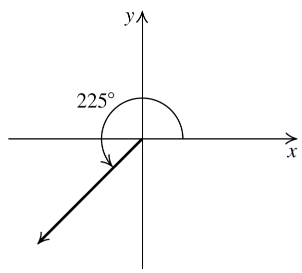
Chapter 5 Trigonometric Functions

5.1 Angles and Their Measure

5.1 Practice Problems

1.



2.a. $13°9'22'' = \left(13 + \frac{9}{60} + \frac{22}{3600}\right)° \approx 13.16°$

b. $41.275° = 41° + 0.275(60') = 41° + 16.5'$
$= 41° + 16' + 0.5(60'') = 41°16'30''$

3. $-45° = -45 \cdot \frac{\pi}{180} = -\frac{\pi}{4}$ radians

4. $\frac{3\pi}{2} = \frac{3\pi}{2} \cdot \frac{180}{\pi} = 270°$

5. complement: 90º – 67° = 23º
supplement: 180º – 67° = 113º

6. First convert the angle measurement from degrees to radians:
$225° = 225°\left(\frac{\pi}{180°}\right) = \frac{5\pi}{4}$ radians .
Then, $s = r\theta = 2\left(\frac{5\pi}{4}\right) = \frac{5\pi}{2} \approx 7.85$ m

7. The difference in the latitudes is
$41°51' - 30°25' = 11°26'$
$= 11° + \left(\frac{26}{60}\right)° \approx 11.43°$
$= 11.43°\left(\frac{\pi}{180°}\right)$
≈ 0.1995 radians
$s = 0.1995 \cdot 3960 \approx 790$ miles.

8. First convert the angle measurement from degrees to radians: $\theta = 60° \cdot \frac{\pi}{180°} = \frac{\pi}{3}$ radians .
$A = \frac{1}{2}r^2\theta = \frac{1}{2} \cdot 10^2 \cdot \frac{\pi}{3} = \frac{50\pi}{3} \approx 52.36 \text{ in.}^2$

9. First convert revolutions per minute into radians per minute:
18 revolutions per minute $= 18 \cdot 2\pi = 36\pi$ radians per minute. Thus, the angular speed $\omega = 36\pi$ radians per minute.
To find the linear speed, use the formula $v = r\omega$: $v = 10 \cdot 36\pi = 360\pi$ radians per minute or about 1131 feet per minute.

5.1 Basic Concepts and Skills

1. A negative angle is formed by rotating the initial side in the <u>clockwise</u> direction.

2. An angle is in standard position if its vertex is at the origin of a coordinate system and its initial side lie on the <u>positive *x*-axis</u>.

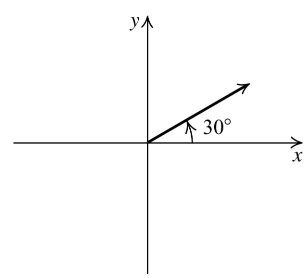
3. One second (1′′) is <u>one-sixtieth</u> of a minute.

4. False. One radian $= \frac{180}{\pi}° \approx 57.3°$.

5. True. The circumference of the circle $= 2\pi r = 2\pi \cdot 1 = 2\pi \approx 6.28$ m.

6. True. Acute angles measure more than 0º and less than 90º.

7.



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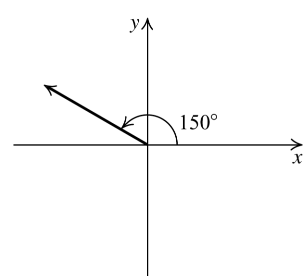

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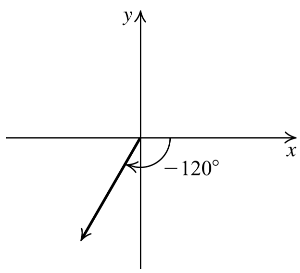

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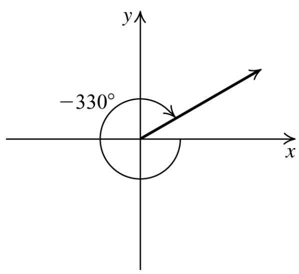

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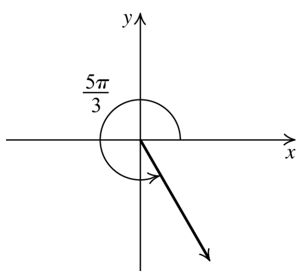

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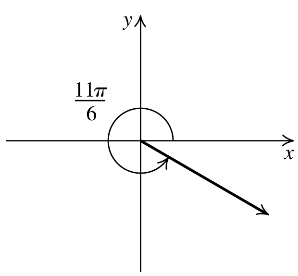

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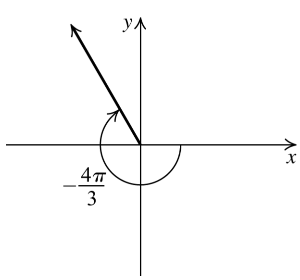

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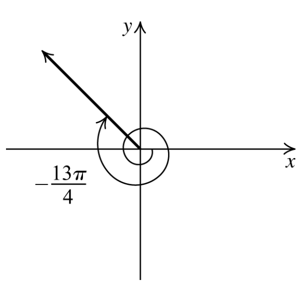


15. $70°45' = \left(70 + \frac{45}{60}\right)° = 70.75°$

16. $38°38' = \left(38 + \frac{38}{60}\right)° \approx 38.63°$

17. $23°42'30'' = \left(23 + \frac{42}{60} + \frac{30}{3600}\right)° \approx 23.71°$

18. $45°50'50'' = \left(45 + \frac{50}{60} + \frac{50}{3600}\right)° \approx 45.85°$

19. $-15°42'57'' = -\left(15 + \frac{42}{60} + \frac{57}{3600}\right)° \approx -15.72°$

20. $-70°18'13'' = -\left(70 + \frac{18}{60} + \frac{13}{3600}\right)° \approx -70.30°$

21. $27.32° = 27° + 0.32(60') = 27° + 19.2'$
$= 27° + 19' + 0.2(60'') = 27°19'12''$

22. $120.64° = 120° + 0.64(60') = 120° + 38.4'$
$= 120° + 38' + 0.4(60'') = 120°38'24''$

23. $13.347° = 13° + 0.347(60') = 13° + 20.82'$
$= 13° + 20' + 0.82(60'') = 13°20'49''$

24. $110.433° = 110° + 0.433(60') = 110° + 25.98'$
$= 110° + 25' + 0.98(60'') = 110°25'59''$

25. $19.0511° = 19° + 0.0511(60') = 19° + 3.066'$
$= 19° + 3' + 0.066(60'') = 19°3'4''$

26. $82.7272° = 82° + 0.7272(60') = 82° + 43.632'$
$= 82° + 43' + 0.632(60'') = 82°43'38''$

27. $20° = 20 \cdot \frac{\pi}{180} = \frac{\pi}{9}$ radian

28. $40° = 40 \cdot \frac{\pi}{180} = \frac{2\pi}{9}$ radian

29. $-180° = -180 \cdot \frac{\pi}{180} = -\pi$ radian

30. $-210° = -210 \cdot \frac{\pi}{180} = -\frac{7\pi}{6}$ radians

31. $315° = 315 \cdot \frac{\pi}{180} = \frac{7\pi}{4}$ radians

32. $330° = 330 \cdot \frac{\pi}{180} = \frac{11\pi}{6}$ radians

33. $480° = 480 \cdot \frac{\pi}{180} = \frac{8\pi}{3}$ radians

34. $450° = 450 \cdot \frac{\pi}{180} = \frac{5\pi}{2}$ radians

35. $-510° = -510 \cdot \frac{\pi}{180} = -\frac{17\pi}{6}$ radians

36. $-420° = -420 \cdot \frac{\pi}{180} = -\frac{7\pi}{3}$ radians

37. $\frac{\pi}{12} = \frac{\pi}{12} \cdot \frac{180}{\pi} = 15°$

38. $\frac{3\pi}{8} = \frac{3\pi}{8} \cdot \frac{180}{\pi} = 67.5°$

39. $-\frac{5\pi}{9} = -\frac{5\pi}{9} \cdot \frac{180}{\pi} = -100°$

40. $-\frac{3\pi}{10} = -\frac{3\pi}{10} \cdot \frac{180}{\pi} = -54°$

41. $\frac{5\pi}{3} = \frac{5\pi}{3} \cdot \frac{180}{\pi} = 300°$

42. $\frac{11\pi}{6} = \frac{11\pi}{6} \cdot \frac{180}{\pi} = 330°$

43. $\frac{5\pi}{2} = \frac{5\pi}{2} \cdot \frac{180}{\pi} = 450°$

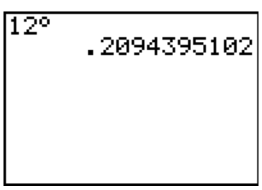
44. $\frac{17\pi}{6} = \frac{17\pi}{6} \cdot \frac{180}{\pi} = 510°$

45. $-\frac{11\pi}{4} = -\frac{11\pi}{4} \cdot \frac{180}{\pi} = -495°$

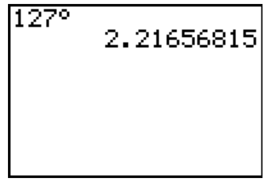
46. $-\frac{7\pi}{3} = -\frac{7\pi}{3} \cdot \frac{180}{\pi} = -420°$

For exercises 47–50, make sure your calculator is in Radian mode.

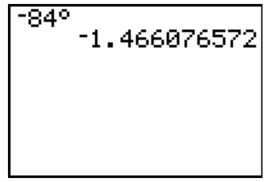
47. $12° = 12°\left(\frac{\pi}{180°}\right) \approx 0.21$ radian



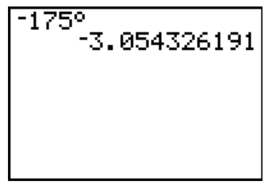
48. $127° = 127°\left(\frac{\pi}{180°}\right) \approx 2.22$ radians



49. $-84° = -84°\left(\frac{\pi}{180°}\right) \approx -1.47$ radians

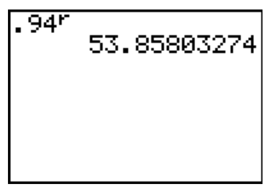


50. $-175° = -175°\left(\frac{\pi}{180°}\right) \approx -3.05$ radians

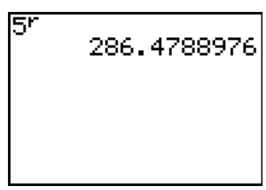


For exercises 51–54, make sure your calculator is in Degree mode.

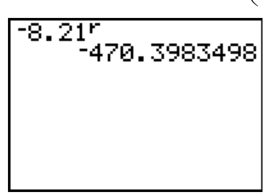
51. $0.94 \text{ radians} = 0.94\left(\frac{180°}{\pi}\right) \approx 53.86°$



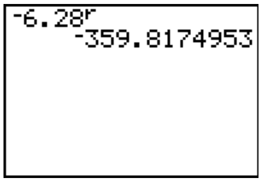
52. $5 \text{ radians} = 5\left(\frac{180°}{\pi}\right) \approx 286.48°$



53. $-8.21 \text{ radians} = -8.21\left(\frac{180°}{\pi}\right) \approx -470.40°$



54. $-6.28 \text{ radians} = -6.28\left(\frac{180°}{\pi}\right) \approx -359.82°$



55. complement: 43°; supplement: 133°

56. complement: 15°; supplement: 105°

57. complement: none because the measure of the angle is greater than 90°; supplement: 60°

58. complement: none because the measure of the angle is greater than 90°; supplement: 20°

59. complement: none because the measure of the angle is greater than 90°; supplement: none because the measure of the angle is greater than 180°

60. complement: none because the measure of the angle is negative; supplement: none because the measure of the angle is negative

In exercises 61−76, use the formulas $s = r\theta, v = \frac{s}{t}$, $\omega = \frac{\theta}{t}, v = r\omega$, and $A = \frac{1}{2}r^2\theta$ where θ is the radian measure of the central angle that intercepts an arc of length *s* in a circle of radius *r*, *v* is the linear velocity, ω is the angular velocity, *A* is the area of a sector of the circle, and *t* is the time.

61. $s = r\theta \Rightarrow 7 = 25\theta \Rightarrow \theta = \frac{7}{25} = 0.28$ radian

62. $s = r\theta \Rightarrow 6 = 5\theta \Rightarrow \theta = \frac{6}{5} = 1.2$ radians

63. $s = r\theta \Rightarrow 22 = 10.5\theta \Rightarrow$
$\theta = \frac{22}{10.5} = \frac{44}{21} \approx 2.095$ radians

64. $s = r\theta \Rightarrow 120 = 60\theta \Rightarrow \theta = \frac{120}{60} = 2$ radians

65. First convert the angle measurement from degrees to radians: $25° = 25°\left(\frac{\pi}{180°}\right) = \frac{5\pi}{36}$.

Then $s = r\theta \Rightarrow s = 3 \cdot \frac{5\pi}{36} = \frac{5\pi}{12} \approx 1.309$ m.

66. First convert the angle measurement from degrees to radians: $357° = 357°\left(\frac{\pi}{180°}\right) = \frac{119\pi}{60}$

Then

$s = r\theta \Rightarrow s = 0.7 \cdot \frac{119\pi}{60} = \frac{833\pi}{60} \approx 4.362$ m

67. $s = r\theta \Rightarrow s = 6.5 \cdot 12 = 78$ m

68. $s = r\theta \Rightarrow s = 6 \cdot \frac{\pi}{6} = \pi \approx 3.142$ m

69. $v = r\omega \Rightarrow v = 6 \cdot 10 = 60$ m/min

70. $v = r\omega \Rightarrow v = 3.2 \cdot 5 = 16$ ft/sec

71. $v = r\omega \Rightarrow 20 = 10\omega \Rightarrow \omega = 2$ radians/sec

72. $v = r\omega \Rightarrow 10 = 6\omega \Rightarrow \omega = \frac{5}{3}$ radians/min

73. $A = \frac{1}{2}r^2\theta \Rightarrow \frac{1}{2} \cdot 10^2 \cdot 4 = 200 \text{ in.}^2$

74. First convert the angle measurement from degrees to radians:

$60° = 60°\left(\frac{\pi}{180°}\right) = \frac{\pi}{3}$ radians

$A = \frac{1}{2}r^2\theta \Rightarrow A = \frac{1}{2}\left(1.5^2\right)\left(\frac{\pi}{3}\right) = \frac{3\pi}{8} \approx 1.178 \text{ ft}^2$

75. First convert the angle measurement from degrees to radians:

$60° = 60°\left(\frac{\pi}{180°}\right) = \frac{\pi}{3}$ radians

$A = \frac{1}{2}r^2\theta \Rightarrow 20 = \frac{1}{2} \cdot \frac{\pi}{3}r^2 \Rightarrow \frac{120}{\pi} = r^2 \Rightarrow$
$r = \sqrt{\frac{120}{\pi}} \approx 6.180$ ft

76. $A = \frac{1}{2}r^2\theta \Rightarrow 60 = \frac{1}{2} \cdot 2r^2 \Rightarrow 60 = r^2 \Rightarrow$
$r = \sqrt{60} = 2\sqrt{15} \approx 7.746$ m

5.1 Applying the Concepts

77. Three-quarters of a revolution is $3\pi/2$ radians. So the arc length (the distance the car moves) is
$s = r\theta = 15\left(\frac{3\pi}{2}\right) \approx 70.69$ inches

78. First convert the angle measurement from degrees to radians:
$\theta = 1' = \frac{1}{60°} = \frac{1}{60} \cdot \frac{\pi}{180} \approx 0.00029$ radians .
Then 1 nautical mile is
$s = r\theta = 3960(0.00029) \approx 1.15$ statute mile .

79. The hands of the clock form an angle of
$\frac{4}{12} \cdot 360° = 120° \cdot \frac{\pi}{180°} = \frac{2\pi}{3}$ radians .

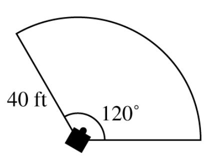
80. The hands of the clock form an angle of
$\frac{7}{12} \cdot 360° = 210° \cdot \frac{\pi}{180°} = \frac{7\pi}{6}$ radians .

81. First convert the angle measurement from degrees to radians:
$\theta = 25° \cdot \frac{\pi}{180°} = \frac{5\pi}{36}$ radians . Using $s = r\theta \Rightarrow$
$\frac{5\pi}{36} = \frac{4}{r} \Rightarrow r \approx 9$ in. $\Rightarrow d \approx 18$ in.

82. First convert the angle measurement from degrees to radians: $\theta = 30° \cdot \frac{\pi}{180°} = \frac{\pi}{6}$ radians .
$\frac{\pi}{6} = \frac{s}{5} \Rightarrow s \approx 2.62$ ft

83. The arc length is 6 inches and the radius is 4 inches, so $\theta = \frac{6}{4} = 1.5$ radians .
1 radian = $\frac{180°}{\pi}$, so $\theta = 1.5\left(\frac{180°}{\pi}\right) \approx 86°$.

84. First convert the angle measurement from degrees to radians:
$\theta = 120° \cdot \frac{\pi}{180°} = \frac{2\pi}{3}$ radians
$\frac{2\pi}{3} = \frac{s}{40} \Rightarrow s \approx 84$ ft



85.a. The linear speed, v, of the larger pulley is 600 inches per minute. To find the angular speed, use the formula $v = r\omega \Rightarrow 600 = 5\omega \Rightarrow$
$\omega = 120$ radians per minute .

b. The linear speed, v, of the smaller pulley is 600 inches per minute. To find the angular speed, use the formula $v = r\omega$.
$600 = 2\omega \Rightarrow \omega = 300$ radians per minute .

86.a. The ferris wheel makes a complete revolution, so $\theta = 2\pi$.
$\omega = \frac{\theta}{t} = \frac{2\pi}{90 \text{ sec}} \approx 0.07$ radians per second.

b. First find the length of the arc traveled in one complete revolution of the wheel:
$\theta = \frac{s}{r} \Rightarrow 2\pi = \frac{s}{30.5} \Rightarrow s \approx 191.64$ m . (Note that this is the circumference of the ferris wheel.) Then $v = \frac{s}{t} = \frac{191.64}{90 \text{ sec}} \approx 2.13$ meters per second.

87. The radius of each wheel is 1 foot, while the linear velocity, v, is 25 miles per hour = $5280 \times 25 = 132{,}000$ feet per hour. $v = r\omega \Rightarrow$ $132{,}000 = 1 \cdot \omega$, so the angular speed is 132,000 radians per hour.

88. The radius of each wheel is 15 inches, and the angular speed, ω , of the wheels is 11 radians per second. So the linear speed, v, of the bicycle is $15 \cdot 11 = 165$ inches per second.

89. The radius of the disk is 1.875 inches. Each revolution is 2π radians, so the disk rotating at 7200 revolutions per minute gives
$\omega = 7200 \frac{\text{revolutions}}{\text{minute}} \cdot \frac{2\pi \text{ radians}}{\text{revolution}} = 14{,}400\pi$
radians per minute.
Then $v = 1.875 \cdot 14{,}400\pi \approx 84{,}823$ inches per minute.

90. $t = 24$ hours, $r = 3960, \theta = 2\pi$
$\omega = \frac{\theta}{t} = \frac{2\pi}{24} = \frac{\pi}{12}$
$v = r\omega = 3960\left(\frac{\pi}{12}\right) \approx 1037$ mph

91. The difference in the latitudes is
$39°44' - 32°23' = 7°21' = 7 + \left(\frac{21}{60}\right)° = 7.35°$
$= 7.35°\left(\frac{\pi}{180°}\right) \approx 0.1283$ radians.
$s = 0.1283 \cdot 3960 \approx 508$ miles.

92. The difference in the latitudes is
$40°30' - 32°54' = 7°36' = 7 + \left(\frac{36}{60}\right)° = 7.6°$.
$s = 7.6°\left(\frac{\pi}{180°}\right) \cdot 3960 \approx 525$ miles.

93. The difference in the latitudes is

$$52°23' - 45°42' = 6°41' = 6 + \left(\frac{41}{60}\right)° = 6.683°.$$

$$s = 6.683°\left(\frac{\pi}{180°}\right) \cdot 3960 \approx 462 \text{ miles.}$$

94. The difference in the latitudes is

$$36°59' - 31°47' = 5°12' = 5 + \left(\frac{12}{60}\right)° = 5.2°.$$

$$s = 5.2°\left(\frac{\pi}{180°}\right) \cdot 3960 \approx 359 \text{ miles.}$$

95. Let the central angle θ = the difference in latitude. Then

$$\theta = \frac{s}{r} = \frac{440}{3960} \approx 0.1111 \text{ radians} \Rightarrow$$

$$\theta \approx 0.1111\left(\frac{180°}{\pi}\right) \approx 6.366°.$$

96. Let the central angle θ = the difference in latitude. Then

$$\theta = \frac{s}{r} = \frac{554}{3960} \approx 0.1399 \text{ radians} \Rightarrow$$

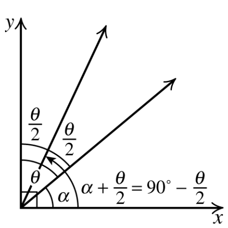
$$\theta \approx 0.1399\left(\frac{180°}{\pi}\right) \approx 8.016°$$

5.1 Beyond the Basics

97. $\dfrac{2\pi}{3} + \pi = \dfrac{5\pi}{3}; \dfrac{2\pi}{3} - \pi = -\dfrac{\pi}{3}$

98. $\alpha + \theta = 90° \Rightarrow \alpha + \theta - \dfrac{\theta}{2} = 90° - \dfrac{\theta}{2} \Rightarrow$

$$\alpha + \frac{\theta}{2} = 90° - \frac{\theta}{2} \Rightarrow \left(\alpha + \frac{\theta}{2}\right) + \frac{\theta}{2} = 90°$$



99. $\theta - \left(\theta - \dfrac{n\pi}{2}\right) = 2\pi \Rightarrow \dfrac{n\pi}{2} = 2\pi \Rightarrow n = 4$

100. $\theta - \left(\theta + \dfrac{n\pi}{3}\right) = -2\pi \Rightarrow -\dfrac{n\pi}{3} = -2\pi \Rightarrow n = 6$

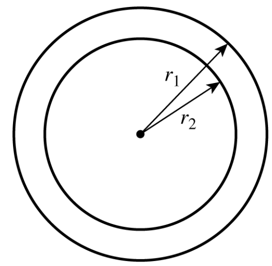
101. $A_{r_1} = \dfrac{1}{2}r_1^2\theta$. If $r_2 = 2r_1$, then

$$A_{r_2} = \frac{1}{2}r_2^2\theta = \frac{1}{2}(2r_1)^2\theta = 4\left(\frac{1}{2}r_1^2\theta\right) = 4A_{r_1}.$$

102. $A_{\theta_1} = \dfrac{1}{2}r^2\theta_1$. If $\theta_2 = \dfrac{1}{2}\theta_1$, then

$$A_{\theta_2} = \frac{1}{2}r^2\theta_2 = \frac{1}{2}r^2\left(\frac{1}{2}\theta_1\right) = \frac{1}{2}\left(\frac{1}{2}r^2\theta_1\right) = \frac{1}{2}A_{\theta_1}$$

103.



We are seeking $r_1 - r_2$. Use the formula $C = 2\pi r$ to find the radius of each circle.

$$396 = 2\pi r_1 \Rightarrow \frac{396}{2\left(\frac{22}{7}\right)} = 63 = r_1$$

$$352 = 2\pi r_2 \Rightarrow \frac{352}{2\left(\frac{22}{7}\right)} = 56 = r_2$$

$$r_1 - r_2 = 63 - 56 = 7$$

The width of the track is 7 m.

104. The circumference of the wheel is

$$\pi d = \frac{22}{7} \cdot 84 = 264 \text{ cm} = 2.64 \text{ m.}$$

The wheel must revolve $\dfrac{792}{2.64} = 300$ times to cover 792 m.

105. $66 \text{ km/hr} = 66 \text{ km/60 min} = 1.1 \text{ km/min.}$
$= 110{,}000 \text{ cm/min.}$

$$C = \pi d = \frac{22}{7} \cdot 140 = 440 \text{ cm}$$

$$\frac{110{,}000 \text{ cm/min}}{440 \text{ cm}} = 250 \text{ rev/min}$$

106. Each wiper sweeps a sector with radius 14 in. and central angle 110°. First convert the angle measurement from degrees to radians:

$$\theta = 110° \cdot \frac{\pi}{180°} = \frac{11\pi}{18} \text{ radians.}$$

$$A = \frac{1}{2}r^2\theta = \frac{1}{2} \cdot 14^2 \cdot \frac{11\pi}{18}$$
$$= \frac{1}{2} \cdot 14^2 \cdot \frac{11}{18} \cdot \frac{22}{7} = \frac{1694}{9} \text{ in.}^2$$

Therefore, the total area swept by the wipers is

$$2 \cdot \frac{1694}{9} = \frac{3388}{9} \approx 376.4 \text{ in.}^2$$

107. We are seeking the area of a sector with radius $\frac{3}{2} = 1.5$ ft and central angle $\frac{2\pi}{8} = \frac{\pi}{4}$.

$A = \frac{1}{2}r^2\theta = \frac{1}{2}\cdot 1.5^2 \cdot \frac{\pi}{4} = \frac{1}{2}\cdot 1.5^2 \cdot \frac{1}{4}\cdot\frac{22}{7}$

$= \frac{99}{112} \approx 0.884 \text{ ft}^2$

108. Since a sector of central angle $\frac{\pi}{4}$ is cut from the circle, the circumference of the cone is $\frac{7}{8}\cdot 2\cdot\frac{22}{7}\cdot 7 = \frac{77}{2}$, so the radius of the cone is $\frac{77}{2} = 2\cdot\frac{22}{7}R \Rightarrow R = \frac{49}{8}$. Find h using the Pythagorean theorem.

$\left(\frac{49}{8}\right)^2 + h^2 = 7^2 \Rightarrow h^2 = 49 - \frac{2401}{64} \Rightarrow$

$h = \sqrt{\frac{735}{64}} = \frac{7\sqrt{15}}{8}$

$V = \frac{1}{3}\cdot\frac{22}{7}\cdot\left(\frac{49}{8}\right)^2\cdot\frac{7\sqrt{15}}{8} \approx 133.19 \text{ cm}^3$

5.1 Critical Thinking/Discussion/Writing

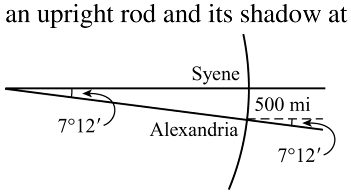
109. 3, π, 4, $\frac{3\pi}{2}$

110. The line of latitude through 40°40′13″ is further north, so its radius is smaller.

111. The length of the arc $\frac{\pi r}{2} - s = \frac{\pi r}{2} - r\theta$

$= r\left(\frac{\pi}{2} - \theta\right)$. So $\frac{\pi}{2} - \theta$ is the complement of θ.

112. Eratosthenes used the fact that the sun's rays are effectively parallel to determine that the central angle that subtends the arc from Syene to Alexandria is the same as the angle formed by an upright rod and its shadow at Alexandria.



Let x = the circumference of the earth, and then solve the proportion $\frac{7.2°}{360°} = \frac{500 \text{ miles}}{x \text{ miles}}$. This gives x = 25,000 miles

5.1 Maintaining Skills

113. $5^2 + 12^2 = c^2 \Rightarrow 169 = c^2 \Rightarrow 13 = c$

114. $12^2 + b^2 = 20^2 \Rightarrow 144 + b^2 = 400 \Rightarrow$
$b^2 = 256 \Rightarrow b = 16$

115. $a^2 + 6^2 = 10^2 \Rightarrow a^2 + 36 = 100 \Rightarrow a^2 = 64 \Rightarrow$
$a = 8$

116. $20^2 + 21^2 = c^2 \Rightarrow 841 = c^2 \Rightarrow 29 = c$

117. $a^2 + 3^2 = 6^2 \Rightarrow a^2 + 9 = 36 \Rightarrow a^2 = 27 \Rightarrow$
$a = \sqrt{27} = 3\sqrt{3}$

118. $5^2 + 10^2 = c^2 \Rightarrow 25 + 100 = c^2 \Rightarrow 125 = c^2 \Rightarrow$
$c = \sqrt{125} = 5\sqrt{5}$

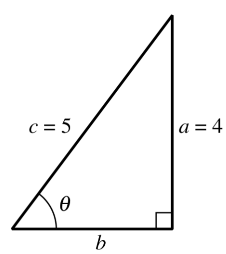
119. $\frac{A}{B} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\frac{1}{2}\cdot 2}{\frac{\sqrt{3}}{2}\cdot 2} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

120. $\frac{A}{B} = \frac{\frac{\sqrt{3}}{2}\cdot\sqrt{\frac{3}{2}}}{\sqrt{\frac{3}{2}}\cdot\sqrt{\frac{3}{2}}} = \frac{\frac{3}{2\sqrt{2}}}{\frac{3}{2}} = \frac{3}{2\sqrt{2}}\cdot\frac{2}{3} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

5.2 Right-Triangle Trigonometry

5.2 Practice Problems

1.



Use the Pythagorean theorem to find the length of the missing leg: $25^2 = 16^2 + b^2 \Rightarrow b = 3$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{4}{5}$ $\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{5}{4}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3}{5}$ $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{5}{3}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{4}{3}$ $\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{3}{4}$

2. $\sin\theta = \frac{3}{5},\ \cos\theta = \frac{4}{5}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

3. $\cos\theta = \frac{1}{3} \Rightarrow b = \text{adjacent} = 1$ and $c = \text{hypotenuse} = 3.$ Using the Pythagorean theorem, we have $a^2 + 1^2 = 3^3 \Rightarrow a = 2\sqrt{2}.$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{2\sqrt{2}}{3}$$

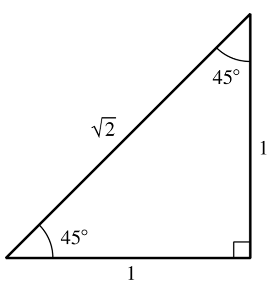
$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = 3$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = 2\sqrt{2}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

4.

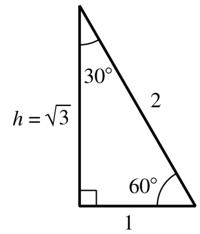


$$\csc 45^\circ = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\sec 45^\circ = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\cot 45^\circ = \frac{\text{adjacent}}{\text{opposite}} = \frac{1}{1} = 1$$

5.



$$\sin 60^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{2}$$

$$\tan 60^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\cot 60^\circ = \frac{\text{adjacent}}{\text{opposite}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

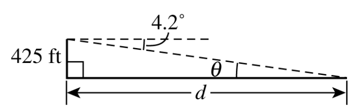
$$\sec 60^\circ = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{2}{1} = 2$$

$$\csc 60^\circ = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

6.a. $\csc 21^\circ \approx 2.7904 \Rightarrow$
$\sec(90^\circ - 21^\circ) = \sec 69^\circ \approx 2.7904$

b. $\tan 75^\circ \approx 3.7321 \Rightarrow$
$\cot(90^\circ - 75^\circ) = \cot 15^\circ \approx 3.7321$

7.



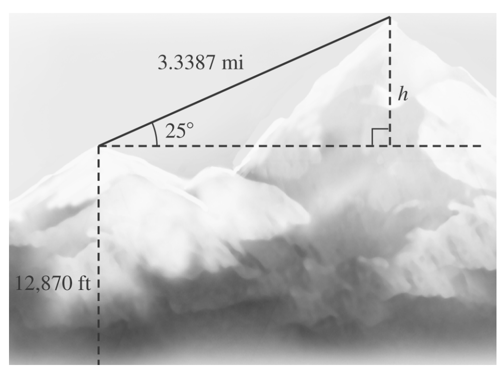
Not drawn to scale

From geometry, we know that $\theta = 4.2^\circ$. Thus,

$$\tan\theta = \frac{425}{d} \Rightarrow \tan 4.2^\circ = \frac{425}{d} \Rightarrow$$

$$d = \frac{425}{\tan 4.2^\circ} \approx 5787.3988 \text{ ft} \approx 1.096 \text{ mi}$$

8.



The sum of the side length h and the location height of 12,870 feet give the approximate height of Mount McKinley.

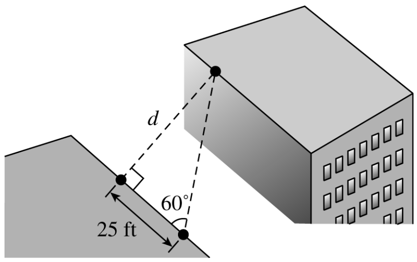
(*continued on next page*)

(*continued*)

$$\sin 25° = \frac{h}{3.3387}$$
$$h = 3.3387 \sin 25° \approx 1.4110 \text{ mi}$$
$$\approx 1.4110(5280 \text{ ft}) \approx 7450 \text{ ft}$$

The height of Mount McKinley is approximately 7450 + 12,870 = 20,320 ft.

9.



$$\tan 60° = \frac{d}{25} \Rightarrow d = 25 \tan 60° \approx 43 \text{ ft}$$

5.2 Basic Concepts and Skills

1. If θ is an acute angle and $\sin\theta = \frac{2\sqrt{2}}{3}$, then $\cos\theta = \underline{\frac{1}{3}}$.

2. In a right triangle with sides of lengths 1, 5, and $\sqrt{26}$, the tangent of the angle opposite the side of length 5 is <u>5</u>.

3. If θ is an acute angle (measured in degrees) in a right triangle and $\cos\theta = 0.7$, then $\sin(90° - \theta) = \underline{0.7}$.

