.** $\cos^2 x = \cos x\cos x = \frac{1}{2}\left[\cos(x+x) + \cos(x-x)\right] = \frac{1}{2}[\cos 2x + \cos 0] = \frac{1}{2}(\cos 2x + 1) = \frac{1+\cos 2x}{2}$

## 6.4 Maintaining Skills

**86.** $\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$
Conditional equation

**87.** $\cos x = 3$
Inconsistent equation

**88.** $\sin x = \cos x \Rightarrow x = \frac{\pi}{4},\ x = \frac{5\pi}{4}$
Conditional equation

**89.** $\cos^2 x = 1 - \sin^2 x \Rightarrow \sin^2 x + \cos^2 x = 1$
Identity

**90.** $\sin 2x = 2\sin x\cos x$
Identity

**91.** $\tan x = \sqrt{\sec^2 x - 1}$
Conditional equation

**92.** $\theta = \frac{5\pi}{3}$ lies in quadrant IV, so the reference angle is $2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$.

**93.** $\theta = \frac{7\pi}{8}$ lies in quadrant II, so the reference angle is $\pi - \frac{7\pi}{8} = \frac{\pi}{8}$.

**94.** $\theta = -\frac{3\pi}{7}$ is coterminal with $-\frac{3\pi}{7} + 2\pi = \frac{11\pi}{7}$. $\theta = \frac{11\pi}{7}$ lies in quadrant IV, so the reference angle is $2\pi - \frac{11\pi}{7} = \frac{3\pi}{7}$.

**95.** $\theta = -\frac{17\pi}{4}$ is coterminal with $-\frac{17\pi}{4} + 6\pi = \frac{7\pi}{4}$. $\theta = \frac{7\pi}{4}$ lies in quadrant IV, so the reference angle is $2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$.

**96.** All angles coterminal with $\theta = -\frac{\pi}{7}$ have the form $-\frac{\pi}{7} + 2n\pi$, for any integer $n$.

**97.** In quadrant III, a reference angle $\theta' = \theta - \pi$, so $\frac{\pi}{5} = \theta - \pi \Rightarrow \theta = \frac{6\pi}{5}$.

## 6.5 Trigonometric Equations I

### 6.5 Practice Problems

**1. a.** $\sin x = 1 \Rightarrow x = \frac{\pi}{2} + 2n\pi$

**b.** $\cos x = 1 \Rightarrow x = 0 + 2n\pi = 2n\pi$

**c.** $\tan x = 1 \Rightarrow x = \frac{\pi}{4} + n\pi$

**2. a.** $\sec x = 1.5 \Rightarrow x$ is in Quadrant I or Quadrant IV.

$x = \sec^{-1}(1.5) \Rightarrow \sec x = 1.5 \Rightarrow \cos x = \frac{2}{3}$

$x = \cos^{-1}\left(\frac{2}{3}\right) \approx 48.2°$ or

$x \approx 360° - 48.2° = 311.8°$

The period of the secant function is 360°, so all solutions are given by $x = 48.2° + n \cdot 360°$ and $x = 311.8° + n \cdot 360°$.

**b.** $\tan x = -2 \Rightarrow x$ is in Quadrant II or Quadrant IV.

$x = \tan^{-1}(-2) \approx \pi - 1.1071 \approx 2.0344$ or

$x \approx 2\pi - 1.1071 \approx 5.1760$

Solution set: {2.0345, 5.1761}

**3.** $2\sec(x-30)° + 1 = \sec(x-30)° + 3 \Rightarrow$

$\sec(x-30)° = 2 \Rightarrow \cos(x-30)° = \frac{1}{2} \Rightarrow$

$x - 30° = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow x = \cos^{-1}\left(\frac{1}{2}\right) + 30°$

$\cos x$ is positive in quadrants I and IV, so

$x = \cos^{-1}\left(\frac{1}{2}\right) + 30° = 60° + 30° = 90°$ or

$x = \cos^{-1}\left(\frac{1}{2}\right) + 30°$

$= (360° - 60°) + 30° = 330°$

Solution set: {90º, 330º).

**4.** $d = tv_0 \cos\theta \Rightarrow 2500 = (1)(3280)\cos\theta \Rightarrow$

$\cos\theta = \frac{2500}{3280} \Rightarrow \theta = \cos^{-1}\left(\frac{2500}{3280}\right) \approx 40°$

**5.** $(\sin x - 1)(\sqrt{3}\tan x + 1) = 0 \Rightarrow$

$\sin x = 1 \Rightarrow x = \frac{\pi}{2}$ or

$\sqrt{3}\tan x + 1 = 0 \Rightarrow \tan x = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3} \Rightarrow$

$x = \frac{5\pi}{6}$ or $x = \frac{11\pi}{6}$

Solution set: $\left\{\frac{\pi}{2}, \frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

**6.** $2\cos^2\theta - \cos\theta - 1 = 0 \Rightarrow$

$(2\cos\theta + 1)(\cos\theta - 1) = 0 \Rightarrow$

$2\cos\theta + 1 = 0 \Rightarrow \cos\theta = -\frac{1}{2}$ or

$\cos\theta - 1 = 0 \Rightarrow \cos\theta = 1$

$\cos\theta = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$ or $\theta = \frac{4\pi}{3}$

$\cos\theta = 1 \Rightarrow \theta = 0$

Solution set: $\left\{0 + 2n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi\right\}$

**7.**
$$2\cos^2\theta+3\sin\theta-3=0$$
$$2\left(1-\sin^2\theta\right)+3\sin\theta-3=0$$
$$2-2\sin^2\theta+3\sin\theta-3=0$$
$$-2\sin^2\theta+3\sin\theta-1=0$$
$$2\sin^2\theta-3\sin\theta+1=0$$
$$(2\sin\theta-1)(\sin\theta-1)=0$$
$$2\sin\theta-1=0 \quad\text{or}\quad \sin\theta-1=0$$
$$\sin\theta=\frac{1}{2} \qquad\qquad \sin\theta=1$$
$$\theta=\frac{\pi}{6}\text{ or }\theta=\frac{5\pi}{6} \qquad\qquad \theta=\frac{\pi}{2}$$
Solution set: $\left\{\frac{\pi}{6},\frac{\pi}{2},\frac{5\pi}{6}\right\}$

**8.**
$$\sqrt{3}\cot\theta+1=\sqrt{3}\csc\theta$$
$$\left(\sqrt{3}\cot\theta+1\right)^2=\left(\sqrt{3}\csc\theta\right)^2$$
$$3\cot^2\theta+2\sqrt{3}\cot\theta+1=3\csc^2\theta$$
$$3\cot^2\theta+2\sqrt{3}\cot\theta+1=3\left(1+\cot^2\theta\right)$$
$$3\cot^2\theta+2\sqrt{3}\cot\theta+1=3+3\cot^2\theta$$
$$2\sqrt{3}\cot\theta=2$$
$$\cot\theta=\frac{\sqrt{3}}{3}\Rightarrow$$
$$\theta=\frac{\pi}{3}\text{ or }\theta=\frac{4\pi}{3}$$
Check each answer:
$$\sqrt{3}\cot\frac{\pi}{3}+1\overset{?}{=}\sqrt{3}\csc\frac{\pi}{3}$$
$$\sqrt{3}\cdot\frac{\sqrt{3}}{3}+1\overset{?}{=}\sqrt{3}\cdot\frac{2}{\sqrt{3}}$$
$$2=2\checkmark$$
$$\sqrt{3}\cot\frac{4\pi}{3}+1\overset{?}{=}\sqrt{3}\csc\frac{4\pi}{3}$$
$$\sqrt{3}\cdot\frac{\sqrt{3}}{3}+1\overset{?}{=}\sqrt{3}\left(-\frac{2}{\sqrt{3}}\right)$$
$$2\neq-2$$
Thus, $\frac{4\pi}{3}$ is extraneous and the solution is $\left\{\frac{\pi}{3}\right\}$.

## 6.5 Basic Concepts and Skills

**1.** The equation $\sin x=\frac{1}{2}$ has <u>two</u> solutions in $[0,2\pi)$.

**2.** All solutions of $\sin x=\frac{1}{2}$ are given by $\underline{\frac{\pi}{6}+2n\pi}$ and $\underline{\frac{5\pi}{6}+2n\pi}$.

**3.** The equation cos $x$ = 1 has <u>one</u> solution in $[0,2\pi)$.

**4.** All solutions of tan $x$ = 1 are given by $\underline{\frac{\pi}{4}+n\pi}$.

**5.** False. The equation $\sec x=\frac{1}{2}$ has no solutions because $\frac{1}{2}$ is not the in range of sec $x$.

**6.** True

**7.** $\cos x=0\Rightarrow x=\frac{\pi}{2}+2n\pi$ or $x=\frac{3\pi}{2}+2n\pi$

**8.** $\sin x=0\Rightarrow x=0+2n\pi$ or $x=\pi+2n\pi$

**9.** $\tan x=-1\Rightarrow x=\frac{3\pi}{4}+n\pi$

**10.** $\cot x=-1\Rightarrow\frac{3\pi}{4}+n\pi$

**11.** $\cos x=\frac{\sqrt{2}}{2}\Rightarrow x=\frac{\pi}{4}+2n\pi$ or $x=\frac{7\pi}{4}+2n\pi$

**12.** $\sin x=\frac{\sqrt{3}}{2}\Rightarrow x=\frac{\pi}{3}+2n\pi$ or $x=\frac{2\pi}{3}+2n\pi$

**13.** $\cot x=\sqrt{3}\Rightarrow x=\frac{\pi}{6}+n\pi$

**14.** $\tan x=-\frac{\sqrt{3}}{3}\Rightarrow\frac{5\pi}{6}+n\pi$

**15.** $\cos x=-\frac{1}{2}\Rightarrow x=\frac{2\pi}{3}+2n\pi$ or $x=\frac{4\pi}{3}+2n\pi$

**16.** $\sin x=-\frac{\sqrt{3}}{2}\Rightarrow x=\frac{4\pi}{3}+2n\pi$ or $\frac{5\pi}{3}+2n\pi$

**17.** $\tan x=\frac{\sqrt{3}}{3}\Rightarrow x=30^\circ+180n$

**18.** $\cot x=1\Rightarrow x=45^\circ+180n$

**19.** $\sin x = -\frac{1}{2} \Rightarrow x = 210^\circ + 360^\circ n$ or $x = 330^\circ + 360^\circ n$

**20.** $\cos x = \frac{1}{2} \Rightarrow x = 60^\circ + 360^\circ n$ or $300^\circ + 360^\circ n$

**21.** $\csc x = 1 \Rightarrow x = 90^\circ + 360^\circ n$

**22.** $\sec x = -1 \Rightarrow x = 180^\circ + 360^\circ n$

**23.** $\sqrt{3}\csc x - 2 = 0 \Rightarrow \csc x = \frac{2\sqrt{3}}{3} \Rightarrow$ $x = 60^\circ + 360^\circ n$ or $120^\circ + 360^\circ n$

**24.** $\sqrt{3}\sec x + 2 = 0 \Rightarrow \sec x = -\frac{2\sqrt{3}}{3} \Rightarrow$ $x = 150^\circ + 360^\circ n$ or $210^\circ + 360^\circ n$

**25.** $2\sec x - 4 = 0 \Rightarrow \sec x = 2 \Rightarrow$ $x = 60^\circ + 360^\circ n$ or $300^\circ + 360^\circ n$

**26.** $2\csc x + 4 = 0 \Rightarrow \csc x = -2 \Rightarrow$ $x = 210^\circ + 360n$ or $x = 330^\circ + 360n$

**27.** $\sin\theta = 0.4 \Rightarrow \theta$ is in Quadrant I or Quadrant II. $\theta = \sin^{-1}(0.4) = 23.6^\circ$ or $\theta = 180^\circ - 23.6^\circ = 156.4^\circ$

**28.** $\cos\theta = 0.6 \Rightarrow \theta$ is in Quadrant I or Quadrant IV. $\theta = \cos^{-1}(0.6) = 53.1^\circ$ or $\theta = 360^\circ - 53.1^\circ = 306.9^\circ$

**29.** $\sec\theta = 7.2 \Rightarrow \theta$ is in Quadrant I or Quadrant IV. $\theta = \sec^{-1}(7.2) = 82.0^\circ$ or $\theta = 360^\circ - 82.0^\circ = 278.0^\circ$

**30.** $\csc\theta = -4.5 \Rightarrow \theta$ is in Quadrant III or Quadrant IV. $\theta = \csc^{-1}(-4.5) = -12.8^\circ \Rightarrow$ $\theta = 180^\circ - (-12.8^\circ) = 192.8^\circ$ or $\theta = 360^\circ + (-12.8^\circ) = 347.2^\circ$

**31.** $\tan(\theta - 30^\circ) = -5 \Rightarrow \theta - 30^\circ$ is in Quadrant II or Quadrant IV. Let $x = \theta - 30^\circ$. Then,
$x = \tan^{-1}(-5) \approx -78.7^\circ \Rightarrow$
$x \approx 180^\circ - 78.7^\circ \approx 101.3$ or
$x \approx 360^\circ - 78.7^\circ \approx 281.3^\circ \Rightarrow \theta \approx 131.3^\circ$ or
$\theta \approx 311.3^\circ$

**32.** $\cot(\theta + 30^\circ) = 6 \Rightarrow \theta + 30^\circ$ is in Quadrant I or Quadrant III. Let $x = \theta + 30^\circ$. Then,
$x = \cot^{-1} 6 \Rightarrow x \approx 9.5^\circ$ or
$x \approx 180^\circ + 9.5^\circ \approx 189.5^\circ$
$\theta + 30^\circ \approx 9.5^\circ \Rightarrow \theta \approx -20.5^\circ = 360^\circ - 20.5^\circ$
$= 339.5^\circ$. $\theta + 30^\circ \approx 189.5^\circ \Rightarrow \theta \approx 159.5^\circ$.

**33.** $\csc x = -3 \Rightarrow x$ is in Quadrant III or Quadrant IV.
$x = \csc^{-1}(-3) \approx -0.3398 \Rightarrow$
$x \approx \pi - (-0.3398) \approx 3.4814$ or
$x \approx 2\pi + (-0.3398) \approx 5.9433$

**34.** $3\sin x - 1 = 0 \Rightarrow \sin x = 1/3 \Rightarrow x$ is in Quadrant I or Quadrant II.
$x = \sin^{-1}(1/3) \Rightarrow x \approx 0.3398$ or
$x \approx \pi - 0.3398 \approx 2.8018$.

**35.** $3\tan x + 4 = 0 \Rightarrow \tan x = -4/3 \Rightarrow x$ is in Quadrant II or Quadrant IV.
$x = \tan^{-1}(-4/3) \approx -0.9273$
$\approx -0.9273 + \pi \approx 2.2143$ or
$x \approx -0.9273 + 2\pi \approx 5.3559$.

**36.** $2\sec x - 7 = 0 \Rightarrow \sec x = 7/2 \Rightarrow x$ is in Quadrant I or Quadrant IV.
$x = \sec^{-1}(7/2) \approx 1.2810$ or
$x \approx 2\pi - 1.2810 \approx 5.0021$.

**37.** $2\csc x + 5 = 0 \Rightarrow \csc x = -5/2 \Rightarrow x$ is in Quadrant III or Quadrant IV.
$x = \csc^{-1}(-5/2)$
$\approx -0.4115 \Rightarrow x \approx \pi + 0.4115 \approx 3.5531$ or
$x \approx 2\pi - 0.4115 \approx 5.8717$.

**38.** $\cos x = 0.1106 \Rightarrow x$ is in Quadrant I or Quadrant IV. $x = \cos^{-1}(0.1106) \Rightarrow$
$x \approx 1.4600$ or $x \approx 2\pi - 1.4600 \approx 4.8232$.

**39.** $\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{2} \Rightarrow x + \frac{\pi}{4} = \sin^{-1}\left(\frac{1}{2}\right) \Rightarrow$
$x + \frac{\pi}{4} = \frac{\pi}{6} \Rightarrow -\frac{\pi}{12} = \frac{23\pi}{12}$ or
$x + \frac{\pi}{4} = \frac{5\pi}{6} \Rightarrow x = \frac{7\pi}{12}$.
The solution is $\left\{\frac{7\pi}{12}, \frac{23\pi}{12}\right\}$.

**40.** $2\cos\left(x-\frac{\pi}{4}\right)+1=0 \Rightarrow \cos\left(x-\frac{\pi}{4}\right)=-\frac{1}{2} \Rightarrow$
$x-\frac{\pi}{4}=\cos^{-1}\left(-\frac{1}{2}\right) \Rightarrow x-\frac{\pi}{4}=\frac{2\pi}{3} \Rightarrow x=\frac{11\pi}{12}$
or $x-\frac{\pi}{4}=\frac{4\pi}{3} \Rightarrow x=\frac{19\pi}{12}$.
The solution is $\left\{\frac{11\pi}{12}, \frac{19\pi}{12}\right\}$.

**41.** $\sec\left(x-\frac{\pi}{8}\right)+2=0 \Rightarrow x-\frac{\pi}{8}=\sec^{-1}(-2) \Rightarrow$
$x-\frac{\pi}{8}=\frac{2\pi}{3} \Rightarrow x=\frac{19\pi}{24}$ or $x-\frac{\pi}{8}=\frac{4\pi}{3} \Rightarrow$
$x=\frac{35\pi}{24}$. The solution is $\left\{\frac{19\pi}{24}, \frac{35\pi}{24}\right\}$.

**42.** $\csc\left(x+\frac{\pi}{8}\right)-2=0 \Rightarrow x+\frac{\pi}{8}=\csc^{-1}(2) \Rightarrow$
$x+\frac{\pi}{8}=\frac{\pi}{6} \Rightarrow x=\frac{\pi}{24}$ or
$x+\frac{\pi}{8}=\frac{5\pi}{6} \Rightarrow x=\frac{17\pi}{24}$.
The solution is $\left\{\frac{\pi}{24}, \frac{17\pi}{24}\right\}$.

**43.** $\sqrt{3}\tan\left(x-\frac{\pi}{6}\right)-1=0 \Rightarrow \tan\left(x-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3} \Rightarrow$
$x-\frac{\pi}{6}=\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) \Rightarrow x-\frac{\pi}{6}=\frac{\pi}{6} \Rightarrow x=\frac{\pi}{3}$
or $x-\frac{\pi}{6}=\frac{7\pi}{6} \Rightarrow x=\frac{4\pi}{3}$.
The solution is $\left\{\frac{\pi}{3}, \frac{4\pi}{3}\right\}$.

**44.** $\cot\left(x+\frac{\pi}{6}\right)+1=0 \Rightarrow \cot\left(x+\frac{\pi}{6}\right)=-1 \Rightarrow$
$x+\frac{\pi}{6}=\cot^{-1}(-1) \Rightarrow x+\frac{\pi}{6}=\frac{3\pi}{4} \Rightarrow x=\frac{7\pi}{12}$
or $x+\frac{\pi}{6}=\frac{7\pi}{4} \Rightarrow x=\frac{19\pi}{12}$.
The solution is $\left\{\frac{7\pi}{12}, \frac{19\pi}{12}\right\}$.

**45.** $2\sin\left(x-\frac{\pi}{3}\right)+1=0 \Rightarrow \sin\left(x-\frac{\pi}{3}\right)=-\frac{1}{2} \Rightarrow$
$x-\frac{\pi}{3}=\sin^{-1}\left(-\frac{1}{2}\right) \Rightarrow x-\frac{\pi}{3}=\frac{7\pi}{6} \Rightarrow x=\frac{3\pi}{2}$
or $x-\frac{\pi}{3}=\frac{11\pi}{6} \Rightarrow x=\frac{13\pi}{6}=\frac{\pi}{6}$.
The solution is $\left\{\frac{\pi}{6}, \frac{3\pi}{2}\right\}$.

**46.** $2\cos\left(x+\frac{\pi}{3}\right)+\sqrt{2}=0 \Rightarrow \cos\left(x+\frac{\pi}{3}\right)=-\frac{\sqrt{2}}{2}$
$x+\frac{\pi}{3}=\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) \Rightarrow x+\frac{\pi}{3}=\frac{3\pi}{4} \Rightarrow$
$x=\frac{5\pi}{12}$ or $x+\frac{\pi}{3}=\frac{5\pi}{4} \Rightarrow x=\frac{11\pi}{12}$.
The solution is $\left\{\frac{5\pi}{12}, \frac{11\pi}{12}\right\}$.

**47.** $(\sin x+1)(\tan x-1)=0 \Rightarrow$
$\sin x=-1$ or $\tan x=1$
$\sin x=-1 \Rightarrow x=\frac{3\pi}{2}$
However, $\tan\frac{3\pi}{2}$ is undefined, so $x=\frac{3\pi}{2}$ is not a solution.
$\tan x=1 \Rightarrow x=\frac{\pi}{4}$ or $x=\frac{5\pi}{4}$.
The solution is $\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$.

**48.** $(2\cos x+1)(\sqrt{3}\tan x-1)=0 \Rightarrow$
$\cos x=-\frac{1}{2}$ or $\tan x=\frac{\sqrt{3}}{3}$
$\cos x=-\frac{1}{2} \Rightarrow x=\frac{2\pi}{3}$ or $x=\frac{4\pi}{3}$
$\tan x=\frac{\sqrt{3}}{3} \Rightarrow x=\frac{\pi}{6}$ or $x=\frac{7\pi}{6}$.
The solution is $\left\{\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}\right\}$.

**49.** $(\csc x-2)(\cot x+1)=0 \Rightarrow \csc x=2$ or
$\cot x=-1$; $\csc x=2 \Rightarrow x=\frac{\pi}{6}$ or $x=\frac{5\pi}{6}$
$\cot x=-1 \Rightarrow x=\frac{3\pi}{4}$ or $x=\frac{7\pi}{4}$.
The solution is $\left\{\frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{4}\right\}$.

**50.** $\left(\sqrt{3}\sec x-2\right)\left(\sqrt{3}\cot x+1\right)=0 \Rightarrow \sec x=\frac{2\sqrt{3}}{3}$ or $\cot x=-\frac{\sqrt{3}}{3}$; $\sec x=\frac{2\sqrt{3}}{3} \Rightarrow x=\frac{\pi}{6}$ or $x=\frac{11\pi}{6}$; $\cot x=-\frac{\sqrt{3}}{3} \Rightarrow x=\frac{2\pi}{3}$ or $x=\frac{5\pi}{3}$
The solution is $\left\{\frac{\pi}{6},\frac{2\pi}{3},\frac{5\pi}{3},\frac{11\pi}{6}\right\}$.

**51.** $(\tan x+1)(2\sin x-1)=0 \Rightarrow \tan x=-1$ or $\sin x=\frac{1}{2}$; $\tan x=-1 \Rightarrow x=\frac{3\pi}{4}$ or $\frac{7\pi}{4}$
$\sin x=\frac{1}{2} \Rightarrow x=\frac{\pi}{6}$ or $x=\frac{5\pi}{6}$
The solution is $\left\{\frac{\pi}{6},\frac{3\pi}{4},\frac{5\pi}{6},\frac{7\pi}{4}\right\}$.

**52.** $\left(2\sin x-\sqrt{3}\right)\left(2\cos x-1\right)=0 \Rightarrow \sin x=\frac{\sqrt{3}}{2}$ or $\cos x=\frac{1}{2}$; $\sin x=\frac{\sqrt{3}}{2} \Rightarrow x=\frac{\pi}{3}$ or $x=\frac{2\pi}{3}$
$\cos x=\frac{1}{2} \Rightarrow x=\frac{\pi}{3}$ or $x=\frac{5\pi}{3}$.
The solution is $\left\{\frac{\pi}{3},\frac{2\pi}{3},\frac{5\pi}{3}\right\}$.

**53.** $\left(\sqrt{2}\sec x-2\right)\left(2\sin x+1\right)=0 \Rightarrow \sec x=\sqrt{2}$ or $\sin x=-\frac{1}{2}$; $\sec x=\sqrt{2} \Rightarrow x=\frac{\pi}{4}$ or $x=\frac{7\pi}{4}$
$\sin x=-\frac{1}{2} \Rightarrow x=\frac{7\pi}{6}$ or $x=\frac{11\pi}{6}$.
The solution is $\left\{\frac{\pi}{4},\frac{7\pi}{6},\frac{7\pi}{4},\frac{11\pi}{6}\right\}$.

**54.** $\left(\cot x-1\right)\left(\sqrt{2}\csc x+2\right)=0 \Rightarrow \cot x=1$ or $\csc x=-\sqrt{2}$; $\cot x=1 \Rightarrow x=\frac{\pi}{4}$ or $x=\frac{5\pi}{4}$
$\csc x=-\sqrt{2} \Rightarrow x=\frac{5\pi}{4}$ or $x=\frac{7\pi}{4}$.
The solution is $\left\{\frac{\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\}$.

**55.** $4\sin^2 x=1 \Rightarrow \sin x=\pm\frac{1}{2} \Rightarrow x=\frac{\pi}{6}$ or $x=\frac{5\pi}{6}$ or $x=\frac{7\pi}{6}$ or $x=\frac{11\pi}{6}$.
The solution is $\left\{\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}\right\}$.

**56.** $4\cos^2 x=1 \Rightarrow \cos x=\pm\frac{1}{2} \Rightarrow x=\frac{\pi}{3}$ or $x=\frac{2\pi}{3}$ or $x=\frac{4\pi}{3}$ or $x=\frac{5\pi}{3}$.
The solution is $\left\{\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}\right\}$.

**57.** $\tan^2 x=1 \Rightarrow \tan x=\pm 1 \Rightarrow x=\frac{\pi}{4}$ or $x=\frac{3\pi}{4}$ or $x=\frac{5\pi}{4}$ or $x=\frac{7\pi}{4}$.
The solution is $\left\{\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\}$.

**58.** $\sec^2 x=2 \Rightarrow \sec x=\pm\sqrt{2} \Rightarrow x=\frac{\pi}{4}$ or $x=\frac{3\pi}{4}$ or $x=\frac{5\pi}{4}$ or $x=\frac{7\pi}{4}$.
The solution is $\left\{\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\}$.