4. False. The length of the leg is a multiple of 2.

5. True

6. True. The hypotenuse of a right triangle is the longest side in the triangle.

7.
$$\sin\theta = \frac{2\sqrt{5}}{25} \qquad \csc\theta = \frac{5\sqrt{5}}{2}$$
$$\cos\theta = \frac{11\sqrt{5}}{25} \qquad \sec\theta = \frac{5\sqrt{5}}{11}$$
$$\tan\theta = \frac{2}{11} \qquad \cot\theta = \frac{11}{2}$$

8.
$$\sin\theta = \frac{\sqrt{2}}{2} \qquad \csc\theta = \sqrt{2}$$
$$\cos\theta = \frac{\sqrt{2}}{2} \qquad \sec\theta = \sqrt{2}$$
$$\tan\theta = 1 \qquad \cot\theta = 1$$

9. Use the Pythagorean theorem to find the length of the hypotenuse: $c = \sqrt{6^2 + 8^2} = 10$.
$$\sin\theta = \frac{6}{10} = \frac{3}{5} \qquad \csc\theta = \frac{10}{6} = \frac{5}{3}$$
$$\cos\theta = \frac{8}{10} = \frac{4}{5} \qquad \sec\theta = \frac{10}{8} = \frac{5}{4}$$
$$\tan\theta = \frac{6}{8} = \frac{3}{4} \qquad \cot\theta = \frac{8}{6} = \frac{4}{3}$$

10. Use the Pythagorean theorem to find the length of the leg: $17^2 = a^2 + 15^2 \Rightarrow a = 8$.
$$\sin\theta = \frac{8}{17} \qquad \csc\theta = \frac{17}{8}$$
$$\cos\theta = \frac{15}{17} \qquad \sec\theta = \frac{17}{15}$$
$$\tan\theta = \frac{8}{15} \qquad \cot\theta = \frac{15}{8}$$

11. Use the Pythagorean theorem to find the length of the hypotenuse: $c = \sqrt{7^2 + 1^2} = 5\sqrt{2}$.
$$\sin\theta = \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{10} \qquad \csc\theta = 5\sqrt{2}$$
$$\cos\theta = \frac{7}{5\sqrt{2}} = \frac{7\sqrt{2}}{10} \qquad \sec\theta = \frac{5\sqrt{2}}{7}$$
$$\tan\theta = \frac{1}{7} \qquad \cot\theta = 7$$

12. Use the Pythagorean theorem to find the length of the leg: $12^2 = a^2 + 4^2 \Rightarrow a = 8\sqrt{2}$.
$$\sin\theta = \frac{4}{12} = \frac{1}{3} \qquad \csc\theta = \frac{12}{4} = 3$$
$$\cos\theta = \frac{8\sqrt{2}}{12} = \frac{2\sqrt{2}}{3} \qquad \sec\theta = \frac{12}{8\sqrt{2}} = \frac{3\sqrt{2}}{4}$$
$$\tan\theta = \frac{4}{8\sqrt{2}} = \frac{\sqrt{2}}{4} \qquad \cot\theta = \frac{8\sqrt{2}}{4} = 2\sqrt{2}$$

13. $\sin\theta = \frac{12}{13}, \cos\theta = \frac{5}{13}$
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}$$
$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{12}{13}} = \frac{13}{12}$$
$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{5}{13}} = \frac{13}{5}$$
$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$$

14. $\sin\theta = \frac{8}{17}$, $\cos\theta = \frac{15}{17}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{8}{17}}{\frac{15}{17}} = \frac{8}{15}$$
$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{8}{17}} = \frac{17}{8}$$
$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{15}{17}} = \frac{17}{15}$$
$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{8}{15}} = \frac{15}{8}$$

15. $\sin\theta = \frac{21}{29}$, $\cos\theta = \frac{20}{29}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{21}{29}}{\frac{20}{29}} = \frac{21}{20}$$
$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{21}{29}} = \frac{29}{21}$$
$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{20}{29}} = \frac{29}{20}$$
$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{21}{20}} = \frac{20}{21}$$

16. $\sin\theta = \frac{40}{41}$, $\cos\theta = \frac{9}{41}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{40}{41}}{\frac{9}{41}} = \frac{40}{9}$$
$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{40}{41}} = \frac{41}{40}$$
$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{9}{41}} = \frac{41}{9}$$
$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{40}{9}} = \frac{9}{40}$$

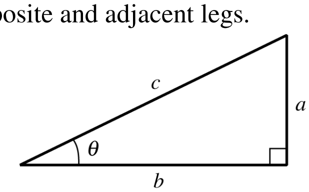
17. $\sin\theta = \frac{2}{5}$, $\cos\theta = \frac{\sqrt{21}}{5}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2}{5}}{\frac{\sqrt{21}}{5}} = \frac{2}{\sqrt{21}} = \frac{2\sqrt{21}}{21}$$
$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{2}{5}} = \frac{5}{2}$$
$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\sqrt{21}}{5}} = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$$
$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{2}{\sqrt{21}}} = \frac{\sqrt{21}}{2}$$

18. $\sin\theta = \frac{2\sqrt{6}}{5}$, $\cos\theta = \frac{1}{5}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2\sqrt{6}}{5}}{\frac{1}{5}} = 2\sqrt{6}$$
$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{2\sqrt{6}}{5}} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$$
$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{1}{5}} = 5$$
$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$$

For exercises 19−24, use this triangle to help identify the opposite and adjacent legs.



19. $\cos\theta = \frac{2}{3} \Rightarrow b = 2, c = 3$.

$3^2 = a^2 + 2^2 \Rightarrow a = \sqrt{5}$

$\sin\theta = \frac{\sqrt{5}}{3}$ $\qquad$ $\csc\theta = \frac{3\sqrt{5}}{5}$

$\cos\theta = \frac{2}{3}$ $\qquad$ $\sec\theta = \frac{3}{2}$

$\tan\theta = \frac{\sqrt{5}}{2}$ $\qquad$ $\cot\theta = \frac{2\sqrt{5}}{5}$

20. $\sin\theta = \frac{3}{4} \Rightarrow a = 3, c = 4$.

$4^2 = 3^2 + b^2 \Rightarrow b = \sqrt{7}$

$\sin\theta = \frac{3}{4}$ $\csc\theta = \frac{4}{3}$

$\cos\theta = \frac{\sqrt{7}}{4}$ $\sec\theta = \frac{4\sqrt{7}}{7}$

$\tan\theta = \frac{3\sqrt{7}}{7}$ $\cot\theta = \frac{\sqrt{7}}{3}$

21. $\tan\theta = \frac{5}{3} \Rightarrow a = 5, b = 3$.

$c^2 = 5^2 + 3^2 \Rightarrow b = \sqrt{34}$

$\sin\theta = \frac{5\sqrt{34}}{34}$ $\csc\theta = \frac{\sqrt{34}}{5}$

$\cos\theta = \frac{3\sqrt{34}}{34}$ $\sec\theta = \frac{\sqrt{34}}{3}$

$\tan\theta = \frac{5}{3}$ $\cot\theta = \frac{3}{5}$

22. $\cot\theta = \frac{6}{11} \Rightarrow a = 11, b = 6$.

$c^2 = 11^2 + 6^2 \Rightarrow b = \sqrt{157}$

$\sin\theta = \frac{11\sqrt{157}}{157}$ $\csc\theta = \frac{\sqrt{157}}{11}$

$\cos\theta = \frac{6\sqrt{157}}{157}$ $\sec\theta = \frac{\sqrt{157}}{6}$

$\tan\theta = \frac{11}{6}$ $\cot\theta = \frac{6}{11}$

23. $\sec\theta = \frac{13}{12} \Rightarrow b = 12, c = 13$.

$13^2 = a^2 + 12^2 \Rightarrow a = 5$

$\sin\theta = \frac{5}{13}$ $\csc\theta = \frac{13}{5}$

$\cos\theta = \frac{12}{13}$ $\sec\theta = \frac{13}{12}$

$\tan\theta = \frac{5}{12}$ $\cot\theta = \frac{12}{5}$

24. $\csc\theta = 4 \Rightarrow a = 1, c = 4$.

$4^2 = 1^2 + b^2 \Rightarrow b = \sqrt{15}$

$\sin\theta = \frac{1}{4}$ $\csc\theta = 4$

$\cos\theta = \frac{\sqrt{15}}{4}$ $\sec\theta = \frac{4\sqrt{15}}{15}$

$\tan\theta = \frac{\sqrt{15}}{15}$ $\cot\theta = \sqrt{15}$

For exercises 25−30, use the fact that the value of any trigonometric function of an acute angle is equal to the co-function of the angle's complement.

25. $\sin 58° \approx 0.8480 \Rightarrow \cos 32° \approx 0.8480$

26. $\cos 37° \approx 0.7986 \Rightarrow \sin 53° \approx 0.7986$

27. $\tan 27° \approx 0.5095 \Rightarrow \cot 63° \approx 0.5095$

28. $\cot 49° \approx 0.8693 \Rightarrow \tan 41° \approx 0.8693$

29. $\sec 65° \approx 2.3662 \Rightarrow \csc 25° \approx 2.3662$

30. $\csc 78° \approx 1.0223 \Rightarrow \sec 12° \approx 1.0223$

31. $\sin 60°\cos 30° + \cos 60°\sin 30°$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1$$

32. $\sin 60°\cos 45° + \cos 60°\sin 45°$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

33. $\cos 60°\cos 30° + \sin 60°\sin 30°$

$$= \frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{1}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

34. $\cos 60°\cos 45° - \sin 60°\sin 45°$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$$

35. $\cot 45° + \csc 30° = 1 + 2 = 3$

36. $\tan 30°\sec 45° + \tan 60°\sec 30°$

$$= \frac{\sqrt{3}}{3}\cdot\sqrt{2} + \sqrt{3}\cdot\frac{2\sqrt{3}}{3} = \frac{\sqrt{6}}{3} + 2 = \frac{\sqrt{6}+6}{3}$$

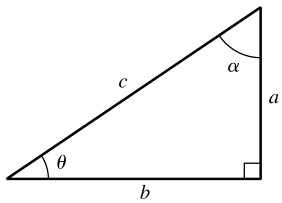
37. $\cos 45°\cos 30° - \sin 45°\sin 30°$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

38. $\sin 30°\cos 45° + \cos 30°\sin 45°$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$$

Use this figure for exercises 39−48.



39. $8^2+10^2=c^2 \Rightarrow c^2=164 \Rightarrow c \approx 12.806$
$\sin\theta = \frac{a}{c} = \frac{8}{12.806} \approx 0.625$
$\tan\theta = \frac{a}{b} = \frac{8}{10} = 0.8$

40. $c^2 = 18^2 + 3^2 \Rightarrow c \approx 18.248$.
$\sin\theta = \frac{18}{18.248} \approx 0.986, \cos\theta = \frac{3}{18.248} \approx 0.164$

41. $c^2 = 23^2 + 7^2 \Rightarrow c \approx 24.042$.
$\cos\theta = \frac{7}{24.042} \approx 0.291, \tan\theta = \frac{23}{7} \approx 3.286$

42. $c^2 = 19^2 + 27^2 \Rightarrow c \approx 33.015$.
$\cos\theta = \frac{27}{33.015} \approx 0.818, \tan\theta = \frac{19}{27} \approx 0.704$

43. $\theta = 30° \Rightarrow \sin\theta = 0.5, \sin 30° = \frac{9}{c} \Rightarrow c = 18$.
Use the Pythagorean theorem to find *b*:
$18^2 = 9^2 + b^2 \Rightarrow b \approx 15.588$

44. $\theta = 30° \Rightarrow \sin\theta = 0.5, \sin 30° = \frac{5}{c} \Rightarrow c = 10$.
Use the Pythagorean theorem to find *b*:
$10^2 = 5^2 + b^2 \Rightarrow b \approx 8.660$

45. $c^2 = 12.5^2 + 6.2^2 \Rightarrow c \approx 13.953$
$\sin\theta = \frac{a}{c} = \frac{12.5}{13.953} \approx 0.896$
$\cos\theta = \frac{b}{c} = \frac{6.2}{13.953} \approx 0.444$

46. $c^2 = 4.3^2 + 8.1^2 \Rightarrow c \approx 9.171$
$\cos\theta = \frac{b}{c} = \frac{8.1}{9.171} \approx 0.883$
$\tan\theta = \frac{a}{b} = \frac{4.3}{8.1} \approx 0.531$

47. $14.5^2 = a^2 + 9.4^2 \Rightarrow a \approx 11.040$
$\sin\theta = \frac{a}{c} \approx \frac{11.040}{14.5} \approx 0.761$
$\tan\theta = \frac{a}{b} = \frac{11.040}{9.4} \approx 1.174$

48. $18.7^2 = 6.2^2 + b^2 \Rightarrow b \approx 17.642$
$\sec\theta = \frac{c}{b} = \frac{18.7}{17.642} \approx 1.060$
$\cot\theta = \frac{b}{a} = \frac{17.642}{6.2} \approx 2.845$

49. $\sin\theta = \frac{1}{3}$

a. $\csc\theta = \frac{1}{\sin\theta} = 3$

b. $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \left(\frac{1}{3}\right)^2 + \cos^2\theta = 1 \Rightarrow$
$\cos^2\theta = \frac{8}{9} \Rightarrow \cos\theta = \frac{2\sqrt{2}}{3}$

c. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1/3}{2\sqrt{2}/3} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

d. $\cot(90° - \theta) = \tan\theta = \frac{\sqrt{2}}{4}$

50. $\cos\theta = \frac{1}{4}$

a. $\sec\theta = \frac{1}{\cos\theta} = 4$

b. $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta + \left(\frac{1}{4}\right)^2 = 1 \Rightarrow$
$\sin^2\theta = \frac{15}{16} \Rightarrow \sin\theta = \frac{\sqrt{15}}{4}$

c. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{15}/4}{1/4} = \sqrt{15}$

d. $\cot(90° - \theta) = \tan\theta = \sqrt{15}$

51. $\sec\theta = 5$

a. $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{5}$

b. $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta + \left(\frac{1}{5}\right)^2 = 1 \Rightarrow$
$\sin^2\theta = \frac{24}{25} \Rightarrow \sin\theta = \frac{2\sqrt{6}}{5}$

c. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2\sqrt{6}/5}{1/5} = 2\sqrt{6}$

d. $\tan(90° - \theta) = \cot\theta = \frac{1}{\tan\theta} = \frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$

52. $\csc\theta = 2$

a. $\sin\theta = \frac{1}{\csc\theta} = \frac{1}{2}$

b. $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1 \Rightarrow$
$\cos^2\theta = \frac{3}{4} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

c. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

d. $\tan(90° - \theta) = \cot\theta = \frac{1}{\tan\theta} = \frac{3}{\sqrt{3}} = \sqrt{3}$

53. $\tan\theta = \frac{5}{2}$

a. $\cot\theta = \frac{1}{\tan\theta} = \frac{2}{5}$

b. $1 + \tan^2\theta = \sec^2\theta \Rightarrow 1 + \left(\frac{5}{2}\right)^2 = \sec^2\theta \Rightarrow$
$\sec^2\theta = \frac{29}{4} \Rightarrow \sec\theta = \frac{\sqrt{29}}{2}$

c. $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{29}/2} = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$

d. $\cos(90° - \theta) = \sin\theta$
$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta + \left(\frac{2\sqrt{29}}{29}\right)^2 = 1$
$\sin^2\theta + \frac{4}{29} = 1 \Rightarrow \sin^2\theta = \frac{25}{29} \Rightarrow$
$\sin\theta = \sqrt{\frac{25}{29}} = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29}$

54. $\cot\theta = \frac{3}{2}$

a. $\tan\theta = \frac{1}{\cot\theta} = \frac{2}{3}$

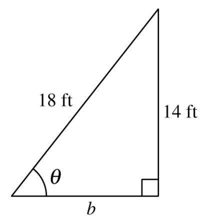
b. $1 + \cot^2\theta = \csc^2\theta \Rightarrow 1 + \left(\frac{3}{2}\right)^2 = \csc^2\theta \Rightarrow$
$\csc^2\theta = \frac{13}{4} \Rightarrow \csc\theta = \frac{\sqrt{13}}{2}$

c. $\sin\theta = \frac{1}{\csc\theta} = \frac{1}{\sqrt{13}/2} = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$

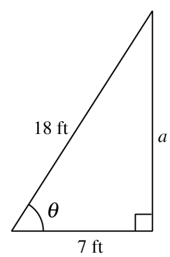
d. $\sin(90° - \theta) = \cos\theta$
$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \left(\frac{2\sqrt{13}}{13}\right)^2 + \cos^2 = 1$
$\frac{4}{13} + \cos^2 = 1 \Rightarrow \cos^2\theta = \frac{9}{13} \Rightarrow$
$\cos\theta = \sqrt{\frac{9}{13}} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$

5.2 Applying the Concepts

55. $18^2 = 14^2 + b^2 \Rightarrow b^2 = 128 \Rightarrow b \approx 11.3$ ft



56. $18^2 = a^2 + 7^2 \Rightarrow a^2 = 275 \Rightarrow a \approx 16.6$ ft



57. $1070^2 = 295^2 + b^2 \Rightarrow b \approx 1029$ m

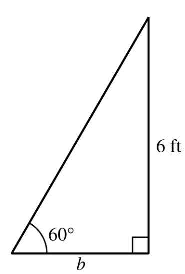
58. $\theta = 16°$ and the hypotenuse is 2050 feet. The vertical rise is the side opposite θ, so

$$\sin 16° = \frac{a}{2050} \Rightarrow 0.2756 = \frac{a}{2050} \Rightarrow a \approx 565 \text{ ft}$$

59. The ski lift is the hypotenuse and $\theta = 30°$. The vertical rise is the side opposite θ, so

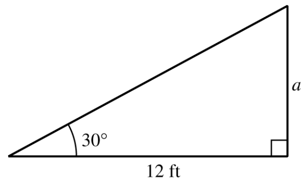
$$\sin 30° = \frac{a}{5000} \Rightarrow 0.5 = \frac{a}{5000} \Rightarrow a = 2500 \text{ ft}$$

60.



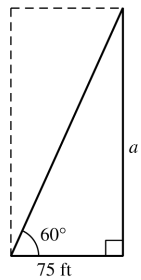
$$\tan 60° = \frac{6}{b} \Rightarrow \sqrt{3} = \frac{6}{b} \Rightarrow b = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ ft}$$

61.



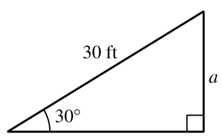
$\tan 30° = \frac{a}{12} \Rightarrow \frac{\sqrt{3}}{3} = \frac{a}{12} \Rightarrow a = 4\sqrt{3} \approx 6.9$ ft

62.



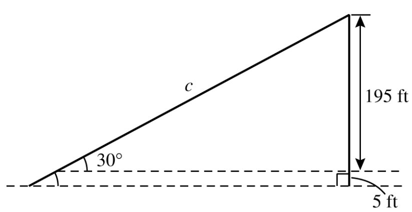
$\tan 60° = \frac{a}{75} \Rightarrow \sqrt{3} = \frac{a}{75} \Rightarrow a = 75\sqrt{3}$ ft

63.



$\sin 30° = \frac{a}{30} \Rightarrow \frac{1}{2} = \frac{a}{30} \Rightarrow a = 15\,\text{ft}$

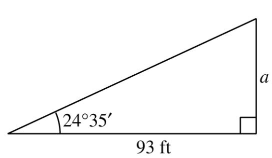
64.



$\sin 30° = \frac{195}{c} \Rightarrow \frac{1}{2} = \frac{195}{c} \Rightarrow c = 390$

The string must be 390 ft.

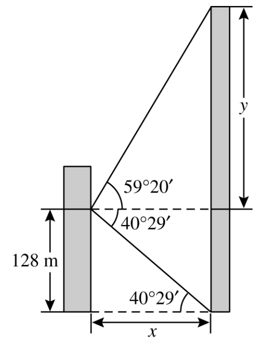
65.



$\tan 24°35' = \frac{a}{93} \Rightarrow 0.4575 \approx \frac{a}{93} \Rightarrow c \approx 43\,\text{ft}$

66. $\tan 73°34' = \frac{a}{10} \Rightarrow 3.3904 \approx \frac{a}{10} \Rightarrow a \approx 34\,\text{ft}$

67.



To find how far the second building is from the Empire State Building, we have

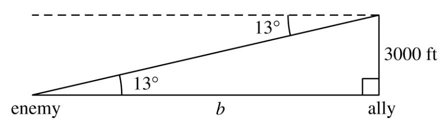
$\tan 40°29' = \frac{128}{x} \Rightarrow 0.8536 \approx \frac{128}{x} \Rightarrow x \approx 150$ m .

To find the height of the Empire State Building, we have height = $y + 128$.

$\tan 59°20' = \frac{y}{150} \Rightarrow 1.6864 \approx \frac{y}{150} \Rightarrow y \approx 253$ m,

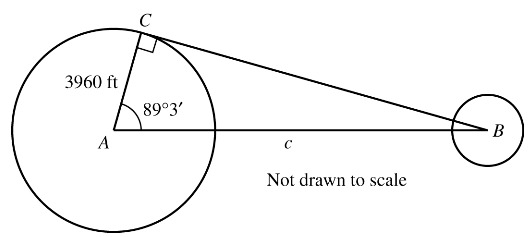
so the height of the Empire State Building is approximately $128 + 253 = 381$ m .

68.



$\tan 13° = \frac{3000}{b} \Rightarrow 0.2309 \approx \frac{3000}{b} \Rightarrow b \approx 12{,}994$ ft

69.

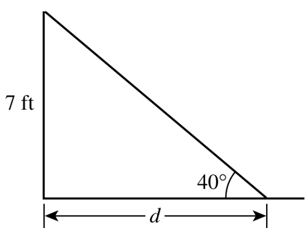


Note that *BC* is perpendicular to *AC* because the tangent drawn at the point of intersection of a radius and a circle is perpendicular to the radius. The latitude of a point on the Earth is the measure of the angle formed by the radius connecting the point with the radius at the equator, so $m\angle A = 89°3'$. Then

$\cos 89°3' = \frac{3960}{c} \Rightarrow 0.0166 \approx \frac{3960}{c} \Rightarrow$

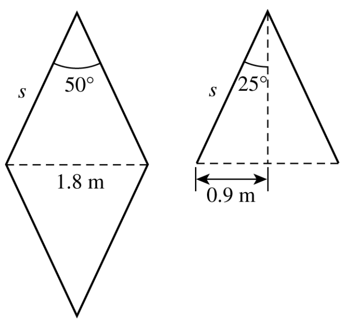
$c \approx 238{,}844$ miles .

70.



$$\tan 40° = \frac{7}{d} \Rightarrow d = \frac{7}{\tan 40°} \approx 8.3 \text{ ft}$$

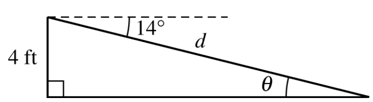
71.



Drop a perpendicular from the top of the sign to form a right triangle as shown.

$$\sin 25° = \frac{0.9}{s} \Rightarrow s = \frac{0.9}{\sin 25°} \approx 2.1 \text{ m}$$

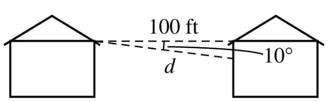
72.



From geometry, we know that $\theta = 14°$. Then,

$$\sin 14° = \frac{4}{d} \Rightarrow d = \frac{4}{\sin 14°} \approx 17 \text{ ft}$$

73.



$$\cos 10° = \frac{100}{d} \Rightarrow d = \frac{100}{\cos 10°} \approx 101.5 \text{ ft}$$

74. If B is at the 50th parallel, then $m\angle CBA = 50°$.

$AB = 3960$, so $\cos 50° = \frac{r}{3960} \Rightarrow$

$0.6428 \approx \frac{r}{3960} \Rightarrow r \approx 2545$ miles.

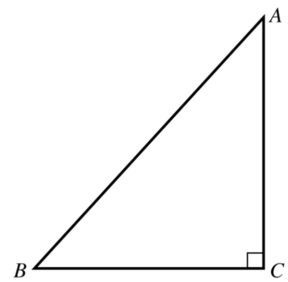
75. If B is at the 53th parallel, then $m\angle CBA = 53°$.

$AB = 3960$, so $\cos 53° = \frac{r}{3960} \Rightarrow$

$0.6018 \approx \frac{r}{3960} \Rightarrow r \approx 2383$ miles.

5.2 Beyond the Basics

76.



Since $\tan A = \frac{1}{2}$, we know that

$$\frac{\text{opposite}}{\text{adjacent}} = \frac{1}{2} \Rightarrow \frac{BC}{AC} = \frac{1}{2} \Rightarrow AC = 2BC.$$

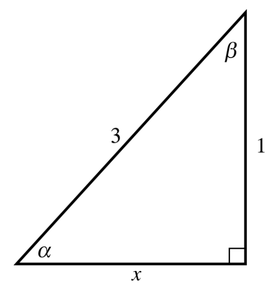
$$AB^2 = AC^2 + BC^2 = (2BC)^2 + BC^2$$
$$= 4BC^2 + BC^2 = 5BC^2 \Rightarrow AB = \sqrt{5}BC$$

So,

$$\sec A = \frac{AB}{AC} = \frac{\sqrt{5}BC}{2BC} = \frac{\sqrt{5}}{2}$$
$$\csc B = \frac{AB}{AC} = \frac{\sqrt{5}BC}{2BC} = \frac{\sqrt{5}}{2}$$

Thus, $\sec A + \csc B = \frac{\sqrt{5}}{2} + \frac{\sqrt{5}}{2} = \sqrt{5}$

77.



$$\sin \alpha = \frac{1}{3}$$

$$3^2 = 1^2 + x^2 \Rightarrow x^2 = 8 \Rightarrow x = 2\sqrt{2}$$

$\cos \alpha = \frac{2\sqrt{2}}{3}$ $\qquad$ $\sec \alpha = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$

$\tan \alpha = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$ $\qquad$ $\csc \alpha = 3$

$$\cos \alpha \csc \alpha + \tan \alpha \sec \alpha = \frac{2\sqrt{2}}{3} \cdot 3 + \frac{\sqrt{2}}{4} \cdot \frac{3\sqrt{2}}{4}$$
$$= 2\sqrt{2} + \frac{3}{8} = \frac{16\sqrt{2} + 3}{8}$$

78. $\sin\theta = \dfrac{m^2 - n^2}{m^2 + n^2} = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\left(m^2 + n^2\right)^2 = \left(m^2 - n^2\right)^2 + \text{adjacent}^2 \Rightarrow$

$$\begin{aligned}\text{adjacent}^2 &= \left(m^2 + n^2\right)^2 - \left(m^2 - n^2\right)^2 \\ &= m^4 + 2m^2n^2 + n^4 \\ &\quad - \left(m^4 - 2m^2n^2 + n^4\right) \\ &= 4m^2n^2 \Rightarrow\end{aligned}$$

adjacent = $2mn$

$\tan\theta = \dfrac{m^2 - n^2}{2mn} \qquad \sec\theta = \dfrac{m^2 + n^2}{2mn}$

$\tan\theta + \sec\theta = \dfrac{m^2 - n^2}{2mn} + \dfrac{m^2 + n^2}{2mn} = \dfrac{2m^2}{2mn} = \dfrac{m}{n}$

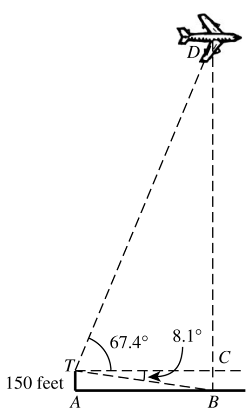
79. $\tan\theta = \dfrac{4}{5} = \dfrac{\text{opposite}}{\text{adjacent}}$

$\text{opposite}^2 + \text{adjacent}^2 = \text{hypotenuse}^2 \Rightarrow$
$4^2 + 5^2 = \text{hypotenuse}^2 \Rightarrow \sqrt{41} = \text{hypotenuse}$

$\cos\theta = \dfrac{5}{\sqrt{41}} = \dfrac{5\sqrt{41}}{41} \qquad \sin\theta = \dfrac{4}{\sqrt{41}} = \dfrac{4\sqrt{41}}{41}$

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{\frac{5\sqrt{41}}{41} - \frac{4\sqrt{41}}{41}}{\frac{5\sqrt{41}}{41} + \frac{4\sqrt{41}}{41}} = \frac{\sqrt{41}}{9\sqrt{41}} = \frac{1}{9}$$

80.

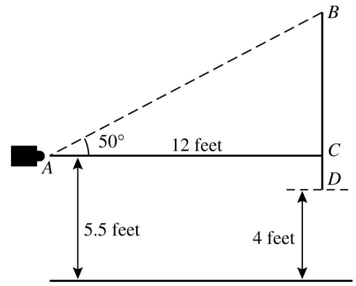


The height of the plane = $BC + CD$. We don't have information about any of the sides in $\triangle TCD$. We know that $TC = AB$ and $m\angle CTB = m\angle ABT = 8.1°$.

Then $\tan 8.1° = \dfrac{150}{AB} \Rightarrow AB \approx 1053.955$. Now find CD:

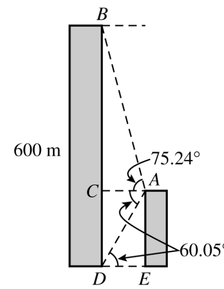
$\tan 67.4° \approx \dfrac{CD}{1053.955} \Rightarrow CD \approx 2531.964$. The height of the plane is
$BC + CD \approx 150 + 2532 \approx 2682$ feet

81.



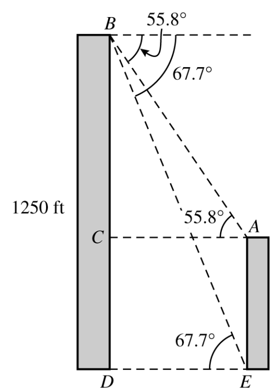
The height of the painting = $BC + CD$.
$CD = 5.5 - 4 = 1.5$ ft. Find BC as follows:
$\tan 50° = BC/12 \Rightarrow BC \approx 14.3$. The height of the painting is 14.3 + 1.5 = 15.8 feet.

82.



The height of the building $AE = 600 - BC$.
$\tan 75.24° = \dfrac{BC}{AC}$ and $\tan 60.05° = \dfrac{600 - BC}{AC}$.
So, $AC = \dfrac{BC}{\tan 75.24°}$ and $AC = \dfrac{600 - BC}{\tan 60.05°} \Rightarrow$.
$\dfrac{BC}{\tan 75.24°} = \dfrac{600 - BC}{\tan 60.05°} \Rightarrow BC \approx 411.7329$
Then $AE \approx 600 - 411.7329 \approx 188.27$ meters.

83.



The height of the building $AE = 1250 - BC$.

In $\triangle BDE$, we have $\tan 67.7° = \dfrac{1250}{DE} \Rightarrow$

$DE = \dfrac{1250}{\tan 67.7°} = AC$.

(*continued on next page*)

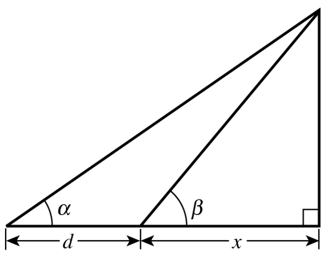
(*continued*)

Then in $\triangle BCA$, we have

$$\tan 55.8° = \frac{BC}{\frac{1250}{\tan 67.7°}} \Rightarrow$$

$BC = \dfrac{1250 \tan 55.8°}{\tan 67.7°} \approx 754$ ft. The height of the building is $AE \approx 1250 - 754 \approx 496$ ft .

84.



$\cot \alpha = \dfrac{d + x}{h} \Rightarrow h \cot \alpha - d = x,$ and

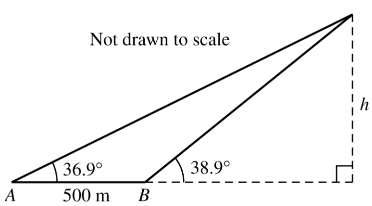
$\cot \beta = \dfrac{x}{h} \Rightarrow h \cot \beta = x.$

$h \cot \alpha - d = h \cot \beta \Rightarrow$

$h \cot \alpha - h \cot \beta = d \Rightarrow$

$$h(\cot \alpha - \cot \beta) = d \Rightarrow h = \frac{d}{\cot \alpha - \cot \beta}$$

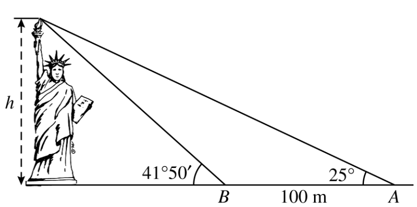
85.



Using the result from exercise 84, we have

$$h = \frac{500}{\cot 36.9° - \cot 38.9°} \approx 5402 \text{ m}$$

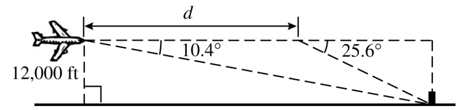
86.



Using the result from exercise 84, we have

$$h = \frac{100}{\cot 25° - \cot 41°50'} \approx 97 \text{ m}$$

87.

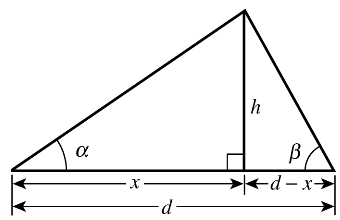


Using the result from exercise 84, we have

$$12,000 = \frac{d}{\cot 10.4° - \cot 25.6°} \approx 40,337 \text{ ft} .$$

Since the plane traveled 40,337 feet in two minutes, its speed is $\dfrac{40,337}{2} = 20,168.5$ feet per minute.

88.

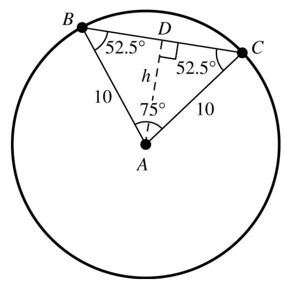


$\cot \alpha = \dfrac{x}{h} \Rightarrow h \cot \alpha = x,$ and $\cot \beta = \dfrac{d - x}{h} \Rightarrow$
$d - h \cot \beta = x.$

$h \cot \alpha = d - h \cot \beta \Rightarrow h \cot \alpha + h \cot \beta = d \Rightarrow$

$$h(\cot \alpha + \cot \beta) = d \Rightarrow h = \frac{d}{\cot \alpha + \cot \beta}$$

89.



$\triangle ABC$ is an isosceles triangle, so
$m\angle B = m\angle C = 52.5°$.

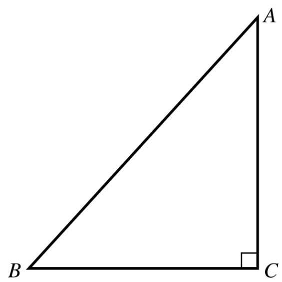
$\sin 52.5° = \dfrac{AD}{10} \Rightarrow AD \approx 7.9$.

$\cos 52.5° = \dfrac{CD}{10} \Rightarrow CD \approx 6.0876 \Rightarrow BC \approx 12.2$

$A_{\triangle ABC} \approx \dfrac{1}{2}(12.2)(7.9) \approx 48$ square units.

5.2 Critical Thinking/Discussion/Writing

90.



a. By the Pythagorean theorem, we have $AC^2 + BC^2 = AB^2$.

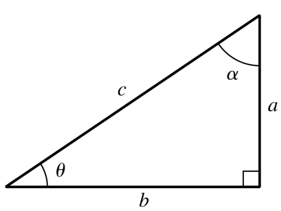
$\sin A = \dfrac{BC}{AB}$ and $\cos A = \dfrac{AC}{AB}$.

$$\sin^2 A + \cos^2 A = \left(\frac{BC}{AB}\right)^2 + \left(\frac{AC}{AB}\right)^2 = \frac{BC^2 + AC^2}{AB^2} = \frac{AB^2}{AB^2} = 1$$

b. From part (a), we have $\sin A = \dfrac{BC}{AB}$ and $\cos A = \dfrac{AC}{AB}$. The cofunction identities give us $\cos B = \sin A = \dfrac{BC}{AB}$ and $\sin B = \cos A = \dfrac{AC}{AB}$.

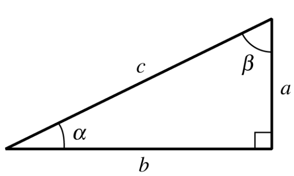
$$\sin A \cos B + \cos A \sin B = \frac{BC}{AB} \cdot \frac{BC}{AB} + \frac{AC}{AB} \cdot \frac{AC}{AB} = \frac{BC^2 + AC^2}{AB^2} = \frac{AB^2}{AB^2} = 1$$

91.



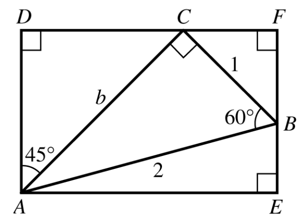
In the triangle, $\tan\theta = a$ if and only if $b = 1$.

92.



From geometry, we know that in any triangle, the longest side is opposite the largest angle and the shortest side is opposite the smallest angle. So, if $\alpha < \beta$, then $a < b$. Using the result from exercise 78, if $\tan\alpha = a$, then $b = 1$, and $\tan\beta = \dfrac{1}{a}$. Therefore $0 < a < 1 \Rightarrow \dfrac{1}{a} > 1$ and $\tan\beta > \tan\alpha$.

93.



(i) $\sin 60° = \dfrac{b}{2} \Rightarrow b = 2\sin 60° = 2\left(\dfrac{\sqrt{3}}{2}\right) = \sqrt{3}$

$\angle CAB = 180° - 90° - 60° = 30°$

(ii) $\cos 45° = \dfrac{AD}{b} \Rightarrow AD = b\cos 45° \Rightarrow$

$AD = \sqrt{3}\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\sqrt{6}}{2}$

Triangle ADC is an isosceles right triangle, so $DC = AD = \dfrac{\sqrt{6}}{2}$ and $m\angle ACD = 45°$.

(iii) $m\angle ACD + m\angle ACB + m\angle BCF = 180° \Rightarrow$

$45° + 90° + m\angle BCF = 180° \Rightarrow m\angle BCF = 45°$

$m\angle CBF = 180° - 45° - 90° = 45°$

(iv) $\cos 45° = \dfrac{CF}{1} \Rightarrow CF = \dfrac{\sqrt{2}}{2}$

Triangle FCB is an isosceles right triangle, so $FB = CF = \dfrac{\sqrt{2}}{2}$.

(v) $AE = DF = DC + CF = \dfrac{\sqrt{6}}{2} + \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{6}+\sqrt{2}}{2}$

$BE = FE - FB = DA - FB = \dfrac{\sqrt{6}}{2} - \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{6}-\sqrt{2}}{2}$

(vi) $m\angle BAE = 90° - (45° + m\angle BAC) = 90° - (45° + 30°) = 15°$

$m\angle ABE + m\angle BAE = 90° \Rightarrow$

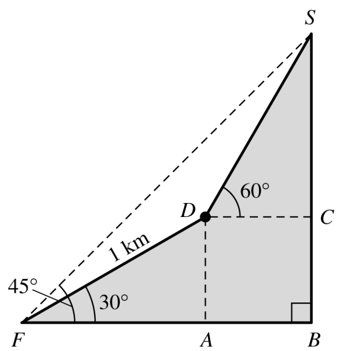
$m\angle ABE + 15° = 90° \Rightarrow m\angle ABE = 75°$

(vii) $\sin\angle BAE = \sin 15° = \dfrac{BE}{BA} = \dfrac{\frac{\sqrt{6}-\sqrt{2}}{2}}{2}$
$= \dfrac{\sqrt{6}-\sqrt{2}}{4}$

$\cos\angle BAE = \cos 15° = \dfrac{AE}{BA} = \dfrac{\frac{\sqrt{6}+\sqrt{2}}{2}}{2}$
$= \dfrac{\sqrt{6}+\sqrt{2}}{4}$

(viii) $\sin 75° = \cos(90° - 75°) = \cos 15° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$
$\cos 75° = \sin(90° - 75°) = \sin 15° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

94.



Since $\angle BFS = 45°$, $FB = SB = x$.

$\sin 30° = \dfrac{AD}{FD} \Rightarrow \dfrac{1}{2} = \dfrac{AD}{1} \Rightarrow$

$AD = \dfrac{1}{2} = BC \Rightarrow SC = x - \dfrac{1}{2}$

$\cos 30° = \dfrac{AF}{FD} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{AF}{1} \Rightarrow AF = \dfrac{\sqrt{3}}{2} \Rightarrow$

$DC = AB = BF - AF = x - \dfrac{\sqrt{3}}{2}$

$\tan 60° = \sqrt{3} = \dfrac{SC}{DC} = \dfrac{x - \frac{1}{2}}{x - \frac{\sqrt{3}}{2}} = \dfrac{2x-1}{2x-\sqrt{3}} \Rightarrow$

$\sqrt{3} = \dfrac{2x-1}{2x-\sqrt{3}} \Rightarrow 2\sqrt{3}x - 3 = 2x - 1 \Rightarrow$

$2\sqrt{3}x - 2x = 2 \Rightarrow \sqrt{3}x - x = 1 \Rightarrow$

$x\left(\sqrt{3}-1\right) = 1 \Rightarrow$

$x = \dfrac{1}{\sqrt{3}-1} \cdot \dfrac{\sqrt{3}+1}{\sqrt{3}+1} = \dfrac{\sqrt{3}+1}{2}$

5.2 Maintaining Skills

95. $\dfrac{\sqrt{3}-1}{\sqrt{3}+1} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} \cdot \dfrac{\sqrt{3}-1}{\sqrt{3}-1} = \dfrac{3-2\sqrt{3}+1}{3-1}$
$= \dfrac{4-2\sqrt{3}}{2} = 2 - \sqrt{3}$

96. $\dfrac{3+\sqrt{7}}{3-\sqrt{7}} = \dfrac{3+\sqrt{7}}{3-\sqrt{7}} \cdot \dfrac{3+\sqrt{7}}{3+\sqrt{7}} = \dfrac{9+6\sqrt{7}+7}{9-7}$
$= \dfrac{16+6\sqrt{7}}{2} = 8 + 3\sqrt{7}$

97. $\dfrac{5+2\sqrt{3}}{7+4\sqrt{3}} = \dfrac{5+2\sqrt{3}}{7+4\sqrt{3}} \cdot \dfrac{7-4\sqrt{3}}{7-4\sqrt{3}} = \dfrac{35-6\sqrt{3}-24}{49-48}$
$= 11 - 6\sqrt{3}$

98. $\dfrac{3}{5-\sqrt{3}} + \dfrac{2}{5+\sqrt{3}}$
$= \dfrac{3}{5-\sqrt{3}} \cdot \dfrac{5+\sqrt{3}}{5+\sqrt{3}} + \dfrac{2}{5+\sqrt{3}} \cdot \dfrac{5-\sqrt{3}}{5-\sqrt{3}}$
$= \dfrac{15+3\sqrt{3}}{25-3} + \dfrac{10-2\sqrt{3}}{25-3} = \dfrac{25+\sqrt{3}}{22}$

99. $75° \cdot \dfrac{\pi}{180°} = \dfrac{5\pi}{12}$

100. $70° \cdot \dfrac{\pi}{180°} = \dfrac{7\pi}{18}$

101. $-510° \cdot \dfrac{\pi}{180°} = -\dfrac{17\pi}{6}$

102. $435° \cdot \dfrac{\pi}{180°} = \dfrac{29\pi}{12}$

103. $\dfrac{1220}{360} = 3\,\text{R}140 \Rightarrow 1220 = 360 \cdot 3 + 140$

104. $\dfrac{2670}{360} = 7\,\text{R}150 \Rightarrow 2670 = 360 \cdot 7 + 150$

105. $\dfrac{-480}{360} = -2\,\text{R}\,240 \Rightarrow -480 = 360 \cdot (-2) + 240$

106. $\dfrac{-2025}{360} = -6\,\text{R}135 \Rightarrow -2025 = 360 \cdot (-6) + 135$

5.3 Trigonometric Functions of Any Angle; The Unit Circle

5.3 Practice Problems

1. $x=2,\ y=-5 \Rightarrow r=\sqrt{2^2+(-5)^2}=\sqrt{29}$

$\sin\theta=\frac{y}{r}=\frac{-5}{\sqrt{29}}=-\frac{5\sqrt{29}}{29}$ $\quad\csc\theta=\frac{r}{y}=-\frac{\sqrt{29}}{5}$

$\cos\theta=\frac{x}{r}=\frac{2}{\sqrt{29}}=\frac{2\sqrt{29}}{29}$ $\quad\sec\theta=\frac{r}{x}=\frac{\sqrt{29}}{2}$

$\tan\theta=\frac{y}{x}=\frac{-5}{2}=-\frac{5}{2}$ $\quad\cot\theta=\frac{x}{y}=-\frac{2}{5}$

2.a. $\theta=180°=\pi\Rightarrow$

$x=-1,\ y=0,\ r=\sqrt{(-1)^2+0^2}=1$

$\sin\theta=\frac{y}{r}=\frac{0}{1}=0$

$\cos\theta=\frac{x}{r}=\frac{-1}{1}=-1$

$\tan\theta=\frac{y}{x}=\frac{0}{-1}=0$

$\cot\theta=\frac{x}{y}=\frac{-1}{0}$, undefined

$\sec\theta=\frac{r}{x}=\frac{1}{-1}=-1$

$\csc\theta=\frac{r}{y}=\frac{1}{0}$, undefined

b. $\theta=270°=\frac{3\pi}{2}\Rightarrow$

$x=0,\ y=-1,\ r=\sqrt{0^2+(-1)^2}=1$

$\sin\theta=\frac{y}{r}=\frac{-1}{1}=-1$

$\cos\theta=\frac{x}{r}=\frac{0}{1}=0$

$\tan\theta=\frac{y}{x}=\frac{-1}{0}$, undefined

$\cot\theta=\frac{x}{y}=\frac{0}{-1}=0$

$\sec\theta=\frac{r}{x}=\frac{1}{0}$, undefined

$\csc\theta=\frac{r}{y}=\frac{1}{-1}=-1$

3.a. $1830°=30°+5\cdot 360°$, so 1830º is coterminal with 30º. Thus,

$\cos 1830°=\cos 30°=\frac{\sqrt{3}}{2}.$

b. $\frac{31\pi}{3}=\frac{\pi}{3}+15(2\pi)$, so $\frac{31\pi}{3}$ is coterminal with $\pi/3$. Thus,

$\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}\Rightarrow\sin\frac{31\pi}{3}=\frac{\sqrt{3}}{2}.$

4. Since $\sin\theta>0$ and $\cos\theta<0$, then θ lies in quadrant II.

5. Because $\tan\theta<0$ and $\cos\theta>0$, θ lies in quadrant IV. $\tan\theta=-\frac{4}{5}$, so let $x = 5$ and $y=-4$. $r=\sqrt{5^2+(-4)^2}=\sqrt{41}$

$\sin\theta=\frac{y}{r}=-\frac{4}{\sqrt{41}}=-\frac{4\sqrt{41}}{41}$;

$\sec\theta=\frac{r}{x}=\frac{\sqrt{41}}{5}$

6.a. $\theta=175°\Rightarrow\theta'=180°-175°=5°$

b. $\theta=\frac{5\pi}{3}\Rightarrow\theta'=2\pi-\frac{5\pi}{3}=\frac{\pi}{3}$

c. $\theta=8.22$ lies in quadrant II and is coterminal with $8.22-2\pi\approx 1.94$. The reference angle $\theta'\approx\pi-1.94\approx 1.20$.

7. $1020°=300°+2\cdot 360°$, so 1020º is coterminal with 300º. Because 300º lies in quadrant IV, $\theta'=360°-300°=60°$. In quadrant IV, $\cos\theta$ is positive, so $\cos 1020°=\cos 60°=0.5$.

8.a. $1035°=315°+2\cdot 360°$, so 1035º is coterminal with 315º. Because 315º lies in quadrant IV, $\theta'=360°-315°=45°$. In quadrant IV, $\csc\theta$ is negative, so $\csc 1035°=-\csc 45°=-\sqrt{2}$.

b. $\frac{17\pi}{6}=\frac{5\pi}{6}+2\pi$, so $\frac{17\pi}{6}$ is coterminal with $5\pi/6$, which lies in quadrant II. Thus, $\theta'=\pi-\frac{5\pi}{6}=\frac{\pi}{6}$. In quadrant II, $\cot\theta$ is negative, so $\cot\frac{17\pi}{6}=-\cot\frac{\pi}{6}=-\sqrt{3}$.

9. $\theta = 600° = 240° + 360°$.
240° lies in quadrant III, thus, $\theta' = 60°$, and $\cos\theta < 0$.

$\cos 60° = \frac{1}{2} \Rightarrow \cos 600° = \cos 240° = -\frac{1}{2}$.

$h = 67.8 - 67.2\left(-\frac{1}{2}\right) = 101.4$ m

10.a. From figure 5.45, we see that the endpoint of the arc of length $t = \frac{7\pi}{6}$ is $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

So, $\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$, $\sin\frac{7\pi}{6} = -\frac{1}{2}$, and

$\tan\frac{7\pi}{6} = \frac{\sin(7\pi/6)}{\cos(7\pi/6)} = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

b. $\theta = \frac{7\pi}{6}$ lies in quadrant III, so its sine and cosine are both negative and its tangent is positive. The reference angle is $\frac{7\pi}{6} - \pi = \frac{\pi}{6}$.

Then $\sin\frac{7\pi}{6} = -\sin\frac{\pi}{6} = -\frac{1}{2}$,

$\cos\frac{7\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$, and

$\tan\frac{7\pi}{6} = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$.

c. $\frac{7\pi}{6} = \frac{7\pi}{6}\cdot\frac{180}{\pi} = 210°$

Because 210° is in quadrant III, its reference angle is 210° − 180° = 30°. Then,

$\cos\frac{7\pi}{6} = \cos 210° = -\cos 30° = -\frac{\sqrt{3}}{2}$,

$\sin\frac{7\pi}{6} = \sin 210° = -\sin 30° = -\frac{1}{2}$, and

$\tan\frac{7\pi}{6} = \tan 210° = \tan 30° = \frac{\sqrt{3}}{3}$.

5.3 Basic Concepts and Skills

1. For a point $P(x, y)$ on the terminal side of an angle θ in standard position, we let $r = \underline{\sqrt{x^2 + y^2}}$. Then, $\sin\theta = \frac{y}{\underline{r}}$, $\cos\theta = \frac{x}{\underline{r}}$, and $\tan\theta = \frac{y}{\underline{x}}$.

2. If $P(x, y)$ is on the terminal side of a quadrantal angle, then either x or y equals <u>0</u>.

3. The reference angle θ' for a nonquadrantal angle θ in standard position is the acute angle formed by the terminal side of θ and the <u>x-axis</u>.

4. If θ_1 and θ_2 are coterminal angles, then $\sin\theta_1$ <u>equals</u> $\sin\theta_2$.

5. False. The value of a trigonometric function for any angle is the same for any point on the terminal side of θ.

6. False. In each quadrant, cosine and secant are either both positive or both negative, and sine and cosecant are either both positive or both negative.

7. $x = -3, y = 4 \Rightarrow r = \sqrt{(-3)^2 + 4^2} = 5$

$\sin\theta = \frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = -\frac{4}{3}$

$\csc\theta = \frac{5}{4}$, $\sec\theta = -\frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

8. $x = 4, y = -3 \Rightarrow r = \sqrt{4^2 + (-3)^2} = 5$

$\sin\theta = -\frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = -\frac{3}{4}$

$\csc\theta = -\frac{5}{3}$, $\sec\theta = \frac{5}{4}$, $\cot\theta = -\frac{4}{3}$

9. $x = 5, y = 12 \Rightarrow r = \sqrt{5^2 + 12^2} = 13$

$\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\tan\theta = \frac{12}{5}$,

$\csc\theta = \frac{13}{12}$, $\sec\theta = \frac{13}{5}$, $\cot\theta = \frac{5}{12}$

10. $x = -12,\ y = 5 \Rightarrow r = \sqrt{(-12)^2 + 5^2} = 13$

$\sin\theta = \frac{5}{13}$, $\cos\theta = -\frac{12}{13}$, $\tan\theta = -\frac{5}{12}$,

$\csc\theta = \frac{13}{5}$, $\sec\theta = -\frac{13}{12}$, $\cot\theta = -\frac{12}{5}$

11. $x = 7,\ y = 24 \Rightarrow r = \sqrt{7^2 + 24^2} = 25$

$\sin\theta = \frac{24}{25}$, $\cos\theta = \frac{7}{25}$, $\tan\theta = \frac{24}{7}$,

$\csc\theta = \frac{25}{24}$, $\sec\theta = \frac{25}{7}$, $\cot\theta = \frac{7}{24}$

12. $x = -24,\ y = 7 \Rightarrow r = \sqrt{(-24)^2 + 7^2} = 25$

$\sin\theta = \frac{7}{25}$, $\cos\theta = -\frac{24}{25}$, $\tan\theta = -\frac{7}{24}$,

$\csc\theta = \frac{25}{7}$, $\sec\theta = -\frac{25}{24}$, $\cot\theta = -\frac{24}{7}$

13. $x=-24,\ y=-7 \Rightarrow r=\sqrt{(-24)^2+(-7)^2}=25$

$\sin\theta=-\frac{7}{25},\ \cos\theta=-\frac{24}{25},\ \tan\theta=\frac{7}{24},$
$\csc\theta=-\frac{25}{7},\ \sec\theta=-\frac{25}{24},\ \cot\theta=\frac{24}{7}$

14. $x=-7,\ y=24 \Rightarrow r=\sqrt{(-7)^2+24^2}=25$

$\sin\theta=\frac{24}{25},\ \cos\theta=-\frac{7}{25},\ \tan\theta=-\frac{24}{7},$
$\csc\theta=\frac{25}{24},\ \sec\theta=-\frac{25}{7},\ \cot\theta=-\frac{7}{24}$

15. $x=1, y=1 \Rightarrow r=\sqrt{1^2+1^2}=\sqrt{2}$

$\sin\theta=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2},\ \cos\theta=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2},\ \tan\theta=1$
$\csc\theta=\sqrt{2},\quad \sec\theta=\sqrt{2},\quad \cot\theta=1$

16. $x=-3, y=-3 \Rightarrow r=\sqrt{(-3)^2+(-3)^2}=3\sqrt{2}$

$\sin\theta=-\frac{3}{3\sqrt{2}}=-\frac{\sqrt{2}}{2},\ \cos\theta=-\frac{3}{3\sqrt{2}}=-\frac{\sqrt{2}}{2},$
$\csc\theta=-\frac{3\sqrt{2}}{3}=-\sqrt{2},\ \sec\theta=-\frac{3\sqrt{2}}{3}=-\sqrt{2},$
$\tan\theta=\frac{-3}{-3}=1,\ \cot\theta=\frac{-3}{-3}=1$

17. $x=\sqrt{2}, y=\sqrt{2} \Rightarrow r=\sqrt{\left(\sqrt{2}\right)^2+\left(\sqrt{2}\right)^2}=2$

$\sin\theta=\frac{\sqrt{2}}{2},\quad \csc\theta=\frac{2}{\sqrt{2}}=\sqrt{2},$
$\cos\theta=\frac{\sqrt{2}}{2},\quad \sec\theta=\frac{2}{\sqrt{2}}=\sqrt{2},$
$\tan\theta=\frac{\sqrt{2}}{\sqrt{2}}=1,\quad \cot\theta=\frac{\sqrt{2}}{\sqrt{2}}=1$

18. $x=-3, y=\sqrt{3} \Rightarrow r=\sqrt{3^2+\left(\sqrt{3}\right)^2}=2\sqrt{3}$

$\sin\theta=\frac{\sqrt{3}}{2\sqrt{3}}=\frac{1}{2},\quad \csc\theta=2$
$\cos\theta=-\frac{3}{2\sqrt{3}}=-\frac{\sqrt{3}}{2},\ \sec\theta=-\frac{2\sqrt{3}}{3},$
$\tan\theta=-\frac{\sqrt{3}}{3},\quad \cot\theta=-\frac{3}{\sqrt{3}}=-\sqrt{3}$

19. $x=\sqrt{3}, y=-1 \Rightarrow r=\sqrt{\left(\sqrt{3}\right)^2+(-1)^2}=2$

$\sin\theta=-\frac{1}{2},\quad \csc\theta=-2,$
$\cos\theta=\frac{\sqrt{3}}{2},\quad \sec\theta=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3},$
$\tan\theta=-\frac{\sqrt{3}}{3},\quad \cot\theta=-\frac{3}{\sqrt{3}}=-\sqrt{3}$

20. $x=\sqrt{13}, y=\sqrt{3} \Rightarrow r=\sqrt{\left(\sqrt{13}\right)^2+\left(\sqrt{3}\right)^2}=4$

$\sin\theta=\frac{\sqrt{3}}{4},\quad \csc\theta=\frac{4}{\sqrt{3}}=\frac{4\sqrt{3}}{3},$
$\cos\theta=\frac{\sqrt{13}}{4},\quad \sec\theta=\frac{4}{\sqrt{13}}=\frac{4\sqrt{13}}{13},$
$\tan\theta=\frac{\sqrt{3}}{\sqrt{13}}=\frac{\sqrt{39}}{13},\quad \cot\theta=\frac{\sqrt{13}}{\sqrt{3}}=\frac{\sqrt{39}}{3}$

21. $x=5, y=-2 \Rightarrow r=\sqrt{5^2+(-2)^2}=\sqrt{29}$

$\sin\theta=-\frac{2}{\sqrt{29}}=-\frac{2\sqrt{29}}{29},\quad \csc\theta=-\frac{\sqrt{29}}{2},$
$\cos\theta=\frac{5}{\sqrt{29}}=\frac{5\sqrt{29}}{29},\quad \sec\theta=\frac{\sqrt{29}}{5},$
$\tan\theta=-\frac{2}{5},\quad \cot\theta=-\frac{5}{2}$

22. $x=-3, y=5 \Rightarrow r=\sqrt{(-3)^2+5^2}=\sqrt{34}$

$\sin\theta=\frac{5\sqrt{34}}{34},\ \cos\theta=-\frac{3\sqrt{34}}{34},\ \tan\theta=-\frac{5}{3},$
$\csc\theta=\frac{\sqrt{34}}{5},\ \sec\theta=-\frac{\sqrt{34}}{3},\ \cot\theta=-\frac{3}{5}$

23. 450° is coterminal with 450° − 360° = 90°. Thus, sin 450° = sin 90° = 1.

24. 450° is coterminal with 450° − 360° = 90°. Thus, cos 450° = cos 90° = 0.

25. −90° is coterminal with 360° − 90° = 270°. Thus, cos (−90°) = cos 270° = 0.

26. −90° is coterminal with 360° − 90° = 270°. Thus sin (−90°) = sin 270° = −1.

27. 450° is coterminal with 450° − 360° = 90°. Thus, tan 450° = tan 90°, which is undefined.

28. 540° is coterminal with 540° − 360° = 180°. Thus, cot 540° = cot 180°, which is undefined.

29. –540° is coterminal with 540° + 2(360°) = 180°. Thus, tan 540° = tan 180° = 0.

30. 1080° is coterminal with 1080° − 3(360°) = 0°. Thus, sec 1080° = sec 0° = 1.

31. 900° is coterminal with 900° − 2(360°) = 180°. Thus, csc 900° = csc 180°, which is undefined.

32. 1080° is coterminal with 1080° − 3(360°) = 0°. Thus, csc 1080° = csc 0°, which is undefined.

33. −1530° is coterminal with 5(360°) − 1530° = 270°. Thus, sin (−1530°) = sin 270° = −1.

34. −2610° is coterminal with 8(360°) − 2610° = 270°. Thus, cos (−2610°) = cos 270° = 0.

35. $\cos(5\pi) = \cos(\pi + 2\cdot 2\pi) = \cos\pi = -1$

36. $\sin(3\pi) = \sin(\pi + 2\pi) = \sin\pi = 0$

37. $\tan(4\pi) = \tan(0 + 2\cdot 2\pi) = \tan 0 = 0$

38. $\cot\left(\frac{7\pi}{2}\right) = \cot\left(\frac{3\pi}{2} + 2\pi\right) = \cot\left(\frac{3\pi}{2}\right) = 0$

39. $\csc\left(\frac{5\pi}{2}\right) = \csc\left(\frac{\pi}{2} + 2\pi\right) = \csc\left(\frac{\pi}{2}\right) = 1$

40. $\sec(7\pi) = \sec(\pi + 3\cdot 2\pi) = \sec\pi = -1$

41. $\sin(-2\pi) = \sin(0 + (-1)2\pi) = \sin 0 = 0$

42. $\cos(-5\pi) = \cos(\pi + (-3)\cdot 2\pi) = \cos\pi = -1$

43. $\cos\left(-\frac{3\pi}{2}\right) = \cos\left(\frac{\pi}{2} + (-1)2\pi\right) = \cos\left(\frac{\pi}{2}\right) = 0$

44. $\sin\left(-\frac{\pi}{2}\right) = \sin\left(\frac{3\pi}{2} + (-1)2\pi\right) = \sin\left(\frac{3\pi}{2}\right) = -1$

45. $\sin\theta < 0$ and $\cos\theta < 0 \Rightarrow \theta$ is in quadrant III.

46. $\sin\theta < 0$ and $\tan\theta > 0 \Rightarrow \theta$ is in quadrant III.

47. $\sin\theta > 0$ and $\cos\theta < 0 \Rightarrow \theta$ is in quadrant II.

48. $\tan\theta > 0$ and $\csc\theta < 0 \Rightarrow \theta$ is in quadrant III.

49. $\cos\theta > 0$ and $\csc\theta < 0 \Rightarrow \theta$ is in quadrant IV.

50. $\cos\theta < 0$ and $\cot\theta > 0 \Rightarrow \theta$ is in quadrant III.

51. $\sec\theta < 0$ and $\csc\theta > 0 \Rightarrow \theta$ is in quadrant II.

52. $\sec\theta < 0$ and $\tan\theta > 0 \Rightarrow \theta$ is in quadrant III.

53. $\sin\theta = -\frac{5}{13} \Rightarrow y = -5,\ r = 13$
$\sqrt{x^2 + (-5)^2} = 13 \Rightarrow x^2 + 25 = 169 \Rightarrow$
$x^2 = 144 \Rightarrow x = \pm 12$
Since θ is in quadrant III, x is negative, so $x = -12$.

54. $\sin\theta = -\frac{5}{13} \Rightarrow y = -5,\ r = 13$
$\sqrt{x^2 + (-5)^2} = 13 \Rightarrow x^2 + 25 = 169 \Rightarrow$
$x^2 = 144 \Rightarrow x = \pm 12$
Since θ is in quadrant IV, x is positive, so $x = 12$.

55. $\cos\theta = \frac{7}{25} \Rightarrow x = 7,\ r = 25$
$\sqrt{7^2 + y^2} = 25 \Rightarrow 49 + y^2 = 625 \Rightarrow$
$y^2 = 576 \Rightarrow y = \pm 24$
Since θ is in quadrant I, y is positive, so $y = 24$.

56. $\cos\theta = \frac{7}{25} \Rightarrow x = 7,\ r = 25$
$\sqrt{7^2 + y^2} = 25 \Rightarrow 49 + y^2 = 625 \Rightarrow$
$y^2 = 576 \Rightarrow y = \pm 24$
Since θ is in quadrant IV, y is negative, so $y = -24$.

57. $\cos\theta = -\frac{5}{13}$, θ in quadrant III $\Rightarrow x < 0, y < 0$
$x = -5, r = 13 \Rightarrow 13^2 = (-5)^2 + y^2 \Rightarrow y = -12$
$\sin\theta = -\frac{12}{13},\ \cos\theta = -\frac{5}{13},\ \tan\theta = \frac{12}{5},$
$\csc\theta = -\frac{13}{12},\ \sec\theta = -\frac{13}{5},\ \cot\theta = \frac{5}{12}$

58. $\tan\theta = -\frac{3}{4}, \theta$ in quadrant IV $\Rightarrow x > 0, y < 0$
$y = -3, x = 4 \Rightarrow r = \sqrt{4^2 + (-3)^2} = 5$
$\sin\theta = -\frac{3}{5},\ \cos\theta = \frac{4}{5},\ \tan\theta = -\frac{3}{4},$
$\csc\theta = -\frac{5}{3},\ \sec\theta = \frac{5}{4},\ \cot\theta = -\frac{4}{3}$

59. $\cot\theta = -\frac{3}{4}, \theta$ in quadrant II $\Rightarrow x < 0, y > 0$
$x = -3, y = 4 \Rightarrow r = \sqrt{(-3)^2 + 4^2} = 5$
$\sin\theta = \frac{4}{5},\ \cos\theta = -\frac{3}{5},\ \tan\theta = -\frac{4}{3},$
$\csc\theta = \frac{5}{4},\ \sec\theta = -\frac{5}{3},\ \cot\theta = -\frac{3}{4}$

60. $\sec\theta = \frac{4}{\sqrt{7}}$, θ in quadrant IV $\Rightarrow x > 0, y < 0$

$x = \sqrt{7}, r = 4 \Rightarrow 4^2 = \left(\sqrt{7}\right)^2 + y^2 \Rightarrow y = -3$

$\sin\theta = -\frac{3}{4}$, $\cos\theta = \frac{\sqrt{7}}{4}$, $\tan\theta = -\frac{3\sqrt{7}}{7}$,

$\csc\theta = -\frac{4}{3}$, $\sec\theta = \frac{4\sqrt{7}}{7}$, $\cot\theta = -\frac{\sqrt{7}}{3}$

61. $\sin\theta = \frac{3}{5}, \tan\theta < 0 \Rightarrow \theta$ is in quadrant II and $x < 0$. $y = 3, r = 5 \Rightarrow 5^2 = x^2 + 3^2 \Rightarrow x = -4$

$\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$,

$\csc\theta = \frac{5}{3}$, $\sec\theta = -\frac{5}{4}$, $\cot\theta = -\frac{4}{3}$

62. $\cot\theta = \frac{3}{2}, \sec\theta > 0 \Rightarrow \theta$ is in quadrant I and $x > 0, y > 0$. $x = 3, y = 2 \Rightarrow r = \sqrt{3^2 + 2^2} = \sqrt{13}$

$\sin\theta = \frac{2\sqrt{13}}{13}$, $\cos\theta = \frac{3\sqrt{13}}{13}$, $\tan\theta = \frac{2}{3}$,

$\csc\theta = \frac{\sqrt{13}}{2}$, $\sec\theta = \frac{\sqrt{13}}{3}$, $\cot\theta = \frac{3}{2}$

63. $\sec\theta = 3, \sin\theta < 0 \Rightarrow \theta$ is in Quadrant IV and $y < 0$. $r = 3, x = 1 \Rightarrow 3^2 = 1^2 + y^2 \Rightarrow y = -2\sqrt{2}$

$\sin\theta = -\frac{2\sqrt{2}}{3}$, $\cos\theta = \frac{1}{3}$, $\tan\theta = -2\sqrt{2}$,

$\csc\theta = -\frac{3\sqrt{2}}{4}$, $\sec\theta = 3$, $\cot\theta = -\frac{\sqrt{2}}{4}$

64. $\tan\theta = -2, \sin\theta > 0 \Rightarrow \theta$ is in Quadrant II and $x < 0$, $y > 0$.

$y = 2, x = -1 \Rightarrow r = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$

$\sin\theta = \frac{2\sqrt{5}}{5}$, $\cos\theta = -\frac{\sqrt{5}}{5}$, $\tan\theta = -2$,

$\csc\theta = \frac{\sqrt{5}}{2}$, $\sec\theta = -\sqrt{5}$, $\cot\theta = -\frac{1}{2}$

65. 1470° is coterminal with $1470° - 4 \cdot 360° = 30°$. Thus, $\sin 1470° = \sin 30° = \frac{1}{2}$.