**59.** $3\csc^2 x=4 \Rightarrow \csc x=\pm\frac{2\sqrt{3}}{3} \Rightarrow x=\frac{\pi}{3}$ or $x=\frac{2\pi}{3}$ or $x=\frac{4\pi}{3}$ or $x=\frac{5\pi}{3}$.
The solution is $\left\{\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}\right\}$.

**60.** $3\cot^2 x=1 \Rightarrow \cot x=\pm\frac{\sqrt{3}}{3} \Rightarrow x=\frac{\pi}{3}$ or $x=\frac{2\pi}{3}$ or $x=\frac{4\pi}{3}$ or $x=\frac{5\pi}{3}$
The solution is $\left\{\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}\right\}$.

**61.** $2\sin^2\theta-\sin\theta-1=0 \Rightarrow$
$(2\sin\theta+1)(\sin\theta-1)=0 \Rightarrow \sin\theta=-\frac{1}{2}$ or $\sin\theta=1$; $\sin\theta=-\frac{1}{2} \Rightarrow \theta=\frac{7\pi}{6}$ or $\theta=\frac{11\theta}{6}$
$\sin\theta=1 \Rightarrow \theta=\frac{\pi}{2}$
The solution is $\left\{\frac{\pi}{2},\frac{7\pi}{6},\frac{11\pi}{6}\right\}$.

**62.** $2\cos^2\theta-5\cos\theta+2=0 \Rightarrow$
$(2\cos\theta-1)(\cos\theta-2)=0 \Rightarrow \cos\theta=1/2$ or $\cos\theta=2$. (Reject this). $\cos\theta=1/2 \Rightarrow$
$\theta=\frac{\pi}{3}$ or $\theta=\frac{5\pi}{3}$. The solution is $\left\{\frac{\pi}{3},\frac{5\pi}{3}\right\}$.

**63.** $\sin x = \cos x \Rightarrow \dfrac{\sin x}{\cos x} = 1 \Rightarrow \tan x = 1 \Rightarrow x = \dfrac{\pi}{4}$ or $x = \dfrac{5\pi}{4}$. The solution is $\left\{\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right\}$.

**64.** $\sqrt{3}\sin x + \cos x = 0 \Rightarrow \dfrac{\cos x}{\sin x} = -\sqrt{3} \Rightarrow$
$\cot x = -\sqrt{3} \Rightarrow x = \dfrac{5\pi}{6}$ or $x = \dfrac{11\pi}{6}$.
The solution is $\left\{\dfrac{5\pi}{6}, \dfrac{11\pi}{6}\right\}$.

**65.** $3\sin^2 x = \cos^2 x \Rightarrow 3 = \cot^2 x \Rightarrow$
$\pm\sqrt{3} = \cot x \Rightarrow x = \dfrac{\pi}{6}$ or $x = \dfrac{5\pi}{6}$ or
$x = \dfrac{7\pi}{6}$ or $x = \dfrac{11\pi}{6}$
The solution is $\left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}\right\}$.

**66.** $3\cos^2 x = \sin^2 x \Rightarrow 3 = \tan^2 x \Rightarrow$
$\pm\sqrt{3} = \tan x \Rightarrow x = \dfrac{\pi}{3}$ or $x = \dfrac{2\pi}{3}$ or
$x = \dfrac{4\pi}{3}$ or $x = \dfrac{5\pi}{3}$
The solution is $\left\{\dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}\right\}$.

**67.** $\cos^2 x - \sin^2 x = 1 \Rightarrow 1 - \sin^2 x - \sin^2 x = 1 \Rightarrow$
$-2\sin^2 x = 0 \Rightarrow \sin x = 0 \Rightarrow x = 0$ or $x = \pi$
The solution is $\{0, \pi\}$.

**68.** $2\sin^2 x + \cos x - 1 = 0 \Rightarrow$
$2(1-\cos^2 x) + \cos x - 1 = 0 \Rightarrow$
$2\cos^2 x - \cos x - 1 = 0 \Rightarrow$
$(2\cos x + 1)(\cos x - 1) = 0 \Rightarrow \cos x = -\dfrac{1}{2}$ or
$\cos x = 1$; $\cos x = -\dfrac{1}{2} \Rightarrow x = \dfrac{2\pi}{3}$ or $x = \dfrac{4\pi}{3}$
$\cos x = 1 \Rightarrow x = 0$.
The solution is $\left\{0, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}\right\}$.

**69.** $2\cos^2 x - 3\sin x - 3 = 0 \Rightarrow$
$2(1-\sin^2 x) - 3\sin x - 3 = 0 \Rightarrow$
$2\sin^2 x + 3\sin x + 1 = 0 \Rightarrow$
$(2\sin x + 1)(\sin x + 1) = 0 \Rightarrow \sin x = -1/2$ or
$\sin x = -1$; $\sin x = -\dfrac{1}{2} \Rightarrow x = \dfrac{7\pi}{6}$ or $x = \dfrac{11\pi}{6}$
$\sin x = -1 \Rightarrow x = \dfrac{3\pi}{2}$.
The solution is $\left\{\dfrac{7\pi}{6}, \dfrac{3\pi}{2}, \dfrac{11\pi}{6}\right\}$.

**70.** $2\sin^2 x - \cos x - 1 = 0 \Rightarrow$
$2(1-\cos^2 x) - \cos x - 1 = 0 \Rightarrow$
$2\cos^2 x + \cos x - 1 = 0 \Rightarrow$
$(2\cos x - 1)(\cos x + 1) = 0 \Rightarrow \cos x = 1/2$ or
$\cos x = -1$; $\cos x = \dfrac{1}{2} \Rightarrow x = \dfrac{\pi}{3}$ or $x = \dfrac{5\pi}{3}$
$\cos x = -1 \Rightarrow x = \pi$.
The solution is $\left\{\dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}\right\}$.

**71.** $\sqrt{3}\sec^2 x - 2\tan x - 2\sqrt{3} = 0 \Rightarrow$
$\sqrt{3}(1+\tan^2 x) - 2\tan x - 2\sqrt{3} = 0 \Rightarrow$
$\sqrt{3}\tan^2 x - 2\tan x - \sqrt{3} = 0 \Rightarrow$
$\left(\sqrt{3}\tan x + 1\right)\left(\tan x - \sqrt{3}\right) = 0 \Rightarrow \tan x = -\dfrac{\sqrt{3}}{3}$
or $\tan x = \sqrt{3}$; $\tan x = -\dfrac{\sqrt{3}}{3} \Rightarrow x = \dfrac{5\pi}{6}$ or
$x = \dfrac{11\pi}{6}$; $\tan x = \sqrt{3} \Rightarrow x = \dfrac{\pi}{3}$ or $x = \dfrac{4\pi}{3}$.
The solution is $\left\{\dfrac{\pi}{3}, \dfrac{5\pi}{6}, \dfrac{4\pi}{3}, \dfrac{11\pi}{6}\right\}$.

**72.** $\csc^2 x - \left(\sqrt{3}+1\right)\cot x + \left(\sqrt{3}-1\right) = 0 \Rightarrow$
$1 + \cot^2 x - \left(\sqrt{3}+1\right)\cot x + \left(\sqrt{3}-1\right) = 0 \Rightarrow$
$\cot^2 x - \left(\sqrt{3}+1\right)\cot x + \sqrt{3} = 0 \Rightarrow$
$\left(\cot x - \sqrt{3}\right)\left(\cot x - 1\right) = 0 \Rightarrow \cot x = \sqrt{3}$ or
$\cot x = 1$; $\cot x = \sqrt{3} \Rightarrow x = \dfrac{\pi}{6}$ or $x = \dfrac{7\pi}{6}$
$\cot x = 1 \Rightarrow x = \dfrac{\pi}{4}$ or $x = \dfrac{5\pi}{4}$.
The solution is $\left\{\dfrac{\pi}{6}, \dfrac{\pi}{4}, \dfrac{7\pi}{6}, \dfrac{5\pi}{4}\right\}$.

**73.** $\sqrt{3}\sin x = 1 + \cos x \Rightarrow$
$\left(\sqrt{3}\sin x\right)^2 = \left(1+\cos x\right)^2 \Rightarrow$
$3\sin^2 x = \cos^2 x + 2\cos x + 1 \Rightarrow$
$3(1-\cos^2 x) - \cos^2 x - 2\cos x - 1 = 0 \Rightarrow$
$-4\cos^2 x - 2\cos x + 2 = 0 \Rightarrow$
$-2(2\cos x - 1)(\cos x + 1) = 0 \Rightarrow \cos x = \dfrac{1}{2} \Rightarrow$

(*continued on next page*)

(*continued*)

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ or $\cos x = -1 \Rightarrow x = \pi$

Checking each answer, we find that $x = \frac{5\pi}{3}$ is extraneous. The solution is $\left\{\frac{\pi}{3}, \pi\right\}$.

**74.** $\tan x + 1 = \sec x \Rightarrow (\tan x + 1)^2 = \sec^2 x \Rightarrow$
$\tan^2 x + 2\tan x + 1 = \tan^2 x + 1 \Rightarrow$
$2\tan x = 0 \Rightarrow x = 0$ or $x = \pi$
Checking each answer, we find that $x = \pi$ is extraneous. The solution is $\{0\}$.

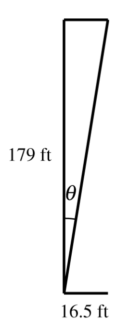
**75.** $\sqrt{3}\tan\theta + 1 = \sqrt{3}\sec\theta \Rightarrow$
$\left(\sqrt{3}\tan\theta + 1\right)^2 = \left(\sqrt{3}\sec\theta\right)^2 \Rightarrow$
$3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 3\sec^2\theta$
$= 3(\tan^2 x + 1) \Rightarrow$
$2\sqrt{3}\tan\theta - 2 = 0 \Rightarrow \tan\theta = \frac{\sqrt{3}}{3} \Rightarrow \theta = \frac{\pi}{6}, \frac{7\pi}{6}$
Checking each answer, we find that $\theta = 7\pi/6$ is extraneous. The solution is $\{\pi/6\}$.

**76.** $\sqrt{3}\cot\theta + 1 = \sqrt{3}\csc\theta \Rightarrow$
$\left(\sqrt{3}\cot\theta + 1\right)^2 = \left(\sqrt{3}\csc\theta\right)^2 \Rightarrow$
$3\cot^2\theta + 2\sqrt{3}\cot\theta + 1 = 3\csc^2\theta$
$= 3(\cot^2 x + 1) \Rightarrow$
$2\sqrt{3}\cot\theta - 2 = 0 \Rightarrow \cot\theta = \frac{\sqrt{3}}{3} \Rightarrow \theta = \frac{\pi}{3}, \frac{4\pi}{3}$
Checking each answer, we find that $\theta = 4\pi/3$ is extraneous. The solution is $\{\pi/3\}$.

### 6.5 Applying the Concepts

**77.** $\tan\theta = \frac{24}{8\sqrt{3}} = \sqrt{3} \Rightarrow \theta = 60°$

**78.**




Not drawn to scale

$\tan\theta = \frac{16.5}{179} \Rightarrow \theta \approx 5.3°$

**79.** $\sin\theta = \frac{60}{605} \Rightarrow \theta \approx 5.7°$

**80.** $d = tv_0\cos\theta \Rightarrow 112 = (3)(78)\cos\theta \Rightarrow$
$\cos\theta = \frac{112}{234} \Rightarrow \theta = \cos^{-1}\left(\frac{112}{234}\right) \approx 61°$

For exercises 81 and 82, from the section opener, we know that the height traveled by a projectile is $h = tv_0\sin\theta - 16t^2$, where $h$ = the height, $t$ = the time in seconds, $\theta =$ the initial angle the projectile makes with the ground, and $v_0$ is the projectile's initial velocity.

**81.** $h = tv_0\sin\theta - 16t^2 \Rightarrow$
$25 = (4)(150)\sin\theta - 16\left(4^2\right) \Rightarrow$
$\sin\theta = \frac{281}{600} \Rightarrow \theta = \sin^{-1}\left(\frac{281}{600}\right) \approx 28°$

**82.** $h = tv_0\sin\theta - 16t^2 \Rightarrow$
$40 = (2)(140)\sin\theta - 16\left(2^2\right) \Rightarrow$
$\sin\theta = \frac{104}{280} \Rightarrow \theta = \sin^{-1}\left(\frac{104}{280}\right) \approx 22°$

### 6.5 Beyond the Basics

**83.** Using factoring by grouping, we have
$\sin x\cos x - \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x - \frac{\sqrt{3}}{4} = 0 \Rightarrow$
$\sin x\left(\cos x - \frac{\sqrt{3}}{2}\right) + \frac{1}{2}\left(\cos x - \frac{\sqrt{3}}{2}\right) = 0 \Rightarrow$
$\left(\sin x + \frac{1}{2}\right)\left(\cos x - \frac{\sqrt{3}}{2}\right) = 0 \Rightarrow \sin x = -\frac{1}{2}$
or $\cos x = \frac{\sqrt{3}}{2}$. If $\sin x = -\frac{1}{2} \Rightarrow$
$x = \frac{7\pi}{6} + 2n\pi \Rightarrow x = \frac{7\pi}{6}$ or
$x = \frac{11\pi}{6} + 2n\pi \Rightarrow x = \frac{11\pi}{6}$
If $\cos x = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6} + 2n\pi \Rightarrow x = \frac{\pi}{6}$ or
$x = \frac{11\pi}{6} + 2n\pi \Rightarrow x = \frac{11\pi}{6}$.
The solution is $\left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$.

**84.** $2\sin x\tan x + 2\sin x + \tan x + 1 = 0 \Rightarrow$
$2\sin x(\tan x + 1) + 1(\tan x + 1) = 0 \Rightarrow$
$(2\sin x + 1)(\tan x + 1) = 0 \Rightarrow 2\sin x + 1 = 0 \Rightarrow$
$\sin x = -\frac{1}{2}$ or $\tan x = -1$.
If $\sin x = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{6} + 2\pi n \Rightarrow x = \frac{7\pi}{6}$ or
$x = \frac{11\pi}{6} + 2\pi n \Rightarrow x = \frac{11\pi}{6}$.
If $\tan x = -1 \Rightarrow x = \frac{3\pi}{4} + \pi n \Rightarrow$
$x = \frac{3\pi}{4}$ or $x = \frac{7\pi}{4}$.
The solution is $\left\{\frac{3\pi}{4}, \frac{7\pi}{6}, \frac{7\pi}{4}, \frac{11\pi}{6}\right\}$.

**85.**
$$5\sin^2\theta + \cos^2\theta - \sec^2\theta = 0$$
$$5\left(1-\cos^2\theta\right) + \cos^2\theta - \frac{1}{\cos^2\theta} = 0$$
$$5 - 4\cos^2\theta - \frac{1}{\cos^2\theta} = 0$$
$$-4\cos^4\theta + 5\cos^2\theta - 1 = 0$$
$$4\cos^4\theta - 5\cos^2\theta + 1 = 0$$
$$\left(4\cos^2\theta - 1\right)\left(\cos^2\theta - 1\right) = 0 \Rightarrow$$
$$(2\cos\theta - 1)(2\cos\theta + 1)(\cos\theta - 1)(\cos\theta + 1) = 0$$
$2\cos\theta - 1 = 0 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$
$2\cos\theta + 1 = 0 \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$
$\cos\theta - 1 = 0 \Rightarrow \cos\theta = 1 \Rightarrow \theta = 0$
$\cos\theta + 1 = 0 \Rightarrow \cos\theta = -1 \Rightarrow \theta = \pi$
Solution set: $\left\{0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

**86.** $2\sin\theta = \cos\theta \Rightarrow 2 = \frac{\cos\theta}{\sin\theta} = \cot\theta \Rightarrow$
$\theta = \cot^{-1} 2 \Rightarrow \theta \approx 0.4636, 3.6052$

**87.**
$$2\cos x = 1 - \sin x$$
$$(2\cos x)^2 = (1-\sin x)^2$$
$$4\cos^2 x = \sin^2 x - 2\sin x + 1$$
$$4\left(1-\sin^2 x\right) = \sin^2 x - 2\sin x + 1$$
$$4 - 4\sin^2 x = \sin^2 x - 2\sin x + 1$$
$$5\sin^2 x - 2\sin x - 3 = 0$$
$$(5\sin x + 3)(\sin x - 1) = 0 \Rightarrow$$
$5\sin x + 3 = 0 \Rightarrow \sin x = -\frac{3}{5} \Rightarrow x \approx 5.6397$ or
$\sin x - 1 = 0 \Rightarrow \sin x = 1 \Rightarrow x = \frac{\pi}{2}$
Solution set: $\left\{\frac{\pi}{2}, 5.6397\right\}$

**88.** $\cos^4 x - \cos^2 x + 1 = 0$
Let $u = \cos^2 x$. Then, we have
$\cos^4 x - \cos^2 x + 1 = 0 \Rightarrow u^2 - u + 1 = 0$.
Using the quadratic formula, we have
$$u = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} = \frac{1 \pm \sqrt{-3}}{2}.$$
There is no real solution for $u$, therefore, there is no solution for the original equation.
Solution set: $\varnothing$

## 6.5 Critical Thinking/Discussion/Writing

**89.** $\tan^2 x = 4 \Rightarrow \tan x = \pm 2$
If $\tan x = 2$, then $x \approx 1.107$ or
$x \approx 1.107 + \pi \approx 4.249$. If $\tan x = -2$, then
$x \approx -1.107$, which is not in $[0, 2\pi)$. So,
$x \approx -1.107 + \pi \approx 2.034$, and
$x \approx -1.107 + 2\pi \approx 5.176$.
Solution: {1.107, 2.034, 4.249, 5.176}

**90.** $3\cos^2 x + \cos x = 0 \Rightarrow \cos x(3\cos x + 1) = 0 \Rightarrow$
$\cos x = 0$ or $\cos x = -\frac{1}{3}$
If $\cos x = 0$, then $x = \frac{\pi}{2} \approx 1.5708$ or
$x = \frac{3\pi}{2} \approx 4.7124$. If $\cos x = -\frac{1}{3}$, then
$x \approx 1.9106$ (which is in quadrant II) or
$x \approx 4.3726$ (which is in quadrant III).
Solution: {1.571, 1.911, 4.373, 4.712}

**91.** $\cos x - \sec x + 1 = 0 \Rightarrow \cos x - \frac{1}{\cos x} + 1 = 0 \Rightarrow$
$\cos^2 x - 1 + \cos x = 0 \Rightarrow$
$\cos^2 x + \cos x + \frac{1}{4} = 1 + \frac{1}{4} \Rightarrow \left(\cos x + \frac{1}{2}\right)^2 = \frac{5}{4} \Rightarrow$
$\cos x + \frac{1}{2} = \pm\frac{\sqrt{5}}{2} \Rightarrow \cos x = -\frac{1}{2} \pm \frac{\sqrt{5}}{2}$
If $\cos x = -\frac{1}{2} + \frac{\sqrt{5}}{2}$, then $x \approx 0.9046$ or
$x \approx 2\pi - 0.9046 \approx 5.3786$.

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$\cos x \neq -\frac{1}{2} - \frac{\sqrt{5}}{2}$, since $-\frac{1}{2} - \frac{\sqrt{5}}{2} \approx -1.6$, which is not in the range of $\cos x$.
Solution: {0.9046, 5.3786}

**92.**
$$3\csc x + 2\cot^2 x = 5$$
$$3\csc x + 2(\csc^2 x - 1) = 5$$
$$2\csc^2 x + 3\csc x - 7 = 0$$
Use the quadratic formula to solve for $\csc x$:
$$\csc x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-7)}}{2(2)} = \frac{-3 \pm \sqrt{65}}{4}$$
If $\csc x = \frac{-3+\sqrt{65}}{4}$, then $x \approx 0.9111$ or $x \approx \pi - 0.9111 \approx 2.2305$. If $\csc x = \frac{-3-\sqrt{65}}{4}$, then $x \approx 3.5116$ or $x \approx 5.9132$.
Solution: {0.911, 2.231, 3.512, 5.913}

**93.** $\sin x \cos x = \frac{1}{2} \Rightarrow 2\sin x \cos x = 1 \Rightarrow$
$\sin 2x = 1 \Rightarrow 2x = \frac{\pi}{2}$ or $2x = \frac{\pi}{2} + 2\pi \Rightarrow$
$x = \frac{\pi}{4}$ or $x = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$
The solution is $\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$.

**94.** $\sin x \cos x = 1 \Rightarrow \sin^2 x \cos^2 x = 1 \Rightarrow$
$\sin^2 x(1 - \sin^2 x) = 1 \Rightarrow$
$\sin^2 x - \sin^4 x - 1 = 0 \Rightarrow \sin^4 x - \sin^2 x + 1 = 0.$
Letting $u = \sin^2 x$, we have $u^2 - u + 1 = 0 \Rightarrow$
$$u = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} = \frac{1 \pm \sqrt{-3}}{2}$$
$$= \frac{1}{2} \pm \frac{i\sqrt{3}}{2} \Rightarrow \sin^2 x = \frac{1}{2} \pm \frac{i\sqrt{3}}{2}.$$
Similarly, we can show that $\cos^2 x = \frac{1}{2} \pm \frac{i\sqrt{3}}{2}$.
So, there are no real roots.

### 6.5 Maintaining Skills

**95.** $\sin x = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$

**96.** $\tan x - 1 = 0 \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$

**97.** $2\cos x = \sqrt{3} \Rightarrow \cos x = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6}, \frac{11\pi}{6}$

**98.** $4\sin^2 x = 1 \Rightarrow \sin^2 x = \frac{1}{4} \Rightarrow \sin x = \pm\frac{1}{2} \Rightarrow$
$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

**99.** Note that the range of $\cos^{-1} y$ is $[0, \pi]$.
$x = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) \Rightarrow \cos x = -\frac{\sqrt{2}}{2} \Rightarrow x = \frac{3\pi}{4}$

**100.** Note that the range of $\sin^{-1} y$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
$x = \sin^{-1}\left(\frac{1}{2}\right) \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$

**101.** Note that the range of $\tan^{-1} y$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.
$x = \tan^{-1}(1) \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$

**102.** Note that the range of $\tan^{-1} y$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.
$x = \tan^{-1}\left(-\sqrt{3}\right) \Rightarrow \tan x = -\sqrt{3} \Rightarrow x = -\frac{\pi}{3}$

**103.**
$$\cos 3x + \cos 5x = 2\cos\left(\frac{3x+5x}{2}\right)\cos\left(\frac{3x-5x}{2}\right)$$
$$= 2\cos 4x \cos(-x)$$
$$= 2\cos 4x \cos x$$

**104.** $\sin^{-1} x = \frac{\pi}{5} \Rightarrow x = \sin\frac{\pi}{5} = \cos\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$

## 6.6 Trigonometric Equations II

### 6.6 Practice Problems

**1.** $\sin 2x = \frac{1}{2} \Rightarrow 2x = \sin^{-1}\left(\frac{1}{2}\right) \Rightarrow$
$2x = \frac{\pi}{6} + 2n\pi$ or $\frac{5\pi}{6} + 2n\pi \Rightarrow$
$x = \frac{\pi}{12} + n\pi$ or $x = \frac{5\pi}{12} + n\pi \Rightarrow$
$x = \frac{\pi}{12}$ or $x = \frac{5\pi}{12}$ or $x = \frac{13\pi}{12}$ or $x = \frac{17\pi}{12}$
Now find the values of $n$ that result in solutions in the interval $[0, 2\pi)$: $n = 0, 1$.
Solution set: $\left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}\right\}$

**2.** $\tan\frac{x}{2}=\frac{1}{\sqrt{3}}\Rightarrow\frac{x}{2}=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\Rightarrow$
$\frac{x}{2}=\frac{\pi}{6}+n\pi\Rightarrow x=\frac{\pi}{3}+2n\pi\Rightarrow x=\frac{\pi}{3}$
Now find the values of $n$ that result in solutions in the interval $[0,2\pi)$: $n=0$

Solution set: $\left\{\frac{\pi}{3}\right\}$

**3.** $0.3=0.5\sin\left[\frac{\pi}{14.75}\left(x-\frac{14.75}{2}\right)\right]+0.5$
$-0.2=0.5\sin\left[\frac{\pi}{14.75}\left(x-\frac{14.75}{2}\right)\right]$
$-0.4=\sin\left[\frac{\pi}{14.75}\left(x-\frac{14.75}{2}\right)\right]$
Thus, $\theta=\sin^{-1}(-0.4)\approx-0.4115$ or
$\theta\approx\pi-0.4115\approx3.5561$
$$-0.4115=\frac{\pi}{14.75}\left(x-\frac{14.75}{2}\right)$$
$$\frac{14.75(-0.4115)}{\pi}=x-\frac{14.75}{2}$$
$$x=-\frac{6.0699}{\pi}+\frac{14.75}{2}\approx5.44$$
$$3.5561=\frac{\pi}{14.75}\left(x-\frac{14.75}{2}\right)$$
$$\frac{14.75(3.5561)}{\pi}=x-\frac{14.75}{2}$$
$$x=16.6821+\frac{14.75}{2}\approx24.06$$
So, 30% of the Moon is visible about 5 days and about 24 days after the new moon.

**4.** $\cot3\theta=1\Rightarrow3\theta=45°+180°(n)\Rightarrow$
$\theta=15°+60°(n)$

| $n$ | $\theta$ |
|---|---|
| 0 | $\theta=15°+60°(0)=15°$ |
| 1 | $\theta=15°+60°(1)=75°$ |
| 2 | $\theta=15°+60°(2)=135°$ |
| 3 | $\theta=15°+60°(3)=195°$ |
| 4 | $\theta=15°+60°(4)=255°$ |
| 5 | $\theta=15°+60°(5)=315°$ |

Solution set: {15°, 75°, 135°, 195°, 255°, 315°}

**5.** $\sin3x+\cos3x=\sqrt{\frac{3}{2}}\Rightarrow$
$(\sin3x+\cos3x)^2=\left(\sqrt{\frac{3}{2}}\right)^2\Rightarrow$
$\sin^2 3x+2\sin3x\cos3x+\cos^2 3x=\frac{3}{2}\Rightarrow$
$1+\sin6x=\frac{3}{2}\Rightarrow\sin6x=\frac{1}{2}$
$\sin6x=\frac{1}{2}\Rightarrow6x=\frac{\pi}{6}+2n\pi\Rightarrow x=\frac{\pi}{36}+\frac{n\pi}{3}$
or $6x=\frac{5\pi}{6}+2n\pi\Rightarrow x=\frac{5\pi}{36}+\frac{n\pi}{3}$

| $n$ | $x$ |
|---|---|
| 0 | $x=\frac{\pi}{36}+\frac{(0)\pi}{3}=\frac{\pi}{36}$ |
| 0 | $x=\frac{5\pi}{36}+\frac{(0)\pi}{3}=\frac{5\pi}{36}$ |
| 1 | $x=\frac{\pi}{36}+\frac{(1)\pi}{3}=\frac{13\pi}{36}$ |
| 1 | $x=\frac{5\pi}{36}+\frac{(1)\pi}{3}=\frac{17\pi}{36}$ |
| 2 | $x=\frac{\pi}{36}+\frac{(2)\pi}{3}=\frac{25\pi}{36}$ |
| 2 | $x=\frac{5\pi}{36}+\frac{(2)\pi}{3}=\frac{29\pi}{36}$ |
| 3 | $x=\frac{\pi}{36}+\frac{(3)\pi}{3}=\frac{37\pi}{36}$ |
| 3 | $x=\frac{5\pi}{36}+\frac{(3)\pi}{3}=\frac{41\pi}{36}$ |
| 4 | $x=\frac{\pi}{36}+\frac{(4)\pi}{3}=\frac{49\pi}{36}$ |
| 4 | $x=\frac{5\pi}{36}+\frac{(4)\pi}{3}=\frac{53\pi}{36}$ |

Since squaring both sides of an equation may result in extraneous solutions, check all possible solutions.

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Check:

| $x$ | $\sin 3x + \cos 3x \overset{?}{=} \sqrt{\frac{3}{2}}$ |
|---|---|
| $\frac{\pi}{36}$ | $\sin\left[3\left(\frac{\pi}{36}\right)\right] + \cos\left[3\left(\frac{\pi}{36}\right)\right] = \sqrt{\frac{3}{2}}$ ✓ |
| $\frac{5\pi}{36}$ | $\sin\left[3\left(\frac{5\pi}{36}\right)\right] + \cos\left[3\left(\frac{5\pi}{36}\right)\right] = \sqrt{\frac{3}{2}}$ ✓ |
| $\frac{13\pi}{36}$ | $\sin\left[3\left(\frac{13\pi}{36}\right)\right] + \cos\left[3\left(\frac{13\pi}{36}\right)\right] = -\sqrt{\frac{3}{2}}$ × |
| $\frac{17\pi}{36}$ | $\sin\left[3\left(\frac{17\pi}{36}\right)\right] + \cos\left[3\left(\frac{17\pi}{36}\right)\right] = -\sqrt{\frac{3}{2}}$ × |
| $\frac{25\pi}{36}$ | $\sin\left[3\left(\frac{25\pi}{36}\right)\right] + \cos\left[3\left(\frac{25\pi}{36}\right)\right] = \sqrt{\frac{3}{2}}$ ✓ |
| $\frac{29\pi}{36}$ | $\sin\left[3\left(\frac{29\pi}{36}\right)\right] + \cos\left[3\left(\frac{29\pi}{36}\right)\right] = \sqrt{\frac{3}{2}}$ ✓ |
| $\frac{37\pi}{36}$ | $\sin\left[3\left(\frac{37\pi}{36}\right)\right] + \cos\left[3\left(\frac{37\pi}{36}\right)\right] = -\sqrt{\frac{3}{2}}$ × |
| $\frac{41\pi}{36}$ | $\sin\left[3\left(\frac{41\pi}{36}\right)\right] + \cos\left[3\left(\frac{41\pi}{36}\right)\right] = -\sqrt{\frac{3}{2}}$ × |
| $\frac{49\pi}{36}$ | $\sin\left[3\left(\frac{49\pi}{36}\right)\right] + \cos\left[3\left(\frac{49\pi}{36}\right)\right] = \sqrt{\frac{3}{2}}$ ✓ |
| $\frac{53\pi}{36}$ | $\sin\left[3\left(\frac{53\pi}{36}\right)\right] + \cos\left[3\left(\frac{53\pi}{36}\right)\right] = \sqrt{\frac{3}{2}}$ ✓ |

Solution set:
$\left\{\frac{\pi}{36}, \frac{5\pi}{36}, \frac{25\pi}{36}, \frac{29\pi}{36}, \frac{49\pi}{36}, \frac{53\pi}{36}\right\}$

**6.** $\sin 2x + \sin 3x = 0$

$$2\sin\frac{5x}{2}\cos\left(-\frac{x}{2}\right) = 0$$
$$2\sin\frac{5x}{2}\cos\frac{x}{2} = 0 \Rightarrow \sin\frac{5x}{2}\cos\frac{x}{2} = 0 \Rightarrow$$
$$\sin\frac{5x}{2} = 0 \text{ or } \cos\frac{x}{2} = 0.$$
$\sin\theta = 0 \Rightarrow \theta = 0 + 2n\pi = 2n\pi$ or
$\theta = \pi + 2n\pi \Rightarrow \frac{5x}{2} = 2n\pi \Rightarrow x = \frac{4n\pi}{5}$ or
$\frac{5x}{2} = \pi + 2n\pi \Rightarrow x = \frac{2\pi + 4n\pi}{5}$

$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2} + 2n\pi$ or $\theta = \frac{3\pi}{2} + 2n\pi.$
$\frac{x}{2} = \frac{\pi}{2} + 2n\pi \Rightarrow x = \pi + 4n\pi$ or
$\frac{x}{2} = \frac{3\pi}{2} + 2n\pi \Rightarrow x = \frac{3\pi}{4} + 4n\pi$

| $n$ | $x$ |
|---|---|
| 0 | $x = \frac{4(0)\pi}{5} = 0$ |
| 0 | $x = \frac{2\pi + 4(0)\pi}{5} = \frac{2\pi}{5}$ |
| 0 | $x = \pi + 4(0)\pi = \pi$ |
| 0 | $x = \frac{3\pi}{4} + 4(0)\pi = \frac{3\pi}{4}$ |
| 1 | $x = \frac{4(1)\pi}{5} = \frac{4\pi}{5}$ |
| 1 | $x = \frac{2\pi + 4(1)\pi}{5} = \frac{6\pi}{5}$ |
| 1 | $x = \pi + 4(1)\pi = 5\pi$ out of range |
| 1 | $x = \frac{3\pi}{4} + 4(1)\pi = \frac{19\pi}{4}$ out of range |
| 2 | $x = \frac{4(2)\pi}{5} = \frac{8\pi}{5}$ |
| 2 | $x = \frac{2\pi + 4(2)\pi}{5} = \frac{10\pi}{5} = 2\pi$ (out of range) |

Check each possible solution.

| $x$ | $\sin 2x + \sin 3x \overset{?}{=} 0$ |
|---|---|
| 0 | $\sin(2\cdot 0) + \sin(3\cdot 0) = 0$ ✓ |
| $\frac{2\pi}{5}$ | $\sin\left(2\cdot\frac{2\pi}{5}\right) + \sin\left(3\cdot\frac{2\pi}{5}\right) = 0$ ✓ |
| $\pi$ | $\sin(2\pi) + \sin(3\pi) = 0$ ✓ |
| $\frac{3\pi}{4}$ | $\sin\left(2\cdot\frac{3\pi}{4}\right) + \sin\left(3\cdot\frac{3\pi}{4}\right) = 0$ × |
| $\frac{4\pi}{5}$ | $\sin\left(2\cdot\frac{4\pi}{5}\right) + \sin\left(3\cdot\frac{4\pi}{5}\right) = 0$ ✓ |

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| $x$ | $\sin 2x + \sin 3x \stackrel{?}{=} 0$ |
|---|---|
| $\frac{6\pi}{5}$ | $\sin\left(2\cdot\frac{6\pi}{5}\right)+\sin\left(3\cdot\frac{6\pi}{5}\right)=0✓$ |
| $\frac{8\pi}{5}$ | $\sin\left(2\cdot\frac{8\pi}{5}\right)+\sin\left(3\cdot\frac{8\pi}{5}\right)=0✓$ |

Solution set: $\left\{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \pi, \frac{6\pi}{5}, \frac{8\pi}{5}\right\}$

**7.** $\frac{\pi}{4}+3\sin^{-1}(x+1)=\frac{5\pi}{4}$

$$3\sin^{-1}(x+1)=\pi$$
$$\sin^{-1}(x+1)=\frac{\pi}{3}\Rightarrow x+1=\sin\frac{\pi}{3}$$
$$x=\sin\frac{\pi}{3}-1=\frac{\sqrt{3}}{2}-1$$
$$=\frac{\sqrt{3}-2}{2}$$

**8. a.** $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$, $x$ in $(-\infty, \infty)$.

Let $\theta=\tan^{-1}x$. Then $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$, so $\frac{\pi}{2}>-\theta>-\frac{\pi}{2}$ and $\pi>\frac{\pi}{2}-\theta>0$ or $0<\frac{\pi}{2}-\theta<\pi$. Then, we have $x=\tan\theta\Rightarrow x=\cot\left(\frac{\pi}{2}-\theta\right)$ (using a cofunction identity).

$$x=\cot\left(\frac{\pi}{2}-\theta\right)\Rightarrow \cot^{-1}x=\frac{\pi}{2}-\theta\Rightarrow$$
$$\theta+\cot^{-1}x=\frac{\pi}{2}\Rightarrow \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$$

**b.** $\tan^{-1}x-\cot^{-1}x=\frac{\pi}{4}$

Using the result from part (a), we have

$$\tan^{-1}x-\left(\frac{\pi}{2}-\tan^{-1}x\right)=\frac{\pi}{4}$$
$$2\tan^{-1}x-\frac{\pi}{2}=\frac{\pi}{4}$$
$$2\tan^{-1}x=\frac{3\pi}{4}\Rightarrow$$
$$\tan^{-1}x=\frac{3\pi}{8}\Rightarrow x=\tan\frac{3\pi}{8}$$

Now use a half-angle formula for $\tan\frac{3\pi}{4}$.

$$x=\tan\left(\frac{\frac{3\pi}{4}}{2}\right)=\frac{\sin\frac{3\pi}{4}}{1+\cos\frac{3\pi}{4}}=\frac{\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}}$$
$$=\frac{\sqrt{2}}{2-\sqrt{2}}=\frac{\sqrt{2}}{2-\sqrt{2}}\cdot\frac{2+\sqrt{2}}{2+\sqrt{2}}=\frac{2\sqrt{2}+2}{2}$$
$$=\sqrt{2}+1$$

Alternatively, we have

$$x=\tan\left(\frac{\frac{3\pi}{4}}{2}\right)=\sqrt{\frac{1-\cos\frac{3\pi}{4}}{1+\cos\frac{3\pi}{4}}}=\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}}}$$
$$=\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}$$

## 6.6 Basic Concepts and Skills

**1.** If $\sin 2x_1=\sin 2x_2$ and $0<x_1<x_2<\frac{\pi}{2}$, then $x_2=\underline{\frac{\pi}{2}-x_1}$.

**2.** If $\cos 2x_1=\cos 2x_2$ and $0<x_1<x_2<\pi$, then $x_2=\underline{\pi-x_1}$.

**3.** If $\tan 2x_1=\tan 2x_2$ and $0<x_1<x_2<\pi$, then $x_2=\underline{\frac{\pi}{2}+x_1}$.

**4.** True

**5.** False. For example, if $x=\frac{\pi}{2}$, then $\frac{\sin 2x}{2}=\frac{\sin\pi}{2}=0$, while $\sin\frac{\pi}{2}=1$.

**6.** False. For example, if $x=\pi$, then $\frac{2\tan\frac{\pi}{2}}{\pi}$ is undefined $\neq 1$.

**7.** $\cos 2x=\frac{1}{2}\Rightarrow 2x=\frac{\pi}{3}+2n\pi\Rightarrow x=\frac{\pi}{6}+n\pi$ or $2x=\frac{5\pi}{3}+2n\pi\Rightarrow x=\frac{5\pi}{6}+n\pi$. Now find the values of $n$ that result in solutions in the interval $[0,2\pi)$: $n=0\Rightarrow x=\frac{\pi}{6}$ or $x=\frac{5\pi}{6}$

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$n = 1 \Rightarrow x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$. If $n > 1$, the solutions are out of the domain, so the solutions are $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$.

**8.** $\cos 2x = 0 \Rightarrow 2x = \frac{\pi}{2} + 2n\pi \Rightarrow x = \frac{\pi}{4} + n\pi$ or $2x = \frac{3\pi}{2} + 2n\pi \Rightarrow x = \frac{3\pi}{4} + n\pi.$
Now find the values of $n$ that result in solutions in the interval $[0, 2\pi)$:

$n = 0 \Rightarrow x = \frac{\pi}{4}$ or $x = \frac{3\pi}{4}$

$n = 1 \Rightarrow x = \frac{5\pi}{4}$ or $x = \frac{7\pi}{4}$. If $n > 1$, the solutions are out of the domain, so the solutions are $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$.

**9.** $\csc 2x = \frac{1}{2}$ has no solution because the range of $\csc x$ is $-\infty < x < -1$ or $1 < x < \infty$.

**10.** $\sec 2x = 0$ has no solution because the range of $\sec x$ is $-\infty < x < -1$ or $1 < x < \infty$.

**11.** $\tan 2x = \frac{\sqrt{3}}{3} \Rightarrow 2x = \frac{\pi}{6} + n\pi \Rightarrow x = \frac{\pi}{12} + \frac{\pi}{2}n$.
Now find the values of $n$ that result in solutions in the interval $[0, 2\pi)$:

$n = 0 \Rightarrow x = \frac{\pi}{12}$, $n = 1 \Rightarrow x = \frac{7\pi}{12}$,

$n = 2 \Rightarrow x = \frac{13\pi}{12}$, $n = 3 \Rightarrow \frac{19\pi}{12}$.

If $n > 3$, the solutions are out of the domain, so the solutions are $\left\{\frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}\right\}$.

**12.** $\cot 2x = \frac{\sqrt{3}}{3} \Rightarrow 2x = \frac{\pi}{3} + n\pi \Rightarrow x = \frac{\pi}{6} + \frac{\pi}{2}n$.
Now find the values of $n$ that result in solutions in the interval $[0, 2\pi)$:

$n = 0 \Rightarrow x = \frac{\pi}{6}$, $n = 1 \Rightarrow x = \frac{2\pi}{3}$,

$n = 2 \Rightarrow x = \frac{7\pi}{6}$, $n = 3 \Rightarrow x = \frac{5\pi}{3}$.

If $n > 3$, the solutions are out of the domain, so the solutions are $\left\{\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}\right\}$.

**13.** $\sin 3x = \frac{1}{2} \Rightarrow 3x = \frac{\pi}{6} + 2n\pi$ or
$3x = \frac{5\pi}{6} + 2n\pi \Rightarrow x = \frac{\pi}{18} + \frac{2n\pi}{3}$
or $x = \frac{5\pi}{18} + \frac{2n\pi}{3}$. Now find the values of $n$ that result in solutions in the interval $[0, 2\pi)$:

| $n$ | $\frac{\pi}{18} + \frac{2n\pi}{3}$ | $\frac{5\pi}{18} + \frac{2n\pi}{3}$ |
|---|---|---|
| 0 | $\frac{\pi}{18}$ | $\frac{5\pi}{18}$ |
| 1 | $\frac{\pi}{18} + \frac{2\pi}{3} = \frac{13\pi}{18}$ | $\frac{5\pi}{18} + \frac{2\pi}{3} = \frac{17\pi}{18}$ |
| 2 | $\frac{\pi}{18} + \frac{4\pi}{3} = \frac{25\pi}{18}$ | $\frac{5\pi}{18} + \frac{4\pi}{3} = \frac{29\pi}{18}$ |

If $n > 3$, the solutions are out of the domain, so the solutions are
$\left\{\frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}\right\}$.

**14.** $\cos 3x = \frac{\sqrt{3}}{2} \Rightarrow 3x = \frac{\pi}{6} + 2n\pi$ or
$3x = \frac{11\pi}{6} + 2n\pi \Rightarrow x = \frac{\pi}{18} + \frac{2n\pi}{3}$ or
$x = \frac{11\pi}{18} + \frac{2n\pi}{3}$. Now find the values of $n$ that result in solutions in the interval $[0, 2\pi)$.

| $n$ | $\frac{\pi}{18} + \frac{2n\pi}{3}$ | $\frac{11\pi}{18} + \frac{2n\pi}{3}$ |
|---|---|---|
| 0 | $\frac{\pi}{18}$ | $\frac{11\pi}{18}$ |
| 1 | $\frac{\pi}{18} + \frac{2\pi}{3} = \frac{13\pi}{18}$ | $\frac{11\pi}{18} + \frac{2\pi}{3} = \frac{23\pi}{18}$ |
| 2 | $\frac{\pi}{18} + \frac{4\pi}{3} = \frac{25\pi}{18}$ | $\frac{11\pi}{18} + \frac{4\pi}{3} = \frac{35\pi}{18}$ |

If $n > 3$, the solutions are out of the domain, so the solutions are
$\left\{\frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{35\pi}{18}\right\}$.

**15.** $\cos\frac{x}{2}=\frac{1}{2}\Rightarrow\frac{x}{2}=\frac{\pi}{3}+2n\pi$ or $\frac{x}{2}=\frac{5\pi}{3}+2n\pi\Rightarrow$ $x=\frac{2\pi}{3}+4n\pi$ or $x=\frac{10\pi}{3}+4n\pi$. (Note that the second value is not in the domain.). The only value of $n$ that results in a solution in the interval $[0,2\pi)$ is $n=0$. The solution is $\left\{\frac{2\pi}{3}\right\}$.

**16.** $\csc\frac{x}{2}=2\Rightarrow\frac{x}{2}=\frac{\pi}{6}+2n\pi$ or $\frac{x}{2}=\frac{5\pi}{6}+2n\pi\Rightarrow$ $x=\frac{\pi}{3}+4n\pi$ or $x=\frac{5\pi}{3}+4n\pi$. The only value of $n$ that results in a solution in the interval $[0,2\pi)$ is $n=0$. The solution is $\left\{\frac{\pi}{3},\frac{5\pi}{3}\right\}$.

**17.** $\tan\frac{x}{3}=1\Rightarrow\frac{x}{3}=\frac{\pi}{4}+n\pi$ or $\frac{x}{3}=\frac{5\pi}{4}+n\pi\Rightarrow$ $x=\frac{3\pi}{4}+3n\pi$. The only value of $n$ that results in a solution in the interval $[0,2\pi)$ is $n=0$. The solution is $\left\{\frac{3\pi}{4}\right\}$.

**18.** $\cot\frac{x}{3}=\sqrt{3}\Rightarrow\frac{x}{3}=\frac{\pi}{6}+n\pi$ or $\frac{x}{3}=\frac{7\pi}{6}+n\pi\Rightarrow$ $x=\frac{\pi}{2}+3n\pi$. The only value of $n$ that results in a solution in the interval $[0,2\pi)$ is $n=0$. The solution is $\left\{\frac{\pi}{2}\right\}$.

**19.** $2\sin 3x=\sqrt{2}\Rightarrow\sin 3x=\frac{\sqrt{2}}{2}\Rightarrow$
$\sin 3x=\sin\left(\frac{\pi}{4}+2n\pi\right)$ or
$\sin 3x=\sin\left(\frac{3\pi}{4}+2n\pi\right)$.

$3x=\frac{\pi}{4}+2n\pi\Rightarrow x=\frac{\pi}{12}+\frac{2n\pi}{3}\Rightarrow$
$x=\frac{\pi}{12},\frac{3\pi}{4},\frac{17\pi}{12}$
$3x=\frac{3\pi}{4}+2n\pi\Rightarrow x=\frac{\pi}{4}+\frac{2n\pi}{3}\Rightarrow$
$x=\frac{\pi}{4},\frac{11\pi}{12},\frac{19\pi}{12}$
Solution set: $\left\{\frac{\pi}{12},\frac{\pi}{4},\frac{3\pi}{4},\frac{11\pi}{12},\frac{17\pi}{12},\frac{19\pi}{12}\right\}$

**20.** $2\cos 3x=\sqrt{2}\Rightarrow\cos 3x=\frac{\sqrt{2}}{2}\Rightarrow$
$\cos 3x=\cos\left(\frac{\pi}{4}+2n\pi\right)$ or
$\cos 3x=\cos\left(\frac{7\pi}{4}+2n\pi\right)$.
$3x=\frac{\pi}{4}+2n\pi\Rightarrow x=\frac{\pi}{12}+\frac{2n\pi}{3}\Rightarrow$
$x=\frac{\pi}{12},\frac{3\pi}{4},\frac{17\pi}{12}$
$3x=\frac{7\pi}{4}+2n\pi\Rightarrow x=\frac{7\pi}{12}+\frac{2n\pi}{3}\Rightarrow$
$x=\frac{7\pi}{12},\frac{5\pi}{4},\frac{23\pi}{12}$
Solution set: $\left\{\frac{\pi}{12},\frac{7\pi}{12},\frac{3\pi}{4},\frac{5\pi}{4},\frac{17\pi}{12},\frac{23\pi}{12}\right\}$

**21.** $2\cos(2x+1)=1\Rightarrow\cos(2x+1)=\frac{1}{2}\Rightarrow$
$2x+1=\frac{\pi}{3}+2n\pi$ or $2x+1=\frac{5\pi}{3}+2n\pi$.
$2x+1=\frac{\pi}{3}+2n\pi\Rightarrow 2x=\frac{\pi}{3}+2n\pi-1\Rightarrow$
$x=\frac{\pi}{6}+n\pi-\frac{1}{2}\Rightarrow x=\frac{\pi}{6}-\frac{1}{2},\frac{7\pi}{6}-\frac{1}{2}$.
$2x+1=\frac{5\pi}{3}+2n\pi\Rightarrow 2x=\frac{5\pi}{3}+2n\pi-1\Rightarrow$
$x=\frac{5\pi}{6}+n\pi-\frac{1}{2}\Rightarrow x=\frac{5\pi}{6}-\frac{1}{2},\frac{11\pi}{6}-\frac{1}{2}$.
Solution set:
$\left\{\frac{\pi}{6}-\frac{1}{2},\frac{5\pi}{6}-\frac{1}{2},\frac{7\pi}{6}-\frac{1}{2},\frac{11\pi}{6}-\frac{1}{2}\right\}$

**22.** $2\sin(2x-1)=\sqrt{3}\Rightarrow\sin(2x-1)=\frac{\sqrt{3}}{2}\Rightarrow$
$2x-1=\frac{\pi}{3}+2n\pi$ or $2x-1=\frac{2\pi}{3}+2n\pi$.
$2x-1=\frac{\pi}{3}+2n\pi\Rightarrow 2x=\frac{\pi}{3}+2n\pi+1\Rightarrow$
$x=\frac{\pi}{6}+n\pi+\frac{1}{2}\Rightarrow x=\frac{\pi}{6}+\frac{1}{2},\frac{7\pi}{6}+\frac{1}{2}$.
$2x-1=\frac{2\pi}{3}+2n\pi\Rightarrow 2x=\frac{2\pi}{3}+2n\pi+1\Rightarrow$
$x=\frac{\pi}{3}+n\pi+\frac{1}{2}\Rightarrow x=\frac{\pi}{3}+\frac{1}{2},\frac{4\pi}{3}+\frac{1}{2}$.
Solution set:
$\left\{\frac{\pi}{6}+\frac{1}{2},\frac{\pi}{3}+\frac{1}{2},\frac{7\pi}{6}+\frac{1}{2},\frac{4\pi}{3}+\frac{1}{2}\right\}$

**23.** $2\sin(4x-1)=1 \Rightarrow \sin(4x-1)=\frac{1}{2} \Rightarrow$
$4x-1=\frac{\pi}{6}+2n\pi$ or $4x-1=\frac{5\pi}{6}+2n\pi$.
$4x-1=\frac{\pi}{6}+2n\pi \Rightarrow 4x=\frac{\pi}{6}+2n\pi+1 \Rightarrow$
$x=\frac{\pi}{24}+\frac{n\pi}{2}+\frac{1}{4} \Rightarrow$
$x=\frac{\pi}{24}+\frac{1}{4}, \frac{13\pi}{24}+\frac{1}{4}, \frac{25\pi}{24}+\frac{1}{4}, \frac{37\pi}{24}+\frac{1}{4}$.
$4x-1=\frac{5\pi}{6}+2n\pi \Rightarrow 4x=\frac{5\pi}{6}+2n\pi+1 \Rightarrow$
$x=\frac{5\pi}{24}+\frac{n\pi}{2}+\frac{1}{4} \Rightarrow$
$x=\frac{5\pi}{24}+\frac{1}{4}, \frac{17\pi}{24}+\frac{1}{4}, \frac{29\pi}{24}+\frac{1}{4}, \frac{41\pi}{24}+\frac{1}{4}$
Solution set:
$\left\{\frac{\pi}{24}+\frac{1}{4}, \frac{5\pi}{24}+\frac{1}{4}, \frac{13\pi}{24}+\frac{1}{4}, \frac{17\pi}{24}+\frac{1}{4}, \frac{25\pi}{24}+\frac{1}{4}, \frac{29\pi}{24}+\frac{1}{4}, \frac{37\pi}{24}+\frac{1}{4}, \frac{41\pi}{24}+\frac{1}{4}\right\}$

**24.** $2\cos(4x+1)=\sqrt{3} \Rightarrow \cos(4x+1)=\frac{\sqrt{3}}{2} \Rightarrow$
$4x+1=\frac{\pi}{6}+2n\pi$ or $4x+1=\frac{11\pi}{6}+2n\pi$.
$4x+1=\frac{\pi}{6}+2n\pi \Rightarrow 4x=\frac{\pi}{6}+2n\pi-1 \Rightarrow$
$x=\frac{\pi}{24}+\frac{n\pi}{2}-\frac{1}{4} \Rightarrow$
$x=\frac{\pi}{24}-\frac{1}{4}, \frac{13\pi}{24}-\frac{1}{4}, \frac{25\pi}{24}-\frac{1}{4}, \frac{37\pi}{24}-\frac{1}{4}, \frac{49\pi}{24}-\frac{1}{4}$.