66. 1860° is coterminal with $1860° - 5 \cdot 360° = 60°$. Thus, $\cos 1860° = \cos 60° = \frac{1}{2}$.

67. 1125° is coterminal with $1125° - 3 \cdot 360° = 45°$. Thus, $\tan 1125° = \tan 45° = 1$.

68. 2190° is coterminal with $2190° - 6 \cdot 360° = 30°$. Thus, $\sec 2190° = \sec 30° = \frac{2\sqrt{3}}{3}$.

69. 2130° is coterminal with $2130° - 5 \cdot 360° = 330°$. 330° lies in quadrant IV with reference angle $\theta' = 360° - 330° = 30°$. In quadrant IV, $\csc\theta < 0$, so $\csc 2130° = -\csc 30° = -2$.

70. sec 1410° is coterminal with $1410° - 3 \cdot 360° = 330°$. 330° lies in quadrant IV with reference angle $\theta' = 360° - 330° = 30°$. In quadrant IV, $\sec\theta > 0$, so $\sec 1410° = \sec 30° = \frac{2\sqrt{3}}{3}$.

71. 690° is coterminal with $690° - 360° = 330°$. 330° lies in quadrant IV with reference angle $\theta' = 360° - 330° = 30°$. In quadrant IV, $\tan\theta < 0$, so $\tan 690° = -\tan 30° = -\frac{\sqrt{3}}{3}$.

72. 2100° is coterminal with $2100° - 5 \cdot 360° = 300°$. 300° lies in quadrant IV with reference angle $\theta' = 360° - 300° = 60°$. In quadrant IV, $\cot\theta < 0$, so $\cot 2100° = -\cot 60° = -\frac{\sqrt{3}}{3}$.

73. $\frac{13\pi}{3} = 4\pi + \frac{\pi}{3} \Rightarrow \frac{13\pi}{3}$ is coterminal with $\frac{\pi}{3}$ and lies in quadrant I. In quadrant I, $\sin\theta > 0$, so $\sin\left(\frac{13\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

74. $\frac{19\pi}{3} = 6\pi + \frac{\pi}{3} \Rightarrow \frac{19\pi}{3}$ is coterminal with $\frac{\pi}{3}$ and lies in quadrant I. In quadrant I, $\cos\theta > 0$, so $\cos\left(\frac{19\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$.

75. $\frac{29\pi}{6} = 4\pi + \frac{5\pi}{6} \Rightarrow \frac{29\pi}{6}$ is coterminal with $\frac{5\pi}{6}$ and lies in quadrant II. The reference angle is $\pi - \frac{5\pi}{6} = \frac{\pi}{6}$. In quadrant II, $\cos\theta < 0$, so $\cos\frac{29\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$.

76. $\frac{31\pi}{6} = 4\pi + \frac{7\pi}{6} \Rightarrow \frac{31\pi}{6}$ is coterminal with $\frac{7\pi}{6}$ and lies in quadrant III. The reference angle is $\frac{7\pi}{6} - \pi = \frac{\pi}{6}$. In quadrant III, $\sin\theta < 0$, so $\sin\frac{31\pi}{6} = -\sin\frac{\pi}{6} = -\frac{1}{2}$.

77. $\frac{37\pi}{4} = 8\pi + \frac{5\pi}{4} \Rightarrow \frac{37\pi}{4}$ is coterminal with $\frac{5\pi}{4}$ and lies in quadrant III. The reference angle is $\frac{5\pi}{4} - \pi = \frac{\pi}{4}$. In quadrant III, $\tan\theta > 0$, so $\tan\frac{37\pi}{4} = \tan\frac{\pi}{4} = 1$.

78. $\frac{35\pi}{4} = 8\pi + \frac{3\pi}{4} \Rightarrow \frac{35\pi}{4}$ is coterminal with $\frac{3\pi}{4}$ and lies in quadrant II. The reference angle is $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$. In quadrant II, $\cot\theta < 0$, so $\tan\frac{35\pi}{4} = -\tan\frac{\pi}{4} = -1$.

79. $\frac{29\pi}{3} = 8\pi + \frac{5\pi}{3} \Rightarrow \frac{29\pi}{3}$ is coterminal with $\frac{5\pi}{3}$ and lies in quadrant IV. In quadrant IV, $\sec\theta > 0$, so $\sec\left(\frac{29\pi}{3}\right) = \sec\frac{5\pi}{3} = 2$.

80. $\frac{43\pi}{6} = 6\pi + \frac{7\pi}{6} \Rightarrow \frac{43\pi}{6}$ is coterminal with $\frac{7\pi}{6}$ and lies in quadrant III. The reference angle is $\frac{7\pi}{6} - \pi = \frac{\pi}{6}$. In quadrant III, $\csc\theta < 0$, so $\csc\frac{43\pi}{6} = -\csc\frac{\pi}{6} = -2$.

81.a. From figure 5.46, we see that the endpoint of the arc of length $s = \frac{2\pi}{3}$ is $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. So,

$\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{2\pi}{3} = -\frac{1}{2}$, and

$\tan\frac{2\pi}{3} = \frac{\sin(2\pi/3)}{\cos(2\pi/3)} = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$.

b. $\theta = \frac{2\pi}{3}$ lies in quadrant II, so its sine is positive and its cosine and tangent are both negative. The reference angle is $\pi - \frac{2\pi}{3} = \frac{\pi}{3}$. Then

$\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$,

$\cos\frac{2\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$, and

$\tan\frac{2\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}$.

c. $\frac{2\pi}{3} = \frac{2\pi}{3} \cdot \frac{180}{\pi} = 120°$

Because 120° is in quadrant II, its reference angle is 180° − 120° = 60°. Then,

$\sin\frac{2\pi}{3} = \sin 60° = \frac{\sqrt{3}}{2}$,

$\cos\frac{2\pi}{3} = -\cos 60° = -\frac{1}{2}$, and

$\tan\frac{2\pi}{3} = \tan 120° = -\tan 60° = -\sqrt{3}$.

82.a. From figure 5.46, we see that the endpoint of the arc of length $s = \frac{3\pi}{4}$ is $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

So, $\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$, $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$, and

$\tan\frac{3\pi}{4} = \frac{\sin(3\pi/4)}{\cos(3\pi/4)} = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$.

b. $\theta = \frac{3\pi}{4}$ lies in quadrant II, so its sine is positive and its cosine and tangent are both negative. The reference angle is $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$. Then $\sin\frac{3\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$,

$\cos\frac{3\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$, and

$\tan\frac{3\pi}{4} = -\tan\frac{\pi}{4} = -1$.

c. $\frac{3\pi}{4} = \frac{3\pi}{4} \cdot \frac{180}{\pi} = 135°$

Because 135° is in quadrant II, its reference angle is 180° − 135° = 45°. Then,

$\sin\frac{3\pi}{4} = \sin 45° = \frac{\sqrt{2}}{2}$,

$\cos\frac{3\pi}{4} = -\cos 45° = -\frac{\sqrt{2}}{2}$, and

$\tan\frac{3\pi}{4} = -\tan 45° = -1$.

83.a. From figure 5.46, we see that the endpoint of the arc of length $s = \frac{11\pi}{6}$ is $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$. So,

$\sin\frac{11\pi}{6} = -\frac{1}{2}$, $\cos\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$, and

$\tan\frac{11\pi}{6} = \frac{\sin(2\pi/3)}{\cos(2\pi/3)} = \frac{-1/2}{\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$.

b. $\theta = \frac{11\pi}{6}$ lies in quadrant IV, so its sine and tangent are both negative and its cosine is positive. The reference angle is $2\pi - \frac{11\pi}{6} = \frac{\pi}{6}$. Then

$\sin\frac{11\pi}{6} = -\sin\frac{\pi}{6} = -\frac{1}{2}$,

$\cos\frac{11\pi}{6} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, and

$\tan\frac{11\pi}{6} = -\tan\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$.

c. $\frac{11\pi}{6} = \frac{11\pi}{6}\cdot\frac{180}{\pi} = 330°$

Because 330° is in quadrant IV, its reference angle is 360° − 330° = 30°. Then,

$\sin\frac{11\pi}{6} = -\sin 30° = -\frac{1}{2}$,

$\cos\frac{11\pi}{6} = \cos 30° = \frac{\sqrt{3}}{2}$, and

$\tan\frac{11\pi}{6} = -\tan 30° = -\frac{\sqrt{3}}{3}$.

84.a. From figure 5.46, we see that the endpoint of the arc of length $s = \frac{4\pi}{3}$ is $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

So, $\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$, $\cos\frac{4\pi}{3} = -\frac{1}{2}$, and

$\tan\frac{4\pi}{3} = \frac{\sin(4\pi/3)}{\cos(4\pi/3)} = \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3}$.

b. $\theta = \frac{4\pi}{3}$ lies in quadrant III, so its sine and cosine are both negative and its tangent is positive. The reference angle is $\frac{4\pi}{3} - \pi = \frac{\pi}{3}$.

Then $\sin\frac{4\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$,

$\cos\frac{4\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$, and

$\tan\frac{4\pi}{3} = \tan\frac{\pi}{3} = \sqrt{3}$.

c. $\frac{4\pi}{3} = \frac{4\pi}{3}\cdot\frac{180}{\pi} = 240°$

Because 240° is in quadrant III, its reference angle is 240° − 180° = 60°. Then,

$\sin\frac{4\pi}{3} = -\sin 60° = -\frac{\sqrt{3}}{2}$,

$\cos\frac{4\pi}{3} = -\cos 60° = -\frac{1}{2}$, and

$\tan\frac{4\pi}{3} = \tan 60° = \sqrt{3}$.

85.a. From figure 5.46, we see that the endpoint of the arc of length $s = \frac{5\pi}{4}$ is $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

So, $\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$, $\cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$, and

$\tan\frac{5\pi}{4} = \frac{\sin(5\pi/4)}{\cos(5\pi/4)} = \frac{-\sqrt{2}/2}{-\sqrt{2}/2} = 1$.

b. $\theta = \frac{5\pi}{4}$ lies in quadrant III, so its sine and cosine are both negative and its tangent is positive. The reference angle is $\frac{5\pi}{4} - \pi = \frac{\pi}{4}$.

Then $\sin\frac{5\pi}{4} = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$,

$\cos\frac{5\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$, and

$\tan\frac{5\pi}{4} = \tan\frac{\pi}{4} = 1$.

c. $\frac{5\pi}{4} = \frac{5\pi}{4}\cdot\frac{180}{\pi} = 225°$

Because 225° is in quadrant III, its reference angle is 225° − 180° = 45°. Then,

$\sin\frac{5\pi}{4} = -\sin 45° = -\frac{\sqrt{2}}{2}$,

$\cos\frac{5\pi}{4} = -\cos 45° = -\frac{\sqrt{2}}{2}$, and

$\tan\frac{5\pi}{4} = \tan 45° = 1$.

86.a. From figure 5.46, we see that the endpoint of the arc of length $s = \dfrac{5\pi}{3}$ is $\left(\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$. So,

$$\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2},\ \cos\frac{5\pi}{3} = \frac{1}{2},\ \text{and}$$

$$\tan\frac{5\pi}{3} = \frac{\sin(5\pi/3)}{\cos(5\pi/3)} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}.$$

b. $\theta = \dfrac{5\pi}{3}$ lies in quadrant IV, so its sine and tangent are both negative and its cosine is positive. The reference angle is $2\pi - \dfrac{5\pi}{3} = \dfrac{\pi}{3}$. Then

$$\sin\frac{5\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2},$$

$$\cos\frac{5\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2},\ \text{and}$$

$$\tan\frac{5\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}.$$

c. $\dfrac{5\pi}{3} = \dfrac{5\pi}{3}\cdot\dfrac{180}{\pi} = 300°$

Because 300° is in quadrant III, its reference angle is 360° − 300° = 60°. Then,

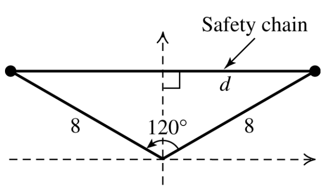
$$\sin\frac{5\pi}{3} = -\sin 60° = -\frac{\sqrt{3}}{2},$$

$$\cos\frac{5\pi}{3} = -\cos 60° = -\frac{1}{2},\ \text{and}$$

$$\tan\frac{5\pi}{3} = \tan 60° = \sqrt{3}.$$

5.3 Applying the Concepts

87.

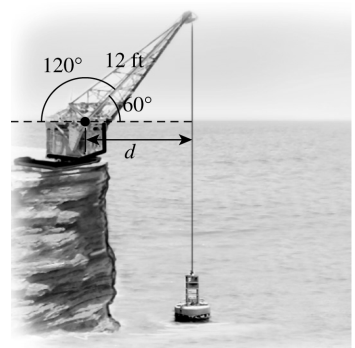


From geometry, we know that an altitude drawn from the vertex of an isosceles triangle bisects the vertex angle and also bisects the base of the triangle. Therefore,

$$\sin 60° = \frac{d}{8} \Rightarrow d = 8\sin 60° = 8\left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3}.$$

The length of the chain is $2\cdot 4\sqrt{3} = 8\sqrt{3} \approx 13.86$ ft.

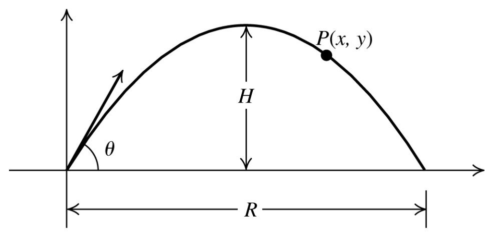
88.



$$\cos 60° = \frac{d}{12} \Rightarrow d = 12\cos 60° = 6$$

The buoy is 6 feet from the cliff.

Use the figure below for exercises 89−96.



89. $H = \dfrac{1}{64}(44\sin 30°)^2 = 7.5625$ ft

$t = \dfrac{44\sin 30°}{16} = 1.375$ sec

$R = \dfrac{44^2\sin 30°\cos 30°}{16} \approx 52.39$ ft

90. $H = \dfrac{1}{64}(44\sin 45°)^2 = 15.125$ ft

$t = \dfrac{44\sin 45°}{16} \approx 1.94$ sec

$R = \dfrac{44^2\sin 45°\cos 45°}{16} = 60.5$ ft

91. $H = \dfrac{1}{64}(44\sin 60°)^2 = 22.6875$ ft

$t = \dfrac{44\sin 60°}{16} \approx 2.38$ sec

$R = \dfrac{44^2\sin 60°\cos 60°}{16} \approx 52.39$ ft

92. $H = \dfrac{1}{64}(44\sin 90°)^2 = 30.25$ ft

$t = \dfrac{44\sin 90°}{16} = 2.75$ sec

$R = \dfrac{44^2\sin 90°\cos 90°}{16} = 0$ ft

93.a. $y = x\tan 45° - \dfrac{16\sec^2 45°}{80^2}x^2$
$= x(1) - \dfrac{16(\sqrt{2})^2}{80^2}x^2 = x - \dfrac{x^2}{200}$

b. $y = 100 - \dfrac{1}{200}(100^2) = 50$ ft

94. $H = \dfrac{1}{64}(80\sin 45°)^2 = 50$ ft

95. $t = \dfrac{80\sin 45°}{16} \approx 3.54$ sec

96. $R = \dfrac{80^2\sin 45°\cos 45°}{16} = 200$ ft

97. $A = \dfrac{3(2)\cos 60°}{\sqrt{7+9\cos^2 60°}} \approx 0.99$ ft

98.a. $\theta = 0°$ for a point due east of the sound source. $D = 25 + 15\cos 0° = 40$ dB

b. $\theta = 180°$ for a point due west of the sound source. $D = 25 + 15\cos 180° = 10$ dB

c. $\theta = 270°$ for a point due south of the sound source. $D = 25 + 15\cos 270° = 25$ dB

99.a. $h = 100\sin 30° = 100 \cdot \dfrac{1}{2} = 50$ ft

b. $h = 100\sin 60° = 100 \cdot \dfrac{\sqrt{3}}{2} = 50\sqrt{3} \approx 86.6$ ft

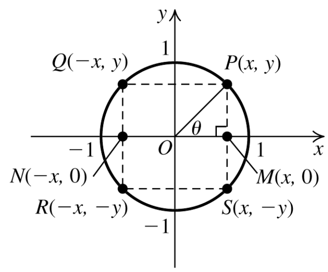
100.a. $V = 166\cos\left(120\pi \cdot \dfrac{1}{120}\right) = 166\cos\pi$
$= 166(-1) = -166$ volts

b. $V = 166\cos\left(120\pi \cdot \dfrac{1}{60}\right) = 166\cos 2\pi$
$= 166(1) = 166$ volts

c. $V = 166\cos\left(120\pi \cdot \dfrac{1}{72}\right) = 166\cos\dfrac{5\pi}{3}$
$= 166\left(\dfrac{1}{2}\right) = 83$ volts

5.3 Beyond the Basics

For exercises 101−104, refer to the following figure.



101. Since $\sin(-\theta) = -\sin\theta$, the y-value of the point on the unit circle on the terminal side of $-\theta$ equals the opposite of y-value of the point on the unit circle on the terminal side of θ. Since $\cos(-\theta) = \cos\theta$, the x-value of the point on the unit circle on the terminal side of $-\theta$ equals the x-value of the point on the unit circle on the terminal side of θ. Thus, the coordinates of the point on the unit circle are $(x, -y)$, and the point on the unit circle is S. The triangle we are seeking is $\triangle SOM$.

102. Since $\sin\theta = \sin(180° - \theta)$, the y-value of the point on the unit circle on the terminal side of θ equals the y-value of the point on the unit circle on the terminal side of $180° - \theta$. Since $\cos\theta = -\cos(180° - \theta)$, the x-value of the point on the unit circle on the terminal side of θ is the opposite of the x-value of the point on the unit circle on the terminal side of $180° - \theta$. Thus, the coordinates of the point on the unit circle are $(-x, y)$, and the point on the unit circle is Q. The triangle we are seeking is $\triangle QON$.

103. Since $\sin\theta = -\sin(180° + \theta)$, the y-value of the point on the unit circle on the terminal side of θ is the opposite the y-value of the point on the unit circle on the terminal side of $180° + \theta$. Since $\cos\theta = -\cos(180° + \theta)$, the x-value of the point on the unit circle on the terminal side of θ is the opposite of the x-value of the point on the unit circle on the terminal side of $180° + \theta$. Thus, the coordinates of the point on the unit circle are $(-x, -y)$, and the point on the unit circle is R. The triangle we are seeking is $\triangle RON$.

104. Since $\sin\theta = -\sin(360° - \theta)$, the y-value of the point on the unit circle on the terminal side of θ is the opposite of the y-value of the point on the unit circle on the terminal side of $360° - \theta$. Since $\cos\theta = \cos(360° - \theta)$, the x-value of the point on the unit circle on the terminal side of θ is equals the x-value of the point on the unit circle on the terminal side of $360° - \theta$. Thus, the coordinates of the point on the unit circle are $(x, -y)$, and the point on the unit circle is S. The triangle we are seeking is $\triangle SOM$.

105. $\sin(-45°) = -\sin 45° = -\frac{\sqrt{2}}{2}$

$\tan(-60°) = -\tan 60° = -\sqrt{3}$

106. $\sin 135° = \sin(180° - 45°) = \sin 45° = \frac{\sqrt{2}}{2}$

$\cos 135° = \cos(180° - 45°) = -\cos 45° = -\frac{\sqrt{2}}{2}$

$\tan 120° = \tan(180° - 60°) = -\tan 60° = -\sqrt{3}$

107. $\sin 225° = \sin(180° + 45°) = -\sin 45° = -\frac{\sqrt{2}}{2}$

$\cos 240° = \cos(180° + 60°) = -\cos 60° = -\frac{1}{2}$

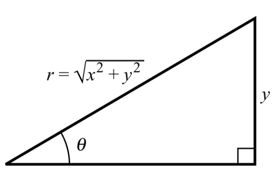
$\tan 210° = \tan(180° + 30°) = \tan 30° = \frac{\sqrt{3}}{3}$

108. $\sin 315° = \sin(360° - 45°) = -\sin 45° = -\frac{\sqrt{2}}{2}$

$\cos 300° = \cos(360° - 300°) = \cos 60° = \frac{1}{2}$

$\tan 330° = \tan(360° - 330°) = -\tan 30° = -\frac{\sqrt{3}}{3}$

109.



For acute angle θ,

$\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2 + y^2}} \Rightarrow \sin^2\theta = \frac{y^2}{x^2 + y^2}$.

For any real numbers x and y, $x^2 + y^2 \geq x^2$, so

$\frac{y^2}{x^2 + y^2} \leq 1 \Rightarrow \sin^2\theta \leq 1 \Rightarrow$

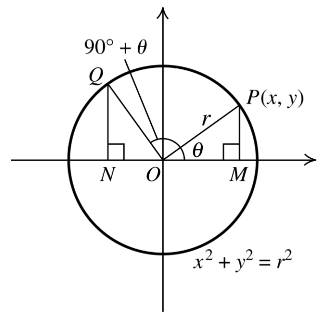
$|\sin\theta| \leq 1 \Rightarrow -1 \leq \sin\theta \leq 1$.

Similarly, we can show that

$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2 + y^2}} \Rightarrow \cos^2\theta = \frac{x^2}{x^2 + y^2} \Rightarrow$

$\cos^2\theta \leq 1 \Rightarrow |\cos\theta| \leq 1 \Rightarrow -1 \leq \cos\theta \leq 1$.

110.



$m\angle QON = 180° - (90° + \theta) = 90° - \theta$

Thus, $m\angle Q = \theta$. $\angle N \cong \angle M$ and

$OQ = OP = r$, so $\triangle POM \cong \triangle NOQ$ by AAS.

a. Q corresponds to M. The coordinates of M are $(x, 0)$, so the y-coordinate of Q is x. Since the coordinates of Q must satisfy the equation $x^2 + y^2 = r^2$, the x-coordinate of Q must be $-y$. Thus, the coordinates of Q are $(-y, x)$.

b. $\sin(\theta + 90°) = \frac{x}{r} = \cos\theta$

$\cos(\theta + 90°) = \frac{-y}{r} = -\frac{y}{r} = -\sin\theta$

$\tan(\theta + 90°) = \frac{x}{-y} = -\frac{x}{y} = -\cot\theta$

111. In triangle ABC, $A + B + C = 180° \Rightarrow$ $A + B = 180° - C$. Substituting we have

$\tan(A + B) + \tan C = \tan(180° - C) + \tan C$
$= -\tan C + \tan C = 0$

112. Since the largest value for $\cos\theta = 1$, the smallest value for $\sec\theta = 1$.

113. cos 105° = cos 60° + cos 45° is false because cos 105° < 0, while cos 60° > 0 and cos 45° > 0.
The sum of two positive numbers is positive.

114. sin 260° = 2 sin 130° is false because sin 260° < 0 while sin 130° > 0.

115. tan 123° = tan 61° + tan 62° is false because tan 123° < 0, while tan 61° > 0 and tan 62° > 0.
The sum of two positive numbers is positive.

116. sec 380° = sec 185° + sec 195° is false because sec 380° > 0 while sec 185° < 0 and sec 195° < 0. The sum of two negative numbers is negative.

5.3 Critical Thinking/Discussion/Writing

117. False. For example, if $\theta = 90°$, then we have $\cos(\sin 90°)° = \cos 1° \approx 0.9998$ while $\sin(\cos 90°)° = \sin 0° = 0$

118. False. $\sec\theta$ and $\tan\theta$ are both undefined for $\theta = 90°$.

5.3 Maintaining Skills

119. $f(x) = 3x - 4 = 0 \Rightarrow x = \dfrac{4}{3}$

120. $g(x) = 2x + 3 = 0 \Rightarrow x = -\dfrac{3}{2}$

121. $g(x) = x^2 + 3x - 10 = 0 \Rightarrow (x+5)(x-2) = 0 \Rightarrow x = -5,\ 2$

122. $g(x) = \dfrac{x^2 - 2x - 3}{x - 4} = 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x+1)(x-3) = 0 \Rightarrow x = -1,\ 3$

For exercises 123−128, recall that $f(-x) = f(x) \Rightarrow f(x)$ is even and $f(-x) = -f(x) \Rightarrow f(x)$ is odd.

123. $f(x) = 2x^3 - 4x$

$f(-x) = 2(-x)^3 - 4(-x) = -2x^3 + 4x = -f(x)$

$f(x)$ is odd.

124. $g(x) = \sqrt{1 - x^2}$

$g(-x) = \sqrt{1 - (-x)^2} = \sqrt{1 - x^2}$

$g(x)$ is even.

125. $h(x) = x^3 - x^2$

$h(-x) = (-x)^3 - (-x)^2 = -x^3 - x^2 \neq h(x)$

$h(-x) = (-x)^3 - (-x)^2 = -x^3 - x^2 \neq -h(x)$

$h(x)$ is neither even nor odd.

126. $p(x) = \sqrt[3]{x^5}$

$p(-x) = \sqrt[3]{(-x)^5} = -\sqrt[3]{x^5}$

$p(x)$ is odd.

127. $g(x) = 3x^4 + 2x^2 - 17$

$g(-x) = 3(-x)^4 + 2(-x)^2 - 17$
$= 3x^4 + 2x^2 - 17$

$g(x)$ is even.

128. $f(x) = 5x + 3$

$f(-x) = 5(-x) + 3 = -5x + 3 \neq f(x)$

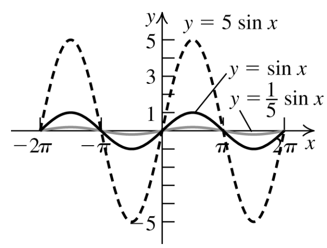
$f(-x) = 5(-x) + 3 = -5x + 3 \neq f(-x)$

$f(x)$ is neither even nor odd.

5.4 Graphs of the Sine and Cosine Functions

5.4 Practice Exercises

1.



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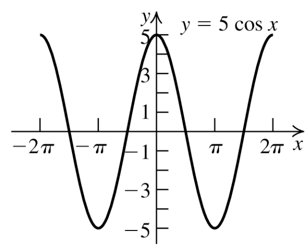
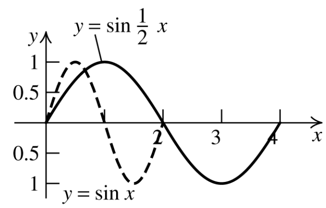


Amplitude = 5, range = [−5, 5]

3.



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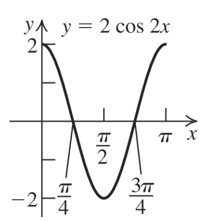
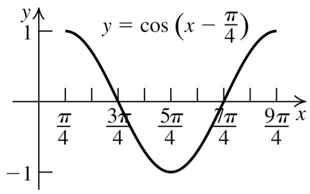


5. Answers will vary.

6.

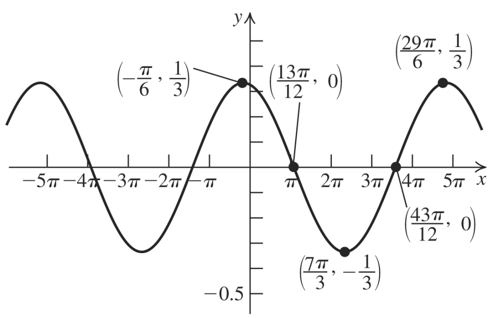


7. $y=\frac{1}{3}\cos\left[\frac{2}{5}\left(x+\frac{\pi}{6}\right)\right]$

$a=\frac{1}{3}\Rightarrow$ the amplitude is $\frac{1}{3}$.

$b=\frac{2}{5}\Rightarrow$ the period is $\frac{2\pi}{\frac{2}{5}}=5\pi$.

$c=-\frac{\pi}{6}\Rightarrow$ the phase shift is $\frac{\pi}{6}$ units left

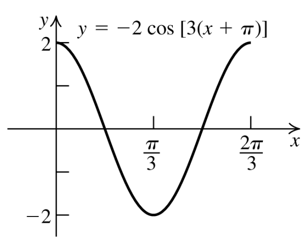


8. $y=-2\cos\left[3(x+\pi)\right]$

$a=-2\Rightarrow$ the amplitude is 2.

$b=3\Rightarrow$ the period is $\frac{2\pi}{3}$.

$c=-\pi\Rightarrow$ the phase shift is π units left



9.a. $y=3\sin\left(2x+\frac{\pi}{4}\right)=3\sin\left[2\left(x+\frac{\pi}{8}\right)\right]$

$=3\sin\left[2\left(x-\left(-\frac{\pi}{8}\right)\right)\right]$

$b=2$, so the period is $\frac{2\pi}{2}=\pi$.

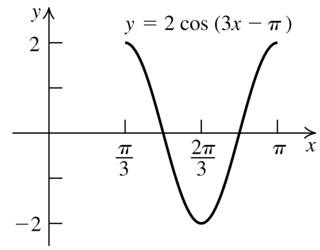
$c=-\frac{\pi}{8}$, so the phase shift is $\frac{\pi}{8}$ units left.

b. $y=\cos(\pi x-1)=\cos\pi\left(x-\frac{1}{\pi}\right)$

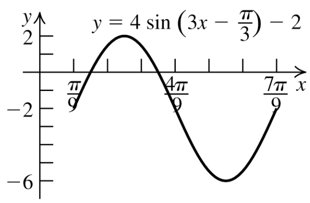
$b=\pi$, so the period is $\frac{2\pi}{\pi}=2$.

$c=\frac{1}{\pi}$, so the phase shift is $\frac{1}{\pi}$ units right.

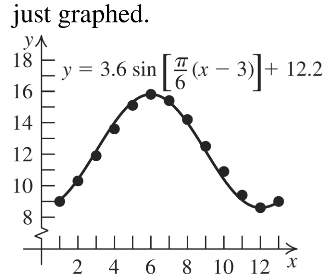
10.



11.



12. Plot the values given in the table by months using January = 1, …, December = 12. Then sketch a function of the form $y=a\sin b(x+c)+d$ that models the points just graphed.



$$\text{amplitude}=a=\frac{\text{highest value}-\text{lowest value}}{2}=\frac{15.8-8.6}{2}=3.6$$

$d=\frac{1}{2}(15.8+8.6)=12.2$

The weather repeats every 12 months, so the period is 12. Then, $12=\frac{2\pi}{b}\Rightarrow b=\frac{\pi}{6}$. The highest point on the graph, 15.8, occurs at $x=6$, so the phase shift is 3. The equation is

$y=3.6\sin\left[\frac{\pi}{6}(x-3)\right]+12.2$

13. Using the reasoning presented in the example in the text, the form of the equation is $y = -4\cos \omega t$. The period is $\frac{2\pi}{\omega} = 3 \Rightarrow$ $\frac{2\pi}{3} = \omega$. So the equation is $y = -4\cos\frac{2\pi}{3}t$.

5.4 Basic Concepts and Skills

1. The lowest point on the graph of $y = \cos x, 0 \le x \le 2\pi$, occurs when $x = \underline{\pi}$.

2. The highest point on the graph of $y = \sin x$, $0 \le x \le 2\pi$, occurs when $x = \underline{\frac{\pi}{2}}$.

3. The range of the cosine function is $\underline{[-1, 1]}$.

4. The maximum value of $y = -2\sin x$ is $\underline{2}$.

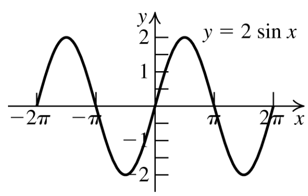
5. False. The amplitude is 3.

6. False. The range is [−1, 1].

7. True

8. True

9.