$4x+1=\frac{11\pi}{6}+2n\pi \Rightarrow 4x=\frac{11\pi}{6}+2n\pi-1 \Rightarrow$
$x=\frac{11\pi}{24}+\frac{n\pi}{2}-\frac{1}{4} \Rightarrow$
$x=\frac{11\pi}{24}-\frac{1}{4}, \frac{23\pi}{24}-\frac{1}{4}, \frac{35\pi}{24}-\frac{1}{4}, \frac{47\pi}{24}-\frac{1}{4}$.
$\frac{\pi}{24}-\frac{1}{4}<0,$ so this is not part of the solution set.
Solution set:
$\left\{\frac{11\pi}{24}-\frac{1}{4}, \frac{13\pi}{24}-\frac{1}{4}, \frac{23\pi}{24}-\frac{1}{4}, \frac{25\pi}{24}-\frac{1}{4}, \frac{35\pi}{24}-\frac{1}{4}, \frac{37\pi}{24}-\frac{1}{4}, \frac{47\pi}{24}-\frac{1}{4}, \frac{49\pi}{24}-\frac{1}{4}\right\}$

**25.** $2\cos\left(\frac{3x}{2}-1\right)=\sqrt{3} \Rightarrow \cos\left(\frac{3x}{2}-1\right)=\frac{\sqrt{3}}{2}$
$\frac{3x}{2}-1=\frac{\pi}{6}+2n\pi$ or $\frac{3x}{2}-1=\frac{11\pi}{6}+2n\pi$.
$\frac{3x}{2}-1=\frac{\pi}{6}+2n\pi \Rightarrow \frac{3x}{2}=\frac{\pi}{6}+2n\pi+1 \Rightarrow$
$x=\frac{\pi}{9}+\frac{4n\pi}{3}+\frac{2}{3} \Rightarrow \frac{\pi}{9}+\frac{2}{3}, \frac{13\pi}{9}+\frac{2}{3}$
$\frac{3x}{2}-1=\frac{11\pi}{6}+2n\pi \Rightarrow \frac{3x}{2}=\frac{11\pi}{6}+2n\pi+1 \Rightarrow$
$x=\frac{11\pi}{9}+\frac{4n\pi}{3}+\frac{2}{3} \Rightarrow \frac{11\pi}{9}+\frac{2}{3}$
Note that we can also include $x=-\frac{\pi}{9}+\frac{2}{3}$ as one of the solutions, since it is in the given interval.
Solution set:
$\left\{-\frac{\pi}{9}+\frac{2}{3}, \frac{\pi}{9}+\frac{2}{3}, \frac{11\pi}{9}+\frac{2}{3}, \frac{13\pi}{9}+\frac{2}{3}\right\}$

**26.** $2\sin\left(\frac{3x}{2}+1\right)=1 \Rightarrow \sin\left(\frac{3x}{2}+1\right)=\frac{1}{2}$
$\frac{3x}{2}+1=\frac{\pi}{6}+2n\pi$ or $\frac{3x}{2}+1=\frac{5\pi}{6}+2n\pi$.
$\frac{3x}{2}+1=\frac{\pi}{6}+2n\pi \Rightarrow \frac{3x}{2}=\frac{\pi}{6}+2n\pi-1 \Rightarrow$
$x=\frac{\pi}{9}+\frac{4n\pi}{3}-\frac{2}{3} \Rightarrow x=\frac{\pi}{9}-\frac{2}{3}$ (out of range), $x=\frac{13\pi}{9}-\frac{2}{3}$
$\frac{3x}{2}+1=\frac{5\pi}{6}+2n\pi \Rightarrow \frac{3x}{2}=\frac{5\pi}{6}+2n\pi-1 \Rightarrow$
$x=\frac{5\pi}{9}+\frac{4n\pi}{3}-\frac{2}{3} \Rightarrow x=\frac{5\pi}{9}-\frac{2}{3}, \frac{17\pi}{9}-\frac{2}{3}$.
$\frac{\pi}{9}-\frac{2}{3}<0,$ so this is not part of the solution set. Solution set:
$\left\{\frac{5\pi}{9}-\frac{2}{3}, \frac{13\pi}{9}-\frac{2}{3}, \frac{17\pi}{9}-\frac{2}{3}\right\}$

**27.** $3\cos 2\theta=1 \Rightarrow \cos 2\theta=\frac{1}{3} \Rightarrow$
$2\theta \approx 70.5288+2n(180°) \Rightarrow \theta \approx 35.3°+180n$
or $2\theta \approx 289.4712+2n(180°) \Rightarrow$
$\theta \approx 144.7°+180n$.
$\theta \approx 35.3°+180n \Rightarrow \theta \approx 35.3°, 215.3°$
$\theta \approx 144.7°+180n \Rightarrow \theta \approx 144.7°, 324.7°$
Solution set: $\{35.3°, 144.7°, 215.3°, 324.7°\}$

**28.** $4\sin 2\theta = 1 \Rightarrow \sin 2\theta = \frac{1}{4} \Rightarrow$
$2\theta \approx 14.4775° + 2n(180°)$ or
$2\theta \approx 165.5225° + 2n(180°)$.
$2\theta \approx 14.4775° + 2n(180°) \Rightarrow$
$\theta \approx 7.2° + n(180°) \Rightarrow \theta \approx 7.2°, 187.2°$
$2\theta \approx 165.5225° + 2n(180°) \Rightarrow$
$\theta \approx 82.8° + n(180°) \Rightarrow \theta \approx 82.8°, 262.8°$
Solution set: {7.2°, 82.8°, 187.2°, 262.8°}

**29.** $2\cos 3\theta + 1 = 2 \Rightarrow \cos 3\theta = \frac{1}{2} \Rightarrow$
$3\theta = 60° + 2n(180°) \Rightarrow \theta = 20° + n(120°)$
or $3\theta = 300° + 2n(180°) \Rightarrow$
$\theta = 100° + n(120°)$
$\theta \approx 20° + n(120°) \Rightarrow \theta = 20°, 140°, 260°$
$\theta \approx 100° + n(120°) \Rightarrow \theta = 100°, 220°, 340°$
Solution set: {20°, 100°, 140°, 220°, 260°, 340°}

**30.** $2\sin 3\theta - 1 = -2 \Rightarrow \sin 3\theta = -\frac{1}{2} \Rightarrow$
$3\theta = 210° + 2n(180°)$ or $3\theta = 330° + 2n(180°)$.
$3\theta = 210° + 2n(180°) \Rightarrow \theta = 70° + n(120°) \Rightarrow$
$\theta = 70°, 190°, 310°$.
$3\theta = 330° + 2n(180°) \Rightarrow \theta = 110° + n(120°) \Rightarrow$
$\theta = 110°, 230°, 350°$
Solution set: {70°, 110°, 190°, 230°, 310°, 350°}

**31.** $3\sin 3\theta + 1 = 0 \Rightarrow \sin 3\theta = -\frac{1}{3} \Rightarrow$
$3\theta \approx -19.4712 + 2n(180°)$ or
$3\theta \approx 199.4712 + 2n(180°)$.
$3\theta \approx -19.4712 + 2n(180°) \Rightarrow$
$\theta \approx -6.5 + (120°)n \Rightarrow$
$\theta \approx 113.5°, 233.5°, 353.5°$.
$3\theta \approx 199.4712 + 2n(180°) \Rightarrow$
$\theta \approx 66.5° + (120°)n \Rightarrow$
$\theta \approx 66.5°, 186.5°, 306.5°$
Solution set: {66.5°, 113.5°, 186.5°, 233.5°, 306.5°, 353.5°}

**32.** $4\cos 3\theta + 1 = 0 \Rightarrow \cos 3\theta = -\frac{1}{4} \Rightarrow$
$3\theta \approx 104.4775° + 2n(180°)$ or
$3\theta \approx 255.5225 + 2n(180°)$
$3\theta \approx 104.4775° + 2n(180°) \Rightarrow$
$\theta \approx 34.8° + (120°)n \Rightarrow$
$\theta \approx 34.8°, 154.8°, 274.8°$
$3\theta \approx 255.5225 + 2n(180°) \Rightarrow$
$\theta \approx 85.2° + (120°)n \Rightarrow \theta \approx 85.2°, 205.2°, 325.2°$
Solution set: {34.8°, 85.2°, 154.8°, 205.2°, 274.8°, 325.2°)

**33.** $2\sin\frac{\theta}{2} + 1 = 0 \Rightarrow \sin\frac{\theta}{2} = -\frac{1}{2} \Rightarrow$
$\frac{\theta}{2} = 210° + 2n(180°)$ or $\frac{\theta}{2} = 330° + 2n(180°)$
$\frac{\theta}{2} = 210° + 2n(180°) \Rightarrow \theta = 420° + 4n(180°)$,
which is outside the desired interval.
$\frac{\theta}{2} = 330° + 2n(180°) \Rightarrow \theta = 660° + 4n(180°)$,
which is also outside the desired interval.
Therefore, the solution set is $\varnothing$.

**34.** $2\cos\frac{\theta}{2} + \sqrt{3} = 0 \Rightarrow \cos\frac{\theta}{2} = -\frac{\sqrt{3}}{2} \Rightarrow$
$\frac{\theta}{2} = 150° + 2n(180°)$ or $\frac{\theta}{2} = 210° + 2n(180°)$.
$\frac{\theta}{2} = 150° + 2n(180°) \Rightarrow$
$\theta = 300° + 4n(180°) \Rightarrow 300°$
$\frac{\theta}{2} = 210° + 2n(180°) \Rightarrow \theta = 420° + 4n(180°)$,
which is outside the desired interval.
Solution set: {300°}

**35.** $2\sec 2\theta + 3 = 7 \Rightarrow \sec 2\theta = 2 \Rightarrow$
$2\theta = 60° + 2n(180°)$ or $2\theta = 300° + 2n(180°)$.
$2\theta = 60° + 2n(180°) \Rightarrow \theta = 30° + n(180°) \Rightarrow$
$\theta = 30°, 210°$
$2\theta = 300° + 2n(180°) \Rightarrow$
$\theta = 150° + n(180°) \Rightarrow \theta = 150°, 330°$
Solution set: {30°, 150°, 210°, 330°}

**36.** $2\csc 2\theta + 1 = 0 \Rightarrow \csc 2\theta = -\frac{1}{2}$
The range of $\csc\theta$ is $(-\infty, 1] \cup [1, \infty)$.
Thus, the solution set is $\varnothing$.

**37.** $3\csc\frac{\theta}{2} - 1 = 5 \Rightarrow \csc\frac{\theta}{2} = 2 \Rightarrow$
$\frac{\theta}{2} = 30° + 2n(180°)$ or $\frac{\theta}{2} = 150° + 2n(180°)$.
$\frac{\theta}{2} = 30° + 2n(180°) \Rightarrow \theta = 60° + 4n(180°) \Rightarrow$
$\theta = 60°$.
$\frac{\theta}{2} = 150° + 2n(180°) \Rightarrow \theta = 300° + 4n(180°) \Rightarrow$
$\theta = 300°$.
Solution set: {60°, 300°}

**38.** $5\sec\frac{\theta}{2} - 3 = 7 \Rightarrow \sec\frac{\theta}{2} = 2 \Rightarrow$

$\frac{\theta}{2} = 60° + 2n(180°)$ or $\frac{\theta}{2} = 300° + 2n(180°)$.

$\frac{\theta}{2} = 60° + 2n(180°) \Rightarrow$

$\theta = 120° + 4n(180°) \Rightarrow \theta = 120°$.

$\frac{\theta}{2} = 300° + 2n(180°) \Rightarrow \theta = 600° + 4n(180°)$,

which is not in the given interval.

Solution set: {120°}

**39.**
$$\sin 2x + \sin x = 0$$
$$2\sin\left(\frac{2x+x}{2}\right)\cos\left(\frac{2x-x}{2}\right) = 0$$
$$\sin\frac{3x}{2}\cos\frac{x}{2} = 0 \Rightarrow$$

$\sin\frac{3x}{2} = 0$ or $\cos\frac{x}{2} = 0$.

$\sin\frac{3x}{2} = 0 \Rightarrow \frac{3x}{2} = 0 + 2n\pi$ or $\frac{3x}{2} = 2n\pi$.

$\frac{3x}{2} = 2n\pi \Rightarrow x = \frac{4n\pi}{3} \Rightarrow x = 0, \frac{4\pi}{3}$.

$\frac{3x}{2} = \pi \Rightarrow x = \frac{2\pi}{3}$

$\cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2} + 2n\pi$ or $\frac{x}{2} = \frac{3\pi}{2} + 2n\pi$

$\frac{x}{2} = \frac{\pi}{2} + 2n\pi \Rightarrow x = \pi + 4n\pi \Rightarrow x = \pi$

$\frac{x}{2} = \frac{3\pi}{2} + 2n\pi \Rightarrow x = 3\pi + 4n\pi$, which is not in the given interval.

Solution set: $\left\{0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$

**40.**
$$\cos 2x + \cos x = 0$$
$$2\cos\left(\frac{2x+x}{2}\right)\cos\left(\frac{2x-x}{2}\right) = 0$$
$$\cos\frac{3x}{2}\cos\frac{x}{2} = 0 \Rightarrow$$

$\cos\frac{3x}{2} = 0$ or $\cos\frac{x}{2} = 0$.

$\cos\frac{3x}{2} = 0 \Rightarrow \frac{3x}{2} = \frac{\pi}{2} + 2n\pi$

or $\frac{3x}{2} = \frac{3\pi}{2} + 2n\pi$.

$\frac{3x}{2} = \frac{\pi}{2} + 2n\pi \Rightarrow x = \frac{\pi}{3} + \frac{4n\pi}{3} \Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$.

$\frac{3x}{2} = \frac{3\pi}{2} + 2n\pi \Rightarrow x = \pi$

$\cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2} + 2n\pi$ or $\frac{x}{2} = \frac{3\pi}{2} + 2n\pi$

$\frac{x}{2} = \frac{\pi}{2} + 2n\pi \Rightarrow x = \pi + 4n\pi \Rightarrow x = \pi$

$\frac{x}{2} = \frac{3\pi}{2} + 2n\pi \Rightarrow x = 3\pi + 4n\pi$, which is not in the given interval.

Solution set: $\left\{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$

**41.**
$$\cos 3x - \cos x = 0$$
$$-2\sin\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right) = 0$$
$$\sin 2x \sin x = 0 \Rightarrow$$

$\sin 2x = 0$ or $\sin x = 0$.

$\sin 2x = 0 \Rightarrow 2x = 0 + 2n\pi$ or $2x = \pi + 2n\pi$.

$2x = 0 + 2n\pi \Rightarrow x = 0 + n\pi \Rightarrow x = 0, \pi$.

$2x = \pi + 2n\pi \Rightarrow x = \frac{\pi}{2} + n\pi \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x = 0 \Rightarrow x = 0 + 2n\pi \Rightarrow x = 0, \pi$

Solution set: $\left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$

**42.**
$$\sin 3x - \sin x = 0$$
$$2\sin\left(\frac{3x-x}{2}\right)\cos\left(\frac{3x+x}{2}\right) = 0$$
$$\sin x \cos 2x = 0 \Rightarrow$$

$\sin x = 0$ or $\cos 2x = 0$.

$\sin x = 0 \Rightarrow x = 0 + 2n\pi \Rightarrow x = 0, \pi$

$\cos 2x = 0 \Rightarrow 2x = \frac{\pi}{2} + 2n\pi$ or $2x = \frac{3\pi}{2} + 2n\pi$.

$2x = \frac{\pi}{2} + 2n\pi \Rightarrow x = \frac{\pi}{4} + n\pi \Rightarrow$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$.

$2x = \frac{3\pi}{2} + 2n\pi \Rightarrow x = \frac{3\pi}{4} + n\pi \Rightarrow$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

Solution set: $\left\{0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

**43.**
$$\cos 3x + \cos 5x = 0$$
$$2\cos\left(\frac{3x+5x}{2}\right)\cos\left(\frac{3x-5x}{2}\right) = 0$$
$$2\cos(4x)\cos(-x) = 0$$
$$2\cos(4x)\cos x = 0 \Rightarrow$$

$\cos 4x = 0$ or $\cos x = 0$.

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(*continued*)

$\cos 4x = 0 \Rightarrow 4x = \frac{\pi}{2} + 2n\pi$ or

$4x = \frac{3\pi}{2} + 2n\pi.$

$4x = \frac{\pi}{2} + 2n\pi \Rightarrow x = \frac{\pi}{8} + \frac{n\pi}{2} \Rightarrow$

$x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}.$

$4x = \frac{3\pi}{2} + 2n\pi \Rightarrow x = \frac{3\pi}{8} + \frac{n\pi}{2} \Rightarrow$

$x = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2} + 2n\pi$ or $x = \frac{3\pi}{2} + 2n\pi.$

$x = \frac{\pi}{2} + 2n\pi \Rightarrow x = \frac{\pi}{2}.$

$x = \frac{3\pi}{2} + 2n\pi \Rightarrow x = \frac{3\pi}{2}.$

Solution set:

$\left\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{3\pi}{2}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\}$

**44.**

$$\cos 3x - \cos 5x = 0$$
$$-2\sin\left(\frac{3x+5x}{2}\right)\sin\left(\frac{3x-5x}{2}\right) = 0$$
$$-2\sin(4x)\sin(-x) = 0$$
$$2\sin(4x)\sin x = 0 \Rightarrow$$

$\sin 4x = 0$ or $\sin x = 0.$

$\sin 4x = 0 \Rightarrow 4x = 0 + 2n\pi$ or

$4x = \pi + 2n\pi.$

$4x = 0 + 2n\pi \Rightarrow x = 0 + \frac{n\pi}{2} \Rightarrow$

$x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}.$

$4x = \pi + 2n\pi \Rightarrow x = \frac{\pi}{4} + \frac{n\pi}{2} \Rightarrow$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\sin x = 0 \Rightarrow x = 0 + 2n\pi \Rightarrow x = 0, \pi$

Solution set: $\left\{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\right\}$

**45.**

$$\sin 3x - \sin 5x = 0$$
$$2\sin\left(\frac{3x-5x}{2}\right)\cos\left(\frac{3x+5x}{2}\right) = 0$$
$$2\sin(-x)\cos(4x) = 0$$
$$-2\sin x\cos(4x) = 0 \Rightarrow$$

$\sin x = 0$ or $\cos 4x = 0.$

$\sin x = 0 \Rightarrow x = 0 + 2n\pi \Rightarrow x = 0, \pi$

$\cos 4x = 0 \Rightarrow 4x = \frac{\pi}{2} + 2n\pi$ or

$4x = \frac{3\pi}{2} + 2n\pi.$

$4x = \frac{\pi}{2} + 2n\pi \Rightarrow x = \frac{\pi}{8} + \frac{n\pi}{2} \Rightarrow$

$x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}.$

$4x = \frac{3\pi}{2} + 2n\pi \Rightarrow x = \frac{3\pi}{8} + \frac{n\pi}{2} \Rightarrow$

$x = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

Solution set:

$\left\{0, \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \pi, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\}$

**46.**

$$\sin 3x + \sin 5x = 0$$
$$2\sin\left(\frac{3x+5x}{2}\right)\cos\left(\frac{3x-5x}{2}\right) = 0$$
$$2\sin(4x)\cos(-x) = 0$$
$$2\sin(4x)\cos x = 0 \Rightarrow$$

$\sin 4x = 0$ or $\cos x = 0.$

$\sin 4x = 0 \Rightarrow 4x = 0 + 2n\pi$ or

$4x = \pi + 2n\pi.$

$4x = 0 + 2n\pi \Rightarrow x = 0 + \frac{n\pi}{2} \Rightarrow$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}.$

$4x = \pi + 2n\pi \Rightarrow x = \frac{\pi}{4} + \frac{n\pi}{2} \Rightarrow$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2} + 2n\pi$ or $x = \frac{3\pi}{2} + 2n\pi.$

$x = \frac{\pi}{2} + 2n\pi \Rightarrow x = \frac{\pi}{2}.$

$x = \frac{3\pi}{2} + 2n\pi \Rightarrow x = \frac{3\pi}{2}.$

Solution set: $\left\{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\right\}$

Use the identity $\sin^{-1} x + \cos^{-1} = \frac{\pi}{2}$, $x$ in $[-1, 1]$ for exercises 47–54.

**47.**
$$\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$$
$$\sin^{-1} x - \left(\frac{\pi}{2} - \sin^{-1} x\right) = \frac{\pi}{6}$$
$$2\sin^{-1} x = \frac{2\pi}{3}$$
$$\sin^{-1} x = \frac{\pi}{3} \Rightarrow x = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

Solution set: $\left\{\frac{\sqrt{3}}{2}\right\}$

**48.**
$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{6}$$
$$\sin^{-1} x + \left(\frac{\pi}{2} - \sin^{-1} x\right) = \frac{\pi}{6} \Rightarrow \frac{\pi}{2} = \frac{\pi}{6}$$

Since this is a false statement, there is no solution. Solution set: $\varnothing$

**49.**
$$\sin^{-1} x + \cos^{-1} x = \frac{3\pi}{4}$$
$$\sin^{-1} x + \left(\frac{\pi}{2} - \sin^{-1} x\right) = \frac{3\pi}{4} \Rightarrow \frac{\pi}{2} = \frac{3\pi}{4}$$

Since this is a false statement, there is no solution. Solution set: $\varnothing$

**50.**
$$\sin^{-1} x - \cos^{-1} x = \frac{\pi}{3}$$
$$\sin^{-1} x - \left(\frac{\pi}{2} - \sin^{-1} x\right) = \frac{\pi}{3}$$
$$2\sin^{-1} x = \frac{5\pi}{6}$$
$$\sin^{-1} x = \frac{5\pi}{12} \Rightarrow x = \sin\frac{5\pi}{12}$$

Using the half-angle formula for $\sin\left(\frac{\frac{5\pi}{6}}{2}\right)$ gives

$$\sin\left(\frac{\frac{5\pi}{6}}{2}\right) = \sqrt{\frac{1-\cos\frac{5\pi}{6}}{2}} = \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}$$
$$= \sqrt{\frac{2+\sqrt{3}}{2}}$$

Solution set: $\left\{\frac{\sqrt{2+\sqrt{3}}}{2}\right\}$

**51.**
$$\sin^{-1} x - \cos^{-1} x = -\frac{\pi}{6}$$
$$\sin^{-1} x - \left(\frac{\pi}{2} - \sin^{-1} x\right) = -\frac{\pi}{6}$$
$$2\sin^{-1} x = \frac{\pi}{3}$$
$$\sin^{-1} x = \frac{\pi}{6} \Rightarrow x = \sin\frac{\pi}{6} = \frac{1}{2}$$

Solution set: $\left\{\frac{1}{2}\right\}$

**52.**
$$\sin^{-1} x + \cos^{-1} x = -\frac{3\pi}{4}$$
$$\sin^{-1} x + \left(\frac{\pi}{2} - \sin^{-1} x\right) = -\frac{3\pi}{4} \Rightarrow \frac{\pi}{2} = -\frac{3\pi}{4}$$

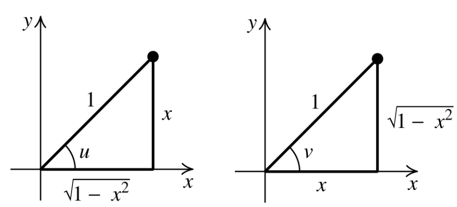
Since this is a false statement, there is no solution. Solution set: $\varnothing$

**53.** $\sin^{-1} x - \tan^{-1}\frac{2}{3} = \frac{\pi}{4}$
$$\sin^{-1} x = \frac{\pi}{4} + \tan^{-1}\frac{2}{3}$$
$$x = \sin\left(\frac{\pi}{4} + \tan^{-1}\frac{2}{3}\right)$$
$$\approx 0.9806$$

Solution set: $\{0.9806\}$

Alternatively,

$$\sin^{-1} x - \tan^{-1}\frac{2}{3} = \frac{\pi}{4}$$




Let $u = \sin^{-1}x$ and $v = \tan^{-1}\frac{2}{3}$. Then $\sin u = x$ and $\tan v = \frac{2}{3}$. From the figure, we have $\cos u = \sqrt{1 - x^2}$. Since $\tan v = \frac{2}{3} = \frac{y}{x}$, we know that $r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 2^2} = \sqrt{13}$. Thus, $\sin v = \frac{2}{\sqrt{13}}$ and $\cos v = \frac{3}{\sqrt{13}}$. Replace the equivalents of the trigonometric functions in the equation.