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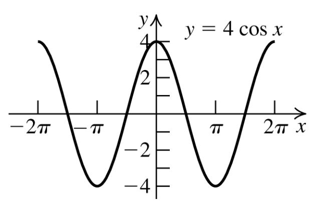

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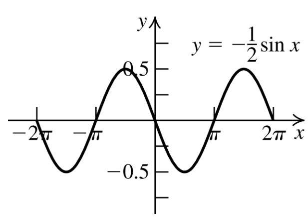

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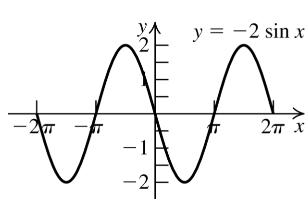

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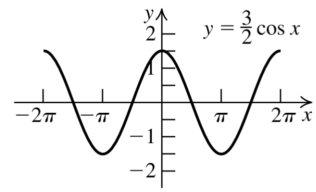

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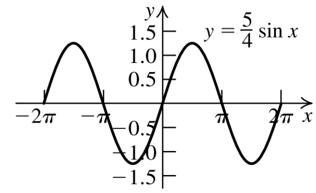

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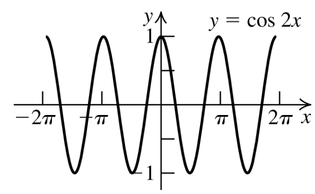

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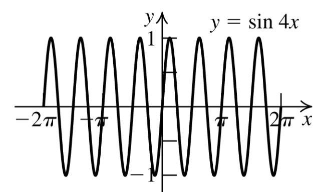

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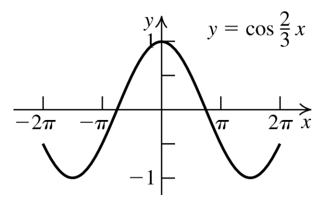

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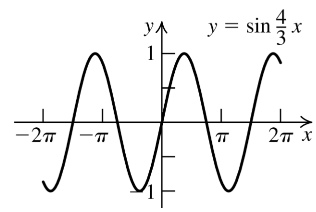

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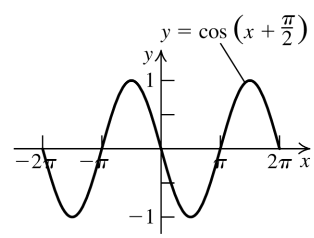

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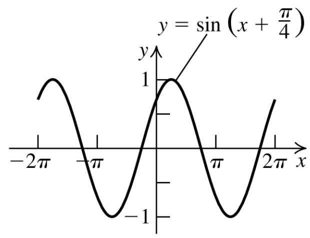

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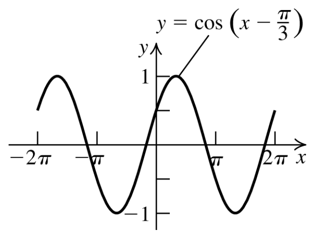

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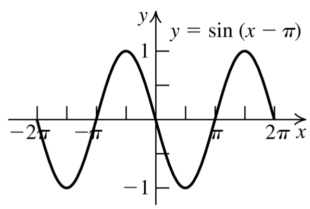

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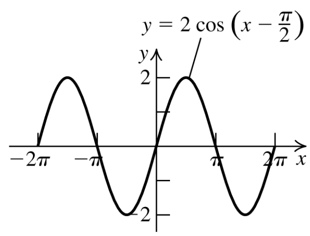

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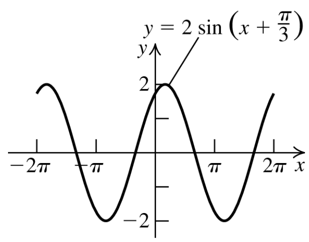

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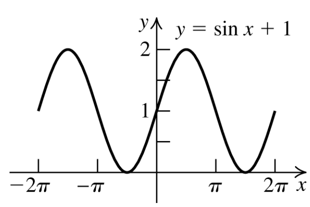

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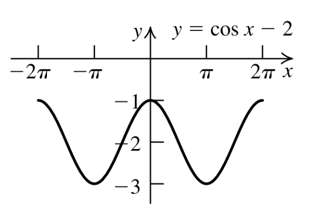

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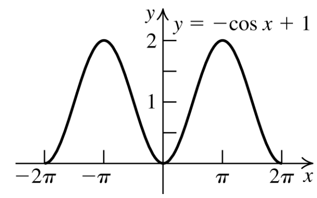

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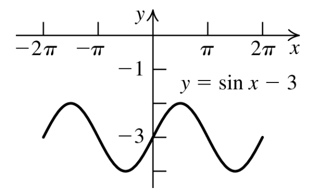


29. $y = 5\cos(x-\pi) \Rightarrow a = 5, b = 1, c = \pi \Rightarrow$
amplitude = 5, period = 2π, phase shift = π

30. $y = 3\sin\left(x - \frac{\pi}{8}\right) \Rightarrow a = 3, b = 1, c = \frac{\pi}{8} \Rightarrow$
amplitude = 3, period = 2π, phase shift = $\frac{\pi}{8}$

31. $y = 7\cos 9\left(x + \frac{\pi}{6}\right) \Rightarrow a = 7, b = 9, c = -\frac{\pi}{6} \Rightarrow$
amplitude = 7, period = $\frac{2\pi}{9}$, phase shift = $-\frac{\pi}{6}$

32. $y = 11\sin 8\left(x + \frac{\pi}{3}\right) \Rightarrow a = 11, b = 8, c = -\frac{\pi}{3} \Rightarrow$
amplitude = 11, period = $\frac{\pi}{4}$, phase shift = $-\frac{\pi}{3}$

33. $y = -6\cos\frac{1}{2}(x+2) \Rightarrow a = -6, b = \frac{1}{2}, c = -2 \Rightarrow$
amplitude = 6, period = 4π, phase shift = −2

34. $y = -8\sin\frac{1}{5}(x+9) \Rightarrow a = -8, b = \frac{1}{5}, c = -9 \Rightarrow$
amplitude = 8, period = 10π, phase shift = −9

35. $y = 0.9\sin 0.25\left(x - \frac{\pi}{4}\right) \Rightarrow a = 0.9, b = 0.25,$
$c = \frac{\pi}{4} \Rightarrow$ amplitude = 0.9, period = 8π,
phase shift = $\frac{\pi}{4}$

36. $y = \sqrt{5}\cos\pi(x+1) \Rightarrow a = \sqrt{5}, b = \pi, c = -1 \Rightarrow$
amplitude = $\sqrt{5}$, period = 2, phase shift = −1

37. $a = \frac{1}{4}$, period $= \frac{\pi}{8} \Rightarrow b = \frac{2\pi}{\pi/8} = 16$, $c = \frac{\pi}{16}$

$y = \frac{1}{4}\sin\left[16\left(x - \frac{\pi}{16}\right)\right]$

38. $a = 2$, period $= \frac{\pi}{6} \Rightarrow b = \frac{2\pi}{\pi/6} = 12$, $c = \frac{\pi}{10}$

$y = 2\sin\left[12\left(x - \frac{\pi}{10}\right)\right]$

39. $a = 6$, period $= 3\pi \Rightarrow b = \frac{2\pi}{3\pi} = \frac{2}{3}$, $c = -\frac{\pi}{4}$

$y = 6\sin\left[\frac{2}{3}\left(x - \left(-\frac{\pi}{4}\right)\right)\right] = 6\sin\left[\frac{2}{3}\left(x + \frac{\pi}{4}\right)\right]$

40. $a = \frac{1}{2}$, period $= 6\pi \Rightarrow b = \frac{2\pi}{6\pi} = \frac{1}{3}$, $c = -\frac{\pi}{2}$

$y = \frac{1}{2}\sin\left[\frac{1}{3}\left(x - \left(-\frac{\pi}{2}\right)\right)\right] = \frac{1}{2}\sin\left[\frac{1}{3}\left(x + \frac{\pi}{2}\right)\right]$

41. $a = 2.4$, period $= 6 \Rightarrow b = \frac{2\pi}{6} = \frac{\pi}{3}$, $c = 3$

$y = 2.4\sin\left[\frac{\pi}{3}(x - 3)\right]$

42. $a = 0.8$, period $= 10 \Rightarrow b = \frac{2\pi}{10} = \frac{\pi}{5}$, $c = 2$

$y = 0.8\sin\left[\frac{\pi}{5}(x - 2)\right]$

43. $a = \frac{\pi}{2}$, period $= \frac{5}{8} \Rightarrow b = \frac{2\pi}{5/8} = \frac{16\pi}{5}$, $c = -\frac{1}{4}$

$y = \frac{\pi}{2}\sin\left[\frac{16\pi}{5}\left(x - \left(-\frac{1}{4}\right)\right)\right]$

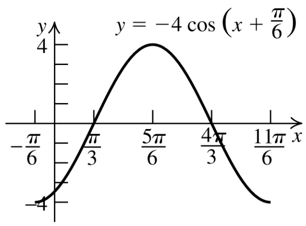
$= \frac{\pi}{2}\sin\left[\frac{16\pi}{5}\left(x + \frac{1}{4}\right)\right]$

44. $a = \frac{2}{\pi}$, period $= \frac{2}{3} \Rightarrow b = \frac{2\pi}{2/3} = 3\pi$, $c = -\frac{1}{6}$

$y = \frac{2}{\pi}\sin\left[3\pi\left(x - \left(-\frac{1}{6}\right)\right)\right]$

$= \frac{2}{\pi}\sin\left[3\pi\left(x + \frac{1}{6}\right)\right]$

45.



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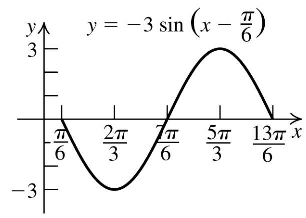

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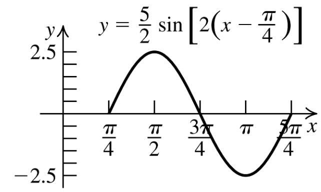

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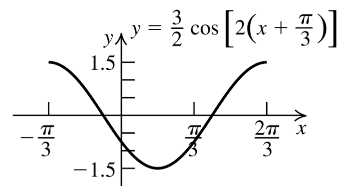

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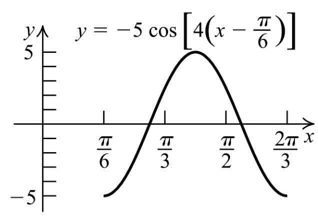

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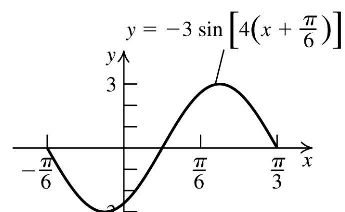

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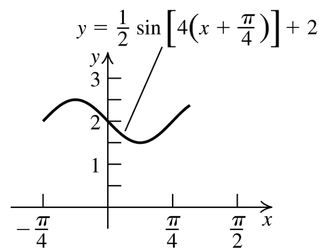

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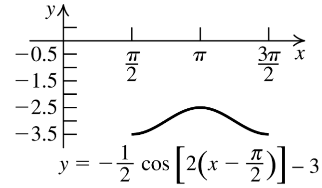


53. $y = 4\cos\left(2x + \frac{\pi}{3}\right) = 4\cos\left[2\left(x + \frac{\pi}{6}\right)\right]$

period $= \pi$; phase shift $= -\pi/6$

54. $y = 5\sin\left(3x + \frac{\pi}{2}\right) = 5\sin\left[3\left(x + \frac{\pi}{6}\right)\right]$

period = $\frac{2\pi}{3}$; phase shift = $-\frac{\pi}{6}$

55. $y = -\frac{3}{2}\sin(2x - \pi) = -\frac{3}{2}\sin\left[2\left(x - \frac{\pi}{2}\right)\right]$

period = π; phase shift = $\pi/2$

56. $y = -2\cos\left(5x - \frac{\pi}{4}\right) = -2\cos\left[5\left(x - \frac{\pi}{20}\right)\right]$

period = $2\pi/5$; phase shift = $\pi/20$

57. $y = 3\cos \pi x$; period = 2; phase shift = 0

58. $y = \sin\left(\frac{\pi x}{3}\right)$; period = 6; phase shift = 0

59. $y = \frac{1}{2}\cos\left(\frac{\pi x}{4} + \frac{\pi}{4}\right) = \frac{1}{2}\cos\left[\frac{\pi}{4}(x+1)\right]$

period = 8; phase shift = -1

60. $y = -\sin\left(\frac{\pi x}{6} + \frac{\pi}{6}\right) = -\sin\left[\frac{\pi}{6}(x+1)\right]$

period = 12; phase shift = -1

61. $y = 2\sin(\pi x + 3) = 2\sin\left[\pi\left(x + \frac{3}{\pi}\right)\right]$

period = 2; phase shift = $-3/\pi$

62. $y = -\cos\left(\pi x - \frac{1}{4}\right) = -\cos\left[\pi\left(x - \frac{1}{4\pi}\right)\right]$

period = 2; phase shift = $1/4\pi$

5.4 Applying the Concepts

63. $P = \sin\left(\frac{2\pi}{23}t\right)$, $E = \sin\left(\frac{2\pi}{28}t\right)$, $I = \sin\left(\frac{2\pi}{33}t\right)$,

where t represents the number of days since birth.
First, calculate t. Number of years from July 22, 1990 to July 22, 2011 = 2011 – 1990 = 21.
Number of leap years = 5 (1992, 1996, 2000, 2004, 2008)
Number of days from April 1, 2001 to July 22, 2001 = 29 (rest of April) + 31 (May) + 30 (June) + 22 (July) = 112
Number of days from July 22, 1990 to July 22, 2001 = (21)(365) + 5 – 112 = 7558

$$P = \sin\left(\frac{2\pi(7558)}{23}\right) \approx -0.63$$
$$E = \sin\left(\frac{2\pi(7558)}{28}\right) \approx -0.43$$
$$I = \sin\left(\frac{2\pi(7558)}{33}\right) \approx 0.19$$

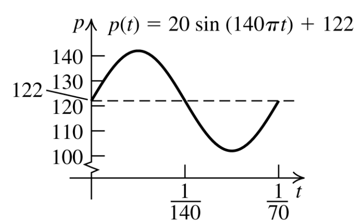
64. $P = \sin\left(\frac{2\pi}{23}t\right)$, $E = \sin\left(\frac{2\pi}{28}t\right)$, $I = \sin\left(\frac{2\pi}{33}t\right)$,

where t represents the number of days since birth.
First, calculate t. Number of years from March 31, 1991 to August 12, 2012 = 2012 – 1991 = 21.
Number of leap years = 5 (1992, 1996, 2000, 2004, 2008)
Number of days from March 31, 2012 to August 12, 2012 = 1 (March) + 30 (April) + 31 (May) + 30 (June) + 31 (July) + 12 (August) = 135
Number of days from March 31, 1991 to August 12, 2012 = (21)(365) + 5 + 135 = 7805

$$P = \sin\left(\frac{2\pi(7805)}{23}\right) \approx 0.82$$
$$E = \sin\left(\frac{2\pi(7805)}{28}\right) = -1$$
$$I = \sin\left(\frac{2\pi(7805)}{33}\right) \approx -0.10$$

65.a. The pulse shows how many times the heart beats in one minute. Therefore, the pulse rate is the frequency of the function. The period is $\frac{2\pi}{140\pi} = \frac{1}{70}$. So the frequency is 70 pulses per minute.

b.

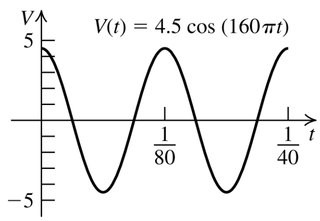


c. The systolic reading is the maximum value of the graph, and the diastolic reading in the minimum value of the graph. The maximum value is 122 + 20 = 142. The minimum value is 122 – 20 = 102. So, Desmond's blood pressure is $\frac{142}{102}$ mm of mercury.

66.a. amplitude = 4.5; period = $\dfrac{2\pi}{160\pi} = \dfrac{1}{80}$

b. frequency = $\dfrac{1}{\text{period}} = 80$ cycles per second

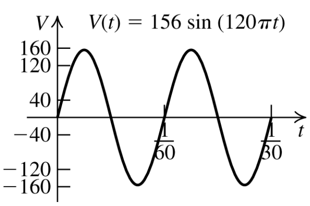
c.



67.a. amplitude = 156; period = $\dfrac{2\pi}{120\pi} = \dfrac{1}{60}$

b. frequency = $\dfrac{1}{\text{period}} = 60$ cycles per second

c.



68. Because the ball was pulled down 5 inches, $a = -5$. The period is given to be 10 seconds, so $\omega = \dfrac{2\pi}{10} = \dfrac{\pi}{5}$.

The equation is $y = -5\cos\left(\dfrac{\pi}{5}t\right)$.

69.a. The largest number of kangaroos is when

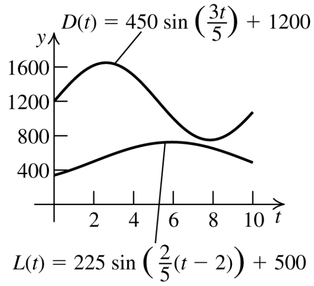
$\sin 2t = 1 \Rightarrow t = \dfrac{\pi}{4} \Rightarrow 650 + 150 = 800.$

b. The smallest number of kangaroos is when

$\sin 2t = -1 \Rightarrow t = \dfrac{3\pi}{4} \Rightarrow 650 - 150 = 500.$

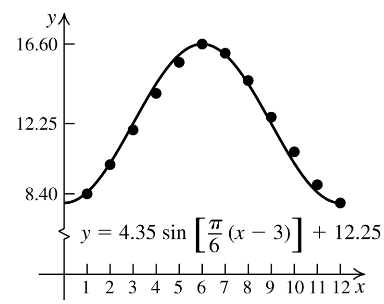
c. The time between occurrences of the largest and the smallest kangaroo population is $1/2$ of the period. The period of the function is $\dfrac{2\pi}{2} = \pi$, so the time between the maximum and the minimum population is $\dfrac{\pi}{2} \approx 1.57$ years.

70.a.



b. On [3, 10], as the lion population increases, the deer population decreases. As the lion population decreases, the deer population increases.

71. Plot the values given in the table by months using January = 1, …, December = 12. Then sketch a function of the form $y = a\sin b(x + c) + d$ that models the points just graphed.



The highest number of daylight hours is 16.6 and the lowest number of daylight hours is 7.9, so $a = \dfrac{16.6 - 7.9}{2} = 4.35$. The vertical shift $d = \dfrac{16.6 + 7.9}{2} = 12.25$. The period is $12 = \dfrac{2\pi}{b} \Rightarrow b = \dfrac{\pi}{6}$. The highest point on the graph, 16.6, occurs at $x = 6$, so the phase shift is 3. The equation is

$y = 4.35\sin\left[\dfrac{\pi}{6}(x - 3)\right] + 12.25$.

72. Plot the values given in the table by months using January = 1, …, December = 12. Then sketch a function of the form $y = a\sin b(x + c) + d$ that models the points just graphed. The maximum number of daylight hours is 14.8 and the minimum number of daylight hours is 9.5, so $a = \dfrac{14.8 - 9.5}{2} = 2.65$.

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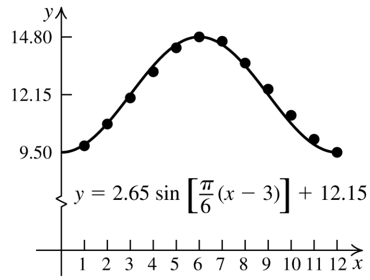
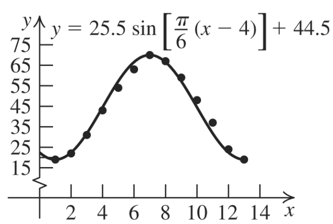


The vertical shift $d = \frac{14.8+9.5}{2} = 12.15$. The period is $12 = \frac{2\pi}{b} \Rightarrow b = \frac{\pi}{6}$. The highest point on the graph, 14.8, occurs at $x = 6$, so the phase shift is 3. The equation is $y = 2.65\sin\left[\frac{\pi}{6}(x-3)\right]+12.15$.

73.a. The highest temperature is 70ºF, and the lowest temperature is 19ºF, so $a = \frac{70-19}{2} = 25.5$. The vertical shift is $d = \frac{70+19}{2} = 44.5$. The period is $12 = \frac{2\pi}{b} \Rightarrow b = \frac{\pi}{6}$. The highest temperature occurs at $x = 7$, so $c = 7 - 3 = 4$. The equation is $y = 25.5\sin\left[\frac{\pi}{6}(x-4)\right]+44.5$.

b.



c. January $= f(1) = 19$; April $= f(4) = 44.5$; July $= f(7) = 70$; October $= f(10) = 44.5$. The computed values are very close to the measured values.

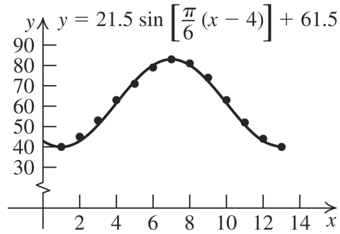
74.a. The highest temperature is 83ºF, and the lowest temperature is 40ºF, so $a = \frac{83-40}{2} = 21.5$.

The vertical shift is $d = \frac{83+40}{2} = 61.5$.

The period is $12 = \frac{2\pi}{b} \Rightarrow b = \frac{\pi}{6}$. The highest temperature occurs at $x = 7$, so $c = 7 - 3 = 4$. The equation is

$y = 21.5\sin\left[\frac{\pi}{6}(x-4)\right]+61.5$.

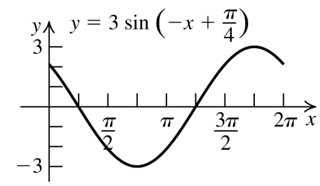
b.



c. January $= f(1) = 40$; April $= f(4) = 61.5$; July $= f(7) = 83$; October $= f(10) = 61.5$. The computed values are very close to the measured values.

5.4 Beyond the Basics

75.



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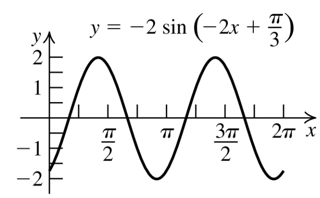

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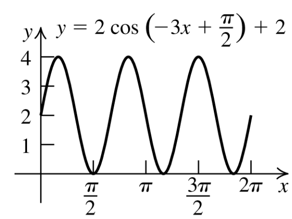

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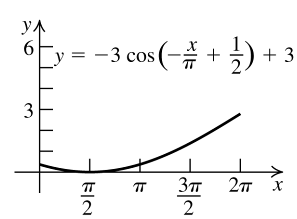

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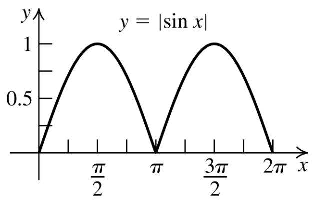
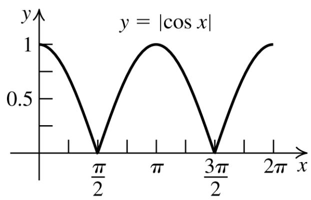


80.



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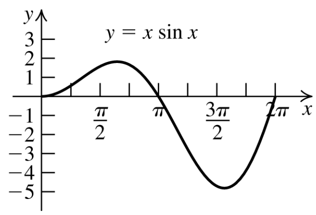
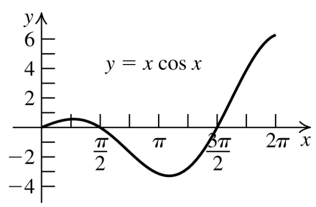
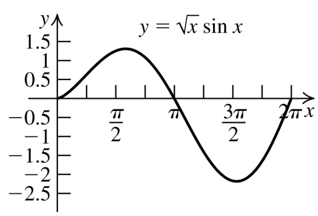


82.



83.



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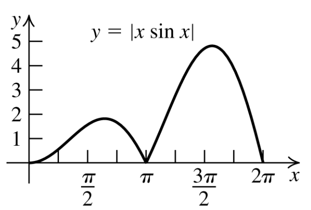

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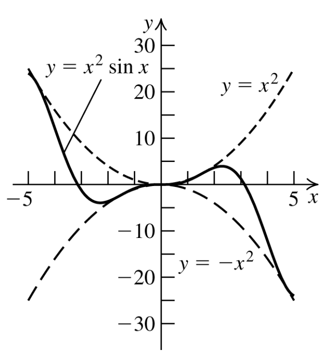


86.a. If $\sin(x+p)=\sin x$ for some p with $0<p<2\pi$ and x a real number, then for $x=0$ we have $\sin(0+p)=\sin 0=0$. But, if $\sin p=0$ and $0<p<2\pi$, then p must equal π, and then $\sin(x+\pi)=\sin x$ for all real numbers x.

b. $\sin\left(\frac{\pi}{2}+\pi\right)=\sin\frac{3\pi}{2}=-1\neq\sin\frac{\pi}{2}$. So $p\neq\pi$. Therefore, $p=2\pi$.

Answers may vary in exercises 87–94. Sample answers are given.

87. The graph is a cosine curve with amplitude 3 and period π, so $a=3$ and $b=\frac{2\pi}{\pi}=2.$
An equation is $y=3\cos 2x.$

88. The graph is a reflection of the sine curve about the x-axis with amplitude 2 and period π, so $a=-2$ and $b=\frac{2\pi}{\pi}=2.$
An equation is $y=-2\sin 2x.$

89. The graph is a sine curve with amplitude 2 and period $\frac{\pi}{2}$, so $a=2$ and $b=\frac{2\pi}{\pi/2}=4.$
An equation is $y=2\sin 4x.$

90. The graph is a reflection of the cosine curve about the x-axis. The amplitude is 2 and the period is $\frac{2\pi}{3}$, so $a=-2$ and $b=\frac{2\pi}{2\pi/3}=3.$
An equation is $y=-2\cos 3x.$

91. The graph is a sine curve with amplitude 3 and period π, so $a=3$ and $b=\frac{2\pi}{\pi}=2.$ The graph is shifted $\frac{\pi}{4}$ units right, so $c=\frac{\pi}{4}.$
An equation is $y=3\sin\left[2\left(x-\frac{\pi}{4}\right)\right].$

92. The graph is a cosine curve with amplitude 2 and period π, so $a=2$ and $b=\frac{2\pi}{\pi}=2.$ The graph is shifted $\frac{\pi}{4}$ units left, so $c=-\frac{\pi}{4}.$
An equation is $y=2\cos\left[2\left(x+\frac{\pi}{4}\right)\right].$

93. The graph is a reflection of the sine curve about the x-axis. The amplitude 2 and the period is $\frac{7\pi}{8}-\left(-\frac{\pi}{8}\right)=\pi,$ so $a=-2$ and $b=\frac{2\pi}{\pi}=2.$

The graph is shifted $\frac{\pi}{8}$ units left, so $c=\frac{\pi}{8}.$

An equation is $y=-2\sin\left[2\left(x-\left(-\frac{\pi}{8}\right)\right)\right]$

$=-2\sin\left[2\left(x+\frac{\pi}{8}\right)\right].$

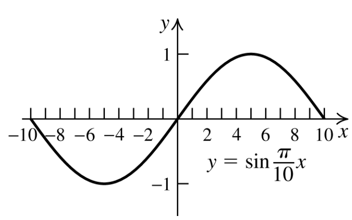
94. The graph is a reflection of the cosine curve about the x-axis. The amplitude is 3 and the period is $\frac{5\pi}{4}-\frac{\pi}{4}=\pi,$ so $a=-3$ and $b=\frac{2\pi}{\pi}=2.$ The graph is shifted $\frac{\pi}{4}$ units right, so $c=\frac{\pi}{4}.$

An equation is $y=-3\cos\left[2\left(x-\frac{\pi}{4}\right)\right].$

5.4 Critical Thinking/Discussion/Writing

95.a. The period is 20. So period $=\frac{2\pi}{b}\Rightarrow$

$20=\frac{2\pi}{b}\Rightarrow b=\frac{\pi}{10}$. Graphing the function $y=\sin\frac{\pi}{10}x$, we find that there are 10 units between the value of x at which the maximum and minimum values are attained.



b. For $y=\sin\frac{\pi}{10}x$, $d=0$. So, from $y=0$ to the maximum of y is 5 units.

96. Since the minimum value occurs at a value of x that is 5 units from the value of x at which the function has the maximum value, the period of the function is 10 units.

5.4 Maintaining Skills

97. $y-(-3)=5(x-2)\Rightarrow y+3=5x-10\Rightarrow$
$y=5x-13$

98. $y-4=\frac{4-2}{1-(-3)}(x-1)\Rightarrow y-4=\frac{2}{4}(x-1)\Rightarrow$

$y-4=\frac{1}{2}(x-1)\Rightarrow 2y-8=x-1\Rightarrow$

$2y=x+7\Rightarrow y=\frac{1}{2}x+\frac{7}{2}$

99. First, find the slope of the given line.

$2x+4y=5\Rightarrow 4y=-2x+5\Rightarrow y=-\frac{1}{2}x+\frac{5}{4}$

Since line we are seeking is parallel to the given line, the slope is the same as the slope of the given line, $m=-\frac{1}{2}.$

$y-4=-\frac{1}{2}(x-(-3))\Rightarrow y-4=-\frac{1}{2}(x+3)\Rightarrow$

$2y-8=-x-3\Rightarrow 2y=-x+5\Rightarrow y=-\frac{1}{2}x+\frac{5}{2}$

100. First, find the slope of the given line.

$3x+6y=7\Rightarrow 6y=-3x+7\Rightarrow y=-\frac{1}{2}x+\frac{7}{6}$

Since line we are seeking is perpendicular to the given line, the slope is the negative reciprocal of the slope of the given line, $m=2.$

$y-(-1)=2(x-3)\Rightarrow y+1=2x-6\Rightarrow$
$y=2x-7$

101. $f(x)=\frac{(x+1)(x-2)}{(x+2)(x-3)}$

There are no common factors in the numerator and denominator, so find the x-intercepts and vertical asymptotes as follows.
x-intercepts: $(x+1)(x-2)=0\Rightarrow x=-1,\ 2$
vertical asymptotes:
$(x+2)(x-3)=0\Rightarrow x=-2,\ x=3$

102. $f(x)=\frac{x^2-3x-10}{x^2-2x-3}=\frac{(x+2)(x-5)}{(x+1)(x-3)}$

There are no common factors in the numerator and denominator, so find the x-intercepts and vertical asymptotes as follows.
x-intercepts:
$x^2-3x-10=0\Rightarrow(x+2)(x-5)=0\Rightarrow$
$x=-2,\ 5$

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vertical asymptotes:

$x^2 - 2x - 3 = 0 \Rightarrow (x+1)(x-3) = 0 \Rightarrow$
$x = -1,\ x = 3$

103. $f(x) = \dfrac{x^2 + 6x + 8}{x^2 + 1} = \dfrac{(x+2)(x+4)}{x^2+1}$

There are no common factors in the numerator and denominator, so find the x-intercepts and vertical asymptotes as follows.
x-intercepts:

$x^2 + 6x + 8 = 0 \Rightarrow (x+2)(x+4) = 0 \Rightarrow$
$x = -2,\ -4$

vertical asymptotes: $x^2 + 1 = 0 \Rightarrow$ there is no real solution. There are no vertical asymptotes.

104. $f(x) = \dfrac{1}{x^2 - 5x - 14} = \dfrac{1}{(x+2)(x-7)}$

The numerator is a constant, so there are no x-intercepts.
vertical asymptotes:

$x^2 - 5x - 14 = (x+2)(x-7) = 0 \Rightarrow$
$x = -2,\ x = 7$

For exercises 105−112, recall that
$f(-x) = f(x) \Rightarrow f(x)$ is even and
$f(-x) = -f(x) \Rightarrow f(x)$ is odd.

105. $f(x) = x^3 - 2x$

$f(-x) = (-x)^3 - 2(-x) = -x^3 + 2x = -f(x)$

$f(x)$ is odd.

106. $g(x) = 3x^2 + 1$

$g(-x) = 3(-x)^2 + 1 = 3x^2 + 1$

$g(x)$ is even.

107. $f(x) + g(x) = x^3 - 2x + 3x^2 + 1$
$= x^3 + 3x^2 - 2x + 1$

$f(-x) + g(-x) = (-x)^3 + 3(-x)^2 - 2(-x) + 1$
$= -x^3 + 3x^2 + 2x + 1$
$\neq f(-x) + g(-x)$
$\neq f(x) + g(x)$

$f(x) + g(x)$ is neither even nor odd.

108. $f(x)g(x) = (x^3 - 2x)(3x^2 + 1)$
$= 3x^5 - 5x^3 - 2x$

$f(-x)g(-x) = 3(-x)^5 - 5(-x)^3 - 2(-x)$
$= -3x^5 + 5x^3 + 2x$
$= f(-x)g(-x)$

$f(x)g(x)$ is odd.

109. $\dfrac{f(x)}{g(x)} = \dfrac{x^3 - 2x}{3x^2 + 1}$

$\dfrac{f(-x)}{g(-x)} = \dfrac{(-x)^3 - 2(-x)}{3(-x)^2 + 1} = \dfrac{-x^3 + 2x}{3x^2 + 1}$
$= \dfrac{f(-x)}{g(-x)}$

$\dfrac{f(x)}{g(x)}$ is odd.

110. $\dfrac{g(x)}{f(x)} = \dfrac{3x^2 + 1}{x^3 - 2x}$

$\dfrac{g(-x)}{f(-x)} = \dfrac{3(-x)^2 + 1}{(-x)^3 - 2(-x)} = \dfrac{3x^2 + 1}{-x^3 + 2x}$
$= \dfrac{g(-x)}{f(-x)}$

$\dfrac{g(x)}{f(x)}$ is odd.

111. $\dfrac{1}{f(x)} = \dfrac{1}{x^3 - 2x}$

$\dfrac{1}{f(-x)} = \dfrac{1}{(-x)^3 - 2(-x)} = \dfrac{1}{-x^3 + 2x}$
$= \dfrac{1}{f(-x)}$

$\dfrac{1}{f(x)}$ is odd.

112. $\dfrac{1}{g(x)} = \dfrac{1}{3x^2 + 1}$

$\dfrac{1}{g(-x)} = \dfrac{1}{3(-x)^2 + 1} = \dfrac{1}{3x^2 + 1}$
$= \dfrac{1}{g(x)}$

$\dfrac{1}{g(x)}$ is even.

5.5 Graphs of the Other Trigonometric Functions

5.5 Practice Problems

1. We have $m = \tan 30° = \frac{\sqrt{3}}{3}$, so the slope of the line is $\frac{\sqrt{3}}{3}$. Using the point-slope form, we have $y - 3 = \frac{\sqrt{3}}{3}(x-2)$ as the equation of the line. In slope-intercept form, the equation is $y = \frac{\sqrt{3}}{3}x + \left(3 - \frac{2\sqrt{3}}{3}\right)$.

2. Step 1: $y = -3\tan\left(x + \frac{\pi}{4}\right)$

$a = -3 \Rightarrow$ the vertical stretch factor is 3.
$b = 1 \Rightarrow$ the period is π.

$c = -\frac{\pi}{4} \Rightarrow$ the phase shift is $\frac{\pi}{4}$ units left.

Step 2: Find vertical asymptotes:

$x + \frac{\pi}{4} = -\frac{\pi}{2} \Rightarrow x = -\frac{3\pi}{4}$ and

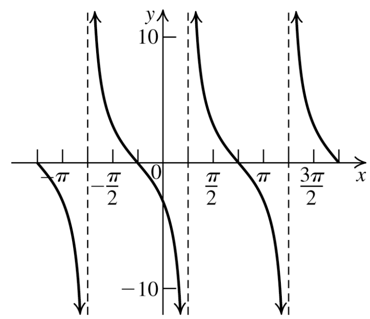
$x + \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{4}$.

Step 3:Divide the interval $\left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$ into four equal parts: $-\frac{\pi}{2}, -\frac{\pi}{4}$, and 0.