(*continued on next page*)

(*continued*)

$$u - v = \frac{\pi}{4} \Rightarrow \sin(u - v) = \sin\left(\frac{\pi}{4}\right) \Rightarrow$$
$$\sin u \cos v - \cos u \sin v = \sin\left(\frac{\pi}{4}\right) \Rightarrow$$
$$x \cdot \frac{3}{\sqrt{13}} - \left(\sqrt{1 - x^2}\right)\left(\frac{2}{\sqrt{13}}\right) = \frac{\sqrt{2}}{2} \Rightarrow$$
$$3x - 2\sqrt{1 - x^2} = \frac{\sqrt{26}}{2} \Rightarrow$$
$$3x - \frac{\sqrt{26}}{2} = 2\sqrt{1 - x^2}$$

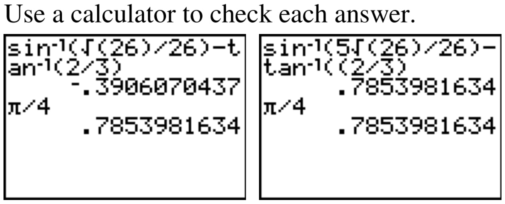
Now square both sides.

$$\left(3x - \frac{\sqrt{26}}{2}\right)^2 = \left(2\sqrt{1 - x^2}\right)^2 \Rightarrow$$
$$9x^2 - 3\sqrt{26}x + \frac{13}{2} = 4\left(1 - x^2\right) \Rightarrow$$
$$9x^2 - 3\sqrt{26}x + \frac{13}{2} = 4 - 4x^2 \Rightarrow$$
$$13x^2 - 3\sqrt{26}x + \frac{5}{2} = 0$$

Use the quadratic formula to solve for *x*.

$$x = \frac{-\left(-3\sqrt{26}\right) \pm \sqrt{\left(-3\sqrt{26}\right)^2 - 4(13)\left(\frac{5}{2}\right)}}{2(13)}$$
$$= \frac{3\sqrt{26} \pm \sqrt{234 - 130}}{26} = \frac{3\sqrt{26} \pm \sqrt{104}}{26}$$
$$= \frac{3\sqrt{26} \pm 2\sqrt{26}}{26} = \frac{5\sqrt{26}}{26} \text{ or } \frac{\sqrt{26}}{26}$$

Use a calculator to check each answer.




Solution set: $\left\{\frac{5\sqrt{26}}{26}\right\}$

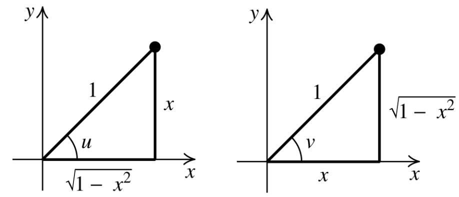
**54.** $\sin^{-1} x - \cos^{-1} \frac{2}{3} = \frac{\pi}{6}$

$$\sin^{-1} x = \frac{\pi}{6} + \cos^{-1} \frac{2}{3}$$
$$x = \sin\left(\frac{\pi}{6} + \cos^{-1} \frac{2}{3}\right)$$
$$\approx 0.9788$$

Solution set: $\{0.9788\}$

Alternatively,

$$\sin^{-1} x - \tan^{-1} \frac{2}{3} = \frac{\pi}{4}$$




Let $u = \sin^{-1} x$ and $v = \cos^{-1} \frac{2}{3}$. Then $\sin u = x$ and $\cos v = \frac{2}{3}$. From the figure, we have $\cos u = \sqrt{1 - x^2}$. Since $\cos v = \frac{2}{3}$, we know that $\sin v = \sqrt{1 - \left(\frac{2}{3}\right)^2} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$. Replace the equivalents of the trigonometric functions in the equation.

$$u - v = \frac{\pi}{6} \Rightarrow \sin(u - v) = \sin\left(\frac{\pi}{6}\right) \Rightarrow$$
$$\sin u \cos v - \cos u \sin v = \sin\left(\frac{\pi}{6}\right) \Rightarrow$$
$$x \cdot \frac{2}{3} - \left(\sqrt{1 - x^2}\right)\left(\frac{\sqrt{5}}{3}\right) = \frac{1}{2} \Rightarrow$$
$$4x - 2\sqrt{5}\sqrt{1 - x^2} = 3 \Rightarrow$$
$$4x - 3 = 2\sqrt{5}\sqrt{1 - x^2}$$

Now square both sides.

$$(4x - 3)^2 = \left(2\sqrt{5}\sqrt{1 - x^2}\right)^2 \Rightarrow$$
$$16x^2 - 24x + 9 = 20\left(1 - x^2\right) \Rightarrow$$
$$16x^2 - 24x + 9 = 20 - 20x^2 \Rightarrow$$
$$36x^2 - 24x - 11 = 0$$

Use the quadratic formula to solve for *x*.

$$x = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(36)(-11)}}{2(36)}$$
$$= \frac{24 \pm \sqrt{2160}}{72} = \frac{24 \pm 12\sqrt{15}}{72} = \frac{2 \pm \sqrt{15}}{6}$$

Since $\frac{2 - \sqrt{15}}{6}$ is negative, we disregard this answer.

Solution set: $\left\{\frac{2 + \sqrt{15}}{6}\right\}$

## 6.6 Applying the Concepts

**55. a.** $30 = 60\sin(120\pi t) \Rightarrow \frac{1}{2} = \sin(120\pi t) \Rightarrow$
$120\pi t = \frac{\pi}{6} \Rightarrow t = \frac{1}{720} \approx 0.0014$ sec or
$120\pi t = \frac{5\pi}{6} \Rightarrow t = \frac{5}{720} > \frac{1}{720}$, so the smallest possible positive value is $t \approx 0.0014$ sec.

**b.** $-20 = 60\sin(120\pi t) \Rightarrow -\frac{1}{3} = \sin(120\pi t) \Rightarrow$
$120\pi t = -0.3398$ (because we are looking for positive values) $\Rightarrow 2\pi - 0.3398 \approx 5.9434 \Rightarrow$
$t \approx 0.0158$ sec or
$120\pi t = -0.3398 = \pi + 0.3398 \approx 3.4814 \Rightarrow$
$t = 0.0092$ sec. The smallest possible positive value is $t \approx 0.0092$ sec.

**56.** $6\sin\left(\frac{\pi}{2}t\right) = 3 \Rightarrow \frac{1}{2} = \sin\left(\frac{\pi}{2}t\right) \Rightarrow$
$\frac{\pi}{2}t = \frac{\pi}{6}$ or $\frac{\pi}{2}t = \frac{5\pi}{6}$. Because we are looking for all positive values, solve
$\frac{\pi}{2}t = \frac{\pi}{6} + 2\pi n \Rightarrow t = \frac{1}{3} + 4n$ sec or
$\frac{\pi}{2}t = \frac{5\pi}{6} + 2\pi n \Rightarrow t = \frac{5}{3} + 4n$ sec

**57.** $6\cos\left(\frac{\pi}{2}t\right) = 3 \Rightarrow \frac{1}{2} = \cos\left(\frac{\pi}{2}t\right) \Rightarrow$
$\frac{\pi}{2}t = \frac{\pi}{3}$ or $\frac{\pi}{2}t = \frac{5\pi}{3}$. Because we are looking for all positive values, solve
$\frac{\pi}{2}t = \frac{\pi}{3} + 2\pi n \Rightarrow t = \frac{2}{3} + 4n$ sec or
$\frac{\pi}{2}t = \frac{5\pi}{3} + 2\pi n \Rightarrow t = \frac{10}{3} + 4n$ sec

**58. a.** $0 = 500 + 500\sin\left(\frac{\pi}{4}(x-2)\right) \Rightarrow$
$\sin\left(\frac{\pi}{4}(x-2)\right) = -1 \Rightarrow$
$\frac{\pi}{4}(x-2) = \frac{3\pi}{2} + 2n\pi \Rightarrow x = 8 + 8n$
When $n = 0$, $x = 8$. When $n = 1$, $x = 16$, which we reject because $x$ cannot be greater than 12. There were 0 overcoats sold in August.

**b.** $1000 = 500 + 500\sin\left(\frac{\pi}{4}(x-2)\right) \Rightarrow$
$\sin\left(\frac{\pi}{4}(x-2)\right) = 1 \Rightarrow$
$\frac{\pi}{4}(x-2) = \frac{\pi}{2} + 2n\pi \Rightarrow x = 4 + 8n$
When $n = 0$, $x = 4$. When $n = 1$, $x = 12$. There were 1000 overcoats sold in April and December.

**c.** $500 = 500 + 500\sin\left(\frac{\pi}{4}(x-2)\right) \Rightarrow$
$\sin\left(\frac{\pi}{4}(x-2)\right) = 0 \Rightarrow$
$\frac{\pi}{4}(x-2) = 0 + 2n\pi \Rightarrow x = 2 + 8n$ or
$\frac{\pi}{4}(x-2) = \pi + 2n\pi \Rightarrow x = 6 + 8n$
For $x = 2 + 8n$, when $n = 0$, $x = 2$. When $n = 1$, $x = 10$. For $x = 6 + 8n$, when $n = 0$, $x = 6$. There were 500 overcoats sold in February, June, and October.

**59. a.** Note that we use 30 instead of 30,000 for $y$ because $y$ is defined as thousands of people.
$30 = 20 + 10\sin\left(\frac{\pi}{26}(x-14)\right) \Rightarrow$
$\sin\left(\frac{\pi}{26}(x-14)\right) = 1 \Rightarrow \frac{\pi}{26}(x-14) = \frac{\pi}{2} \Rightarrow$
$x = 27$

**b.** $25 = 20 + 10\sin\left(\frac{\pi}{26}(x-14)\right) \Rightarrow$
$\sin\left(\frac{\pi}{26}(x-14)\right) = \frac{1}{2} \Rightarrow \frac{\pi}{26}(x-14) = \frac{\pi}{6} \Rightarrow$
$x \approx 18.3$ (the 18th week) or
$\frac{\pi}{26}(x-14) = \frac{5\pi}{6} \Rightarrow x \approx 35.6$ (which is during the 36th week).

**c.** $15 = 20 + 10\sin\left(\frac{\pi}{26}(x-14)\right) \Rightarrow$
$\sin\left(\frac{\pi}{26}(x-14)\right) = -\frac{1}{2} \Rightarrow \frac{\pi}{26}(x-14) = \frac{7\pi}{6} \Rightarrow$
$x \approx 44.3 \approx$ the 44th week or $\frac{\pi}{26}(x-14)$
$= -\frac{\pi}{6} \Rightarrow x \approx 9.7$ (which is during the 10th week).

**60. a.** $78.75° = 10.5\sin\left(\frac{\pi}{6}(x-5)\right) + 73.5 \Rightarrow$

$\sin\left(\frac{\pi}{6}(x-5)\right) = 0.5 \Rightarrow \frac{\pi}{6}(x-5) = \frac{\pi}{6} \Rightarrow$

$x = 6$ (June) or $\frac{\pi}{6}(x-5) = \frac{5\pi}{6} \Rightarrow x = 10$ (October)

**b.** $84° = 10.5\sin\left(\frac{\pi}{6}(x-5)\right) + 73.5° \Rightarrow$

$\sin\left(\frac{\pi}{6}(x-5)\right) = 1 \Rightarrow \frac{\pi}{6}(x-5) = \frac{\pi}{2} \Rightarrow$

$x = 8$ (August)

**61. a.** $24 = 12 + 12\sin\left(\frac{\pi}{4}(x-2)\right) \Rightarrow$

$\sin\left(\frac{\pi}{4}(x-2)\right) = 1 \Rightarrow \frac{\pi}{4}(x-2) = \frac{\pi}{2} \Rightarrow$

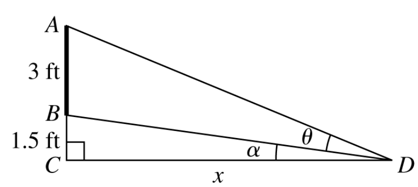
$x = 4$ (January)

**b.** $18 = 12 + 12\sin\left(\frac{\pi}{4}(x-2)\right) \Rightarrow$

$\sin\left(\frac{\pi}{4}(x-2)\right) = \frac{1}{2} \Rightarrow \frac{\pi}{4}(x-2) = \frac{\pi}{6} \Rightarrow$

$x \approx 2.7$ (which is during December) or $\frac{\pi}{4}(x-2) = \frac{5\pi}{6} \Rightarrow x \approx 5.3$ (which is during February).

**62. a.**




$\tan(\angle ADC) = \tan(\theta + \alpha) = \frac{3 + 1.5}{x} = \frac{9}{2x} \Rightarrow$

$(\theta + \alpha) = \tan^{-1}\left(\frac{9}{2x}\right).$

$\tan\alpha = \frac{1.5}{x} = \frac{3}{2x} \Rightarrow \alpha = \tan^{-1}\left(\frac{3}{2x}\right).$ Thus,

$\theta = (\theta + \alpha) - \alpha = \tan^{-1}\left(\frac{9}{2x}\right) - \tan^{-1}\left(\frac{3}{2x}\right).$

**b.** Let $u = \tan^{-1}\left(\frac{9}{2x}\right)$ and $v = \tan^{-1}\left(\frac{3}{2x}\right).$

Then, $\tan u = \frac{9}{2x}$ and $\tan v = \frac{3}{2x}.$

$\tan\theta = \frac{1}{4} \Rightarrow$

$\tan\left[\tan^{-1}\left(\frac{9}{2x}\right) - \tan^{-1}\left(\frac{3}{2x}\right)\right] = \frac{1}{4} \Rightarrow$

$\tan(u - v) = \frac{1}{4}$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v} = \frac{\frac{9}{2x} - \frac{3}{2x}}{1 + \left(\frac{9}{2x}\cdot\frac{3}{2x}\right)}$$

$$= \frac{\frac{3}{x}}{1 + \left(\frac{27}{4x^2}\right)} = \frac{\frac{3}{x}}{\frac{4x^2+27}{4x^2}}$$

$$= \frac{12x}{4x^2 + 27} = \frac{1}{4} \Rightarrow$$

$48x = 4x^2 + 27 \Rightarrow 4x^2 - 48x + 27 = 0$

Solve for $x$ using the quadratic formula.

$$x = \frac{-(-48) \pm \sqrt{(-48)^2 - 4(4)(27)}}{2(4)}$$

$$= \frac{48 \pm \sqrt{1872}}{8} \approx 11.4 \text{ ft}$$

## 6.6 Beyond the Basics

**63.** $\sin^4 2x = 1 \Rightarrow \sin 2x = \pm 1 \Rightarrow 2x = \frac{\pi}{2} + 2\pi n \Rightarrow$

$x = \frac{\pi}{4} + \pi n$ or $2x = \frac{3\pi}{2} + 2\pi n \Rightarrow x = \frac{3\pi}{4} + \pi n$

If $x = \frac{\pi}{4} + \pi n \Rightarrow x = \frac{\pi}{4}$ or $x = \frac{5\pi}{4}$

If $x = \frac{3\pi}{4} + \pi n \Rightarrow x = \frac{3\pi}{4}$ or $x = \frac{7\pi}{4}$

Solution set: $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}.$

**64.** $\cos^4 2x = 1 \Rightarrow \cos 2x = \pm 1$

$\cos 2x = 1 \Rightarrow 2x = 0 + 2\pi n \Rightarrow$

$x = 0 + \pi n \Rightarrow x = 0$ or $x = \pi.$

$\cos 2x = -1 \Rightarrow 2x = \pi + 2\pi n \Rightarrow x = \frac{\pi}{2} + \pi n \Rightarrow$

$x = \frac{\pi}{2}$ or $\frac{3\pi}{2}.$

Solution set: $\left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}.$

**65.** $4\cos^2 \frac{x}{2} = 3 \Rightarrow \cos^2 \frac{x}{2} = \frac{3}{4} \Rightarrow \cos \frac{x}{2} = \pm \frac{\sqrt{3}}{2}$

$\cos \frac{x}{2} = \frac{\sqrt{3}}{2} \Rightarrow \frac{x}{2} = \frac{\pi}{6} + 2n\pi$ or $\frac{x}{2} = \frac{11\pi}{6} + 2n\pi$

$\frac{x}{2} = \frac{\pi}{6} + 2n\pi \Rightarrow x = \frac{\pi}{3} + 4n\pi \Rightarrow x = \frac{\pi}{3}$

$\frac{x}{2} = \frac{11\pi}{6} + 2n\pi \Rightarrow x = \frac{11\pi}{3} + 4n\pi,$ which is outside the required interval.

$\cos \frac{x}{2} = -\frac{\sqrt{3}}{2} \Rightarrow \frac{x}{2} = \frac{5\pi}{6} + 2n\pi$ or $\frac{x}{2} = \frac{7\pi}{6} + 2n\pi$

$\frac{x}{2} = \frac{5\pi}{6} + 2n\pi \Rightarrow x = \frac{5\pi}{3} + 4n\pi \Rightarrow x = \frac{5\pi}{3}$

$\frac{x}{2} = \frac{7\pi}{6} + 2n\pi \Rightarrow x = \frac{7\pi}{3} + 4n\pi,$ which is outside the required interval.

Solution set: $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

**66.** $4\sin^2 \frac{x}{2} = 1 \Rightarrow \sin^2 \frac{x}{2} = \frac{1}{4} \Rightarrow \sin \frac{x}{2} = \pm \frac{1}{2}$

$\sin \frac{x}{2} = \frac{1}{2} \Rightarrow \frac{x}{2} = \frac{\pi}{6} + 2n\pi$ or $\frac{x}{2} = \frac{5\pi}{6} + 2n\pi$

$\frac{x}{2} = \frac{\pi}{6} + 2n\pi \Rightarrow x = \frac{\pi}{3} + 4n\pi \Rightarrow x = \frac{\pi}{3}$

$\frac{x}{2} = \frac{5\pi}{6} + 2n\pi \Rightarrow x = \frac{5\pi}{3} + 4n\pi \Rightarrow x = \frac{5\pi}{3}$

$\sin \frac{x}{2} = -\frac{1}{2} \Rightarrow \frac{x}{2} = \frac{7\pi}{6} + 2n\pi$ or $\frac{x}{2} = \frac{11\pi}{6} + 2n\pi.$

$\frac{x}{2} = \frac{7\pi}{6} + 2n\pi \Rightarrow x = \frac{7\pi}{3} + 4n\pi \Rightarrow x = \frac{7\pi}{3}$

$\frac{x}{2} = \frac{11\pi}{6} + 2n\pi \Rightarrow x = \frac{11\pi}{3} + 4n\pi.$ Both of these values are outside the required interval.

Solution set: $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

**67.** $\tan^2 2x = 1 \Rightarrow \tan 2x = \pm 1 \Rightarrow 2x = \frac{\pi}{4} + \pi n \Rightarrow$

$x = \frac{\pi}{8} + \frac{\pi}{2}n$ or $2x = \frac{5\pi}{4} + \pi n \Rightarrow x = \frac{5\pi}{8} + \frac{\pi}{2}n$

or $2x = \frac{3\pi}{4} + \pi n \Rightarrow x = \frac{3\pi}{8} + \frac{\pi}{2}n$ or

$2x = \frac{7\pi}{4} + \pi n \Rightarrow x = \frac{7\pi}{8} + \frac{\pi}{2}n$

If $x = \frac{\pi}{8} + \frac{\pi}{2}n \Rightarrow x = \frac{\pi}{8}$ or $x = \frac{5\pi}{8}$ or $x = \frac{9\pi}{8}$ or $x = \frac{13\pi}{8}$. If $x = \frac{5\pi}{8} + \frac{\pi}{2}n \Rightarrow x = \frac{5\pi}{8}$ or $x = \frac{9\pi}{8}$ or $x = \frac{13\pi}{8}$. If $x = \frac{3\pi}{8} + \frac{\pi}{2}n \Rightarrow$ $x = \frac{3\pi}{8}$ or $x = \frac{7\pi}{8}$ or $x = \frac{11\pi}{8}$ or $x = \frac{15\pi}{8}$.

If $x = \frac{7\pi}{8} + \frac{\pi}{2}n \Rightarrow \frac{7\pi}{8}$ or $x = \frac{11\pi}{8}$ or $x = \frac{15\pi}{8}$.

$\left\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\}$.

**68.** $\cot^2 \frac{x}{2} = 1 \Rightarrow \cot \frac{x}{2} = \pm 1$

$\cot \frac{x}{2} = 1 \Rightarrow \frac{x}{2} = \frac{\pi}{4} + \pi n \Rightarrow x = \frac{\pi}{2} + 2n\pi \Rightarrow$ $x = \frac{\pi}{2}$ or $\frac{x}{2} = \frac{5\pi}{4} + \pi n \Rightarrow x = \frac{5\pi}{2} + 2n\pi,$ which is outside the required interval.

$\cot \frac{x}{2} = -1 \Rightarrow \frac{x}{2} = \frac{3\pi}{4} + \pi n \Rightarrow$

$x = \frac{3\pi}{2} + 2n\pi \Rightarrow x = \frac{3\pi}{2}$ or

$\frac{x}{2} = \frac{7\pi}{4} + \pi n \Rightarrow x = \frac{7\pi}{2} + 2n\pi,$ which is outside the required interval.

Solution set: $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

**69.** $3\sec^2 \frac{x}{2} = 12 \Rightarrow \sec^2 \frac{x}{2} = 4 \Rightarrow \sec \frac{x}{2} = \pm 2$

$\sec \frac{x}{2} = 2 \Rightarrow \frac{x}{2} = \frac{\pi}{3} + 2n\pi \Rightarrow$

$x = \frac{2\pi}{3} + 4n\pi \Rightarrow x = \frac{2\pi}{3}$ or $\frac{x}{2} = \frac{5\pi}{3} + 2n\pi \Rightarrow$ $x = \frac{10\pi}{3} + 2n\pi,$ which is outside the required interval.

$\sec \frac{x}{2} = -2 \Rightarrow \frac{x}{2} = \frac{2\pi}{3} + 2n\pi \Rightarrow$

$x = \frac{4\pi}{3} + 4n\pi \Rightarrow x = \frac{4\pi}{3}$ or

$\frac{x}{2} = \frac{4\pi}{3} + 2n\pi \Rightarrow x = \frac{8\pi}{3} + 4n\pi,$ which is outside the required interval.

Solution set: $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

**70.** $5\csc^2\frac{x}{2}-11=9\Rightarrow\csc^2\frac{x}{2}=4\Rightarrow\csc\frac{x}{2}=\pm2$

$\csc\frac{x}{2}=2\Rightarrow\frac{x}{2}=\frac{\pi}{6}+2n\pi\Rightarrow x=\frac{\pi}{3}+4n\pi\Rightarrow x=\frac{\pi}{3}$ or $\frac{x}{2}=\frac{5\pi}{6}+2n\pi\Rightarrow x=\frac{5\pi}{3}+4n\pi\Rightarrow x=\frac{5\pi}{3}$

$\csc\frac{x}{2}=-2\Rightarrow\frac{x}{2}=\frac{7\pi}{6}+2n\pi\Rightarrow x=\frac{7\pi}{3}+4n\pi\Rightarrow x=\frac{7\pi}{3}$ or $\frac{x}{2}=\frac{11\pi}{6}+2n\pi\Rightarrow x=\frac{11\pi}{3}+4n\pi.$

Both of these values are outside the required interval.

Solution set: $\left\{\frac{\pi}{3},\frac{5\pi}{3}\right\}$

**71.** $(\tan 2x-1)(\sin x+1)=0\Rightarrow\tan 2x=1\Rightarrow 2x=\frac{\pi}{4}+n\pi\Rightarrow x=\frac{\pi}{8}+\frac{\pi}{2}n$ or $\sin x=-1\Rightarrow$

$x=\frac{3\pi}{2}$. If $x=\frac{\pi}{8}+\frac{\pi}{2}n\Rightarrow x=\frac{\pi}{8}$ or $x=\frac{5\pi}{8}$ or $x=\frac{9\pi}{8}$ or $x=\frac{13\pi}{8}$.

The solution is $\left\{\frac{\pi}{8},\frac{5\pi}{8},\frac{9\pi}{8},\frac{3\pi}{2},\frac{13\pi}{8}\right\}$.

**72.** $\left(\sqrt{3}\cot 2x-1\right)\left(2\cos x-1\right)=0\Rightarrow\sqrt{3}\cot 2x-1=0\Rightarrow\cot 2x=\frac{\sqrt{3}}{3}\Rightarrow 2x=\frac{\pi}{3}+n\pi\Rightarrow$

$x=\frac{\pi}{6}+\frac{\pi}{2}n$ or $2\cos x-1=0\Rightarrow\cos x=\frac{1}{2}\Rightarrow x=\frac{\pi}{3}$ or $x=\frac{5\pi}{3}$.

If $x=\frac{\pi}{6}+\frac{\pi}{2}n$ then $x=\frac{\pi}{6}$ or $x=\frac{2\pi}{3}$ or $x=\frac{7\pi}{6}$ or $x=\frac{5\pi}{3}$.

The solution is $\left\{\frac{\pi}{6},\frac{\pi}{3},\frac{2\pi}{3},\frac{7\pi}{6},\frac{5\pi}{3}\right\}$.

**73.** $\left(1-2\sin 2x-1\right)\left(\sqrt{3}+2\cos x\right)=0\Rightarrow 1-2\sin 2x-1=0$ or $\sqrt{3}+2\cos x=0.$

$1-2\sin 2x-1=0\Rightarrow\sin 2x=0\Rightarrow 2x=0+2n\pi\Rightarrow x=0+n\pi\Rightarrow x=0,\ \pi$ or $2x=\pi+2n\pi\Rightarrow$

$x=\frac{\pi}{2}+n\pi\Rightarrow x=\frac{\pi}{2},\frac{3\pi}{2}.\sqrt{3}+2\cos x=0\Rightarrow\cos x=-\frac{\sqrt{3}}{2}\Rightarrow x=\frac{5\pi}{6}+2n\pi\Rightarrow x=\frac{5\pi}{6}$ or

$x=\frac{7\pi}{6}+2n\pi\Rightarrow x=\frac{7\pi}{6}.$

Solution set: $\left\{0,\frac{\pi}{2},\frac{5\pi}{6},\pi,\frac{7\pi}{6},\frac{3\pi}{2}\right\}$

**74.** $\left(\sqrt{3}\tan 2x+1\right)\left(2\cos x+1\right)=0\Rightarrow\sqrt{3}\tan 2x+1=0$ or $2\cos x+1=0.$

$\sqrt{3}\tan 2x+1=0\Rightarrow\tan 2x=-\frac{\sqrt{3}}{3}\Rightarrow 2x=\frac{5\pi}{6}+n\pi\Rightarrow x=\frac{5\pi}{12}+\frac{n\pi}{2}\Rightarrow x=\frac{5\pi}{12},\frac{11\pi}{12},\frac{17\pi}{12},\frac{23\pi}{12}.$

$2\cos x+1=0\Rightarrow\cos x=-\frac{1}{2}\Rightarrow x=\frac{2\pi}{3},\frac{4\pi}{3}$

Solution set: $\left\{\frac{5\pi}{12},\frac{2\pi}{3},\frac{11\pi}{12},\frac{4\pi}{3},\frac{17\pi}{12},\frac{23\pi}{12}\right\}$

**75.**
$$\sin 2x\cos 2x + \frac{\sqrt{3}}{2}\sin 2x + \frac{1}{2}\cos 2x + \frac{\sqrt{3}}{4} = 0$$
$$\frac{1}{2}(2\sin 2x\cos 2x) + \left[\left(\cos\frac{\pi}{6}\right)\sin 2x + \left(\sin\frac{\pi}{6}\right)\cos 2x\right] + \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) = 0$$
$$\frac{1}{2}\sin 4x + \sin\left(2x + \frac{\pi}{6}\right) + \frac{1}{2}\left(\sin\frac{\pi}{3}\right) = 0$$
$$\frac{1}{2}\left(\sin 4x + \sin\frac{\pi}{3}\right) + \sin\left(2x + \frac{\pi}{6}\right) = 0$$
$$\sin\left(\frac{4x + \frac{\pi}{3}}{2}\right)\cos\left(\frac{4x - \frac{\pi}{3}}{2}\right) + \sin\left(2x + \frac{\pi}{6}\right) = 0$$
$$\sin\left(2x + \frac{\pi}{6}\right)\cos\left(2x - \frac{\pi}{6}\right) + \sin\left(2x + \frac{\pi}{6}\right) = 0$$
$$\sin\left(2x + \frac{\pi}{6}\right)\left[\cos\left(2x - \frac{\pi}{6}\right) + 1\right] = 0 \Rightarrow$$
$$\sin\left(2x + \frac{\pi}{6}\right) = 0 \text{ or } \left[\cos\left(2x - \frac{\pi}{6}\right) + 1\right] = 0.$$

$\sin\left(2x + \frac{\pi}{6}\right) = 0 \Rightarrow 2x + \frac{\pi}{6} = 0 + 2n\pi \Rightarrow 2x = 0 - \frac{\pi}{6} + 2n\pi \Rightarrow x = -\frac{\pi}{12} + n\pi \Rightarrow x = \frac{11\pi}{12}, \frac{23\pi}{12}$ or
$2x = \pi - \frac{\pi}{6} + 2n\pi \Rightarrow x = \frac{5\pi}{12} + n\pi \Rightarrow x = \frac{5\pi}{12}, \frac{17\pi}{12}$
$\cos\left(2x - \frac{\pi}{6}\right) + 1 = 0 \Rightarrow \cos\left(2x - \frac{\pi}{6}\right) = -1 \Rightarrow 2x - \frac{\pi}{6} = \pi + 2n\pi \Rightarrow 2x = \frac{7\pi}{6} + 2n\pi \Rightarrow x = \frac{7\pi}{12} + n\pi \Rightarrow$
$x = \frac{7\pi}{12}, \frac{19\pi}{12}$

Solution set: $\left\{\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}\right\}$

**76.** $2\sin 3x\tan 2x - 2\sin 3x + \tan 2x - 1 = 0 \Rightarrow \tan 2x(2\sin 3x + 1) - (2\sin 3x + 1) = 0 \Rightarrow$
$(\tan 2x - 1)(2\sin 3x + 1) = 0 \Rightarrow \tan 2x - 1 = 0$ or $2\sin 3x + 1 = 0$.
$\tan 2x - 1 = 0 \Rightarrow \tan 2x = 1 \Rightarrow 2x = \frac{\pi}{4} + n\pi$ or $2x = \frac{5\pi}{4} + n\pi$.
$2x = \frac{\pi}{4} + n\pi \Rightarrow x = \frac{\pi}{8} + \frac{n\pi}{2} \Rightarrow x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$
$2x = \frac{5\pi}{4} + n\pi \Rightarrow x = \frac{5\pi}{8} + \frac{n\pi}{2} \Rightarrow x = \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$
$2\sin 3x + 1 = 0 \Rightarrow \sin 3x = -\frac{1}{2} \Rightarrow 3x = \frac{7\pi}{6} + 2n\pi$ or $3x = \frac{11\pi}{6} + 2n\pi$.
$3x = \frac{7\pi}{6} + 2n\pi \Rightarrow x = \frac{7\pi}{18} + \frac{2n\pi}{3} \Rightarrow x = \frac{7\pi}{18}, \frac{19\pi}{18}, \frac{31\pi}{18}$
$3x = \frac{11\pi}{6} + 2n\pi \Rightarrow x = \frac{11\pi}{18} + \frac{2n\pi}{3} \Rightarrow x = \frac{11\pi}{18}, \frac{23\pi}{18}, \frac{35\pi}{18}$

Solution set: $\left\{\frac{\pi}{8}, \frac{7\pi}{18}, \frac{11\pi}{18}, \frac{5\pi}{8}, \frac{19\pi}{18}, \frac{9\pi}{8}, \frac{23\pi}{18}, \frac{13\pi}{8}, \frac{31\pi}{18}, \frac{35\pi}{18}\right\}$

**77.** $\sin 2x + \sin 4x = 2\sin 3x \Rightarrow 2\sin\left(\frac{2x+4x}{2}\right)\cos\left(\frac{2x-4x}{2}\right) - 2\sin 3x = 0 \Rightarrow 2\sin 3x\cos(-x) - 2\sin 3x = 0 \Rightarrow$

$2\sin 3x\cos x - 2\sin 3x = 0 \Rightarrow (2\sin 3x)(\cos x - 1) = 0 \Rightarrow 2\sin 3x = 0$ or $\cos x = 1$

$2\sin 3x = 0 \Rightarrow \sin 3x = 0 \Rightarrow 3x = 0 + 2n\pi$ or $3x = \pi + 2n\pi$.

$3x = 0 + 2n\pi \Rightarrow x = \frac{2n\pi}{3} \Rightarrow x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}.$

$3x = \pi + 2n\pi \Rightarrow x = \frac{\pi}{3} + \frac{2n\pi}{3} \Rightarrow x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}.$

$\cos x = 1 \Rightarrow x = 0$

Solution set: $\left\{0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

**78.** $\cos x + \cos 5x = 2\cos 2x \Rightarrow 2\cos\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right) - 2\cos 2x = 0 \Rightarrow$

$2\cos 3x\cos(-2x) - 2\cos 2x = 0 \Rightarrow 2\cos 3x\cos 2x - 2\cos 2x = 0 \Rightarrow$

$2\cos 2x(\cos 3x - 1) = 0 \Rightarrow 2\cos 2x = 0$ or $\cos 3x - 1 = 0.$

$2\cos 2x = 0 \Rightarrow \cos 2x = 0 \Rightarrow 2x = \frac{\pi}{2} + 2n\pi$ or $2x = \frac{3\pi}{2} + 2n\pi.$

$2x = \frac{\pi}{2} + 2n\pi \Rightarrow x = \frac{\pi}{4} + n\pi \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}.$

$2x = \frac{3\pi}{2} + 2n\pi \Rightarrow x = \frac{3\pi}{4} + n\pi \Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}.$

$\cos 3x - 1 = 0 \Rightarrow \cos 3x = 1 \Rightarrow 3x = 0 + 2n\pi \Rightarrow x = \frac{2n\pi}{3} \Rightarrow x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}.$

Solution set: $\left\{0, \frac{\pi}{4}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{7\pi}{4}\right\}$

**79.** $\sin x + \sin 3x = \cos x + \cos 3x \Rightarrow 2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right) = 2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right) \Rightarrow$

$2\sin 2x\cos(-x) = 2\cos 2x\cos(-x) \Rightarrow 2\sin 2x\cos x = 2\cos 2x\cos x \Rightarrow 2\sin 2x\cos x - 2\cos 2x\cos x = 0 \Rightarrow$

$2\cos x(\sin 2x - \cos 2x) = 0 \Rightarrow \cos x = 0$ or $\sin 2x - \cos 2x = 0.$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2} + 2n\pi$ or $x = \frac{3\pi}{2} + 2n\pi.$

$x = \frac{\pi}{2} + 2n\pi \Rightarrow x = \frac{\pi}{2};\ x = \frac{3\pi}{2} + 2n\pi \Rightarrow x = \frac{3\pi}{2}.$

$\sin 2x - \cos 2x = 0 \Rightarrow \sin 2x - \sin\left(\frac{\pi}{2} - 2x\right) = 0 \Rightarrow 2\sin\left(\frac{2x - \left(\frac{\pi}{2} - 2x\right)}{2}\right)\cos\left(\frac{2x + \left(\frac{\pi}{2} - 2x\right)}{2}\right) = 0 \Rightarrow$

$2\sin\left(2x - \frac{\pi}{4}\right)\cos\frac{\pi}{4} = 0 \Rightarrow \sin\left(2x - \frac{\pi}{4}\right) = 0 \Rightarrow 2x - \frac{\pi}{4} = 0 + 2n\pi$ or $2x - \frac{\pi}{4} = \pi + 2n\pi$

$2x - \frac{\pi}{4} = 0 + 2n\pi \Rightarrow 2x = \frac{\pi}{4} + 2n\pi \Rightarrow x = \frac{\pi}{8} + n\pi \Rightarrow x = \frac{\pi}{8}, \frac{9\pi}{8}$

$2x - \frac{\pi}{4} = \pi + 2n\pi \Rightarrow 2x = \frac{5\pi}{4} + 2n\pi \Rightarrow x = \frac{5\pi}{8} + n\pi \Rightarrow x = \frac{5\pi}{8}, \frac{13\pi}{8}$

Solution set: $\left\{\frac{\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{3\pi}{2}, \frac{13\pi}{8}\right\}$

**80.** $\sin 2x + \sin 4x = \cos 2x + \cos 4x \Rightarrow 2\sin\left(\frac{2x+4x}{2}\right)\cos\left(\frac{2x-4x}{2}\right) = 2\cos\left(\frac{2x+4x}{2}\right)\cos\left(\frac{2x-4x}{2}\right) \Rightarrow$
$2\sin 3x\cos(-x) = 2\cos 3x\cos(-x) \Rightarrow 2\sin 3x\cos x = 2\cos 3x\cos x \Rightarrow 2\sin 3x\cos x - 2\cos 3x\cos x = 0 \Rightarrow$
$2\cos x(\sin 3x - \cos 3x) = 0 \Rightarrow \cos x = 0$ or $\sin 3x - \cos 3x = 0.$
$\cos x = 0 \Rightarrow x = \frac{\pi}{2} + 2n\pi$ or $x = \frac{3\pi}{2} + 2n\pi.$
$x = \frac{\pi}{2} + 2n\pi \Rightarrow x = \frac{\pi}{2};\ x = \frac{3\pi}{2} + 2n\pi \Rightarrow x = \frac{3\pi}{2}.$
$\sin 3x - \cos 3x = 0 \Rightarrow \sin 3x - \sin\left(\frac{\pi}{2} - 3x\right) = 0 \Rightarrow 2\sin\left(\frac{3x - \left(\frac{\pi}{2} - 3x\right)}{2}\right)\cos\left(\frac{3x + \left(\frac{\pi}{2} - 3x\right)}{2}\right) = 0 \Rightarrow$
$2\sin\left(3x - \frac{\pi}{4}\right)\cos\frac{\pi}{4} = 0 \Rightarrow \sin\left(3x - \frac{\pi}{4}\right) = 0 \Rightarrow 3x - \frac{\pi}{4} = 0 + 2n\pi$ or $3x - \frac{\pi}{4} = \pi + 2n\pi$
$3x - \frac{\pi}{4} = 0 + 2n\pi \Rightarrow 3x = \frac{\pi}{4} + 2n\pi \Rightarrow x = \frac{\pi}{12} + \frac{2n\pi}{3} \Rightarrow x = \frac{\pi}{12}, \frac{3\pi}{4}, \frac{17\pi}{12}$
$3x - \frac{\pi}{4} = \pi + 2n\pi \Rightarrow 3x = \frac{5\pi}{4} + 2n\pi \Rightarrow x = \frac{5\pi}{12} + \frac{2n\pi}{3} \Rightarrow x = \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{7\pi}{4}$
Solution set: $\left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{2}, \frac{7\pi}{4}\right\}$

For exercises 81–84, recall that if $(a, b)$ is any point on the terminal side of an angle $\theta$ in standard position, then $a\sin x + b\cos x = \sqrt{a^2 + b^2}\sin(x + \theta)$ for any real number $x$, and $\theta = \tan^{-1}\left(\frac{b}{a}\right)$.

**81. a.** $3\sin 2x + 4\cos 2x = 0$
Using the reduction formula, we have
$3\sin 2x + 4\cos 2x = \sqrt{3^2 + 4^2}\sin\left(2x + \tan^{-1}\left(\frac{4}{3}\right)\right) = 5\sin\left(2x + \tan^{-1}\left(\frac{4}{3}\right)\right).$

**b.** $5\sin\left(2x + \tan^{-1}\left(\frac{4}{3}\right)\right) = 0 \Rightarrow 2x + \tan^{-1}\left(\frac{4}{3}\right) = 0 + 2n\pi$ or $2x + \tan^{-1}\left(\frac{4}{3}\right) = \pi + 2n\pi.$
$2x + \tan^{-1}\left(\frac{4}{3}\right) = 0 + 2n\pi \Rightarrow 2x = -\tan^{-1}\left(\frac{4}{3}\right) + 2n\pi \Rightarrow x = \frac{-\tan^{-1}\left(\frac{4}{3}\right)}{2} + n\pi \Rightarrow x \approx 2.678, 5.820.$
$2x + \tan^{-1}\left(\frac{4}{3}\right) = \pi + 2n\pi \Rightarrow 2x = \pi - \tan^{-1}\left(\frac{4}{3}\right) + 2n\pi \Rightarrow x = \frac{\pi}{2} - \frac{\tan^{-1}\left(\frac{4}{3}\right)}{2} + n\pi \Rightarrow x \approx 1.107, 4.249.$
Solution set: {1.107, 2.678, 4.249, 5.820}

**82. a.** $3\cos 2x - 4\sin 2x = \frac{5}{2} \Rightarrow -(3\cos 2x - 4\sin 2x) = -\frac{5}{2} \Rightarrow 4\sin 2x - 3\cos 2x = -\frac{5}{2}$
Using the reduction formula, we have
$4\sin 2x - 3\cos 2x = \sqrt{4^2 + (-3)^2}\sin\left(2x + \tan^{-1}\left(\frac{-3}{4}\right)\right) = 5\sin\left(2x + \tan^{-1}\left(-\frac{3}{4}\right)\right).$

**b.** $5\sin\left(2x+\tan^{-1}\left(-\frac{3}{4}\right)\right)=-\frac{5}{2}\Rightarrow \sin\left(2x+\tan^{-1}\left(-\frac{3}{4}\right)\right)=-\frac{1}{2}\Rightarrow$

$2x+\tan^{-1}\left(-\frac{3}{4}\right)=\frac{7\pi}{6}+2n\pi$ or $2x+\tan^{-1}\left(-\frac{3}{4}\right)=\frac{11\pi}{6}+2n\pi$

$2x=-\tan^{-1}\left(-\frac{3}{4}\right)+\frac{7\pi}{6}+2n\pi\Rightarrow x=\frac{-\tan^{-1}\left(-\frac{3}{4}\right)}{2}+\frac{7\pi}{12}+n\pi\approx 2.154,\ 5.296$

$2x+\tan^{-1}\left(-\frac{3}{4}\right)=\frac{11\pi}{6}+2n\pi\Rightarrow x=\frac{-\tan^{-1}\left(-\frac{3}{4}\right)}{2}+\frac{11\pi}{12}+n\pi\approx 0.060,\ 3.202$

Solution set: {0.060, 2.154, 3.202, 5.296}

**83. a.** $5\sin 3x-12\cos 3x=\frac{13\sqrt{3}}{2}$

Using the reduction formula, we have

$5\sin 3x-12\cos 3x=\sqrt{5^2+(-12)^2}\sin\left(3x+\tan^{-1}\left(-\frac{12}{5}\right)\right)=13\sin\left(3x+\tan^{-1}\left(-\frac{12}{5}\right)\right).$

**b.** $13\sin\left(3x+\tan^{-1}\left(-\frac{12}{5}\right)\right)=\frac{13\sqrt{3}}{2}\Rightarrow \sin\left(3x+\tan^{-1}\left(-\frac{12}{5}\right)\right)=\frac{\sqrt{3}}{2}\Rightarrow$

$3x-\tan^{-1}\left(\frac{12}{5}\right)=\frac{\pi}{3}+2n\pi$ or $3x-\tan^{-1}\left(\frac{12}{5}\right)=\frac{2\pi}{3}+2n\pi.$

$3x-\tan^{-1}\left(\frac{12}{5}\right)=\frac{\pi}{3}+2n\pi\Rightarrow 3x=\frac{\pi}{3}+\tan^{-1}\left(\frac{12}{5}\right)+2n\pi\Rightarrow x=\frac{\frac{\pi}{3}+\tan^{-1}\left(\frac{12}{5}\right)+2n\pi}{3}\Rightarrow$

$x\approx 0.741,\ 2.835,\ 4.930.$

$3x-\tan^{-1}\left(\frac{12}{5}\right)=\frac{2\pi}{3}+2n\pi\Rightarrow 3x=\frac{2\pi}{3}+\tan^{-1}\left(\frac{12}{5}\right)+2n\pi\Rightarrow x=\frac{\frac{2\pi}{3}+\tan^{-1}\left(\frac{12}{5}\right)+2n\pi}{3}\Rightarrow$

$x\approx 1.090,\ 3.185,\ 5.279$

Solution set: {0.741, 1.090, 2.835, 3.185, 4.930, 5.279}

**84. a.** $12\sin 3x+5\cos 3x=\frac{13}{2}$

Using the reduction formula, we have

$12\sin 3x+5\cos 3x=\sqrt{12^2+5^2}\sin\left(3x+\tan^{-1}\left(\frac{5}{12}\right)\right)=13\sin\left(3x+\tan^{-1}\left(\frac{5}{12}\right)\right).$

**b.** $13\sin\left(3x+\tan^{-1}\left(\frac{5}{12}\right)\right)=\frac{13}{2}\Rightarrow \sin\left(3x+\tan^{-1}\left(\frac{5}{12}\right)\right)=\frac{1}{2}\Rightarrow$

$3x+\tan^{-1}\left(\frac{5}{12}\right)=\frac{\pi}{6}+2n\pi$ or $3x+\tan^{-1}\left(\frac{5}{12}\right)=\frac{5\pi}{6}+2n\pi.$

$3x+\tan^{-1}\left(\frac{5}{12}\right)=\frac{\pi}{6}+2n\pi\Rightarrow 3x=\frac{\pi}{6}-\tan^{-1}\left(\frac{5}{12}\right)+2n\pi\Rightarrow x=\frac{\frac{\pi}{6}-\tan^{-1}\left(\frac{5}{12}\right)+2n\pi}{3}\Rightarrow$

$x\approx 0.043,\ 2.137,\ 4.232.$

$3x+\tan^{-1}\left(\frac{5}{12}\right)=\frac{5\pi}{6}+2n\pi\Rightarrow 3x=\frac{5\pi}{6}-\tan^{-1}\left(\frac{5}{12}\right)+2n\pi\Rightarrow x=\frac{\frac{5\pi}{6}-\tan^{-1}\left(\frac{5}{12}\right)+2n\pi}{3}\Rightarrow$

$x\approx 0.741,\ 2.835,\ 4.930.$

Solution set: {0.043, 0.741, 2.137, 2.835, 4.232, 4.930}

### 6.6 Critical Thinking/Discussion/Writing

**85.** Let $u = \sin^{-1} x$. Then $\sin u = x$.

$\cos\left(2\sin^{-1} x\right) = \frac{1}{2} \Rightarrow \cos(2u) = \frac{1}{2} \Rightarrow 1 - 2\sin^2 u = \frac{1}{2} \Rightarrow 1 - 2x^2 = \frac{1}{2} \Rightarrow 2x^2 = \frac{1}{2} \Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm\frac{1}{2}.$

**86.** Let $u = \tan^{-1} x$. Then $\tan u = x$. $\tan^{-1} 1 = \frac{\pi}{4}$.

$\tan\left(\tan^{-1} x + \tan^{-1} 1\right) = 5 \Rightarrow \tan\left(u + \frac{\pi}{4}\right) = 5 \Rightarrow \dfrac{\tan u + \tan\frac{\pi}{4}}{1 - (\tan u)\left(\tan\frac{\pi}{4}\right)} = 5 \Rightarrow \frac{x+1}{1-x} = 5 \Rightarrow$

$x + 1 = 5 - 5x \Rightarrow 6x = 4 \Rightarrow x = \frac{2}{3}$

### 6.6 Maintaining Skills

**87.** $\frac{3}{x} = \frac{5}{\pi} \Rightarrow \frac{x}{3} = \frac{\pi}{5}$

**88.** $\frac{\sin 12°}{4} = \frac{\sqrt{2}}{2x} \Rightarrow \frac{4}{\sin 12°} = \frac{2x}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}x$

**89.** $\frac{12}{x} = \frac{3}{4} \Rightarrow 3x = 48 \Rightarrow x = 16$

**90.** $\frac{7}{9} = \frac{2}{3x} \Rightarrow 21x = 18 \Rightarrow x = \frac{6}{7}$

**91.** $\sin 60° = \frac{\sqrt{3}}{2}$

**92.** $\sin 45° = \frac{\sqrt{2}}{2}$

**93.** $\sin(180° - 30°)$
$= \sin 180° \cos 30° - \cos 180° \sin 30°$
$= 0 - (-1)\left(\frac{1}{2}\right) = \frac{1}{2}$

**94.** $\sin(180° - 45°)$
$= \sin 180° \cos 45° - \cos 180° \sin 45°$
$= 0 - (-1)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}$

**95.** True

**96.** False

## Chapter 6 Review Exercises

### Basic Concepts and Skills

**1.** $\sin\theta = -\frac{2}{3}$ and $\cos\theta < 0 \Rightarrow \theta$ is in Quadrant III.

$\left(-\frac{2}{3}\right)^2 + \left(\cos^2\theta\right) = 1 \Rightarrow \cos\theta = -\frac{\sqrt{5}}{3}.$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-2/3}{-\sqrt{5}/3} = \frac{2\sqrt{5}}{5},\ \cot\theta = \frac{1}{\tan\theta} = \frac{\sqrt{5}}{2}, \sec\theta = \frac{1}{\cos\theta} = -\frac{3\sqrt{5}}{5},\ \csc\theta = \frac{1}{\sin\theta} = -\frac{3}{2}.$

**2.** $\tan\theta = -\frac{1}{2}$ and $\csc\theta > 0 \Rightarrow \theta$ is in Quadrant II.

$\cot\theta = \frac{1}{\tan\theta} = -2; (-2)^2 + 1 = \csc^2\theta \Rightarrow \csc\theta = \sqrt{5};\ \sin\theta = \frac{1}{\csc\theta} = \frac{\sqrt{5}}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow -\frac{1}{2} = \frac{\sqrt{5}/5}{\cos\theta} \Rightarrow \cos\theta = -\frac{2\sqrt{5}}{5};\quad \sec\theta = \frac{1}{\cos\theta} = -\frac{\sqrt{5}}{2}$

**3.** $\sec\theta = 3$ and $\tan\theta < 0 \Rightarrow \theta$ is in Quadrant IV.

$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{3}; \sin^2\theta + \left(\frac{1}{3}\right)^2 = 1 \Rightarrow \sin\theta = -\frac{2\sqrt{2}}{3};\ \tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{2\sqrt{2}/3}{1/3} \Rightarrow \tan\theta = -2\sqrt{2};$

$\cot\theta = \frac{1}{\tan\theta} = -\frac{\sqrt{2}}{4};\ \csc\theta = \frac{1}{\sin\theta} = -\frac{3\sqrt{2}}{4}$

**4.** $\csc\theta = 5$ and $\cot\theta < 0 \Rightarrow \theta$ is in Quadrant II.

$\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{1}{5}$; $\left(\dfrac{1}{5}\right)^2 + \cos^2\theta = 1 \Rightarrow \cos\theta = -\dfrac{2\sqrt{6}}{5}$; $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{1/5}{-2\sqrt{6}/5} = -\dfrac{\sqrt{6}}{12}$

$\cot\theta = \dfrac{1}{\tan\theta} = -2\sqrt{6}$; $\sec\theta = \dfrac{1}{\cos\theta} = -\dfrac{5\sqrt{6}}{12}$

**5.**
$$\begin{aligned}(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2 &= \sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta+(\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta)\\ &= (\sin^2\theta+\cos^2\theta)+2\sin\theta\cos\theta-2\sin\theta\cos\theta+(\sin^2\theta+\cos^2\theta)\\ &= 2(\sin^2\theta+\cos^2\theta)=2(1)=2\end{aligned}$$

**6.**
$$\begin{aligned}(1-\tan x)^2+(1+\tan x)^2 &= (1-2\tan x+\tan^2 x)+(1+2\tan x+\tan^2 x)\\ &= (1+\tan^2 x)-2\tan x+2\tan x+(1+\tan^2 x)=2(1+\tan^2 x)=2\sec^2 x\end{aligned}$$

**7.**
$$\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}}=\frac{\dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}=\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}=\frac{\cos^2\theta-\sin^2\theta}{1}=\cos^2\theta-\sin^2\theta$$