Step 4: Evaluate the function at the three x-values found in Step 3.

x	$y = -3\tan\left(x + \frac{\pi}{4}\right)$
$-\frac{\pi}{2}$	3
$-\frac{\pi}{4}$	0
0	−3

Step 5: Sketch the vertical asymptotes and the points $\left(-\frac{\pi}{2}, 3\right), \left(-\frac{\pi}{4}, 0\right)$, and $(0, -3)$. Connect the points with a smooth curve. Repeat the graph to the left and right over intervals of length π.



3. Step 1: $y = -3\cot\left(x + \frac{\pi}{4}\right)$

$a = -3 \Rightarrow$ the vertical stretch factor is 3.
$b = 1 \Rightarrow$ the period is π.

$c = -\frac{\pi}{4} \Rightarrow$ the phase shift is $\frac{\pi}{4}$ units left.

Step 2: Find the vertical asymptotes:

$x + \frac{\pi}{4} = 0 \Rightarrow x = -\frac{\pi}{4}$ and

$x + \frac{\pi}{4} = \pi \Rightarrow x = \frac{3\pi}{4}$.

Step 3: Divide the interval $\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$ into four equal parts: $0, \frac{\pi}{4}$, and $\frac{\pi}{2}$.

Step 4: Evaluate the function at the three x-values found in Step 3.

x	$y = -3\cot\left(x + \frac{\pi}{4}\right)$
0	−3
$\frac{\pi}{4}$	0
$\frac{\pi}{2}$	3

Step 5: Sketch the vertical asymptotes and the points $(0, -3), \left(\frac{\pi}{4}, 0\right)$, and $\left(\frac{\pi}{2}, 3\right)$. Connect the points with a smooth curve. Repeat the graph to the left and right over intervals of length π to graph over the interval $\left(-\frac{3\pi}{4}, \frac{5\pi}{4}\right)$.

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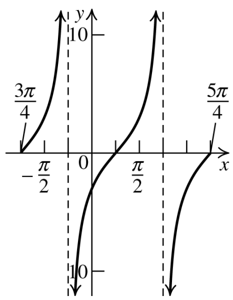
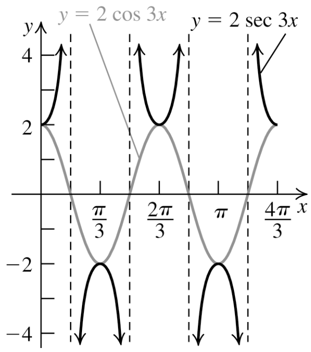


4. Steps 1, 2: $y = 2\sec 3x$

Because $\sec 3x = \frac{1}{\cos 3x}$, $2\sec 3x = 2\cos 3x$ when $\cos 3x = \pm 1$. So first graph $y = 2\cos 3x$.
$a = 2 \Rightarrow$ the vertical stretch factor is 3.
$b = 3 \Rightarrow$ the period is $\frac{2\pi}{3}$. There is no phase shift, so the graph starts at $x = 0$.
Step 3: Divide the interval $\left(0, \frac{2\pi}{3}\right)$ into four equal parts: $\frac{\pi}{6}, \frac{\pi}{3}$, and $\frac{\pi}{2}$.
Step 4: Evaluate the function at the three x-values found in Step 3.

x	$y = 2\cos 3x$
$\frac{\pi}{6}$	0
$\frac{\pi}{3}$	−2
$\frac{\pi}{2}$	0

Step 5: Sketch the graph of $y = 2\cos 3x$ through the points $\left(\frac{\pi}{6}, 0\right), \left(\frac{\pi}{3}, -2\right)$, and $\left(\frac{\pi}{2}, 0\right)$. Repeat the graph to the right over an interval of length $\frac{2\pi}{3}$ to graph a two-period interval. Then use the reciprocal relationship to draw the graph of $y = 2\sec 3x$.



5. For $x = \frac{\pi}{8}, y = \csc\frac{x}{2} = \csc\frac{\pi}{16} \approx 5.1$ and for $x = \frac{\pi}{4}, y = \csc\frac{x}{2} = \csc\frac{\pi}{8} \approx 2.6$. The range of Mach numbers associated with the interval $\left[\frac{\pi}{8}, \frac{\pi}{4}\right]$ is about [2.6, 5.1].

5.5 Basic Concepts and Skills

1. The tangent function has period $\underline{\pi}$.

2. The value of x with $0 \le x \le \pi$ for which the tangent function is undefined is $\underline{\frac{\pi}{2}}$.

3. If $0 \le x \le \pi$ and $\cot x = 0$, then $x = \underline{\frac{\pi}{2}}$.

4. The maximum value of $y = \csc x$ for $\pi \le x \le 2\pi$ is $\underline{-1}$.

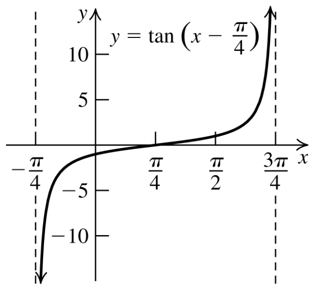
5. $\theta = 45° \Rightarrow \tan 45° = 1 =$ slope of the line
$y - 3 = 1(x - (-2)) \Rightarrow y - 3 = x + 2 \Rightarrow y = x + 5$

6. $\theta = 60° \Rightarrow \tan 60° = \sqrt{3} =$ slope of the line
$y - (-1) = \sqrt{3}(x - 3) \Rightarrow y + 1 = \sqrt{3}x - 3\sqrt{3} \Rightarrow$
$y = \sqrt{3}x - 3\sqrt{3} - 1 = \sqrt{3}x - (3\sqrt{3} + 1)$

7. $\theta = 120° \Rightarrow \tan 60° = -\sqrt{3} =$ slope of the line
$y - (-2) = -\sqrt{3}(x - (-3)) \Rightarrow$
$y + 2 = -\sqrt{3}(x + 3) \Rightarrow y + 2 = -\sqrt{3}x - 3\sqrt{3} \Rightarrow$
$y = -\sqrt{3}x - 3\sqrt{3} - 2 = -\sqrt{3}x - (3\sqrt{3} + 2)$

8. $\theta = 135° \Rightarrow \tan 135° = -1 =$ slope of the line
$y - 5 = -1(x - 2) \Rightarrow y - 5 = -x + 2 \Rightarrow$
$y = -x + 7$

9.



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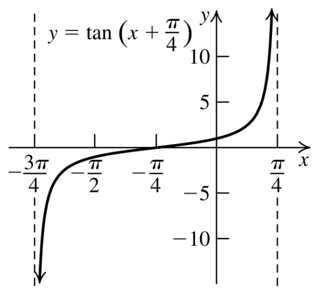

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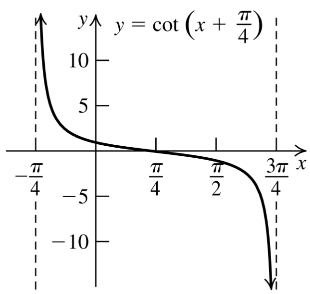

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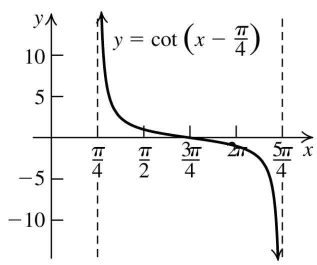

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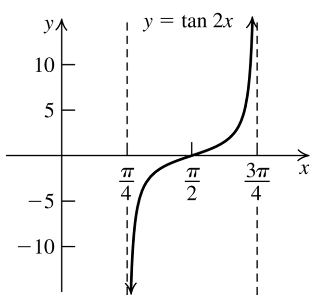

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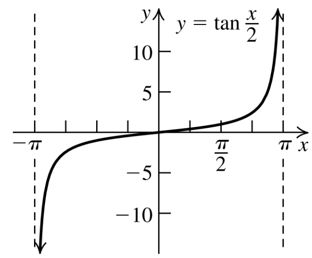

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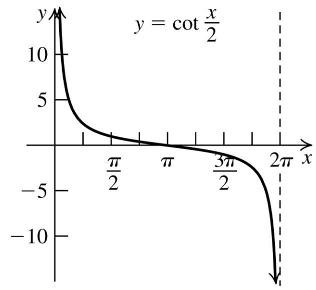

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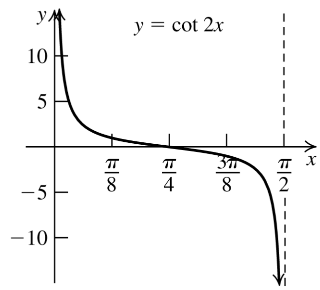

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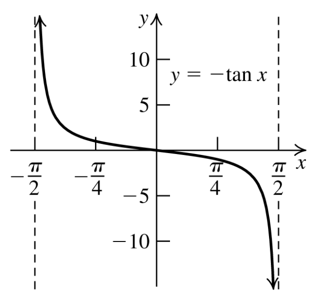

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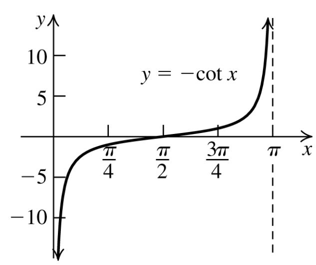

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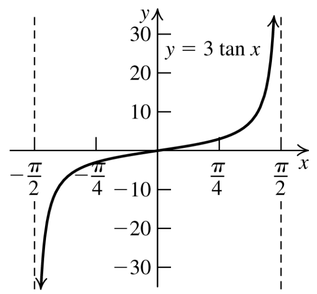

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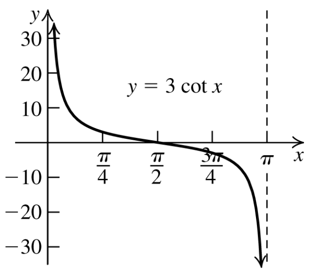

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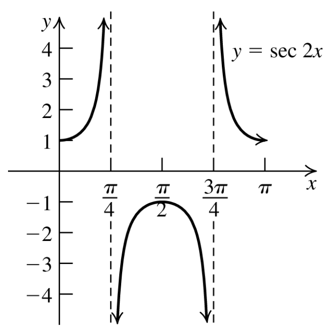

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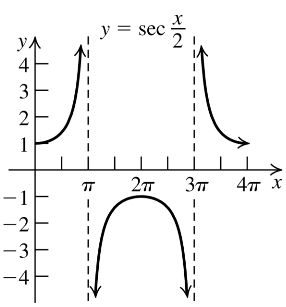

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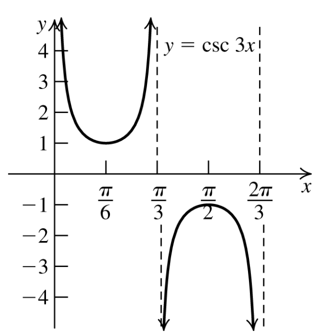

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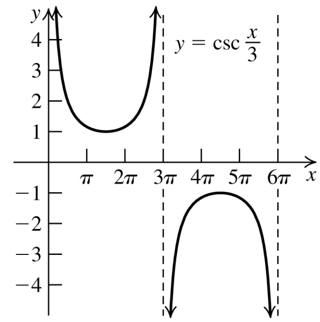

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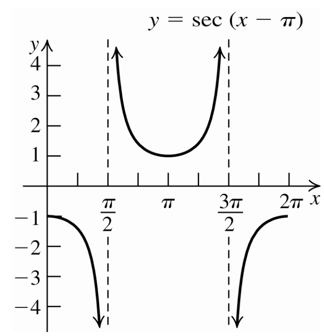

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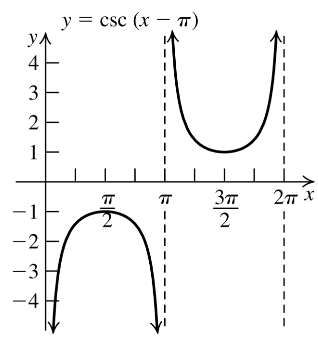

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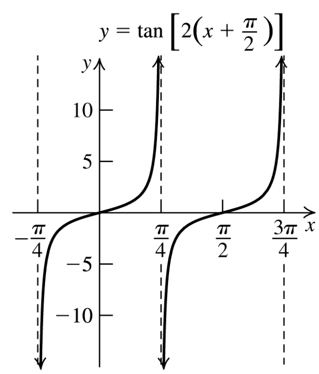
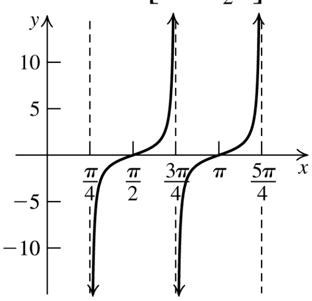
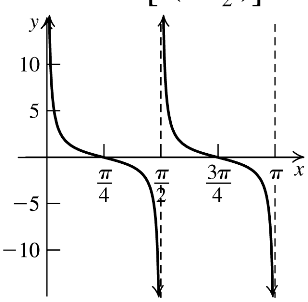


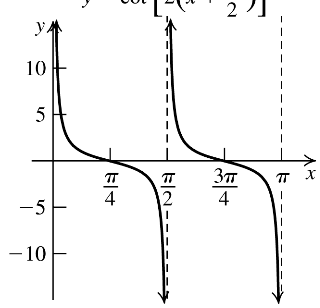
28. $y = \tan\left[2\left(x - \frac{\pi}{2}\right)\right]$



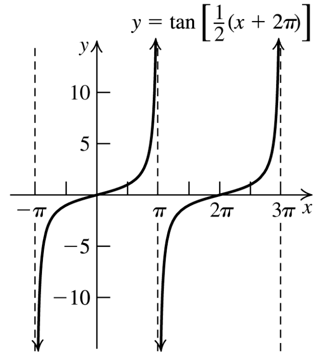
29. $y = \cot\left[2\left(x - \frac{\pi}{2}\right)\right]$



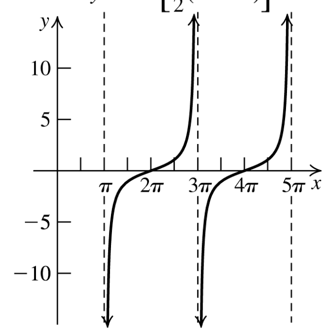
30. $y = \cot\left[2\left(x + \frac{\pi}{2}\right)\right]$



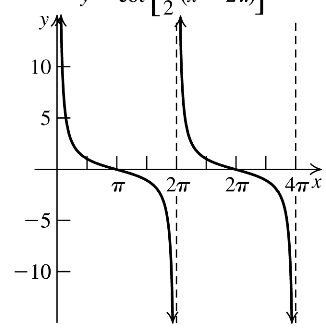
31. $y = \tan\left[\frac{1}{2}(x + 2\pi)\right]$



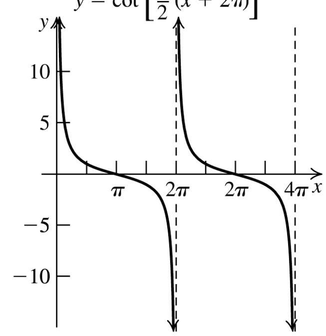
32. $y = \tan\left[\frac{1}{2}(x - 2\pi)\right]$



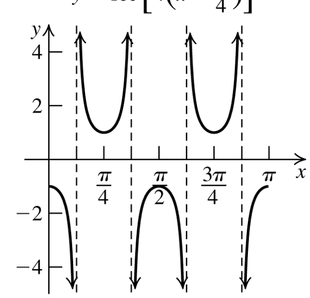
33. $y = \cot\left[\frac{1}{2}(x - 2\pi)\right]$



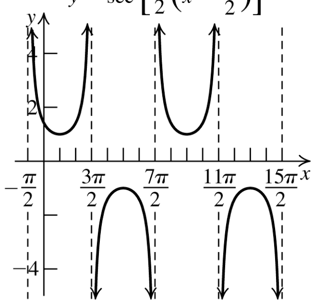
34. $y = \cot\left[\frac{1}{2}(x + 2\pi)\right]$



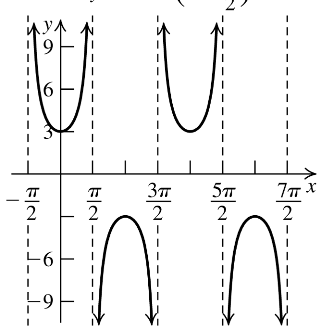
35. $y = \sec\left[4\left(x - \frac{\pi}{4}\right)\right]$



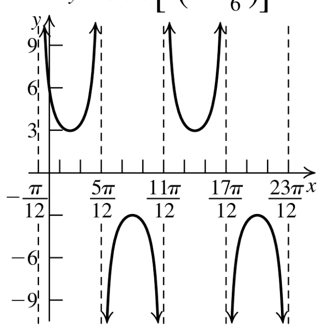
36. $y = \sec\left[\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right]$



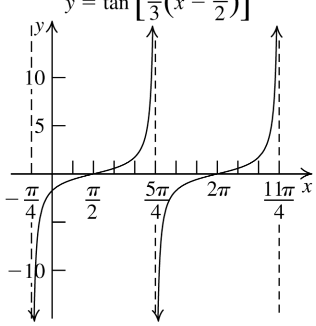
37. $y = 3\csc\left(x + \frac{\pi}{2}\right)$



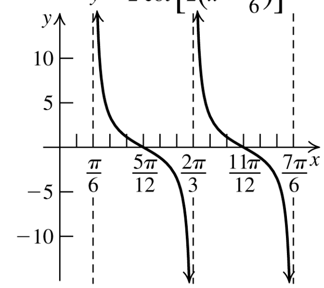
38. $y = 3\sec\left[2\left(x - \frac{\pi}{6}\right)\right]$



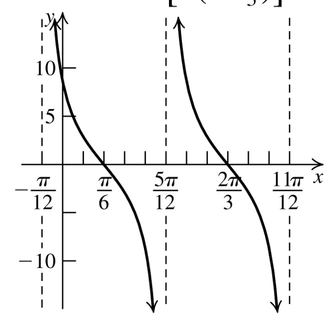
39. $y = \tan\left[\frac{2}{3}\left(x - \frac{\pi}{2}\right)\right]$



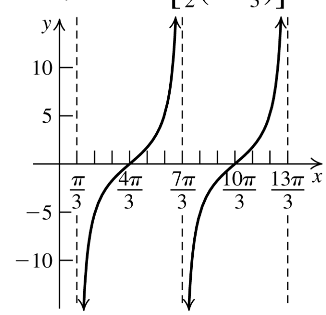
40. $y = 2\cot\left[2\left(x - \frac{\pi}{6}\right)\right]$



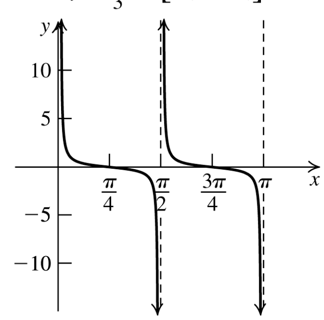
41. $y = -5\tan\left[2\left(x + \frac{\pi}{3}\right)\right]$



42. $y = -3\cot\left[\frac{1}{2}\left(x - \frac{\pi}{3}\right)\right]$



43. $y = \frac{1}{3}\cot\left[2(x - \pi)\right]$



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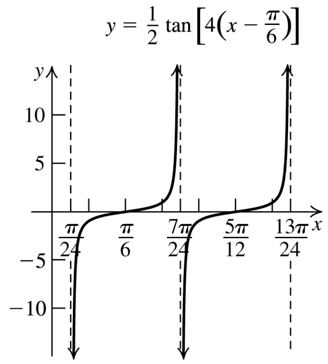
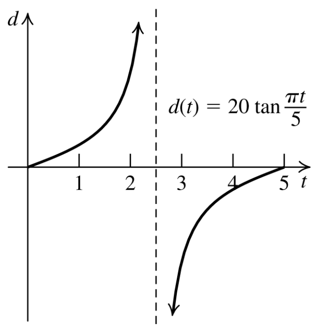


5.5 Applying the Concepts

45.a.



b. When $t = 2.5$, the light is pointing parallel to the wall.

46. The period $b = \frac{\pi}{10}$.

47. Solve the system of equations.

$$-7 = a\tan\left[b(-3-c)\right] \quad (1)$$
$$0 = a\tan\left[b(-1-c)\right] \quad (2)$$
$$7 = a\tan\left[b(1-c)\right] \quad (3)$$

Examining equation (2), either $a = 0$ or
$\tan\left[b(-1-c)\right] = 0 \Rightarrow b(-1-c) = 0 \Rightarrow$
$b = 0$ or $c = -1$

If $a = 0$ or $b = 0$, then equations (1) and (3) are false, so we disregard these solutions. Thus, $c = -1$. Substitute this value into equations (1) and (3).

$$-7 = a\tan\left[b(-3-(-1))\right] = a\tan(-2b) \quad (4)$$
$$7 = a\tan\left[b(1-(-1))\right] = a\tan(2b) \quad (5)$$

If we let $a = 7$, then

$$\tan(2b) = 1 \Rightarrow 2b = \frac{\pi}{4} \Rightarrow b = \frac{\pi}{8}.$$

Verify that $a = 7,\ b = \frac{\pi}{8},\ c = -1$ makes equations (1)–(3) true.

The model is $y = 7\tan\left[\frac{\pi}{8}(x+1)\right]$.

$$y(-1.5) = 7\tan\left[\frac{\pi}{8}(-1.5+1)\right] \approx -1.3924$$
$$y(1.5) = 7\tan\left[\frac{\pi}{8}(1.5+1)\right] \approx 10.4762$$

48. Solve the system of equations.

$$6 = a\tan\left[b(-0.5-c)\right] \quad (1)$$
$$0 = a\tan\left[b(2-c)\right] \quad (2)$$
$$-6 = a\tan\left[b(4.5-c)\right] \quad (3)$$

Examining equation (2), either $a = 0$ or
$\tan\left[b(2-c)\right] = 0 \Rightarrow b(2-c) = 0 \Rightarrow$
$b = 0$ or $c = 2$

If $a = 0$ or $b = 0$, then equations (1) and (3) are false, so we disregard these solutions. Thus, $c = 2$. Substitute this value into equations (1) and (3).

$$6 = a\tan\left[b(-0.5-2)\right] = a\tan(-2.5b) \quad (4)$$
$$-6 = a\tan\left[b(4.5.-2)\right] = a\tan(2.5b) \quad (5)$$

If we let $a = -6$, then

$$\tan(2.5b) = 1 \Rightarrow 2.5b = \frac{\pi}{4} \Rightarrow b = \frac{\pi}{10}.$$

Verify that $a = -6,\ b = \frac{\pi}{10},\ c = 2$ makes equations (1)–(3) true.

The model is $y = -6\tan\left[\frac{\pi}{10}(x-2)\right]$.

$$y(-1.5) = -6\tan\left[\frac{\pi}{10}(x-2)\right] \approx 11.7757$$
$$y(1.5) = -6\tan\left[\frac{\pi}{10}(x-2)\right] \approx 0.9503$$

49. The asymptotes are $x = 1$ and $x = 5$, so the period is 4 and $b = \frac{\pi}{4}$.

Now solve the system of equations

$$2 = a\cot\left[\frac{\pi}{4}(2-c)\right] \quad (1)$$
$$0 = a\cot\left[\frac{\pi}{4}(3-c)\right] \quad (2)$$
$$-2 = a\cot\left[\frac{\pi}{4}(4-c)\right] \quad (3)$$

Examining equation (2), either $a = 0$ or
$\cot\left[\frac{\pi}{4}(3-c)\right] = 0 \Rightarrow \frac{\pi}{4}(3-c) = \frac{\pi}{2} \Rightarrow$
$3 - c = 2 \Rightarrow c = 1$

(*continued on next page*)

(*continued*)

If $a = 0$, then equations (1) and (3) are false, so we disregard this solution. Substitute $c = 1$ into equation (1) and solve for a.

$$2 = a\cot\left[\frac{\pi}{4}(2-1)\right] \Rightarrow 2 = a\cot\frac{\pi}{4} \Rightarrow 2 = a$$

Verify that $a = 2$ and $c = 1$ makes equation (3) true. The model is $y = 2\cot\left[\frac{\pi}{4}(x-1)\right]$.

50. The asymptotes are $x = 2$ and $x = 8$, so the period is 6 and $b = \frac{\pi}{6}$.

Now solve the system of equations

$$-3 = a\cot\left[\frac{\pi}{6}(3.5-c)\right] \quad (1)$$
$$0 = a\cot\left[\frac{\pi}{6}(5-c)\right] \quad (2)$$
$$3 = a\cot\left[\frac{\pi}{6}(6.5-c)\right] \quad (3)$$

Examining equation (2), either $a = 0$ or

$$\cot\left[\frac{\pi}{6}(5-c)\right] = 0 \Rightarrow \frac{\pi}{6}(5-c) = \frac{\pi}{2} \Rightarrow$$
$$5 - c = 3 \Rightarrow c = 2$$

If $a = 0$, then equations (1) and (3) are false, so we disregard this solution. Substitute $c = 2$ into equation (1) and solve for a.

$$-3 = a\cot\left[\frac{\pi}{6}(3.5-2)\right] \Rightarrow -3 = a\cot\frac{\pi}{4} \Rightarrow$$
$$-3 = a$$

Verify that $a = -3$ and $c = 2$ makes equation (3) true. The model is $y = -3\cot\left[\frac{\pi}{6}(x-2)\right]$.

51. The asymptotes are $x = -2$ and $x = 0$, so the period is 2 and $b = \frac{\pi}{2}$.

Now solve the system of equations

$$2 = a\sec\left[\frac{\pi}{2}(-3-c)\right] \quad (1)$$
$$-2 = a\sec\left[\frac{\pi}{2}(-1-c)\right] \quad (2)$$
$$2 = a\cot\left[\frac{\pi}{2}(1-c)\right] \quad (3)$$

Notice that the values of y are 2, −2, and 2, so let $a = 2$ or $a = -2$.

Using equation (3), if $a = 2$, then

$$2 = 2\sec\left[\frac{\pi}{2}(1-c)\right] \Rightarrow 1 = \sec\left[\frac{\pi}{2}(1-c)\right] \Rightarrow$$
$$\frac{\pi}{2}(1-c) = 0 \Rightarrow c = 1$$

Checking $a = 2$ and $c = 1$ in equation (2) gives

$$-2 = 2\sec\left[\frac{\pi}{2}(-1-1)\right] \Rightarrow -2 = 2\sec[-2\pi] \Rightarrow$$

$2 = -2$, which is false. So, let $a = -2$, then use equation (3) to solve for c.

$$2 = -2\sec\left[\frac{\pi}{2}(1-c)\right] \Rightarrow -1 = \sec\left[\frac{\pi}{2}(1-c)\right] \Rightarrow$$
$$\frac{\pi}{2}(1-c) = \pi \Rightarrow c = -1$$

Verify $a = -2$ and $c = -1$ in equations (1) and (2). Thus, the model is

$$y = -2\sec\left[\frac{\pi}{2}(x-(-1))\right] = -2\sec\left[\frac{\pi}{2}(x+1)\right].$$

When $x = \frac{1}{3}$, the model gives

$$y = -2\sec\left[\frac{\pi}{2}\left(\frac{1}{3}+1\right)\right] = -2\sec\left[\frac{2\pi}{3}\right]$$
$$= -2(-2) = 4$$

52. The asymptotes are $x = -2$ and $x = 0$, so the period is 2 and $b = \frac{\pi}{2}$.

Now solve the system of equations

$$2 = a\csc\left[\frac{\pi}{2}(-3-c)\right] \quad (1)$$
$$-2 = a\csc\left[\frac{\pi}{2}(-1-c)\right] \quad (2)$$
$$2 = a\csc\left[\frac{\pi}{2}(1-c)\right] \quad (3)$$

Notice that the values of y are 2, −2, and 2, so let $a = 2$ or $a = -2$. Using equation (3), if $a = 2$, then

$$2 = 2\csc\left[\frac{\pi}{2}(1-c)\right] \Rightarrow 1 = \csc\left[\frac{\pi}{2}(1-c)\right] \Rightarrow$$
$$\frac{\pi}{2} = \frac{\pi}{2}(1-c) \Rightarrow c = 0$$

Verify $a = 2$ and $c = 0$ in equations (1) and (2). Thus, the model is $y = 2\csc\left(\frac{\pi}{2}x\right)$.

5.5 Beyond the Basics

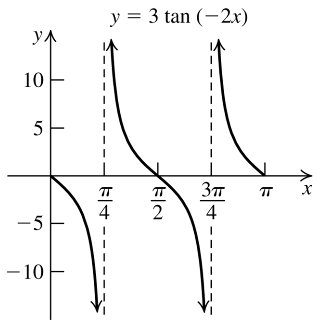
53. For any x,

$$\frac{1}{f}(x+p) = \frac{1}{f(x+p)} = \frac{1}{f(x)} = \frac{1}{f}(x)$$

54. If f is an odd function, then $f(-x) = -f(x)$. So $\frac{1}{f}(-x) = \frac{1}{f(-x)} = \frac{1}{-f(x)} = -\frac{1}{f(x)} = -\frac{1}{f}(x)$.

55. If f is an even function, then $f(-x) = f(x)$. So we have $\frac{1}{f}(-x) = \frac{1}{f(-x)} = \frac{1}{f(x)} = \frac{1}{f}(x)$.

56.



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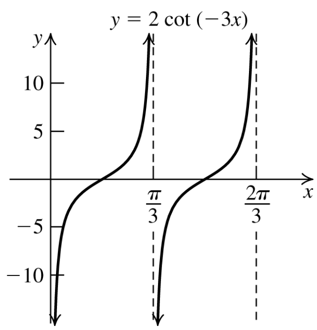

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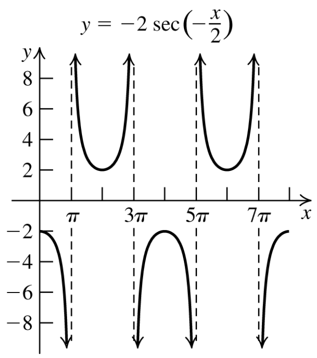
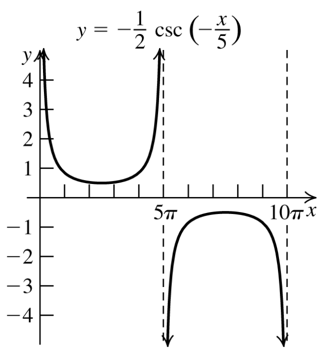


59.



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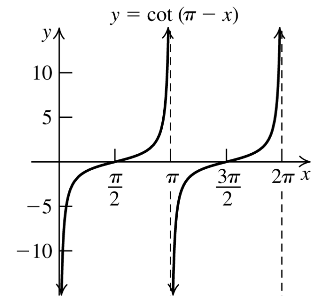

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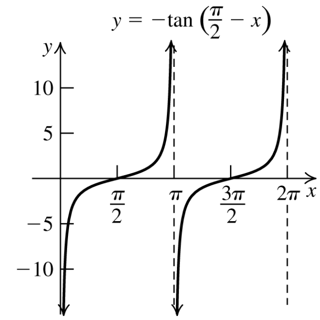
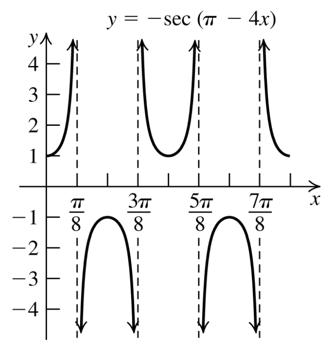


62.



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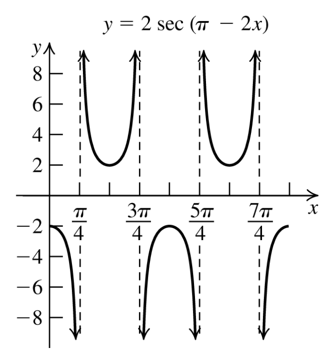
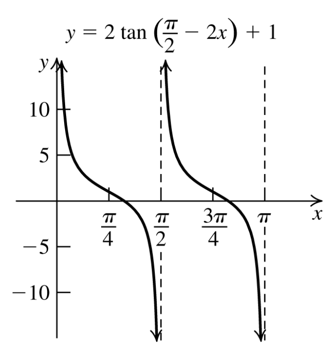


64.



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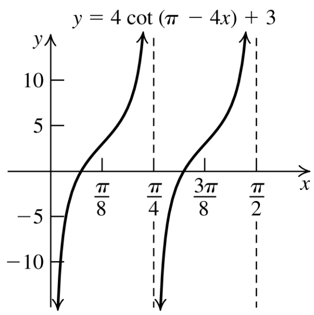


Answers may vary in exercises 66–69.

66. The graph is a cotangent, shifted $\pi/2$ units right, so $c = \frac{\pi}{2}$. The period $= 2\pi \Rightarrow$ $b = \frac{\pi}{2\pi} = \frac{1}{2}$. $a = 5$, so the equation is $y = 5\cot\left(\frac{x}{2} - \frac{\pi}{2}\right)$.

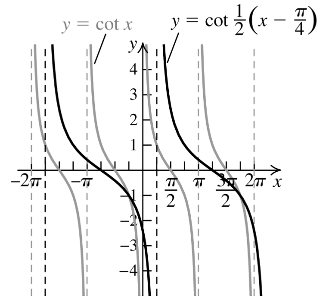
67. The graph is a tangent, shifted $\pi/2$ units left, so $c = -\frac{\pi}{2}$. The period $= \pi \Rightarrow b = \frac{\pi}{\pi} = 1$. $a = 2$, so the equation is $y = 2\tan\left(x + \frac{\pi}{2}\right)$.

68. The graph is a cosecant with no phase shift. Its minimum is 2, so $a = 2$. The period $= \pi \Rightarrow$ $b = \frac{2\pi}{\pi} = 2$. The equation is $y = 2\csc 2x$.