**8.**
$$\begin{aligned}\frac{\sin x+\tan x}{\csc x+\cot x}&=\frac{\sin x+\dfrac{\sin x}{\cos x}}{\dfrac{1}{\sin x}+\dfrac{\cos x}{\sin x}}=\frac{\dfrac{\sin x\cos x+\sin x}{\cos x}}{\dfrac{1+\cos x}{\sin x}}=\frac{\dfrac{\sin x(1+\cos x)}{\cos x}}{\dfrac{1+\cos x}{\sin x}}=\frac{\sin x(1+\cos x)}{\cos x}\cdot\frac{\sin x}{1+\cos x}\\ &=\frac{\sin^2 x}{\cos x}=\sin^2 x\sec x\end{aligned}$$

**9.**
$$\begin{aligned}\frac{\sin\theta}{1+\cos\theta}+\frac{\sin\theta}{1-\cos\theta}&=\frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}+\frac{\sin\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}=\frac{\sin\theta-\sin\theta\cos\theta+\sin\theta+\sin\theta\cos\theta}{1-\cos^2\theta}\\ &=\frac{2\sin\theta}{\sin^2\theta}=\frac{2}{\sin\theta}=2\csc\theta\end{aligned}$$

**10.** Start with the right side.

$$\tan^2 x-\sin^2 x=\frac{\sin^2 x}{\cos^2 x}-\sin^2 x=\frac{\sin^2 x-\sin^2 x\cos^2 x}{\cos^2 x}=\frac{\sin^2 x(1-\cos^2 x)}{\cos^2 x}=\frac{\sin^2 x\sin^2 x}{\cos^2 x}=\tan^2 x\sin^2 x$$

**11.**
$$\begin{aligned}\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta}&=\frac{\sin\theta}{1-\dfrac{\cos\theta}{\sin\theta}}+\frac{\cos\theta}{1-\dfrac{\sin\theta}{\cos\theta}}=\frac{\sin\theta}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\cos\theta}{\dfrac{\cos\theta-\sin\theta}{\cos\theta}}=\frac{\sin^2\theta}{\sin\theta-\cos\theta}+\frac{\cos^2\theta}{\cos\theta-\sin\theta}\\ &=\frac{\sin^2\theta}{\sin\theta-\cos\theta}-\frac{\cos^2\theta}{\sin\theta-\cos\theta}=\frac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}=\frac{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}{\sin\theta-\cos\theta}\\ &=\sin\theta+\cos\theta\end{aligned}$$

**12.**
$$\begin{aligned}\frac{\tan\theta-\sin\theta}{\sin^3\theta}&=\frac{\dfrac{\sin\theta}{\cos\theta}-\sin\theta}{\sin^3\theta}=\frac{\dfrac{\sin\theta-\sin\theta\cos\theta}{\cos\theta}}{\sin^3\theta}=\frac{\sin\theta(1-\cos\theta)}{\cos\theta}\cdot\frac{1}{\sin^3\theta}=\frac{1-\cos\theta}{\sin^2\theta\cos\theta}=\frac{1-\cos\theta}{(1-\cos^2\theta)\cos\theta}\\ &=\frac{1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)\cos\theta}=\frac{1}{(1+\cos\theta)\cos\theta}=\frac{\sec\theta}{1+\cos\theta}\end{aligned}$$

**13.** $\dfrac{\tan x}{\sec x-1}+\dfrac{\tan x}{\sec x+1}=\dfrac{\tan x(\sec x+1)+\tan x(\sec x-1)}{\sec^2 x-1}=\dfrac{\tan x\sec x+\tan x+\tan x\sec x-\tan x}{\sec^2 x-1}$

$=\dfrac{2\tan x\sec x}{\tan^2 x}=\dfrac{2\sec x}{\tan x}=\dfrac{\frac{2}{\cos x}}{\frac{\sin x}{\cos x}}=\dfrac{2}{\sin x}=2\csc x$

**14.** $\dfrac{1}{\csc x-\cot x}-\dfrac{1}{\cot x+\csc x}=\dfrac{\cot x+\csc x-(\csc x-\cot x)}{\csc^2 x-\cot^2 x}=\dfrac{2\cot x}{1}=2\cot x$

**15.** $\dfrac{1+\sin\theta}{1-\sin\theta}=\dfrac{1+\sin\theta}{1-\sin\theta}\cdot\dfrac{1+\sin\theta}{1+\sin\theta}=\dfrac{(1+\sin\theta)^2}{1-\sin^2\theta}=\dfrac{(1+\sin\theta)^2}{\cos^2\theta}=\left(\dfrac{1+\sin\theta}{\cos\theta}\right)^2=\left(\dfrac{1}{\cos\theta}+\dfrac{\sin\theta}{\cos\theta}\right)^2=(\sec\theta+\tan\theta)^2$

**16.** $\dfrac{1-\cos\theta}{1+\cos\theta}=\dfrac{1-\cos\theta}{1+\cos\theta}\cdot\dfrac{1-\cos\theta}{1-\cos\theta}=\dfrac{(1-\cos\theta)^2}{1-\cos^2\theta}=\dfrac{(1-\cos\theta)^2}{\sin^2\theta}=\left(\dfrac{1-\cos\theta}{\sin\theta}\right)^2=\left(\dfrac{1}{\sin\theta}-\dfrac{\cos\theta}{\sin\theta}\right)^2=(\csc\theta-\cot\theta)^2$

**17.** $\dfrac{\sec x-\tan x}{\sec x+\tan x}=\dfrac{\sec x-\tan x}{\sec x+\tan x}\cdot\dfrac{\sec x-\tan x}{\sec x-\tan x}=\dfrac{(\sec x-\tan x)^2}{\sec^2 x-\tan^2 x}=\dfrac{(\sec x-\tan x)^2}{1}=(\sec x-\tan x)^2$

**18.** $\dfrac{\csc x+\cot x}{\csc x-\cot x}=\dfrac{\csc x+\cot x}{\csc x-\cot x}\cdot\dfrac{\csc x+\cot x}{\csc x+\cot x}=\dfrac{(\csc x+\cot x)^2}{\csc^2 x-\cot^2 x}=\dfrac{(\csc x+\cot x)^2}{1}=(\csc x+\cot x)^2$

**19.** $\dfrac{\sin x-\cos x+1}{\sin x+\cos x-1}=\dfrac{\sin x+(1-\cos x)}{\sin x-(1-\cos x)}=\dfrac{\sin x+(1-\cos x)}{\sin x-(1-\cos x)}\cdot\dfrac{\sin x+(1-\cos x)}{\sin x+(1-\cos x)}$

$=\dfrac{\sin^2 x+2\sin x(1-\cos x)+(1-\cos x)^2}{\sin^2 x-(1-\cos x)^2}=\dfrac{\sin^2 x+2\sin x-2\sin x\cos x+1-2\cos x+\cos^2 x}{\sin^2 x-(1-2\cos x+\cos^2 x)}$

$=\dfrac{\sin^2 x+\cos^2 x+2\sin x-2\sin x\cos x-2\cos x+1}{\sin^2 x-1+2\cos x-\cos^2 x}=\dfrac{2-2\cos x-2\sin x\cos x+2\sin x}{\sin^2 x-1+2\cos x-\cos^2 x}$

$=\dfrac{2(1-\cos x)+2\sin x(1-\cos x)}{1-\cos^2 x-1+2\cos x-\cos^2 x}=\dfrac{(2+2\sin x)(1-\cos x)}{-2\cos^2 x-2\cos x}=\dfrac{2(1+\sin x)(1-\cos x)}{-2\cos x(\cos x-1)}$

$=\dfrac{2(1+\sin x)(1-\cos x)}{2\cos x(1-\cos x)}=\dfrac{1+\sin x}{\cos x}$

**20.** Start with the right side.

$\dfrac{2\cos x}{\sin x+\csc x+\cos^2 x\csc x}=\dfrac{2\cos x}{\sin x+\frac{1}{\sin x}+\frac{\cos^2 x}{\sin x}}=\dfrac{2\cos x}{\frac{\sin^2 x+1+\cos^2 x}{\sin x}}=\dfrac{2\cos x}{\frac{2}{\sin x}}=\cos x\sin x$

$=\cos x\sin x\left(\dfrac{\sin x}{\sin x}\right)=\sin^2 x\left(\dfrac{\cos x}{\sin x}\right)=\sin^2 x\cot x$

**21.** $\cos 15^\circ=\cos\dfrac{30^\circ}{2}=\sqrt{\dfrac{1+\cos 30^\circ}{2}}=\sqrt{\dfrac{1+\sqrt{3}/2}{2}}=\dfrac{\sqrt{2+\sqrt{3}}}{2}$

**22.** $\sin 105^\circ=\sin(60^\circ+45^\circ)=\sin 60^\circ\cos 45^\circ+\cos 60^\circ\sin 45^\circ=\dfrac{\sqrt{3}}{2}\left(\dfrac{\sqrt{2}}{2}\right)+\dfrac{1}{2}\left(\dfrac{\sqrt{2}}{2}\right)=\dfrac{\sqrt{6}+\sqrt{2}}{4}$

**23.** $\csc 75^\circ=\dfrac{1}{\sin(45^\circ+30^\circ)}=\dfrac{1}{\sin 45^\circ\cos 30^\circ+\cos 45^\circ\sin 30^\circ}=\dfrac{1}{\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)+\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)}=\dfrac{1}{\frac{\sqrt{6}+\sqrt{2}}{4}}=\sqrt{6}-\sqrt{2}$

**24.** $\tan 75° = \tan(45° + 30°) = \dfrac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°} = \dfrac{1 + \sqrt{3}/3}{1 - \sqrt{3}/3} = \dfrac{3 + \sqrt{3}}{3 - \sqrt{3}}$

**25.** $\sin 41° \cos 49° + \cos 41° \sin 49° = \sin(41° + 49°) = \sin 90° = 1$

**26.** $\cos 50° \cos 10° - \sin 50° \sin 10° = \cos(50° + 10°) = \cos 60° = \dfrac{1}{2}$

**27.** $\dfrac{\tan 69° + \tan 66°}{1 - \tan 69° \tan 66} = \tan(69° + 66°) = \tan 135° = -1$

**28.**
$$\begin{aligned} 2\cos 75° \cos 15° &= 2\cos(45° + 30°)\cos(45° - 30°) \\ &= 2(\cos 45° \cos 30° - \sin 45° \sin 30°) \cdot (\cos 45° \cos 30° + \sin 45° \sin 30°) \\ &= 2\left(\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)\right) \cdot \left(\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)\right) = \frac{1}{2} \end{aligned}$$

In exercises 29–32, $\sin u = \dfrac{4}{5}$ *and* $0 < u \le \dfrac{\pi}{2} \Rightarrow \cos u = \dfrac{3}{5}, \tan u = \dfrac{4}{3}$ and $\cos v = \dfrac{5}{13}$ and $0 < v \le \dfrac{\pi}{2} \Rightarrow \sin v = \dfrac{12}{13}, \tan v = \dfrac{12}{5}$.

**29.** $\sin(u - v) = \sin u \cos v - \cos u \sin v = \dfrac{4}{5} \cdot \dfrac{5}{13} - \dfrac{3}{5} \cdot \dfrac{12}{13} = -\dfrac{16}{65}$

**30.** $\cos(u + v) = \cos u \cos v - \sin u \sin v = \dfrac{3}{5} \cdot \dfrac{5}{13} - \dfrac{4}{5} \cdot \dfrac{12}{13} = -\dfrac{33}{65}$

**31.** $\cos(u - v) = \cos u \cos v + \sin u \sin v = \dfrac{3}{5} \cdot \dfrac{5}{13} + \dfrac{4}{5} \cdot \dfrac{12}{13} = \dfrac{63}{65}$

**32.** $\tan(u - v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v} = \dfrac{4/3 - 12/5}{1 + (4/3)(12/5)} = -\dfrac{16}{63}$

**33.**
$$\begin{aligned} \sin(x - y)\cos y + \cos(x - y)\sin y &= \cos y(\sin x \cos y - \sin y \cos x) + \sin y(\cos x \cos y + \sin x \sin y) \\ &= \sin x \cos^2 y - \sin y \cos x \cos y + \sin y \cos x \cos y + \sin x \sin^2 y \\ &= \sin x \cos^2 y + \sin x \sin^2 y = \sin x(\cos^2 y + \sin^2 y) = \sin x \end{aligned}$$

**34.**
$$\begin{aligned} \cos(x - y)\cos y - \sin(x - y)\sin y &= \cos y(\cos x \cos y + \sin x \sin y) - \sin y(\sin x \cos y - \sin y \cos x) \\ &= \cos x \cos^2 y + \sin x \sin y \cos y - \sin x \sin y \cos y + \sin^2 y \cos x \\ &= \cos x \cos^2 y + \sin^2 y \cos x = \cos x(\cos^2 y + \sin^2 y) = \cos x \end{aligned}$$

**35.** Start with the right side.

$$\frac{\tan u + \tan v}{\tan u - \tan v} = \frac{\frac{\sin u}{\cos u} + \frac{\sin v}{\cos v}}{\frac{\sin u}{\cos u} - \frac{\sin v}{\cos v}} = \frac{\frac{\sin u \cos v + \sin v \cos u}{\cos u \cos v}}{\frac{\sin u \cos v - \sin v \cos u}{\cos u \cos v}} = \frac{\sin u \cos v + \sin v \cos u}{\sin u \cos v - \sin v \cos u} = \frac{\sin(u + v)}{\sin(u - v)}$$

**36.** $\dfrac{\tan(x + y)}{\cot(x - y)} = \tan(x + y)\tan(x - y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y} \cdot \dfrac{\tan x - \tan y}{1 + \tan x \tan y} = \dfrac{\tan^2 x - \tan^2 y}{1 - \tan^2 x \tan^2 y}$

**37.** $\dfrac{\sin 4x}{\sin 2x} - \dfrac{\cos 4x}{\cos 2x} = \dfrac{2\sin 2x \cos 2x}{\sin 2x} - \dfrac{2\cos^2 2x - 1}{\cos 2x} = 2\cos 2x - 2\cos 2x + \dfrac{1}{\cos 2x} = \sec 2x$

**38.** See section 6.3 example 3 and practice problem 3

$$\frac{\sin 3x}{\sin x}-\frac{\cos 3x}{\cos x}=\frac{3\sin x-4\sin^3 x}{\sin x}-\frac{4\cos^3 x-3\cos x}{\cos x}$$
$$=3-4\sin^2 x-4\cos^2 x+3=6-4(\sin^2 x+\cos^2 x)=6-4=2$$

Alternatively,

$$\frac{\sin 3x}{\sin x}-\frac{\cos 3x}{\cos x}=\frac{\sin 3x\cos x-\cos 3x\sin x}{\sin x\cos x}=\frac{\frac{1}{2}\left[\sin(3x+x)+\sin(3x-x)\right]-\frac{1}{2}\left[\sin(3x+x)-\sin(3x-x)\right]}{\sin x\cos x}$$
$$=\frac{\sin 4x+\sin 2x-\sin 4x+\sin 2x}{2\sin x\cos x}=\frac{2\sin 2x}{\sin 2x}=2$$

**39.** $$\sin\left(x-\frac{\pi}{6}\right)+\cos\left(x+\frac{\pi}{3}\right)=\sin x\cos\frac{\pi}{6}-\sin\frac{\pi}{6}\cos x+\cos x\cos\frac{\pi}{3}-\sin x\sin\frac{\pi}{3}$$
$$=\frac{\sqrt{3}}{2}\sin x-\frac{1}{2}\cos x+\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x=0$$

**40.** Start with the right side.

$$\cos^4 x-\sin^4 x=(\cos^2 x+\sin^2 x)(\cos^2 x-\sin^2 x)=\cos 2x$$

**41.** $$1+\tan\theta\tan 2\theta=1+\frac{\sin\theta}{\cos\theta}\cdot\frac{\sin 2\theta}{\cos 2\theta}=1+\frac{2\sin^2\theta\cos\theta}{\cos\theta(\cos^2\theta-\sin^2\theta)}=1+\frac{2\sin^2\theta}{\cos^2\theta-\sin^2\theta}$$
$$=\frac{\cos^2\theta-\sin^2\theta+2\sin^2\theta}{\cos^2\theta-\sin^2\theta}=\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta-\sin^2\theta}=\frac{1}{\cos 2\theta}=\sec 2\theta$$

**42.** $$\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}=\frac{\sin\theta+2\sin\theta\cos\theta}{1+\cos\theta+2\cos^2\theta-1}=\frac{\sin\theta(1+2\cos\theta)}{\cos\theta(1+2\cos\theta)}=\tan\theta$$

**43.** $$\frac{\sin\theta+\sin 2\theta}{\cos\theta+\cos 2\theta}=\frac{2\sin\frac{3\theta}{2}\cos\left(-\frac{\theta}{2}\right)}{2\cos\frac{3\theta}{2}\cos\left(-\frac{\theta}{2}\right)}=\tan\frac{3\theta}{2}$$

**44.** $$\frac{\sin\theta-\sin 2\theta}{\cos\theta-\cos 2\theta}=\frac{2\sin\left(-\frac{\theta}{2}\right)\cos\frac{3\theta}{2}}{-2\sin\frac{3\theta}{2}\sin\left(-\frac{\theta}{2}\right)}=-\cot\frac{3\theta}{2}$$

**45.** $$\frac{\sin 5x-\sin 3x}{\sin 5x+\sin 3x}=\frac{2\sin x\cos 4x}{2\sin 4x\cos x}=\tan x\cot 4x=\frac{\tan x}{\tan 4x}$$

**46.** $$\frac{\cos 5x-\cos 3x}{\cos 5x+\cos 3x}=\frac{-2\sin x\sin 4x}{2\cos 4x\cos x}=-\tan x\tan 4x$$

**47.** $$\frac{\tan 3x+\tan x}{\tan 3x-\tan x}=\frac{\frac{\sin 3x}{\cos 3x}+\frac{\sin x}{\cos x}}{\frac{\sin 3x}{\cos 3x}-\frac{\sin x}{\cos x}}=\frac{\frac{\sin 3x\cos x+\sin x\cos 3x}{\cos 3x\cos x}}{\frac{\sin 3x\cos x-\sin x\cos 3x}{\cos 3x\cos x}}=\frac{\sin 3x\cos x+\sin x\cos 3x}{\sin 3x\cos x-\sin x\cos 3x}$$
$$=\frac{\sin 4x}{\sin 2x}=\frac{2\sin 2x\cos 2x}{\sin 2x}=2\cos 2x$$

**48.** $\dfrac{\sin 4x}{2(1+\cos 4x)} = \dfrac{2\sin 2x\cos 2x}{2(1+2\cos^2 2x-1)} = \dfrac{2\sin 2x\cos 2x}{4\cos^2 2x} = \dfrac{\sin 2x}{2\cos 2x} = \dfrac{1}{2}\tan 2x = \dfrac{1}{2}\left(\dfrac{2\tan x}{1-\tan^2 x}\right) = \dfrac{\tan x}{1-\tan^2 x}$

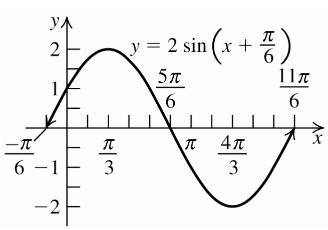
**49.** $\dfrac{\sin 3x+\sin 5x+\sin 7x+\sin 9x}{\cos 3x+\cos 5x+\cos 7x+\cos 9x} = \dfrac{(\sin 3x+\sin 5x)+(\sin 7x+\sin 9x)}{(\cos 3x+\cos 5x)+(\cos 7x+\cos 9x)} = \dfrac{2\sin 4x\cos x+2\sin 8x\cos x}{2\cos 4x\cos x+2\cos 8x\cos x}$

$= \dfrac{2\cos x(\sin 4x+\sin 8x)}{2\cos x(\cos 4x+\cos 8x)} = \dfrac{2\sin 6x\cos(-2x)}{2\cos 6x\cos(-2x)} = \dfrac{\sin 6x}{\cos 6x} = \tan 6x$

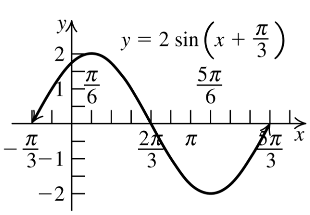
**50.** $\dfrac{\cos x+\cos 3x+\cos 5x+\cos 7x}{\sin x+\sin 3x+\sin 5x+\sin 7x} = \dfrac{(\cos x+\cos 3x)+(\cos 5x+\cos 7x)}{(\sin x+\sin 3x)+(\sin 5x+\sin 7x)} = \dfrac{2\cos 2x\cos(-x)+2\cos 6x\cos(-x)}{2\sin 2x\cos(-x)+2\sin 6x\cos(-x)}$

$= \dfrac{2\cos x(\cos 2x+\cos 6x)}{2\cos x(\sin 2x+\sin 6x)} = \dfrac{2\cos 4x\cos(-2x)}{2\sin 4x\cos(-2x)} = \dfrac{\cos 4x}{\sin 4x} = \cot 4x$

**51.** Using the reduction formula, we have $y=\sqrt{3}\sin x+\cos x = a\sin x+b\cos x \Rightarrow$
$a=\sqrt{3}, b=1 \Rightarrow \sqrt{a^2+b^2}=2=A.$ So, $\theta$ is any angle in standard position that has $\left(\sqrt{3},1\right)$ on its terminal side $\Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = \dfrac{\pi}{6}$. So $y=\sqrt{3}\sin x+\cos x = 2\sin\left(x+\dfrac{\pi}{6}\right)$.




**52.** Using the reduction formula, we have $y=\sin x+\sqrt{3}\cos x = a\sin x+b\cos x \Rightarrow$
$a=1, b=\sqrt{3} \Rightarrow \sqrt{a^2+b^2}=2=A.$ So, $\theta$ is any angle in standard position that has $\left(1,\sqrt{3}\right)$ on its terminal side $\Rightarrow \tan\theta=\sqrt{3} \Rightarrow \theta=\dfrac{\pi}{3}$. So $y=\sin x+\sqrt{3}\cos x = 2\sin\left(x+\dfrac{\pi}{3}\right)$.




**53.** $\sin 2A+\sin 2B-\sin 2C = (\sin 2A+\sin 2B)-\sin 2C = 2\sin(A+B)\cos(A-B)-2\sin C\cos C$
$A+B+C=180° \Rightarrow C=180°-(A+B)$ or $A+B=180°-C$, so
$2\sin(A+B)\cos(A-B)-2\sin C\cos C = 2\sin(180°-C)\cos(A-B)-2\sin C\cos C$
$= 2\sin C\cos(A-B)-2\sin C\cos C = 2\sin C[\cos(A-B)-\cos C]$
$= 2\sin C[\cos(A-B)-\cos(180°-(A+B))$
$= 2\sin C[\cos(A-B)+\cos(A+B)]$
$= 2\sin C[\cos A\cos B+\sin A\sin B+\cos A\cos B-\sin A\sin B]$
$= 2\sin C[2\cos A\cos B] = 4\cos A\cos B\sin C$

**54.** $A+B+C=180° \Rightarrow A+B=180°-C \Rightarrow \tan(A+B)=\tan(180°-C) \Rightarrow$
$\dfrac{\tan A+\tan B}{1-\tan A\tan B}=-\tan C \Rightarrow \tan A+\tan B=-\tan C(1-\tan A\tan B) \Rightarrow$
$\tan A+\tan B=-\tan C+\tan A\tan B\tan C \Rightarrow \tan A+\tan B+\tan C=\tan A\tan B\tan C$

**55.** $2\cos^2 x-1=0 \Rightarrow \cos x=\pm\dfrac{\sqrt{2}}{2} \Rightarrow x\in\left\{\dfrac{\pi}{4},\dfrac{3\pi}{4},\dfrac{5\pi}{4},\dfrac{7\pi}{4}\right\}$

**56.** $3\tan^2 x-1=0 \Rightarrow \tan x=\pm\dfrac{\sqrt{3}}{3} \Rightarrow x\in\left\{\dfrac{\pi}{6},\dfrac{5\pi}{6},\dfrac{7\pi}{6},\dfrac{11\pi}{6}\right\}$

**57.** $2\cos^2 x-\cos x-1=0 \Rightarrow (2\cos x+1)(\cos x-1)=0 \Rightarrow \cos x=-\dfrac{1}{2}$ or $\cos x=1.$
If $\cos x=-\dfrac{1}{2} \Rightarrow x=\dfrac{2\pi}{3}$ or $x=\dfrac{4\pi}{3}.$ If $\cos x=1 \Rightarrow x=0.$ The solution is $\left\{0,\dfrac{2\pi}{3},\dfrac{4\pi}{3}\right\}.$

**58.** $2\sin^2 x-5\sin x-3=0 \Rightarrow (2\sin x+1)(\sin x-3)=0 \Rightarrow \sin x=-\dfrac{1}{2}$ or $\sin x=3$ (reject this) $\Rightarrow x\in\left\{\dfrac{7\pi}{6},\dfrac{11\pi}{6}\right\}$

**59.** $2\sin 3x-1=0 \Rightarrow \sin 3x=\dfrac{1}{2} \Rightarrow 3x\in\left\{\dfrac{\pi}{6},\dfrac{5\pi}{6},\dfrac{13\pi}{6},\dfrac{17\pi}{6},\dfrac{25\pi}{6},\dfrac{29\pi}{6}\right\} \Rightarrow x\in\left\{\dfrac{\pi}{18},\dfrac{5\pi}{18},\dfrac{13\pi}{18},\dfrac{17\pi}{18},\dfrac{25\pi}{18},\dfrac{29\pi}{18}\right\}$

**60.** $2\cos(2x-1)-1=0 \Rightarrow \cos(2x-1)=1/2 \Rightarrow x\in\left\{\dfrac{\pi+3}{6},\dfrac{5\pi+3}{6},\dfrac{7\pi+3}{6},\dfrac{11\pi+3}{6}\right\}$

**61.** $3\sin\theta-1=0 \Rightarrow \sin\theta=\dfrac{1}{3} \Rightarrow \theta\in\{19.5°,160.5°\}$

**62.** $4\cos\theta+1=0 \Rightarrow \cos\theta=-\dfrac{1}{4} \Rightarrow \theta\in\{104.5°,255.5°\}$

**63.** $2\sqrt{3}\cos 2\theta-3=0 \Rightarrow \cos 2\theta=\dfrac{\sqrt{3}}{2} \Rightarrow 2\theta\in\{30°,330°,390°,690°\} \Rightarrow \theta\in\{15°,165°,195°,345°\}$

**64.** $(1-2\sin 2\theta)\left(\sqrt{3}+2\cos 2\theta\right)=0 \Rightarrow \sin 2\theta=\dfrac{1}{2}$ or $\cos 2\theta=-\dfrac{\sqrt{3}}{2}.$
If $\sin 2\theta=\dfrac{1}{2} \Rightarrow 2\theta\in\{30°,150°,390°,510°\} \Rightarrow \theta\in\{15°,75°,195°,255°\}.$
If $\cos 2\theta=-\dfrac{\sqrt{3}}{2} \Rightarrow \theta\in\{150°,210°,510°,570°\} \Rightarrow \theta\in\{75°,105°,255°,285°\}.$
The solution is $\{15°,75°,105°,195°,255°,285°\}.$

**65.** $\left(\sqrt{3}\tan\theta+1\right)(2\cos\theta+1)=0 \Rightarrow \tan\theta=-\dfrac{\sqrt{3}}{3}$ or $\cos\theta=-\dfrac{1}{2}.$ $\tan\theta=-\dfrac{\sqrt{3}}{3} \Rightarrow \theta\in\{150°,330°\}.$
$\cos\theta=-\dfrac{1}{2} \Rightarrow \theta\in\{120°,240°\}.$ The solution is $\{120°,150°,240°,330°\}.$

**66.** $\sqrt{3}\cos\theta=\sin\theta+1 \Rightarrow 3\cos^2\theta=\sin^2\theta+2\sin\theta+1 \Rightarrow 3(1-\sin^2\theta)=\sin^2\theta+2\sin\theta+1 \Rightarrow$
$2(2\sin\theta-1)(\sin\theta+1)=0 \Rightarrow \sin\theta=\dfrac{1}{2} \Rightarrow \theta\in\{30°,150°\}$ or $\sin\theta=-1 \Rightarrow \theta=270°.$
Checking each solution in the original equation, we find that $\theta=150°$ is extraneous.
The solution is $\{30°,270°\}$.

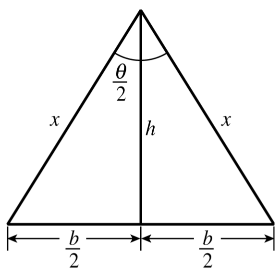
### Applying the Concepts

**67.** $\frac{2}{3} = \frac{4}{3}\cos\alpha \Rightarrow \cos\alpha = \frac{1}{2} \Rightarrow \alpha = 60°$

**68.** $1900 = 2400 + 500\sin\pi t \Rightarrow \sin\pi t = -1 \Rightarrow \pi t = \sin^{-1}(-1) = \frac{3\pi}{2} \Rightarrow t = 1.5$ years

**69.** $80 = 160\cos 120\pi t \Rightarrow \cos 120\pi t = \frac{1}{2} \Rightarrow 120\pi t = \cos^{-1}\frac{1}{2} = \frac{\pi}{3} \Rightarrow t = \frac{1}{360}$ sec

**70.**




$h = x\cos\frac{\theta}{2}$ and $\frac{b}{2} = x\sin\frac{\theta}{2} \Rightarrow b = 2x\sin\frac{\theta}{2}$

$A = \frac{1}{2}bh = \frac{1}{2}\left(2x\sin\frac{\theta}{2}\right)\left(x\cos\frac{\theta}{2}\right) = x^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

**71.** $A = 1$ and $x = 2 \Rightarrow 4\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 1 \Rightarrow 2\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = 2\sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{2}\right) = 30°$

## Chapter 6 Practice Test A

**1.** $\sin\theta = \frac{3}{5} = \frac{y}{r}$ and $\cos\theta < 0 \Rightarrow \cos\theta = \frac{x}{r} = -\frac{4}{5} \Rightarrow \tan\theta = -\frac{3}{4}$

**2.** $\tan x = \frac{2}{3} = \frac{y}{x}$ and $\csc x < 0 \Rightarrow$

$\sin x < 0$ and $\cos x < 0 \Rightarrow \cos x = \frac{x}{r} = -\frac{3\sqrt{13}}{13}$

**3.** $\frac{1 - \sin^2 x}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x} = \cot^2 x$

**4.** Let $u = \sin x + \cos x$. Starting with the right side, we have

$$(\sin x + \cos x + 1)(\sin x + \cos x - 1) = (u+1)(u-1) = u^2 - 1 = (\sin x + \cos x)^2 - 1$$
$$= \sin^2 x + 2\sin x\cos x + \cos^2 x - 1$$
$$= (\sin^2 x + \cos^2 x) - 1 + 2\sin x\cos x = 2\sin x\cos x$$

**5.** $\sin x\sin\left(\frac{\pi}{2} - x\right) = \sin x\left(\sin\frac{\pi}{2}\cos x - \sin x\cos\frac{\pi}{2}\right) = \sin x\cos x = \frac{2\sin x\cos x}{2} = \frac{\sin 2x}{2}$

**6.** Start with the right side.

$$\frac{\sin x}{1+\tan x}=\frac{\sin x}{1+\frac{\sin x}{\cos x}}=\frac{\sin x}{\frac{\cos x+\sin x}{\cos x}}=\frac{\sin x\cos x}{\cos x+\sin x}=\frac{2\sin x\cos x}{2(\cos x+\sin x)}=\frac{\sin 2x}{2(\cos x+\sin x)}$$

**7.** $\dfrac{\sin 2x+\sin 4x}{\cos 2x+\cos 4x}=\dfrac{2\sin\left(\frac{2x+4x}{2}\right)\cos\left(\frac{2x-4x}{2}\right)}{2\cos\left(\frac{2x+4x}{2}\right)\cos\left(\frac{2x-4x}{2}\right)}=\dfrac{2\sin 3x\cos(-x)}{2\cos 3x\cos(-x)}=\dfrac{\sin 3x}{\cos 3x}=\tan 3x$

**8.** $\cos(x+y)\cos(x-y)=\frac{1}{2}\left(\cos\left[(x+y)-(x-y)\right]+\cos\left[(x+y)+(x-y)\right]\right)=\frac{1}{2}\left(\cos 2y+\cos 2x\right)$

$$=\frac{1}{2}\left(2\cos^2 y-1+2\cos^2 x-1\right)=\cos^2 x+\cos^2 y-1$$

**9.** $\tan(-x)=1\Rightarrow -x=\tan^{-1}1\Rightarrow -x=\frac{\pi}{4}\Rightarrow x=-\frac{\pi}{4}+n\pi,$ where $n$ is any integer.

**10.** $\sin 4x=\frac{1}{2}\Rightarrow 4x=\sin^{-1}\left(\frac{1}{2}\right)\Rightarrow 4x=\frac{\pi}{6}+2\pi$ or $4x=\frac{5\pi}{6}+2\pi\Rightarrow x=\frac{\pi}{24}+\frac{\pi}{2}n$ or $x=\frac{5\pi}{24}+\frac{\pi}{2}n,$ where $n$ is any integer.

**11.** $\cos\frac{x}{3}=\frac{\sqrt{2}}{2}\Rightarrow\frac{x}{3}=\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\Rightarrow\frac{x}{3}\in\left\{\frac{\pi}{4},\frac{7\pi}{4}\right\}\Rightarrow x=\frac{3\pi}{4}$. (The other solution is not in the domain.)

**12.** $\sin 2x+\cos x=0\Rightarrow 2\sin x\cos x+\cos x=0\Rightarrow\cos x(2\sin x+1)=0\Rightarrow\cos x=0$ or $\sin x=-\frac{1}{2}$

If $\cos x=0\Rightarrow x=\frac{\pi}{2}$ or $x=\frac{3\pi}{2}$. If $\sin x=-\frac{1}{2}\Rightarrow x=\frac{7\pi}{6}$ or $x=\frac{11\pi}{6}$.

The solution is $\left\{\frac{\pi}{2},\frac{7\pi}{6},\frac{3\pi}{2},\frac{11\pi}{6}\right\}$.

**13.** $\sin 56°+\cos 146°=\sin(90°-34°)+\cos(180°-34°)$

$$=\sin 90°\cos 34°-\sin 34°\cos 90°+\cos 180°\cos 34°+\sin 180°\sin 34°$$
$$=\cos 34°-0-\cos 34°+0=0$$

**14.** $\cos 48°\cos 12°-\sin 48°\sin 12°=\cos(48°+12°)=\cos 60°=\frac{1}{2}$

**15.** $\sin\frac{5\pi}{12}=\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)=\sin\frac{\pi}{6}\cos\frac{\pi}{4}+\cos\frac{\pi}{6}\sin\frac{\pi}{4}=\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)+\frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{6}+\sqrt{2}}{4}$

**16.** $\cos 2\theta=1-2\sin^2\theta=1-2\left(\frac{4}{5}\right)^2=-\frac{7}{25}$

**17.** $\tan\theta=\frac{3}{4}\Rightarrow\sin\theta=\frac{3}{5},\cos\theta=\frac{4}{5}$

$\sin 2\theta=2\sin\theta\cos\theta=2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)=\frac{24}{25}$

**18.** $y=\cos\left(\frac{\pi}{2}-x\right)+\tan x=\sin x+\tan x$

$f(-x)=\sin(-x)+\tan(-x)=-\sin x-\tan x=-(\sin x+\tan x)\Rightarrow f(x)$ is odd.

**19.** If $x = y = \frac{\pi}{2}$, $\sin x + \sin y = 1 + 1 = 2 \neq \sin \pi$, so $\sin x + \sin y = \sin(x + y)$ is not an identity.

**20.** $\frac{128^2 \sin 2\theta}{32} = 512 \Rightarrow \sin 2\theta = \frac{512 \cdot 32}{128^2} = 1 \Rightarrow 2\theta = 90° \Rightarrow \theta = 45°$

## Chapter 6 Practice Test B

**1.** $\sin\theta = -\frac{12}{13}$ and $\pi < \theta < \frac{3\pi}{2} \Rightarrow \cos\theta = -\frac{5}{13} \Rightarrow \sec\theta = -\frac{13}{5}$
The answer is A.

**2.** $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = \sin^2 x + 2\sin x\cos x + \cos^2 x + \sin^2 x - 2\sin x\cos x + \cos^2 x$
$= 2(\sin^2 x + \cos^2 x) = 2$. The answer is B.

**3.** $\frac{\sin x}{1+\cos x} + \frac{\sin x}{1-\cos x} = \frac{\sin x(1-\cos x) + \sin x(1+\cos x)}{1-\cos^2 x} = \frac{2\sin x}{\sin^2 x} = \frac{2}{\sin x} = 2\csc x$. The answer is D.

**4.** $\frac{\tan x + \tan y}{\cot x + \cot y} = \frac{\tan x + \tan y}{\frac{1}{\tan x} + \frac{1}{\tan y}} = \frac{\tan x + \tan y}{\frac{\tan y + \tan x}{\tan x \tan y}} = \tan x \tan y$. The answer is B.

**5.** $\frac{\sin x + \sin y}{\cos x + \cos y} + \frac{\cos x - \cos y}{\sin x - \sin y} = \frac{(\sin x + \sin y)(\sin x - \sin y) + (\cos x - \cos y)(\cos x + \cos y)}{(\cos x + \cos y)(\sin x - \sin y)}$
$= \frac{\sin^2 x - \sin^2 y + \cos^2 x - \cos^2 y}{(\cos x + \cos y)(\sin x - \sin y)} = \frac{(\sin^2 x + \cos^2 x) - (\sin^2 y + \cos^2 y)}{(\cos x + \cos y)(\sin x - \sin y)} = 0$
The answer is A.

**6.** $\frac{\sin 2x}{1+\cos 2x} = \frac{2\sin x\cos x}{1+2\cos^2 x - 1} = \frac{2\sin x\cos x}{2\cos^2 x}$
$= \frac{\sin x}{\cos x} = \tan x$. The answer is B.

**7.** $\frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x$. The answer is C.

**8.** $\frac{\sin 3x - \sin x}{\cos x - \cos 3x} = \frac{3\sin x - 4\sin^3 x - \sin x}{\cos x - (4\cos^3 x - 3\cos x)}$
$= \frac{2\sin x(1-2\sin^2 x)}{4\cos x(1-\cos^2 x)}$
$= \frac{2\sin x\cos 2x}{4\cos x\sin^2 x} = \frac{\cos 2x}{2\sin x\cos x}$
$= \frac{\cos 2x}{\sin 2x} = \cot 2x$. The answer is D.

**9.** $\cot(-x) = -1 \Rightarrow -\cot x = -1 \Rightarrow \cot x = 1 \Rightarrow$
$x = \pi/4 + \pi n$. The answer is C.

**10.** $2\cos 4x = -1 \Rightarrow \cos 4x = -\frac{1}{2} \Rightarrow$
$4x = \frac{2\pi}{3} + 2n\pi \Rightarrow x = \frac{\pi}{6} + \frac{n\pi}{2}$
or $4x = \frac{4\pi}{3} + 2n\pi \Rightarrow x = \frac{\pi}{3} + \frac{n\pi}{2}$, where $n$ is any integer. The answer is C.

**11.** $\sin\frac{x}{3} = \frac{\sqrt{2}}{2} \Rightarrow \frac{x}{3} = \frac{\pi}{4} + 2n\pi \Rightarrow x = \frac{3\pi}{4} + 6n\pi$
or $\frac{x}{3} = \frac{3\pi}{4} + 2n\pi \Rightarrow x = \frac{9\pi}{4} + 6n\pi$, where $n$ is any integer. The answer is C.

**12.** $\cos x - \sin 2x = 0 \Rightarrow \cos x - 2\sin x\cos x = 0 \Rightarrow$
$\cos x(1-2\sin x) = 0 \Rightarrow \cos x = 0$ or $\sin x = \frac{1}{2}$.
If $\cos x = 0 \Rightarrow x = \frac{\pi}{2}$ or $x = \frac{3\pi}{2}$.
If $\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$.
The answer is A.

**13.** $\sin 71° + \cos 161°$
$= \sin(90° - 19°) + \cos(180° - 19°)$
$= \sin 90° \cos 19° - \cos 90° \sin 19° + \cos 180° \cos 19° + \sin 180° \sin 19°$
$= \cos 19° - 0 - \cos 19° + 0 = 0$
The answer is D.

**14.** $\sin 53° \cos 37° + \cos 53° \sin 37° = \sin(53° + 37°) = \sin 90° = 1$
The answer is D.

**15.** $\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2(2/3)^2 = 1/9$
The answer is B.

**16.** $\tan\theta = \frac{4}{3} = \frac{y}{x} \Rightarrow \sin\theta = \frac{4}{5}, \cos\theta = \frac{3}{5}.$
$\sin 2\theta = 2\sin\theta\cos\theta = 2(4/5)(3/5) = 24/25$
The answer is A.

**17.** $f(-x) = \sin\left(\frac{\pi}{2} - (-x)\right) + \cot(-x)$
$= \cos(-x) + \cot(-x) = \cos x - \cot x$
$\neq f(x) \Rightarrow f(x)$ is not even (not symmetric with respect to the $y$-axis)
$\neq -f(x) \Rightarrow f(x)$ is not odd (not symmetric with respect to the origin)
$-y = -\left(\sin\left(\frac{\pi}{2} - (-x)\right) + \cot(-x)\right)$
$= -(\cos(-x) + \cot(-x))$
$= -(\cos x - \cot x) = -\cos x + \cot x \Rightarrow$
$y = \cos x - \cot x = \sin\left(\frac{\pi}{2} - x\right) - \cot x \Rightarrow$
the equation is not symmetric with respect to the origin. The answer is D.

**18.** $\tan\left(\frac{\pi}{4} + \frac{5\pi}{4}\right) = \tan\frac{3\pi}{2}$ (undefined)
$\tan\left(\frac{\pi}{4}\right) + \tan\left(\frac{5\pi}{4}\right) = 1 + 1 = 2.$ The answer is C.

**19.** $2\cos^2 x = \frac{3}{2} \Rightarrow \cos^2 x = \frac{3}{4} \Rightarrow \cos x = \pm\frac{\sqrt{3}}{2} \Rightarrow$
$x \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}.$ The answer is D.

**20.** $\frac{128^2 \sin 2\theta}{32} = 256\sqrt{3} \Rightarrow$
$\sin 2\theta = \frac{256\sqrt{3} \cdot 32}{128^2} = \frac{\sqrt{3}}{2} \Rightarrow 2\theta = 60°, 120° \Rightarrow$
$\theta = 30°, 60°$
The answer is B.

## Cumulative Review Exercises Chapters P–6

**1.** $x^2 + x - 1 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} \Rightarrow$
$x = \frac{-1 \pm \sqrt{5}}{2}$

**2.** $\log_2(x+1) + \log_2(x-1) = 1 \Rightarrow$
$\log_2\left((x+1)(x-1)\right) = 1 \Rightarrow$
$x^2 - 1 = 2^1 \Rightarrow x^2 = 3 \Rightarrow x = \sqrt{3}$ (Reject the negative solution; logarithms are not defined for negative numbers.)

**3.** $5^{-x} = 9 \Rightarrow -x\log 5 = \log 9 \Rightarrow x = -\frac{\log 9}{\log 5}$

**4.** $\sin 2x - \cos x = 0 \Rightarrow 2\sin x\cos x - \cos x = 0 \Rightarrow$
$\cos x(2\sin x - 1) = 0 \Rightarrow \cos x = 0$ or $\sin x = \frac{1}{2}.$
If $\cos x = 0 \Rightarrow x = \frac{\pi}{2}$ or $x = \frac{3\pi}{2}.$
If $\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}.$
The solution is $\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}.$

**5.** Solve the associated equation:
$x^3 - 4x = 0 \Rightarrow x(x-2)(x+2) = 0 \Rightarrow x = 0$ or $x = 2$ or $x = -2.$ The intervals to be tested are $(-\infty, -2)$, $(-2, 0)$, $(0, 2)$, and $(2, \infty)$.

| Interval | Test point | Value of $x^3 - 4x$ | Result |
|---|---|---|---|
| $(-\infty, -2)$ | −3 | −15 | − |
| $(-2, 0)$ | −1 | 3 | + |
| $(0, 2)$ | 1 | −3 | − |
| $(2, \infty)$ | 3 | 15 | + |

The solution set is $(-2, 0) \cup (2, \infty)$.

**6.** $\frac{2x-3}{x+2} - 1 < 0 \Rightarrow \frac{2x - 3 - (x+2)}{x+2} < 0 \Rightarrow$
$\frac{x-5}{x+2} < 0$. Now solve $x - 5 = 0 \Rightarrow x = 5$ and $x + 2 = 0 \Rightarrow x = -2$. The intervals to be tested are $(-\infty, -2), (-2, 5)$, and $(5, \infty)$.

(*continued on next page*)

(*continued*)

| Interval | Test point | Value of $\frac{2x-3}{x+2}-1$ | Result |
|---|---|---|---|
| $(-\infty,-2)$ | –3 | 8 | + |
| $(-2,5)$ | 0 | –5/2 | – |
| $(5,\infty)$ | 6 | 1/8 | + |

The solution set is $(-2,5)$.

**7.** $\frac{f(x+h)-f(x)}{h}$

$$= \frac{\left(2(x+h)^2-(x+h)+3\right)-\left(2x^2-x+3\right)}{h}$$
$$= \frac{2x^2+4xh+2h^2-x-h+3-2x^2+x-3}{h}$$
$$= \frac{2h^2+4xh-h}{h} = \frac{h(4x+2h-1)}{h} = 4x-1+2h$$

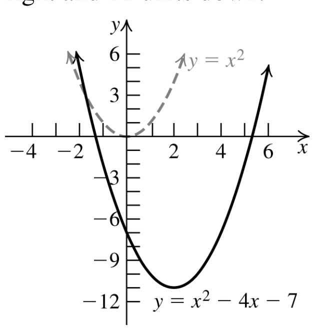
**8.** $f(x) = y = \frac{x+2}{2x-1}$. Switch the variables, and then solve for $y$ to find $f^{-1}(x)$:

$$x = \frac{y+2}{2y-1} \Rightarrow 2xy - x = y+2 \Rightarrow$$
$$2xy - y = x+2 \Rightarrow y(2x-1) = x+2 \Rightarrow$$
$$y = f^{-1}(x) = \frac{x+2}{2x-1}$$

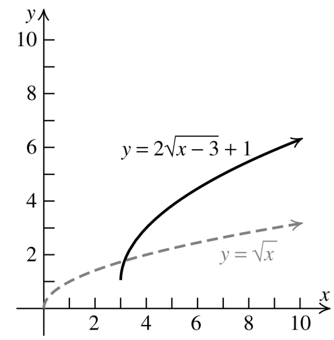
**9.** First, complete the square to find the transformations needed:

$$y = x^2-4x-7 \Rightarrow y+7+4 = x^2-4x+4 \Rightarrow$$
$$y+11 = (x-2)^2 \Rightarrow y = (x-2)^2-11$$

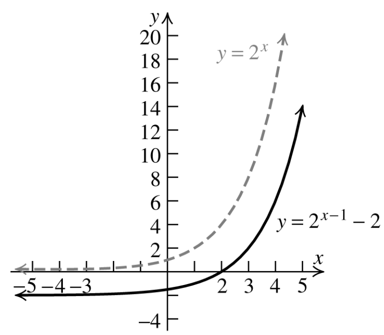
Shift the graph of $y = x^2$ two units to the right and 11 units down.



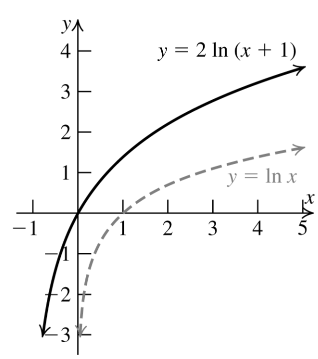

**10.** Shift the graph of $y = \sqrt{x}$ three units to the right, then stretch the graph vertically by a factor of 2, then shift the resulting graph one unit up.



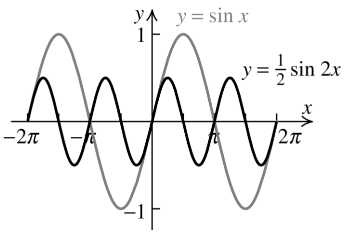

**11.** Shift the graph of $y = 2^x$ one unit to the right and two units down.




**12.** Shift the graph of $y = \ln x$ one unit to the left and then stretch the graph vertically by a factor of 2.

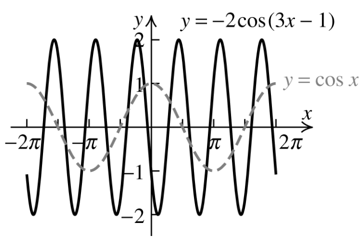


**13.** The period of $y = \frac{1}{2}\sin 2x$ is $\frac{2\pi}{2} = \pi$ and its amplitude is $1/2$. Compress the graph of $y = \sin x$ horizontally by a factor of 2 and vertically by a factor of 2.

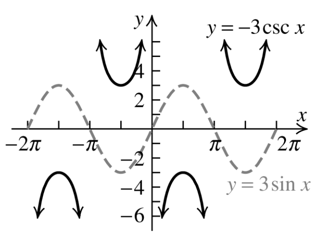



**14.** The amplitude of

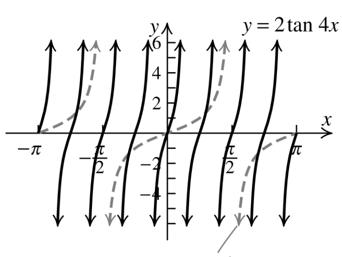
$y = -2\cos(3x - 1) = -2\cos 3\left(x - \frac{1}{3}\right)$ is 2, the period is $2\pi/3$, and the phase shift is $1/3$ to the right.




**15.**




**16.**

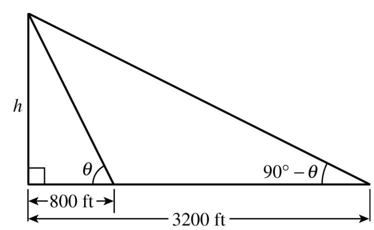



**17.**
$$\frac{2\csc^2 x - 5\cot x - 5}{2\cot x + 1} = \frac{2(1+\cot^2 x) - 5\cot x - 5}{2\cot x + 1} = \frac{2\cot^2 x - 5\cot x - 3}{2\cot x + 1} = \frac{(\cot x - 3)(2\cot x + 1)}{2\cot x + 1} = \cot x - 3$$

**18.**
$$\sec 15° = \sec(45° - 30°) = \frac{1}{\cos(45° - 30°)} = \frac{1}{\cos 45° \cos 30° + \sin 45° \sin 30°} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)} = \frac{4}{\sqrt{6}+\sqrt{2}}$$
$$= \frac{4}{\sqrt{6}+\sqrt{2}} \cdot \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \sqrt{6} - \sqrt{2}$$

**19.** $\sin\left(x - \frac{3\pi}{2}\right)$
$$= \sin x \cos\frac{3\pi}{2} - \sin\frac{3\pi}{2}\cos x = 0 - (-1)\cos x = \cos x$$

**20.**




$\tan\theta = \frac{h}{800} \Rightarrow h = 800\tan\theta$

$\tan(90° - \theta) = \cot\theta = \frac{h}{3200} \Rightarrow h = 3200\cot\theta$

$800\tan\theta = 3200\cot\theta \Rightarrow \frac{\tan\theta}{\cot\theta} = 4 \Rightarrow$

$\tan^2\theta = 4 \Rightarrow \tan\theta = \pm 2$

(Reject the negative solution.)

$2 = \frac{h}{800} \Rightarrow h = 1600.$

The tower is 1600 feet tall.