69. The graph is a secant, shifted π units to the right, so $c = \pi$. Its maximum is −3, so $a = -3$. The period $= 4\pi \Rightarrow b = \frac{2\pi}{4\pi} = \frac{1}{2}$. The equation is $y = -3\sec\left(\frac{x}{2} - \pi\right)$.

5.5 Critical Thinking/Discussion/Writing

70. The equation represents a cotangent curve with $b = \frac{1}{2} \Rightarrow$ period $= \frac{\pi}{1/2} = 2\pi$. Since the period of a cotangent curve is π, this curve has been stretched horizontally by a factor of 2. The curve is then shifted $\frac{\pi}{8}(2) = \frac{\pi}{4}$ units to the right. The cotangent curve has an asymptote at $x = -2\pi$, so the asymptote for this curve is also shifted $\pi/4$ units to the right. Therefore, $k = -2\pi + \frac{\pi}{4} = -\frac{7\pi}{4}$.



71. period $= \frac{\pi}{3} = \frac{2\pi}{b} \Rightarrow b = 6$

72. $y = -2\sec x = -2\csc\left(x + \frac{\pi}{2}\right)$

5.5 Maintaining Skills

73. $f(x) = 3x + 7$ is a linear function that is not horizontal, so it's a one-to-one function. Find the inverse by interchanging x and y, and then solving for y.

$f(x) = y = 3x + 7$

$x = 3y + 7 \Rightarrow x - 7 = 3y \Rightarrow \frac{x-7}{3} = y = f^{-1}(x)$

74. $f(x) = -2x + 5$ is a linear function that is not horizontal, so it's a one-to-one function. Find the inverse by interchanging x and y, and then solving for y.

$f(x) = y = -2x + 5$

$x = -2y + 5 \Rightarrow x - 5 = -2y \Rightarrow$

$-\frac{x-5}{2} = \frac{5-x}{2} = y = f^{-1}(x)$

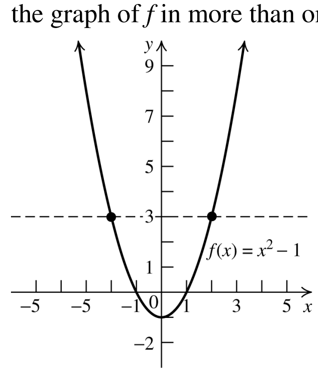
75. True

76. True

77. True

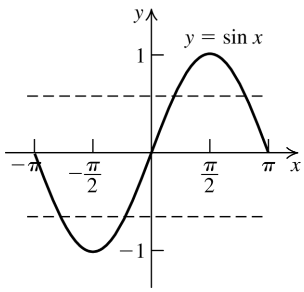
78. True

79. Using the horizontal line test (see Section 2.9), we see that any horizontal line $y = n$ intersects the graph of f in more than one point.



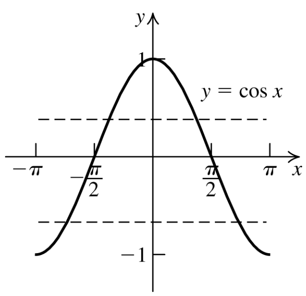
If the domain is restricted to $[0, \infty)$, then f is one-to-one.

80. Using the horizontal line test (see Section 2.9), we see that any horizontal line $y = n$ intersects the graph of f in more than one point.



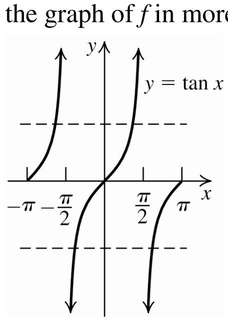
If the domain is restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then f is one-to-one.

81. Using the horizontal line test (see Section 2.9), we see that any horizontal line $y = n$ intersects the graph of f in more than one point.



If the domain is restricted to $[0, \pi]$, then f is one-to-one.

82. Using the horizontal line test (see Section 2.9), we see that any horizontal line $y = n$ intersects the graph of f in more than one point.



If the domain is restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then f is one-to-one.

5.6 Inverse Trigonometric Functions

5.6 Practice Problems

1.a. $y = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) \Rightarrow \sin y = -\frac{\sqrt{3}}{2}$ and

$-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = -\frac{\pi}{3}$

b. $y = \sin^{-1}(-1) \Rightarrow \sin y = -1$ and

$-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = -\frac{\pi}{2}$

2.a. $y = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) \Rightarrow \cos y = -\frac{\sqrt{2}}{2}$ and

$0 \le y \le \pi \Rightarrow y = \frac{3\pi}{4}$

b. $y = \cos^{-1}\frac{1}{2} \Rightarrow \cos y = \frac{1}{2}$ and

$0 \le y \le \pi \Rightarrow y = \frac{\pi}{3}$

3. $y = \tan^{-1}\frac{\sqrt{3}}{3} \Rightarrow \tan y = \frac{\sqrt{3}}{3}$ and

$-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = \frac{\pi}{6}$

4. $y = \sec^{-1} 2 \Rightarrow \sec y = 2$ and

$0 \le y \le \pi \Rightarrow y = \frac{\pi}{3}$

5.a. For $|x| \le 1$, let $\theta = \sin^{-1} x$ with θ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Then, $\sin\theta = x$. For θ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, we have $(2\pi - \theta)$ is also in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, and

$\sin(2\pi - \theta) = -\sin\theta \Rightarrow$
$\sin(2\pi - \theta) = -x \Rightarrow$
$2\pi - \theta = \sin^{-1}(-x) \Rightarrow$
$2\pi - \sin^{-1} x = \sin^{-1}(-x)$

b. $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]=\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$
$=\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)$
$=\sin\left(\frac{\pi}{2}\right)=1$

6.a. $y=\cos^{-1}0.22\approx 1.3490$

b. $y=\csc^{-1}3.5=\sin^{-1}\frac{1}{3.5}\approx 0.2898$

c. $y=\cot^{-1}(-4.7)=\pi+\tan^{-1}\left(-\frac{1}{4.7}\right)\approx 2.9320$

7.a. $y=\cot^{-1}(0.75)=\tan^{-1}\left(\frac{1}{0.75}\right)\approx 53.1301°$

b. $y=\csc^{-1}(13)=\sin^{-1}\left(\frac{1}{13}\right)\approx 4.4117°$

c. $y=\tan^{-1}(-12)\approx -85.2364°$

8. We cannot use the formula $\sin^{-1}(\sin x)=x$ for $x=\frac{3\pi}{2}$ because $\frac{3\pi}{2}$ is not in the interval $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$ for which $\sin^{-1}x$ is defined.

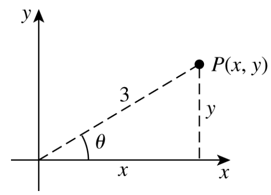
However, $\sin\frac{3\pi}{2}=\sin\left(\frac{3\pi}{2}-2\pi\right)=\sin\left(-\frac{\pi}{2}\right)$, so $\sin^{-1}\left(\sin\frac{3\pi}{2}\right)=-\frac{\pi}{2}$.

9. $\cos\left[\sin^{-1}\left(-\frac{1}{3}\right)\right]$

Let $\theta=\sin^{-1}\left(-\frac{1}{3}\right)$. Then $\sin\theta=-\frac{1}{3}$. Using the Pythagorean theorem with $y=-1$ and $r=3$, we have $x=\sqrt{3^2-(-1)^2}=\sqrt{8}=2\sqrt{2}$. So,

$\cos\left[\sin^{-1}\left(-\frac{1}{3}\right)\right]=\cos\theta=\frac{2\sqrt{2}}{3}$.

10. Let $\theta=\cos^{-1}\frac{x}{3}$. Then $\cos\theta=\frac{x}{3}$, and θ is in the interval $[0,\pi]$. The figure shows angle θ in standard position with a point $P(x, y)$ on the terminal side of θ.



Using the Pythagorean theorem, we have
$x^2+y^2=3^2\Rightarrow y^2=9-x^2\Rightarrow y=\sqrt{9-x^2}$

Substituting, we have $y=\tan\theta\Rightarrow y=\frac{\sqrt{9-x^2}}{x}$

11. $y=\frac{x}{\sqrt{7-x^2}}=\frac{\sqrt{7}\cos\theta}{\sqrt{7-\left(\sqrt{7}\cos\theta\right)^2}}$
$=\frac{\sqrt{7}\cos\theta}{\sqrt{7-7\cos^2\theta}}=\frac{\sqrt{7}\cos\theta}{\sqrt{7\left(1-\cos^2\theta\right)}}$
$=\frac{\sqrt{7}\cos\theta}{\sqrt{7\sin^2\theta}}=\frac{\cos\theta}{\sin\theta}=\cot\theta$

12. $\tan\frac{\theta}{2}=\frac{10}{12}=\frac{5}{6}$
$\frac{\theta}{2}=\tan^{-1}\left(\frac{5}{6}\right)\approx 39.81°$
$\theta\approx 79.62°$
Set the camera to rotate through 80º to scan the entire counter.

5.6 Basic Concepts and Skills

1. The domain of $f(x)=\sin^{-1}x$ is $\underline{[-1, 1]}$.

2. The range of $f(x)=\tan^{-1}x$ is $\underline{\left(-\frac{\pi}{2},\frac{\pi}{2}\right)}$.

3. The exact value of $y=\cos^{-1}\frac{1}{2}$ is $\underline{\frac{\pi}{3}}$.

4. $\sin^{-1}(\sin\pi)=\underline{0}$.

5. True

6. False. If $-1\le x\le 0$, then $\cos^{-1}x\ge 0$.

7. False. The domain of $\cos^{-1}x$ is $[-1, 1]$.

8. False. The value of $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

9. $y=\arcsin 0=0$

10. $y=\cos^{-1}0=\frac{\pi}{2}$

11. $y = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

12. $y = \arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

13. $y = \cos^{-1}(-1) = \pi$

14. $y = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

15. $y = \arccos\left(\frac{\pi}{2}\right)$ is not defined

16. $y = \sin^{-1}\pi$ is not defined.

17. $y = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

18. $y = \arctan 1 = \frac{\pi}{4}$

19. $y = \tan^{-1}(-1) = -\frac{\pi}{4}$

20. $y = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$

21. $y = \cot^{-1}(-1) = \frac{3\pi}{4}$

22. $y = \arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

23. $y = \arccos(-2)$ is not defined.

24. $y = \sin^{-1}\sqrt{3}$ is not defined.

25. $y = \sec^{-1}(-2) = \frac{2\pi}{3}$

26. $y = \csc^{-1}(-2) = -\frac{\pi}{6}$

27. $y = \sin\left(\sin^{-1}\frac{1}{8}\right) = \frac{1}{8}$

28. $y = \cos\left(\cos^{-1}\frac{1}{5}\right) = \frac{1}{5}$

29. $y = \arctan\left(\tan\frac{\pi}{7}\right) = \frac{\pi}{7}$

30. $y = \arctan\left(\tan\frac{\pi}{4}\right) = \frac{\pi}{4}$

31. $y = \tan\left(\tan^{-1} 247\right) = 247$

32. $y = \tan\left(\tan^{-1} 7\right) = 7$

33. $y = \arcsin\left(\sin\frac{4\pi}{3}\right) = \arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

34. $y = \arccos\left(\cos\frac{5\pi}{3}\right) = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

35. $y = \tan^{-1}\left(\tan\frac{2\pi}{3}\right) = \tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$

36. $y = \tan\left(\tan^{-1}\frac{2\pi}{3}\right) = \frac{2\pi}{3}$

37. $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = \tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$

38. $\sec^{-1}\left(\sqrt{2}\right) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

39. $\csc^{-1}(2) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

40. $\csc^{-1}\left(\frac{2\sqrt{3}}{3}\right) = \sin^{-1}\left(\frac{3}{2\sqrt{3}}\right) = \sin^{-1}\left(\frac{3\sqrt{3}}{6}\right)$
$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

41. $\operatorname{arccot}\left(-\frac{1}{\sqrt{3}}\right) = \arctan\left(-\sqrt{3}\right) = \frac{2\pi}{3}$

42. $\cot^{-1}\left(-\sqrt{3}\right) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$
$= \frac{5\pi}{6}$

43. $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\arcsin\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

44. $\csc^{-1}\left(-\frac{2}{\sqrt{3}}\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
$= -\frac{\pi}{3}$

45. $\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

46. $\operatorname{arcsec}\left(-\frac{2}{\sqrt{3}}\right) = \arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

47. $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right] = \sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$
$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\frac{\pi}{2}$
$= 1$

48. $\cos\left[\frac{\pi}{6} + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] = \cos\left(\frac{\pi}{6} + \frac{5\pi}{6}\right)$
$= \cos\pi = -1$

49. $\sin\left[\frac{\pi}{2} - \cos^{-1}(-1)\right] = \sin\left(\frac{\pi}{2} - \pi\right)$
$= \sin\left(-\frac{\pi}{2}\right) = -\sin\frac{\pi}{2}$
$= -1$

50. $\tan\left[\frac{\pi}{6} + \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right] = \tan\left[\frac{\pi}{6} + \tan^{-1}\left(-\sqrt{3}\right)\right]$
$= \tan\left(\frac{\pi}{6} + \frac{2\pi}{3}\right)$
$= \tan\frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$

51. $\sin\left[\tan^{-1}\left(-\sqrt{3}\right) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$
$= \sin\left(-\frac{\pi}{3} + \frac{5\pi}{6}\right) = \sin\frac{\pi}{2} = 1$

52. $\cos\left[\cot^{-1}\left(-\sqrt{3}\right) + \sin^{-1}\left(-\frac{1}{2}\right)\right]$
$= \cos\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

53. $y = \cos^{-1} 0.6 \approx 53.13°$

54. $y = \sin^{-1} 0.23 \approx 13.30°$

55. $y = \sin^{-1}(-0.69) \approx -43.63°$

56. $y = \cos^{-1}(-0.57) \approx 124.75°$

57. $y = \sec^{-1}(3.5) \approx 73.40°$

58. $y = \csc^{-1}(6.8) \approx 8.46°$

59. $y = \tan^{-1} 14 \approx 85.91°$

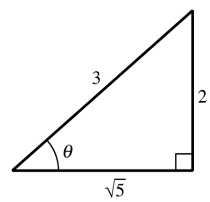
60. $y = \tan^{-1} 50 \approx 88.85°$

61. $y = \tan^{-1}(-42.147) \approx -88.64°$

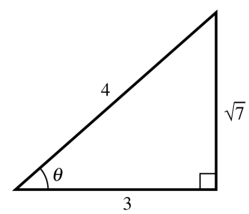
62. $y = \tan^{-1}(-0.3863) \approx -21.12°$

In exercises 63−86, use the Pythagorean theorem to find the length of the third side of the triangle.

63. $y = \cos\left(\sin^{-1}\frac{2}{3}\right) = \frac{\sqrt{5}}{3}$



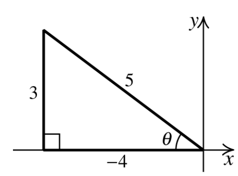
Exercise 63



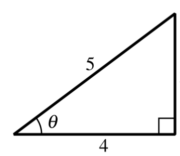
Exercise 64

64. $y = \sin\left(\cos^{-1}\frac{3}{4}\right) = \frac{\sqrt{7}}{4}$

65. $y = \sin\left(\cos^{-1}\left(-\frac{4}{5}\right)\right) = \frac{3}{5}$



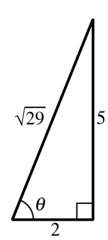
Exercise 65



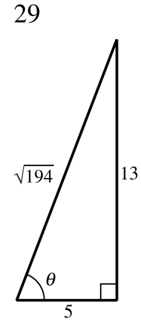
Exercise 66

66. $y = \cos\left(\sin^{-1}\frac{3}{5}\right) = \frac{4}{5}$

67. $y = \cos\left(\tan^{-1}\frac{5}{2}\right) = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$



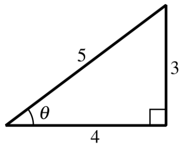
Exercise 67



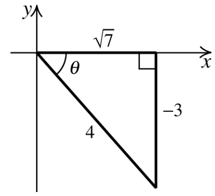
Exercise 68

68. $y = \sin\left(\tan^{-1}\frac{13}{5}\right) = \frac{13\sqrt{194}}{194}$

69. $y = \tan\left(\cos^{-1}\frac{4}{5}\right) = \frac{3}{4}$



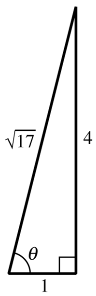
Exercise 69



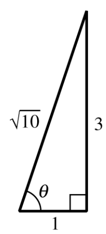
Exercise 70

70. $y = \tan\left(\sin^{-1}\left(-\frac{3}{4}\right)\right) = -\frac{3\sqrt{7}}{7}$

71. $y = \sin\left(\tan^{-1} 4\right) = \frac{4\sqrt{17}}{17}$



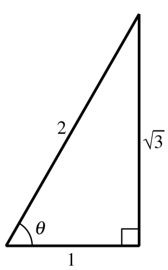
Exercise 71



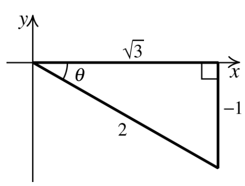
Exercise 72

72. $y = \cos\left(\tan^{-1} 3\right) = \frac{\sqrt{10}}{10}$

73. $y = \tan\left(\sec^{-1} 2\right) = \sqrt{3}$



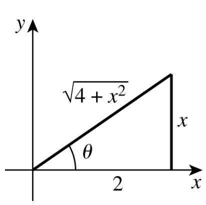
Exercise 73



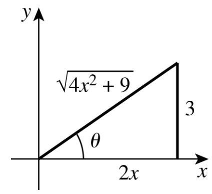
Exercise 74

74. $y = \tan\left(\csc^{-1}(-2)\right) = -\frac{\sqrt{3}}{3}$

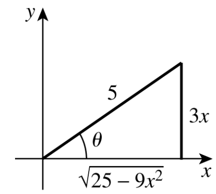
75. $y = \sin\left(\tan^{-1}\frac{x}{2}\right) = \frac{x}{\sqrt{4+x^2}}$



76. $y = \sin\left(\cot^{-1}\frac{2x}{3}\right) = \frac{3}{\sqrt{4x^2+9}}$



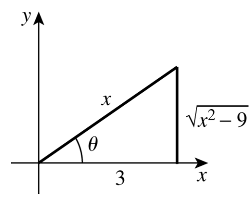
Exercise 76



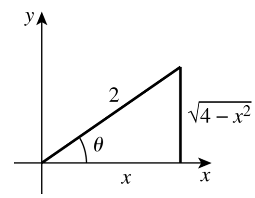
Exercise 77

77. $y = \cot\left(\sin^{-1}\frac{3x}{5}\right) = \frac{\sqrt{25-9x^2}}{3x}$

78. $y = \sin\left(\sec^{-1}\frac{x}{3}\right) = \frac{\sqrt{x^2-9}}{x}$



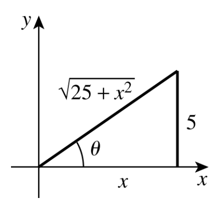
Exercise 78



Exercise 79

79. $y = \cot\left(\cos^{-1}\frac{x}{2}\right) = \frac{x}{\sqrt{4-x^2}}$

80. $y = \sin\left(\cot^{-1}\frac{x}{5}\right) = \frac{5}{\sqrt{25+x^2}}$



81. $$y = \frac{x}{\sqrt{9-x^2}} = \frac{3\cos\theta}{\sqrt{9-(3\cos\theta)^2}} = \frac{3\cos\theta}{\sqrt{9-9\cos^2\theta}} = \frac{3\cos\theta}{\sqrt{9\left(1-\cos^2\theta\right)}} = \frac{3\cos\theta}{\sqrt{9\sin^2\theta}} = \frac{3\cos\theta}{3\sin\theta} = \cot\theta$$

82. $$y = \frac{\sqrt{16-x^2}}{x} = \frac{\sqrt{16-(4\sin\theta)^2}}{4\sin\theta} = \frac{\sqrt{16-16\sin^2\theta}}{4\sin\theta} = \frac{\sqrt{16\left(1-\sin^2\theta\right)}}{4\sin\theta} = \frac{\sqrt{16\cos^2\theta}}{4\sin\theta} = \frac{4\cos\theta}{4\sin\theta} = \cot\theta$$

83. $y = \frac{x}{\sqrt{4+x^2}} = \frac{2\tan\theta}{\sqrt{4+(2\tan\theta)^2}} = \frac{2\tan\theta}{\sqrt{4+4\tan^2\theta}}$
$= \frac{2\tan\theta}{\sqrt{4(1+\tan^2\theta)}} = \frac{2\tan\theta}{\sqrt{4\sec^2\theta}} = \frac{2\tan\theta}{2\sec\theta}$
$= \frac{\frac{2\sin\theta}{\cos\theta}}{\frac{2}{\cos\theta}} = \sin\theta$

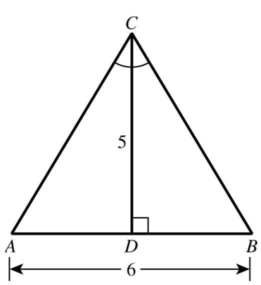
84. $y = \frac{\sqrt{9+x^2}}{x} = \frac{\sqrt{9+(3\cot\theta)^2}}{3\cot\theta} = \frac{\sqrt{9+9\cot^2\theta}}{3\cot\theta}$
$= \frac{\sqrt{9(1+\cot^2\theta)}}{3\cot\theta} = \frac{\sqrt{9\csc^2\theta}}{3\cot\theta} = \frac{3\csc\theta}{3\cot\theta}$
$= \frac{\frac{1}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}} = \frac{1}{\cos\theta} = \sec\theta$

85. $y = \frac{x}{\sqrt{x^2-25}} = \frac{5\sec\theta}{\sqrt{(5\sec\theta)^2-25}}$
$= \frac{5\sec\theta}{\sqrt{25\sec^2\theta-25}} = \frac{5\sec\theta}{\sqrt{25(\sec^2\theta-1)}}$
$= \frac{5\sec\theta}{\sqrt{25\tan^2\theta}} = \frac{5\sec\theta}{5\tan\theta} = \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \frac{1}{\sin\theta} = \csc\theta$

86. $y = \frac{\sqrt{x^2-36}}{x} = \frac{\sqrt{(6\csc\theta)^2-36}}{6\csc\theta}$
$= \frac{\sqrt{36\csc^2\theta-36}}{6\csc\theta} = \frac{\sqrt{36(\csc^2\theta-1)}}{6\csc\theta}$
$= \frac{\sqrt{36\cot^2\theta}}{6\csc\theta} = \frac{6\cot\theta}{6\csc\theta} = \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} = \cos\theta$

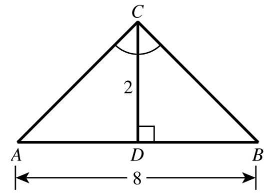
5.6 Applying the Concepts

87.



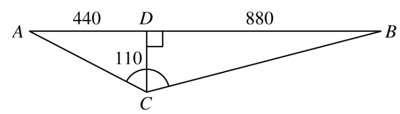
$\triangle ABC$ is isosceles, so CD bisects AB and $\angle ACB$. Then $BD = 3$ and
$\angle BCD = \tan^{-1}\left(\frac{3}{5}\right) \approx 31° \Rightarrow m\angle ACB \approx 62°$

88.



$\triangle ABC$ is isosceles, so CD bisects AB and $\angle ACB$. Then $BD = 4$ and
$\angle BCD = \tan^{-1}\left(\frac{4}{2}\right) \approx 63.4° \Rightarrow m\angle ACB \approx 127°$

89.



Not drawn to scale

$m\angle ACB = m\angle ACD + m\angle BCD$
$m\angle ACB = \tan^{-1}\left(\frac{440}{110}\right) + \tan^{-1}\left(\frac{880}{110}\right) \approx 159°$

90.a. $\theta = 2\tan^{-1}\frac{18}{50} \approx 39.6°$

b. $\theta = 2\tan^{-1}\frac{18}{200} \approx 10.3°$

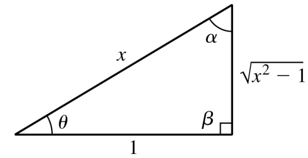
5.6 Beyond the Basics

91. Examining the graph of $y = \sin^{-1} x$ in the text (Figure 5.76), we see that the function is increasing on its domain.

92. Examining the graph of $y = \cos^{-1} x$ in the text (Figure 5.78), we see that the function is decreasing on its domain.

93. Examining the graph of $y = \tan^{-1} x$ in the text (Figure 5.80), we see that the function is increasing on its domain.

94.



a. Using the figure, we have $\theta = \sec^{-1} x$.

(*continued on next page*)

(*continued*)

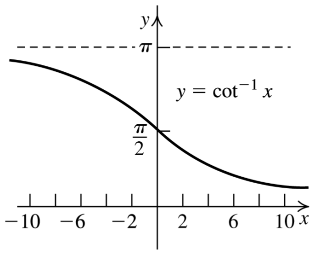
There is no angle θ' such that $\theta' = \cos^{-1} x$

$\left(\text{note that} \cos\alpha = \dfrac{\sqrt{x^2 - 1}}{x} \text{ and } \cos\beta = 0 \right),$

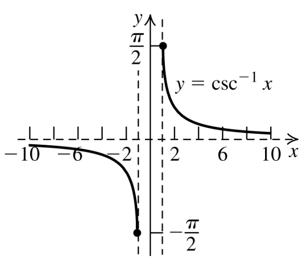
so $\dfrac{1}{\cos^{-1} x}$ doesn't exist for $\theta = \sec^{-1} x$.

b. Using the triangle, we have $\theta = \sec^{-1} x$ and $\theta = \cos^{-1} \dfrac{1}{x}$. So $\sec^{-1} x = \cos^{-1} \dfrac{1}{x}$.

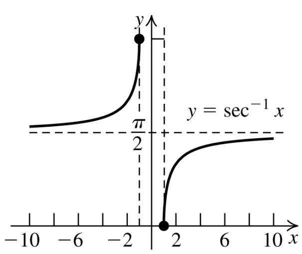
95.



96.



97.



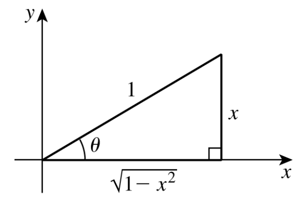
98. $\cot 0$ is undefined so $\cot^{-1}(\cot x) = x$ is true for $(0, \pi)$

99. $\sec\dfrac{\pi}{2}$ is undefined, so $\sec^{-1}(\sec x) = x$ is true for $\left[0, \dfrac{\pi}{2}\right) \cup \left(\dfrac{\pi}{2}, \pi\right]$

100. $\csc 0$ is undefined so $\csc^{-1}(\csc x) = x$ is true for $\left[-\dfrac{\pi}{2}, 0\right) \cup \left(0, \dfrac{\pi}{2}\right]$

5.6 Critical Thinking/Discussion/Writing

101.



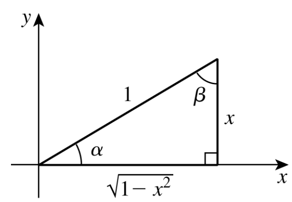
From the figure, we see that

$\sin^{-1} x = \cos^{-1} \sqrt{1 - x^2}$ if $0 \le x \le 1$.

For $-1 \le x < 0$, $\sin^{-1} x = -\cos^{-1} \sqrt{1 - x^2}$ since $\sin^{-1}(-x) = -\sin^{-1} x$.

102. Let $\sin^{-1} x = \alpha$ and let $\cos^{-1} x = \beta$.

By definition $\sin\alpha = x$, $-\dfrac{\pi}{2} < \alpha \le \dfrac{\pi}{2}$, and $\cos\beta = x$, $0 \le \beta < \pi$. Therefore $\sin\alpha = \cos\beta$ for $0 \le \alpha, \beta < \dfrac{\pi}{2}$.



From the diagram, we see that $\beta = \dfrac{\pi}{2} - \alpha$, or $\alpha + \beta = \dfrac{\pi}{2}$. Substituting gives us

$\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$.

5.6 Maintaining Skills

103.
$$\begin{aligned} 5(x+3) &= 8x - 24 \\ 5x + 15 &= 8x - 24 \\ -3x &= -39 \\ x &= 13 \end{aligned}$$
Conditional equation

104.
$$\begin{aligned} x(20-3) - 1 &= 10x(2-1) + 7x - 1 \\ 17x - 1 &= 17x - 1 \end{aligned}$$
Identity

105.
$$\begin{aligned} 3x + 1 &= 5(x+1) - 2x \\ 3x + 1 &= 3x + 5 \end{aligned}$$
Neither a conditional equation nor an identity

106. $x^2 - 4 = 2x^2 + 3x - 2$
$0 = x^2 + 3x + 2$
$0 = (x+1)(x+2) \Rightarrow$
$x = -1, -2$
Conditional equation

107. $\dfrac{1}{x^2 - 9} = \dfrac{1}{(x+3)(x-3)}$
$\dfrac{1}{x^2 - 9} = \dfrac{1}{x^2 - 9}$
Identity

108. $x^2 - 5x + 6 = 0$
$(x-2)(x-3) = 0 \Rightarrow x = 2, 3$
Conditional equation

109. $x^2 + 5x = 8x - 2x^2 + 1$
$3x^2 - 3x - 1 = 0$
$x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-1)}}{2(-3)}$
$= \dfrac{3 \pm \sqrt{9+12}}{-6} = \dfrac{3 \pm \sqrt{21}}{-6}$
Conditional equation

110. $\dfrac{1}{x^2 - 3x + 2} = \dfrac{1}{(x-1)(x-2)}$
$\dfrac{1}{x^2 - 3x + 2} = \dfrac{1}{x^2 - 3x + 2}$
Identity

111. $(x-2)(x+2) + 5x - 2 = x^2 + 5x - 5$
$x^2 - 5 + 5x - 2 = x^2 + 5x - 5$
$-7 = -5$
Neither a conditional equation nor an identity

112. $x(x-2) + 3 = x(1-x) + 4$
$x^2 - 2x + 3 = x - x^2 + 4$
$2x^2 - 3x - 1 = 0$
$x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-1)}}{2(2)}$
$= \dfrac{3 \pm \sqrt{9+8}}{4} = \dfrac{3 \pm \sqrt{17}}{4}$
Conditional equation

113. $x^2 + 2x + 9 = (x-1)(x-2) + 5x + 7$
$x^2 + 2x + 9 = x^2 - 3x + 2 + 5x + 7$
$x^2 + 2x + 9 = x^2 + 2x + 9$
Identity

114. $3x(x+2) = x - 1$
$3x^2 + 6x = x - 1$
$3x^2 + 5x + 1 = 0$
$x = \dfrac{-5 \pm \sqrt{5^2 - 4(3)(1)}}{2(3)} = \dfrac{-5 \pm \sqrt{13}}{6}$
Conditional equation

115. $\dfrac{1}{x-1} - \dfrac{1}{x+1} = \dfrac{(x+1)-(x-1)}{(x-1)(x+1)} = \dfrac{2}{x^2 - 1}$

116. $\dfrac{1}{2-x} + \dfrac{1}{2+x} = \dfrac{(2+x)+(2-x)}{(2-x)(2+x)} = \dfrac{4}{4-x^2}$

117. $\dfrac{1}{x-1} - \dfrac{1}{x^2-1} = \dfrac{1}{x-1} - \dfrac{1}{(x-1)(x+1)}$
$= \dfrac{(x+1)-1}{(x-1)(x+1)} = \dfrac{x}{x^2-1}$

118. $\dfrac{1}{x+2} - \dfrac{1}{x+3} = \dfrac{(x+3)-(x+2)}{(x+2)(x+3)}$
$= \dfrac{1}{x^2 + 5x + 6}$

119. $\dfrac{1}{\sqrt{2}+1} + \dfrac{1}{\sqrt{3}+\sqrt{2}}$
$= \dfrac{1}{\sqrt{2}+1} \cdot \dfrac{\sqrt{2}-1}{\sqrt{2}-1} + \dfrac{1}{\sqrt{3}+\sqrt{2}} \cdot \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$
$= \dfrac{\sqrt{2}-1}{2-1} + \dfrac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{2} - 1 + \sqrt{3} - \sqrt{2}$
$= \sqrt{3} - 1$

120. $\dfrac{x}{\sqrt{x+2}-\sqrt{2}} = \dfrac{x}{\sqrt{x+2}-\sqrt{2}} \cdot \dfrac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}$
$= \dfrac{x\left(\sqrt{x+2}+\sqrt{2}\right)}{x+2-2}$
$= \dfrac{x\left(\sqrt{x+2}+\sqrt{2}\right)}{x} = \sqrt{x+2} + \sqrt{2}$

121. $\dfrac{x}{1+y} - \dfrac{1-y}{x} = \dfrac{x^2 - (1-y)(1+y)}{(1+y)x}$
$= \dfrac{x^2 - \left(1-y^2\right)}{(1+y)x}$
$= \dfrac{x^2 + y^2 - 1}{(1+y)x}$

We are given $x^2 + y^2 = 1$, so substituting in the numerator gives $\dfrac{x^2 + y^2 - 1}{(1+y)x} = \dfrac{1-1}{(1+y)x} = 0.$

122. $\left(x+\frac{1}{y}\right)\left(y+\frac{1}{x}\right)=xy+1+1+\frac{1}{xy}$

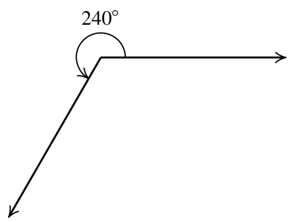
$$=xy+2+\frac{1}{xy}$$

We are given $xy=1$, so substituting gives

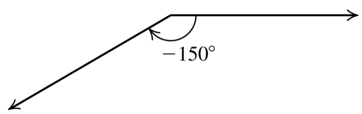
$$xy+2+\frac{1}{xy}=1+2+\frac{1}{1}=4.$$

Chapter 5 Review Exercises

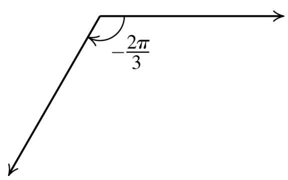
1.



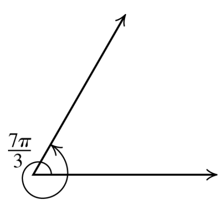
2.



3.



4.



5. $20°=20\cdot\frac{\pi}{180}=\frac{\pi}{9}$ radians

6. $36°=36\cdot\frac{\pi}{180}=\frac{\pi}{5}$ radians

7. $-60°=-60\cdot\frac{\pi}{180}=-\frac{\pi}{3}$ radians

8. $\frac{7\pi}{10}=\frac{7\pi}{10}\cdot\frac{180}{\pi}=126°$

9. $\frac{5\pi}{18}=\frac{5\pi}{18}\cdot\frac{180}{\pi}=50°$

10. $-\frac{4\pi}{9}=-\frac{4\pi}{9}\cdot\frac{180}{\pi}=-80°$

11. $\theta=\frac{s}{r}=\frac{40}{15}=2.667$ radians

12. $\theta=\frac{s}{r}=\frac{17}{9}=1.889$ radians

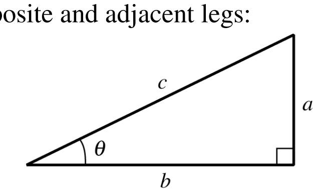
13. Convert the angle measurement from degrees to radians: $36°=36°\left(\frac{\pi}{180°}\right)=\frac{s}{2}\Rightarrow s=1.257\text{ m}$.

14. Convert the angle measurement from degrees to radians: $12°=12°\left(\frac{\pi}{180°}\right)=\frac{s}{0.9}\Rightarrow s=0.188\text{ m}$.

15. $A=\frac{1}{2}r^2\theta=\frac{1}{2}\cdot 3^2\cdot 32°\cdot\frac{\pi}{180°}\approx 2.513\text{ ft}^2$

16. $A=\frac{1}{2}r^2\theta=\frac{1}{2}\cdot 4^2\cdot 47°\cdot\frac{\pi}{180°}\approx 6.562\text{ m}^2$

For exercises 17–20, use this triangle to help identify the opposite and adjacent legs:



17. $\cos\theta=\frac{2}{9}\Rightarrow b=2, c=9$

$9^2=a^2+2^2\Rightarrow a=\sqrt{77}$

$\sin\theta=\frac{\sqrt{77}}{9}, \tan\theta=\frac{\sqrt{77}}{2}, \cot\theta=\frac{2\sqrt{77}}{77},$

$\sec\theta=\frac{9}{2}, \csc\theta=\frac{9\sqrt{77}}{77}$

18. $\sin\theta=\frac{1}{5}\Rightarrow a=1, c=5$

$5^2=1^1+b^2\Rightarrow b=2\sqrt{6}$

$\cos\theta=\frac{2\sqrt{6}}{5}, \tan\theta=\frac{\sqrt{6}}{12}, \cot\theta=2\sqrt{6},$

$\sec\theta=\frac{5\sqrt{6}}{12}, \csc\theta=5$

19. $\tan\theta=\frac{5}{3}\Rightarrow a=5, b=3$

$c^2=5^2+3^2\Rightarrow c=\sqrt{34}$

$\sin\theta=\frac{5\sqrt{34}}{34}, \cos\theta=\frac{3\sqrt{34}}{34}, \cot\theta=\frac{3}{5},$

$\sec\theta=\frac{\sqrt{34}}{3}, \csc\theta=\frac{\sqrt{34}}{5}$

20. $\cot\theta = \frac{5}{4} \Rightarrow a = 4, b = 5$
$c^2 = 4^2 + 5^2 \Rightarrow c = \sqrt{41}$
$\sin\theta = \frac{4\sqrt{41}}{41}, \cos\theta = \frac{5\sqrt{41}}{41}, \tan\theta = \frac{4}{5},$
$\sec\theta = \frac{\sqrt{41}}{5}, \csc\theta = \frac{\sqrt{41}}{4}$

21. $x = 2, y = 8 \Rightarrow r^2 = 8^2 + 2^2 \Rightarrow r = 2\sqrt{17}$
$\sin\theta = \frac{4\sqrt{17}}{17}, \cos\theta = \frac{\sqrt{17}}{17}, \tan\theta = 4,$
$\cot\theta = \frac{1}{4}, \sec\theta = \sqrt{17}, \csc\theta = \frac{\sqrt{17}}{4}$

22. $x = -3, y = 7 \Rightarrow r^2 = 7^2 + (-3)^2 \Rightarrow r = \sqrt{58}$
$\sin\theta = \frac{7\sqrt{58}}{58}, \cos\theta = -\frac{3\sqrt{58}}{58}, \tan\theta = -\frac{7}{3},$
$\cot\theta = -\frac{3}{7}, \sec\theta = -\frac{\sqrt{58}}{3}, \csc\theta = \frac{\sqrt{58}}{7}$

23. $x = -\sqrt{5}, y = 2 \Rightarrow r^2 = 2^2 + \left(-\sqrt{5}\right)^2 \Rightarrow r = 3$
$\sin\theta = \frac{2}{3}, \cos\theta = -\frac{\sqrt{5}}{3}, \tan\theta = -\frac{2\sqrt{5}}{5},$
$\cot\theta = -\frac{\sqrt{5}}{2}, \sec\theta = -\frac{3\sqrt{5}}{5}, \csc\theta = \frac{3}{2}$

24. $x = -\sqrt{3}, y = -\sqrt{6} \Rightarrow r^2 = \left(-\sqrt{6}\right)^2 + \left(-\sqrt{3}\right)^2 \Rightarrow$
$r = 3$
$\sin\theta = -\frac{\sqrt{6}}{3}, \cos\theta = -\frac{\sqrt{3}}{3}, \tan\theta = \sqrt{2},$
$\cot\theta = \frac{\sqrt{2}}{2}, \sec\theta = -\sqrt{3}, \csc\theta = -\frac{\sqrt{6}}{2}$

25. Quadrant IV **26.** Quadrant I

27. Quadrant III **28.** Quadrant II

29. $\cos\theta = -\frac{4}{5}$, θ in Quadrant III $\Rightarrow x < 0, y < 0$
$x = -4, r = 5 \Rightarrow 5^2 = (-4)^2 + y^2 \Rightarrow y = -3$
$\sin\theta = -\frac{3}{5}, \tan\theta = \frac{3}{4}, \cot\theta = \frac{4}{3}, \sec\theta = -\frac{5}{4},$
$\csc\theta = -\frac{5}{3}$

30. $\tan\theta = -\frac{5}{12}$, θ in Quadrant IV $\Rightarrow x > 0, y < 0$
$y = -5, x = 12 \Rightarrow r = \sqrt{12^2 + (-5)^2} = 13$
$\sin\theta = -\frac{5}{13}, \cos\theta = \frac{12}{13}, \cot\theta = -\frac{12}{5},$
$\sec\theta = \frac{13}{12}, \csc\theta = -\frac{13}{5}$

31. $\sin\theta = \frac{3}{5}$, θ in Quadrant II $\Rightarrow x < 0, y > 0$
$y = 3, r = 5 \Rightarrow 5^2 = x^2 + 3^2 \Rightarrow x = -4$
$\cos\theta = -\frac{4}{5}, \tan\theta = -\frac{3}{4}, \cot\theta = -\frac{4}{3},$
$\sec\theta = -\frac{5}{4}, \csc\theta = \frac{5}{3}$

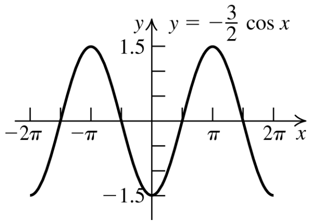
32. $\csc\theta = -\frac{5}{4}$, θ in Quadrant III $\Rightarrow x < 0, y < 0$
$y = -4, r = 5 \Rightarrow 5^2 = x^2 + (-4)^2 \Rightarrow x = -3$
$\sin\theta = -\frac{4}{5}, \cos\theta = -\frac{3}{5}, \tan\theta = \frac{4}{3}, \cot\theta = \frac{3}{4},$
$\sec\theta = -\frac{5}{3}$

33. Because $150°$ lies in Quadrant II, the reference angle is $180° - \theta = 180° - 150° = 30°$. In Quadrant II, the cosine is negative, so
$\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$.

34. $-300°$ is co-terminal with $60°$. Because $60°$ lies in Quadrant I, the reference angle is $60°$. The sine is positive in Quadrant I, so
$\sin(-300°) = \sin 60° = \frac{\sqrt{3}}{2}$.

35. $390°$ is co-terminal with $30°$. Because $30°$ lies in Quadrant I, the reference angle is $30°$. The tangent is positive in Quadrant I, so
$\tan 390° = \tan 30° = \frac{\sqrt{3}}{3}$.

36. $-405°$ is co-terminal with $315°$. Because $315°$ lies in Quadrant IV, the reference angle is $360° - \theta = 360° - 315° = 45°$. The cotangent is negative in Quadrant IV, so
$\cot(-405°) = -\cot 45° = -1$.

37.


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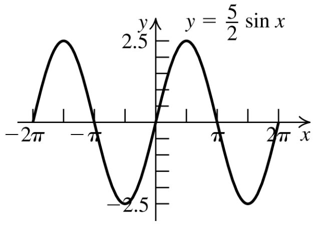

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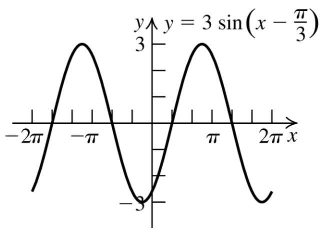

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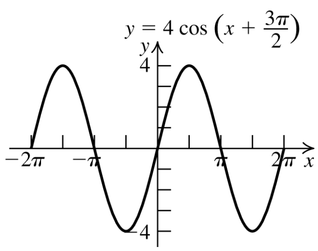
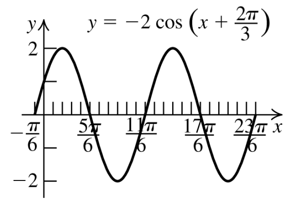


41. $y = 14\sin\left(2x + \frac{\pi}{7}\right) = 14\sin\left[2\left(x + \frac{\pi}{14}\right)\right] \Rightarrow$
$a = 14, b = 2, c = -\frac{\pi}{14} \Rightarrow$ amplitude $= 14$,
period $= \frac{2\pi}{2} = \pi$, phase shift $= -\frac{\pi}{14}$

42. $y = 21\cos\left(8x + \frac{\pi}{9}\right) = 21\cos\left[8\left(x + \frac{\pi}{72}\right)\right] \Rightarrow$
$a = 21, b = 8, c = -\frac{\pi}{72} \Rightarrow$ amplitude $= 21$,
period $= \frac{2\pi}{8} = \frac{\pi}{4}$, phase shift $= -\frac{\pi}{72}$

43. $y = 6\tan\left(2x + \frac{\pi}{5}\right) = 6\tan\left[2\left(x + \frac{\pi}{10}\right)\right] \Rightarrow$
$a = 6, b = 2, c = -\frac{\pi}{10} \Rightarrow$ vertical stretch $= 6$,
period $= \frac{\pi}{2}$, phase shift $= -\frac{\pi}{10}$

44. $y = -11\cot\left(6x + \frac{\pi}{12}\right) = -11\cot\left[6\left(x + \frac{\pi}{72}\right)\right] \Rightarrow$
$a = -11, b = 6, c = -\frac{\pi}{72} \Rightarrow$ vertical stretch $= 11$,
period $= \frac{\pi}{6}$, phase shift $= -\frac{\pi}{72}$

45.


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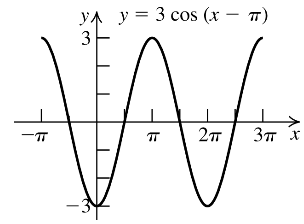

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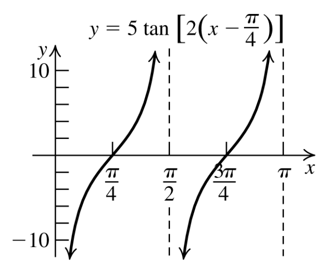

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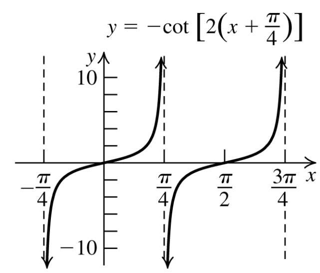

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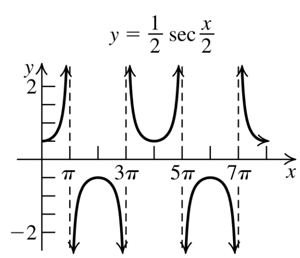

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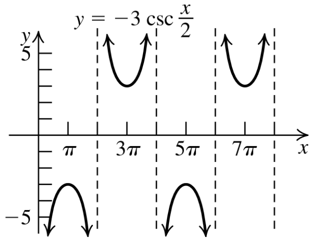
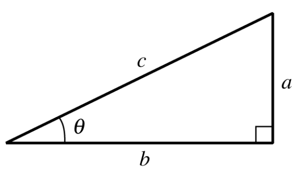


51. $y = \cos^{-1}\left(\cos\frac{5\pi}{8}\right) = \frac{5\pi}{8}$

52. $y = \sin^{-1}\left(\sin\frac{7\pi}{6}\right) = -\frac{\pi}{6}$

53. $y = \tan^{-1}\left(\tan\left(-\frac{2\pi}{3}\right)\right) = \frac{\pi}{3}$

In exercises 54–58, use the Pythagorean theorem and the triangle below to find the length of the third side of the triangle.



54. $\cos^{-1}\frac{1}{2} \Rightarrow b = 1, c = 2 \Rightarrow 2^2 = a^2 + 1^2 \Rightarrow a = \sqrt{3}$

$y = \sin\left(\cos^{-1}\frac{1}{2}\right) = \frac{a}{c} = \frac{\sqrt{3}}{2}$

55. $\sin^{-1}\frac{\sqrt{2}}{2} \Rightarrow a = \sqrt{2}, c = 2 \Rightarrow$

$2^2 = \left(\sqrt{2}\right)^2 + b^2 \Rightarrow b = \sqrt{2}$

$y = \cos\left(\sin^{-1}\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}$

56. $\tan^{-1}\frac{3}{4} \Rightarrow a = 3, b = 4 \Rightarrow c^2 = 3^2 + 4^2 \Rightarrow c = 5$

$y = \cos\left(\tan^{-1}\frac{3}{4}\right) = \frac{4}{5}$

57. $\cos^{-1}\left(-\frac{1}{2}\right) \Rightarrow b = -1, c = 2 \Rightarrow$

$2^2 = a^2 + (-1)^2 \Rightarrow a = \sqrt{3}$

$y = \tan\left(\cos^{-1}\left(-\frac{1}{2}\right)\right) = -\sqrt{3}$

58. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) \Rightarrow a = -\sqrt{3}, c = 2 \Rightarrow$

$2^2 = \left(-\sqrt{3}\right)^2 + b^2 \Rightarrow b = 1$

$y = \tan\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = -\sqrt{3}$

59. The wheels turn a quarter of a revolution, so $\theta = \frac{\pi}{2}$. $\theta = \frac{s}{r} \Rightarrow \frac{\pi}{2} = \frac{s}{28} \Rightarrow s \approx 44$ inches

60. The smaller central angle formed by the hands of a clock at 5:00 = $\frac{5}{12}(2\pi) = \frac{5\pi}{6}$.

61. The difference in the latitudes is

$42°31' - 29°59' = 12°32' = 12 + \frac{32}{60}° \approx 12.533°$

$= 12.533°\left(\frac{\pi}{180°}\right) \approx 0.2187$ radians.

$s = 0.2187 \cdot 3960 \approx 866$ miles.

62. $v = r\omega \Rightarrow 314 = 20\omega \Rightarrow \omega = 15.7$ radians per minute

63. $d = 28$ in. $\Rightarrow r = 14$ in.
$v = r\omega \Rightarrow 21 = 14\omega \Rightarrow \omega = 1.5$ radians per second

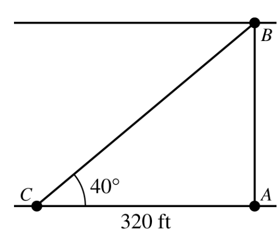
64. The distance of the satellite from the center of Mars is about 6780 + 400 = 7180 km. $t = 2$ hr
In one orbit, $\theta = 2\pi$ and
$s = r\theta \Rightarrow s = 7180 \cdot 2\pi \Rightarrow s \approx 45{,}113.3$ km .

Then $v = \frac{s}{t} = \frac{45{,}113.3 \text{ km}}{2 \text{ hr}} \approx 22{,}557$ km per hour.

65. The distance of the satellite from the center of Earth is about 36,000 + 6400 = 42,400 km.
In one orbit, $\theta = 2\pi$ and
$s = r\theta \Rightarrow s = 42{,}400 \cdot 2\pi \Rightarrow s \approx 266{,}407$ km .

$t = 24$ hr, so $v = \frac{s}{t} = \frac{266{,}407 \text{ km}}{24 \text{ hr}} \approx 11{,}100$ km per hour.

66.



$\tan 40° = \frac{AB}{320} \Rightarrow AB \approx 268.5$ ft

67. The highest number of daylight hours is 14.3 and the lowest number of daylight hours is 9.8, so $a = \frac{14.3 - 9.8}{2} = 2.25$.

The vertical shift $d = \frac{14.3 + 9.8}{2} = 12.05$.

The period is $12 = \frac{2\pi}{b} \Rightarrow b = \frac{\pi}{6}$.

The highest point on the graph, 14.3, occurs at $x = 7$, so the phase shift $c = 7 - 3 = 4$. The equation is $y = 2.25\sin\left[\frac{\pi}{6}(x - 4)\right] + 12.05$.

68. Because the ball was pulled down 11 inches, $a = -11$. The period is 5, so $\omega = 2\pi/5$. The equation is $y = -11\cos\left(\frac{2\pi}{5}t\right)$.

Chapter 5 Practice Test A

1. $140° = 140 \cdot \frac{\pi}{180} = \frac{7\pi}{9} \approx 2.4435$ radians

2. $\frac{7\pi}{5} = \frac{7\pi}{5} \cdot \frac{180}{\pi} = 252°$

3. $x = 2, y = -5 \Rightarrow r^2 = 2^2 + (-5)^2 \Rightarrow r = \sqrt{29}$

$\cos\theta = \frac{x}{r} = \frac{2\sqrt{29}}{29}$

4. $\theta = \frac{s}{r} = \frac{15}{10} = 1.5$ radians

5. From exercise 4, we have $\theta = 1.5$.

$A = \frac{1}{2}r^2\theta = \frac{1}{2} \cdot 10^2 \cdot 1.5 = 75 \text{ cm}^2$

6. $\cos\theta = \frac{2}{7} \Rightarrow x = 2, r = 7 \Rightarrow 7^2 = 2^2 + y^2 \Rightarrow$

$y = 3\sqrt{5} \Rightarrow \sin\theta = \frac{3\sqrt{5}}{7}$

7. Quadrant II

8. $\cot\theta = -\frac{5}{12}, \theta$ in Quadrant IV $\Rightarrow x > 0, y < 0$

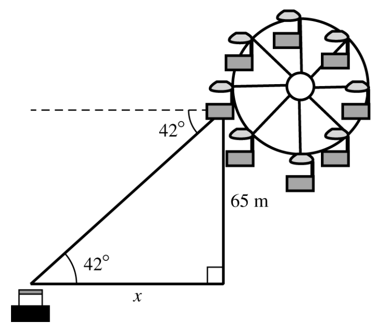
$x = 5, y = -12 \Rightarrow r = \sqrt{5^2 + (-12)^2} = 13 \Rightarrow$

$\sec\theta = \frac{r}{x} = \frac{13}{5}$

9. Because $217°$ lies in Quadrant III, the reference angle is $\theta - 180° = 217° - 180° = 37°$.

10. $0.223 = \cos\frac{3\pi}{7} = \sin\left(\frac{\pi}{2} - \frac{3\pi}{7}\right) = \sin\frac{\pi}{14} = 0.223$

11.



$\cot 42° = \frac{x}{65} \Rightarrow x = 65\cot 42° \Rightarrow x \approx 72$ m

12. Amplitude = 7, range = [−7, 7]

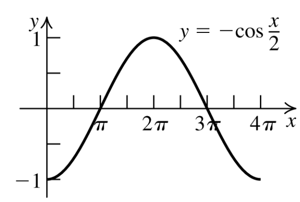
13. $y = 19\sin[12(x + 3\pi)] \Rightarrow a = 19, b = 12,$

$c = -3\pi \Rightarrow \text{amplitude} = 19, \text{ period} = \frac{2\pi}{12} = \frac{\pi}{6},$

$\text{phase shift} = -3\pi$

14. Because the ball was pulled down 7 inches $a = -7$. The period is 4, so $\omega = \frac{2\pi}{4} = \frac{\pi}{2}$. The equation is $y = -7\cos\left(\frac{\pi}{2}t\right)$.

15.



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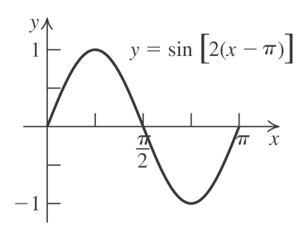
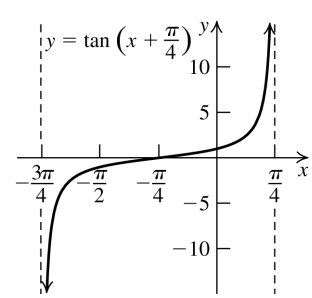


17.



18. $y = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

18. $y = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

19. $y = \cos^{-1}\left[\cos\frac{4\pi}{3}\right] = \frac{2\pi}{3}$

Note that $\frac{4\pi}{3}$ is not in the range of $\cos^{-1}\theta$.

20. $\sin^{-1}\left(-\frac{1}{3}\right) \Rightarrow a = -1, c = 3 \Rightarrow 3^2 = (-1)^2 + b^2$

$\Rightarrow b = 2\sqrt{2}$

$y = \tan\left(\sin^{-1}\left(-\frac{1}{3}\right)\right) = \frac{a}{b} = -\frac{1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}$

Chapter 5 Practice Test B

1. $160° = 160 \cdot \frac{\pi}{180} = \frac{8\pi}{9}$. The answer is B.

2. $\frac{13\pi}{5} = \frac{13\pi}{5} \cdot \frac{180}{\pi} = 468°$. The answer is D.

3. $x = -3, y = 1 \Rightarrow r^2 = (-3)^2 + 1^2 \Rightarrow r = \sqrt{10}$

$\cos\theta = \frac{x}{r} = -\frac{3\sqrt{10}}{10}$. The answer is A.

4. $\theta = \frac{s}{r} = \frac{4}{8} = \frac{1}{2}$ radian. The answer is A.

5. $A = \frac{1}{2}r^2\theta = \frac{1}{2} \cdot 6^2 \cdot \frac{3}{6} = 9 \text{ cm}^2$.
The answer is D.

6. $\cos\theta = \frac{3}{4} \Rightarrow x = 3, r = 4 \Rightarrow 4^2 = 3^2 + y^2 \Rightarrow$

$y = \sqrt{7} \Rightarrow \sin\theta = \sqrt{7}/4$. The answer is B.

7. The answer is D.

8. $\tan\theta = 12/5, \theta$ in Quadrant III $\Rightarrow x < 0, y < 0$

$x = -5, y = -12 \Rightarrow r = \sqrt{(-5)^2 + (-12)^2} = 13 \Rightarrow$

$\csc\theta = -13/12$. The answer is B.

9. $640°$ is co-terminal with $280°$. Because $280°$ lies in Quadrant IV, the reference angle is $360° - \theta = 360° - 280° = 80°$. The answer is C.

10. The answer is A.

11. $v = r\omega = 50(0.8) = 40$ cm/sec. The answer is C.

12. $a = \frac{1}{2}, 4\pi = \frac{2\pi}{b} \Rightarrow b = \frac{1}{2}$. The answer is A.

13. The answer is B.

14. $y = -5\cos\left(\frac{\pi}{4}t\right) \Rightarrow$ period $= \frac{2\pi}{\pi/4} = 8$.
The answer is D.

15. $y = -7\cos\left[\frac{1}{6}\left(x - \frac{\pi}{12}\right)\right] \Rightarrow a = -7, b = \frac{1}{6},$

$c = \frac{\pi}{12} \Rightarrow$ amplitude $= 7$, period $= \frac{2\pi}{1/6} = 12\pi,$

phase shift $= \pi/12$.
The answer is D.

16. The answer is C.

17. $\cos^{-1}\left(\cos\frac{11\pi}{3}\right) = \frac{\pi}{3}$. The answer is C.

18. $\sin^{-1}\left(-\frac{4}{5}\right) \Rightarrow y = -4, r = 5 \Rightarrow$

$5^2 = x^2 + (-4)^2 \Rightarrow x = 3$

$\cos\left(\sin^{-1}\left(-\frac{4}{5}\right)\right) = \frac{3}{5}$. The answer is A.

19. $y = \cos^{-1}\left(\cos\frac{7\pi}{4}\right) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$.

The answer is D.

20. $\tan^{-1}\left(\frac{5}{3}\right) \Rightarrow x = 3, y = 5 \Rightarrow r^2 = 3^2 + 5^2 \Rightarrow$

$r = \sqrt{34}$

$\cos\left(\tan^{-1}\left(\frac{5}{3}\right)\right) = \frac{3\sqrt{34}}{34}$.

The answer is D.

Cumulative Review Exercises (Chapters P–5)

1. $3x^2 - 30x + 50 = -24 \Rightarrow 3x^2 - 30x + 74 = 0 \Rightarrow$

$x = \frac{30 \pm \sqrt{(-30)^2 - 4(3)(74)}}{2(3)} = 5 \pm \frac{\sqrt{3}}{3}$

2. Let $u = \sqrt{t} + 1$. Then

$\left(\sqrt{t} + 1\right)^2 + 2\left(\sqrt{t} + 1\right) = 3 \Rightarrow$

$u^2 + 2u - 3 = 0 \Rightarrow (u + 3)(u - 1) = 0 \Rightarrow$

$u = -3$ or $u = 1$. Reject the negative root.

$1 = \sqrt{t} + 1 \Rightarrow t = 0$

3. $\dfrac{4-x}{2x-4} > 0.$ $4 - x = 0 \Rightarrow x = 4$
and $2x - 4 = 0 \Rightarrow x = 2$. The intervals to be tested are $(-\infty, 2), (2, 4),$ and $(4, \infty)$.

Interval	Test point	Value of $\frac{4-x}{2x-4}$	Result
$(-\infty, 2)$	0	−1	−
$(2, 4)$	3	1/2	+
$(4, \infty)$	5	−1/6	−

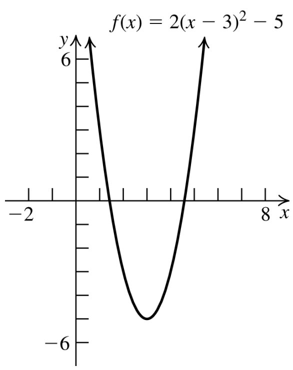
The solution set is (2, 4).

4. $m = \dfrac{-1-5}{3-(-6)} = -\dfrac{2}{3}; -1 = -\dfrac{2}{3}(3) + b \Rightarrow b = 1$

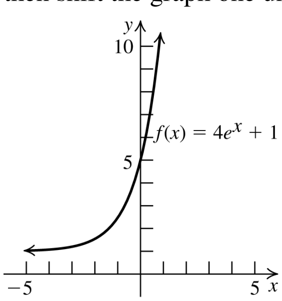
$y = -\dfrac{2}{3}x + 1$

5. Logarithms are defined only for positive numbers, so $\dfrac{x}{2-x} > 0 \Rightarrow 0 < x < 2$.
The domain is (0, 2).

6. This is a parabola. Shift the graph of $y = x^2$ three units to the right, stretch vertically by a factor of 2, then shift the graph down five units.



7. Stretch the graph of $y = e^x$ by a factor of 4, then shift the graph one unit up.



8. $(f \circ g)(x) = \ln\left(-\dfrac{1}{x}\right)$. Since the natural logarithm is defined only for positive numbers, $-\dfrac{1}{x} > 0 \Rightarrow$ the domain is $(-\infty, 0)$.

9.a. $f(x) = 3x + 7$ is a one-to-one function, so an inverse exists. Find the inverse by interchanging the variables and solving for *y*:

$$y = 3x + 7 : x = 3y + 7 \Rightarrow y = \frac{x-7}{3} = f^{-1}(x)$$

b. $f(x) = |x|$ is not one-to-one, so no inverse exists.

10.

$\underline{-2}|$	2	4	1	1	−2
		−4	0	−2	2
	2	0	1	−1	0

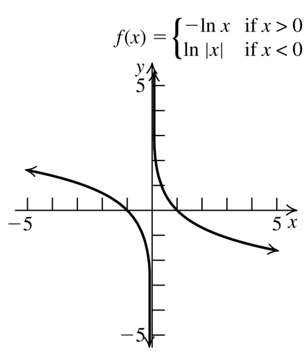
$$\frac{2x^4 + 4x^3 + x^2 + x - 2}{x+2} = 2x^3 + x - 1$$

11. Solve $x^2 + 4x - 5 = 0$ to find the vertical asymptotes: $x^2 + 4x - 5 = 0 \Rightarrow$ $(x+5)(x-1) = 0 \Rightarrow x = -5$ or $x = 1$. These are the vertical asymptotes. The numerator and denominator both have degree 2, so the horizontal asymptote is $y = -3$.

12.a.
$$\log\sqrt[3]{\frac{x^2y}{z}} = \frac{1}{3}\log\left(\frac{x^2y}{z}\right) = \frac{1}{3}\left(\log(x^2) + \log y - \log z\right) = \frac{2}{3}\log x + \frac{1}{3}\log y - \frac{1}{3}\log z$$

b.
$$\ln\left(\frac{7x^4}{5\sqrt{y}}\right) = \ln\left(\frac{7}{5}\right) + \ln x^4 - \ln\sqrt{y} = \ln 1.4 + 4\ln x - \frac{1}{2}\ln y$$

13.



14. $\log(x^2+9x)=1 \Rightarrow x^2+9x=10^1 \Rightarrow$
$x^2+9x-10=0 \Rightarrow (x+10)(x-1)=0 \Rightarrow$
$x=-10$ or $x=1$

15. The possible rational zeros are $\left\{\pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2, \pm 3, \pm 6\right\}$.

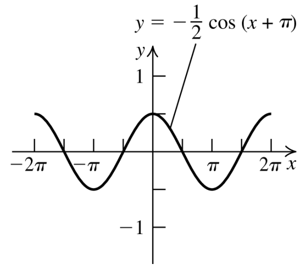
16.a. $\cos\theta=\frac{3}{7} \Rightarrow x=3, r=7 \Rightarrow 7^2=3^2+y^2 \Rightarrow$
$y=2\sqrt{10}$
$\sin\theta=\frac{2\sqrt{10}}{7}, \tan\theta=\frac{2\sqrt{10}}{3}, \cot\theta=\frac{3\sqrt{10}}{20},$
$\sec\theta=\frac{7}{3}, \csc\theta=\frac{7\sqrt{10}}{20}$

b. $\sin\theta=\frac{2}{11} \Rightarrow y=2, r=11 \Rightarrow 11^2=x^2+2^2 \Rightarrow$
$x=\sqrt{117}=3\sqrt{13}$
$\cos\theta=\frac{3\sqrt{13}}{11}, \tan\theta=\frac{2\sqrt{13}}{39}, \cot\theta=\frac{3\sqrt{13}}{2},$
$\sec\theta=\frac{11\sqrt{13}}{39}, \csc\theta=\frac{11}{2}$

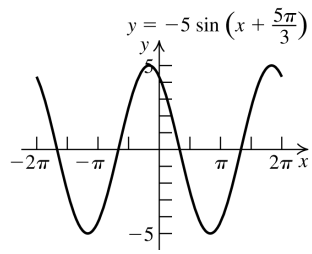
17.a. $\cos\theta=-\frac{4}{5}, \theta$ in Quadrant II $\Rightarrow x=-4, r=5$
$5^2=(-4)^2+y^2 \Rightarrow y=3$
$\sin\theta=\frac{3}{5}, \tan\theta=-\frac{3}{4}, \cot\theta=-\frac{4}{3},$
$\sec\theta=-\frac{5}{4}, \csc\theta=\frac{5}{3}$

b. $\cot\theta=\frac{5}{12}, \theta$ is Quadrant III $\Rightarrow x=-5, y=-12$
$r^2=(-5)^2+(-12)^2 \Rightarrow r=13$
$\sin\theta=-\frac{12}{13}, \cos\theta=-\frac{5}{13}, \tan\theta=\frac{12}{5},$
$\sec\theta=-\frac{13}{5}, \csc\theta=-\frac{13}{12}$

18.a.



b.



19.a. $\sin^{-1}\left(-\frac{1}{5}\right) \Rightarrow y=-1, r=5 \Rightarrow$
$5^2=x^2+(-1)^2 \Rightarrow x=2\sqrt{6}$
$\cos\left(\sin^{-1}\left(-\frac{1}{5}\right)\right)=\frac{2\sqrt{6}}{5}$

b. $\tan^{-1}\frac{7}{2} \Rightarrow y=7, x=2 \Rightarrow r^2=2^2+7^2 \Rightarrow$
$r=\sqrt{53}$
$\sin\left(\tan^{-1}\frac{7}{2}\right)=\frac{7}{\sqrt{53}}=\frac{7\sqrt{53}}{53}$

20. $y=\cos^{-1}\left(\cos\frac{5\pi}{3}\right)=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$